# Generalized Uncertainty Principle: from the harmonic oscillator to a QFT toy model

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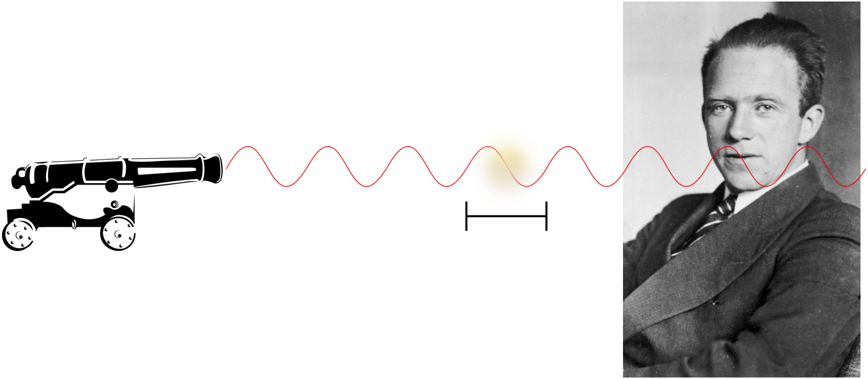
# Heisenberg's Uncertainty Relation



Scardigli, PLB 452, 39-44 (1999)

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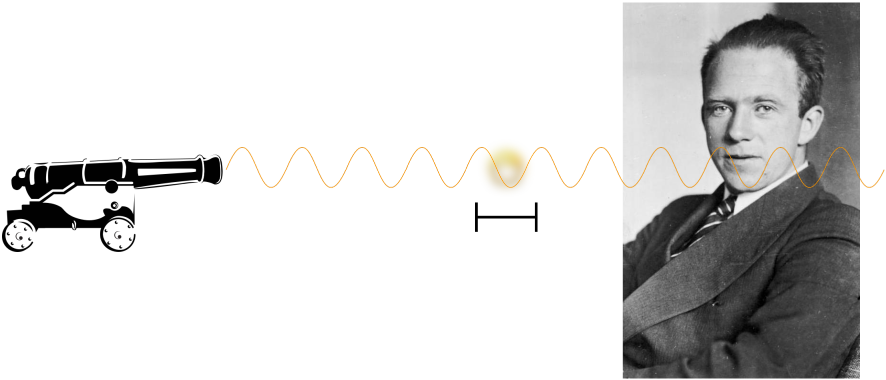

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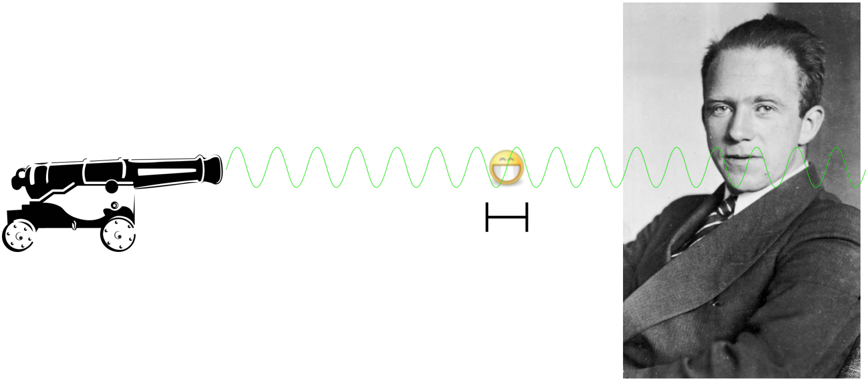

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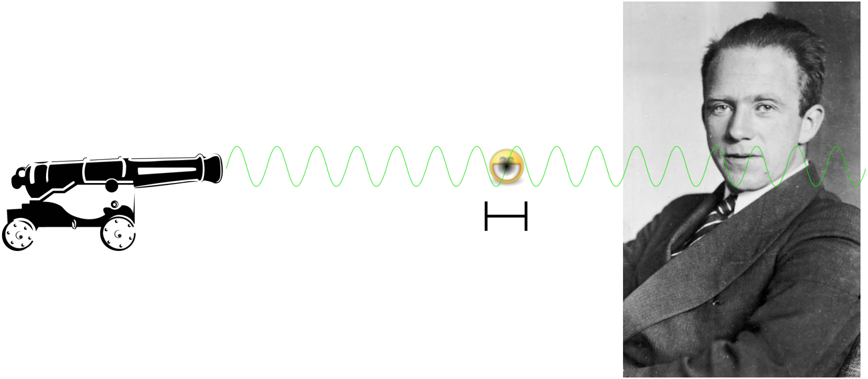

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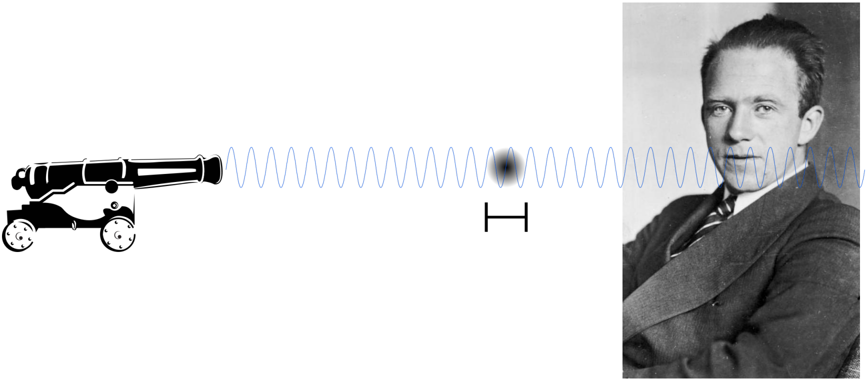

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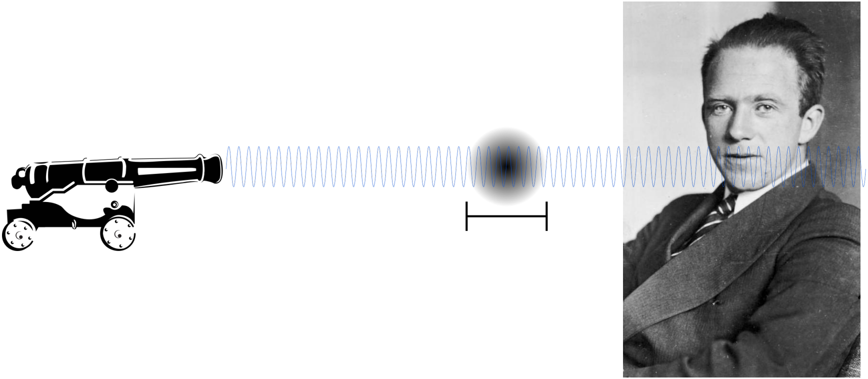

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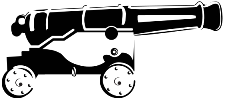

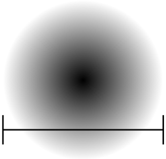

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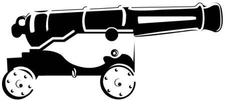

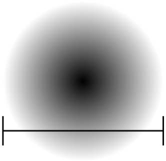

Heisenberg

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# Modified Uncertainty Relation

## Robertson–Schrödinger relation

$$(\Delta A)^2(\Delta B)^2 \geqslant \left|\frac{\langle AB + BA\rangle}{2} - \langle A\rangle\langle B\rangle\right|^2 + \left|\frac{\langle[A, B]\rangle}{2}\right|^2$$

$$[\hat{q}, \hat{p}] = i\hbar(1 + \gamma^2\hat{p}^2),$$

Kempf, Mangano, Mann, PRD 52, 1108-1118 (1995)

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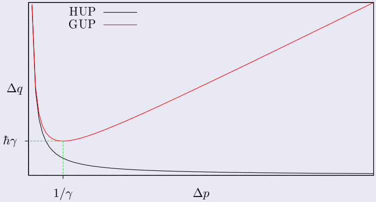


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# Modified Uncertainty Relation

$$[\hat{q},\hat{p}] = i\hbar[1 - 2\delta\hat{p} + (\delta^2+\epsilon)\hat{p}^2]$$

### Momentum space

$$\hat{p} = p$$

$$\hat{q} = i\hbar[1 - 2\delta\hat{p} + (\delta^2+\epsilon)\hat{p}^2]\frac{\mathrm{d}}{\mathrm{d}p}$$

### (Quasi-)position space

$$\hat{p} = \frac{\tan\left(-i\hbar\sqrt{\epsilon}\frac{\mathrm{d}}{\mathrm{d}\xi}\right)}{\sqrt{\epsilon} + \delta\tan\left(-i\hbar\sqrt{\epsilon}\frac{\mathrm{d}}{\mathrm{d}\xi}\right)}$$

$$\hat{q} = \xi + i\hbar[-\delta + (\delta^2+\epsilon)\hat{p}]$$

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PB: Clas.Quan.Grav., **38**, 145006, [arXiv:2012.04795 [gr-qc]] (2021).

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# Harmonic Oscillator with GUP (non-perturbative)

## HO Schrödinger Equation

$$\frac{\mathrm{d}}{\mathrm{d}p}\left[f(p)\frac{\mathrm{d}}{\mathrm{d}p}\psi_n(p)\right] + \frac{2}{\hbar^2 m\omega^2}\frac{1}{f(p)}\left(E_n - \frac{p^2}{2m}\right)\psi_n(p) = 0$$

$$f(p) = 1 - 2\delta p + (\delta^2 + \epsilon)p^2$$

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## Solution

$$\psi_n(\zeta) = \zeta^{\rho_n}(1-\zeta)^{\rho_n^*}\,{}_2F_1(a_n, b_n, c_n; \zeta)$$

$$\zeta = \frac{1}{2}\left[1 - i\frac{(\delta^2+\epsilon)p - \delta}{\sqrt{\epsilon}}\right], \qquad \rho_n = -\frac{\sqrt{2E_n m - \frac{1}{(\delta+i\sqrt{\epsilon})^2}}}{2m\omega\hbar\sqrt{\epsilon}},$$

$$a_n = -n, \qquad b_n = -n - \frac{\sqrt{m^2\omega^2\hbar^2(\delta^2+\epsilon)^2+4}}{m\omega\hbar(\delta^2+\epsilon)},$$

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# Hypergeometric functions

$$\text{Combinations of } \zeta\cdot \text{ and } \frac{\mathrm{d}}{\mathrm{d}\zeta}$$
$$\downarrow$$
$${}_2F_1(a_n, b_n, c_n; \zeta) \to {}_2F_1(a_n+k, b_n+l, c_n+m; \zeta) \qquad \text{with } k, l, m \in \mathbb{Z}.$$

$$a_n + k = a_{n-k} [= -(n-k)] \quad ✓$$
$$b_n + l = b_{n-l} \quad ✓$$
$$c_n + m = 1 + 2\Re(\rho_{n-m}) + i\Im(\rho_n) \quad ✗$$

No ladder operator made out of $\zeta\cdot$ and $\frac{\mathrm{d}}{\mathrm{d}\zeta}$ if $\delta \neq 0$.

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# GUP Ladder Operators ($\delta = 0$)

$$A^{\pm} = \frac{\omega\hbar\sqrt{\epsilon m}}{2}\left[\pm\frac{2ip\sqrt{\epsilon}}{\sqrt{1+\epsilon p^2}}\left(\rho_0 - \frac{N}{2}\right) + \frac{i\sqrt{1+\epsilon p^2}}{\sqrt{\epsilon}}\frac{\mathrm{d}}{\mathrm{d}p}\right]$$

$$\begin{aligned} A^{+}\psi_n(p) &= \frac{\omega\hbar\sqrt{\epsilon m}}{2}(2\rho_0 - n)\psi_{n+1}(p), \\ A^{-}\psi_n(p) &= \frac{\omega\hbar\sqrt{\epsilon m}}{2}\frac{n(1+4\rho_0-n)}{n-2\rho_0-1}\psi_{n-1}(p), \\ N\psi_n(p) &= n\psi_n(p) \end{aligned}$$

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# Better Ladder Operators

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\end{aligned}
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(C^{\pm})^{\dagger} = C^{\mp} \qquad C^{-} = C \qquad H = C^{\dagger}C + E_0\mathbb{I}
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C|n\rangle = \sqrt{E_n - E_0}\,|n-1\rangle, \qquad C^{\dagger}|n\rangle = \sqrt{E_{n+1} - E_0}\,|n+1\rangle
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# Better Ladder Operators

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$$C^- = \sqrt{2}\sqrt{\frac{N-2\rho_0}{N-4\rho_0}}A^-\sqrt{\frac{N-4\rho_0}{N-2\rho_0}}$$

$$(C^\pm)^\dagger = C^\mp \qquad C^- = C \qquad H = C^\dagger C + E_0\mathbb{I}$$

$$C|n\rangle = \sqrt{E_n - E_0}|n-1\rangle, \qquad C^\dagger|n\rangle = \sqrt{E_{n+1} - E_0}|n+1\rangle$$

PB, G.G. Luciano: Eur.Phys.J.C 81 (2021) 11, 982, [arXiv:2109.15259 [hep-th]].

# QFT with GUP

## Fields as anti-transforms of ladder operators

$$\phi(\xi) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\pi/2\sqrt{\epsilon}}^{+\pi/2\sqrt{\epsilon}} \mathrm{d}P \mathcal{N}_P \cos(\sqrt{\epsilon}P) e^{i\frac{\xi}{\hbar}P} (C_P + C^\dagger_{-P})$$

$$\langle p|\phi|0\rangle \propto \cos(\sqrt{\epsilon}P) e^{-i\frac{\xi}{\hbar}P}$$

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## Time-dependent fields

$$\phi(p,t) = e^{-i\Omega_p t} C_p + C^\dagger_{-p} e^{+i\Omega_p t}$$

$$\Omega_p = \frac{1}{\hbar}[C_p, C_p^\dagger] = \frac{\hbar\omega}{2}\left(2\sqrt{1+2\epsilon m H} + \hbar\omega\epsilon m\right)$$

$$\ddot{\phi}(p,t) = -\Omega_p \phi(p,t)\Omega_p + \frac{1}{2}\left[\Omega_p\phi(p,t) + \phi(p,t)\Omega_p\right] - \frac{1}{2}\phi(p,t)(\alpha_p\omega_p\epsilon m)^2$$

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# Conclusion

- GUP as a phenomenological model for a minimal length.
- Harmonic oscillator
  - Eigenfunction (linear and quadratic)
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# Grazie!