General relativity as an effective field theory of quantum gravity

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Contents for this talk

This talk will cover

- Overview of popular action formulations for GR
- Trinity of gravity
- York-Gibbons-Hawking boundary term
- Inertial frame description for Energy in GR
- Feasible action formulations from an EFT point of view

Conventions: A circle denotes zero torsion (or Levi-Civita connection) for instance the Ricci scalar in GR is denoted by $\mathring{R}$

Disclaimer

I am new to the field of quantum gravity, and this is work in progress

I assume the following for treating GR as an EFT for quantum gravity:

- GR have an action that gives rise to Einstein field equations
- The quantum theory has higher order invariances which **preserves the symmetries of Einstein field equations**

$$G_{\mu\nu} = \kappa T_{\mu\nu} \tag{1}$$

Some history

Einstein gravity

$$S_{\mathrm{E}} = \frac{1}{2\kappa}\int \mathrm{d}^4x\sqrt{-g}g^{\mu\nu}\left(\Gamma^{\alpha}{}_{\mu\beta}\Gamma^{\beta}{}_{\alpha\nu} - \Gamma^{\alpha}{}_{\mu\nu}\Gamma^{\beta}{}_{\alpha\beta}\right) + S_{\mathrm{matter}} \tag{2}$$

Hilberts correction

$$S_{\mathrm{EH}} = \frac{1}{2\kappa}\int \mathrm{d}^4x\sqrt{-g}\overset{\circ}{R} + S_{\mathrm{matter}}. \tag{3}$$

The 2 formulations differ by a boundary term which is only invariant under diffeomorphisms after Hilbert's correction.

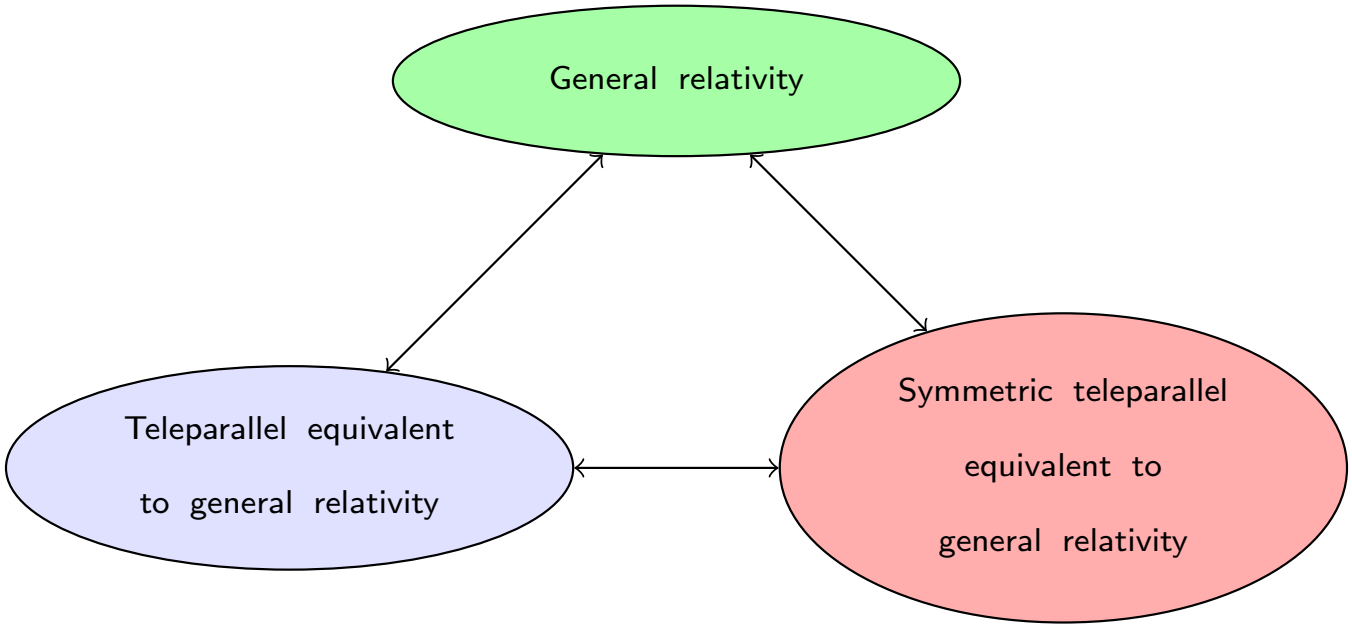
EH-action as an EFT for Quantum gravity

The action is not renormalizable, but quantum gravity works as an EFT up to some scale where the theory breaks down

$$
\begin{aligned}
S_{\mathrm{EFT}} = & \int \mathrm{d}^4x\sqrt{-g}\left(\frac{R}{16\pi G} + c_1(\mu)R^2 + c_2(\mu)R_{\mu\nu}R^{\mu\nu}\right. \\
& \left. + c_3(\mu)R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\right) \\
& - \int \mathrm{d}^4x\sqrt{-g}\left[\alpha R \ln\left(\frac{\Box}{\mu^2}\right)R + \beta R_{\mu\nu}\ln\left(\frac{\Box}{\mu^2}\right)R^{\mu\nu}\right. \\
& \left. + \gamma R_{\mu\nu\rho\sigma}\ln\left(\frac{\Box}{\mu^2}\right)R^{\mu\nu\rho\sigma}\right]
\end{aligned}
\tag{4}
$$

$$
R_{\mu\nu\rho\sigma} = \mathring{R}_{\mu\nu\rho\sigma}
$$

Alternative theories to general relativity



Ricci scalar of a general metric affine connection is given by:

$$R(\Gamma) = \overset{\circ}{R} + \mathbb{T} + \mathbb{Q} - Q_\mu T^\mu + \overline{Q}_\mu T^\mu + \overset{\circ}{\nabla}\left(Q^\mu - \overline{Q}^\mu + 2T^\mu\right) \quad (5)$$

1. J. Beltrán Jiménez et. al Phys. Lett. B. **805**. arxiv:1909.09045

Teleparallism

Teleparallism means that $R(\Gamma) = 0$

This implies

$$\overset{\circ}{R} = -\mathbb{T} - \mathbb{Q} + Q_\mu T^\mu - \overline{Q}_\mu T^\mu - \overset{\circ}{\nabla}\left(Q^\mu - \overline{Q}^\mu + 2T^\mu\right) \tag{6}$$

The Einstein-Hilbert action is given by

$$S_{\mathrm{EH}} = \frac{1}{2\kappa}\int \mathrm{d}^4x\sqrt{-g}\overset{\circ}{R} + S_{\mathrm{matter}} \tag{7}$$

We can use equation (6) to substitute $\overset{\circ}{R}$ in equation (7)

On the equivalence

$-\mathring{\nabla}\left(Q^{\mu}-\overline{Q}^{\mu}+2T^{\mu}\right)$ is a total derivative and becomes a boundary term which we can "classically" neglect.

Assuming vanishing torsion $\Gamma^{\rho}{}_{\nu\mu}-\Gamma^{\rho}{}_{\mu\nu}=T^{\rho}{}_{\mu\nu}=0$ in addition to flatness we get symmetric teleparallel equivalent to general relativity

$$S_{\mathrm{STEGR}}=-\frac{1}{2\kappa}\int \mathrm{d}^4x\sqrt{-g}\mathbb{Q}+S_{\mathrm{matter}} \tag{8}$$

Assuming vanishing non-metricity ($\nabla g_{\mu\nu}=0$) instead we get teleparallel equivalent to general relativity

$$S_{\mathrm{TEGR}}=-\frac{1}{2\kappa}\int \mathrm{d}^4x\theta\mathbb{T}+S_{\mathrm{matter}} \tag{9}$$

York-Gibbons-Hawking boundary term

Goal: To make sense of mass in GR.

1. Integrate by parts to get well-defined momenta and Hamiltonian (This gives a boundary term)
2. Evaluating the Hamiltonian for a BH gives infinity
3. Subtract the vacuum Minkowski value (another boundary term)
4. The result is the ADM-mass which comes from adding YGH-term to EH-action

Criticism:

- Ad-hoc
- Is it really consistent with quantum gravity (my criticism)

Mass in trinity of gravity

In TEGR and STEGR mass is dependent on the gauge

Prescription *inertial frame* $\Longrightarrow$ better prescription for mass and energy in gravity.[2]

2. D.A. Gomes et. al arxiv:2205.09716

Modified teleparallel gravity

In the spirit of $f(\overset{\circ}{R})$ theories of gravity we can create $f(T)$

Note that $f(\mathbb{T}) \neq f(\overset{\circ}{R}) = f(-\mathbb{T} + B)$

The Torsion scalar T can be written out as

$$\mathbb{T} = \frac{1}{4} T^{\rho}{}_{\mu\nu} T_{\rho}{}^{\mu\nu} + \frac{1}{2} T^{\rho}{}_{\mu\nu} T^{\nu\mu}{}_{\rho} - T^{\rho}{}_{\mu\rho} T^{\sigma\mu}{}_{\sigma}$$

Changing the fixed parameters yields 2 parameter space of new theories historically named "new general relativity"

$$\mathbb{T}_{\mathrm{NGR}} = c_1 T^{\rho}{}_{\mu\nu} T_{\rho}{}^{\mu\nu} + c_2 T^{\rho}{}_{\mu\nu} T^{\nu\mu}{}_{\rho} + c_3 T^{\rho}{}_{\mu\rho} T^{\sigma\mu}{}_{\sigma}$$

Constraints

The primary constraints related to Lorentz invariance of type II in TEGR, $f(T)$-gravity and 1-parameter NGR:

$$ {}^{\mathcal{V}}C^{i} = \frac{{}^{\mathcal{V}}\pi^{i}{}_{\kappa}}{\phi\sqrt{\gamma}} - T^{B}{}_{kl}\gamma^{il}\theta_{B}{}^{k} \approx 0 \tag{10} $$

$$ {}^{\mathscr{A}}C^{ij} = \frac{{}^{\mathscr{A}}\pi^{ij}{}_{\kappa}}{\phi\sqrt{\gamma}} - \frac{1}{2}T^{A}{}_{kl}\gamma^{il}\gamma^{jk}n_{A} \approx 0 \tag{11} $$

- $f(T) = T \implies \phi = 1$
- 1-parameter NGR does not satisfy ${}^{\mathscr{A}}C^{ij} \approx 0$

Lorentz invariance of type II is broken by

- $f(T) \implies \phi \neq 1$ which spoils the Lorentz algebra
- 1-parameter NGR missing ${}^{\mathscr{A}}C^{ij} \approx 0$

Summary

- Classically the Einstein-Hilbert action is not the unique action description for GR
- Trinity of gravity contain improved descriptions for considering mass in GR
- However, the boundary terms break symmetries which restricts the possibility in treating GR as and EFT for quantum gravity.

Outlook

- Modified gravity as an EFT for quantum gravity
- Explore truly equivalent gauge theories for EH (tetrad GR with spin connection, Lagrange multipliers, etc)
- Other prescriptions to quantize gravity?

Boundary terms are important to discuss in the context of treating GR as an EFT for quantum gravity

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