Gravitational-wave interferometers and possible quantum gravity signatures

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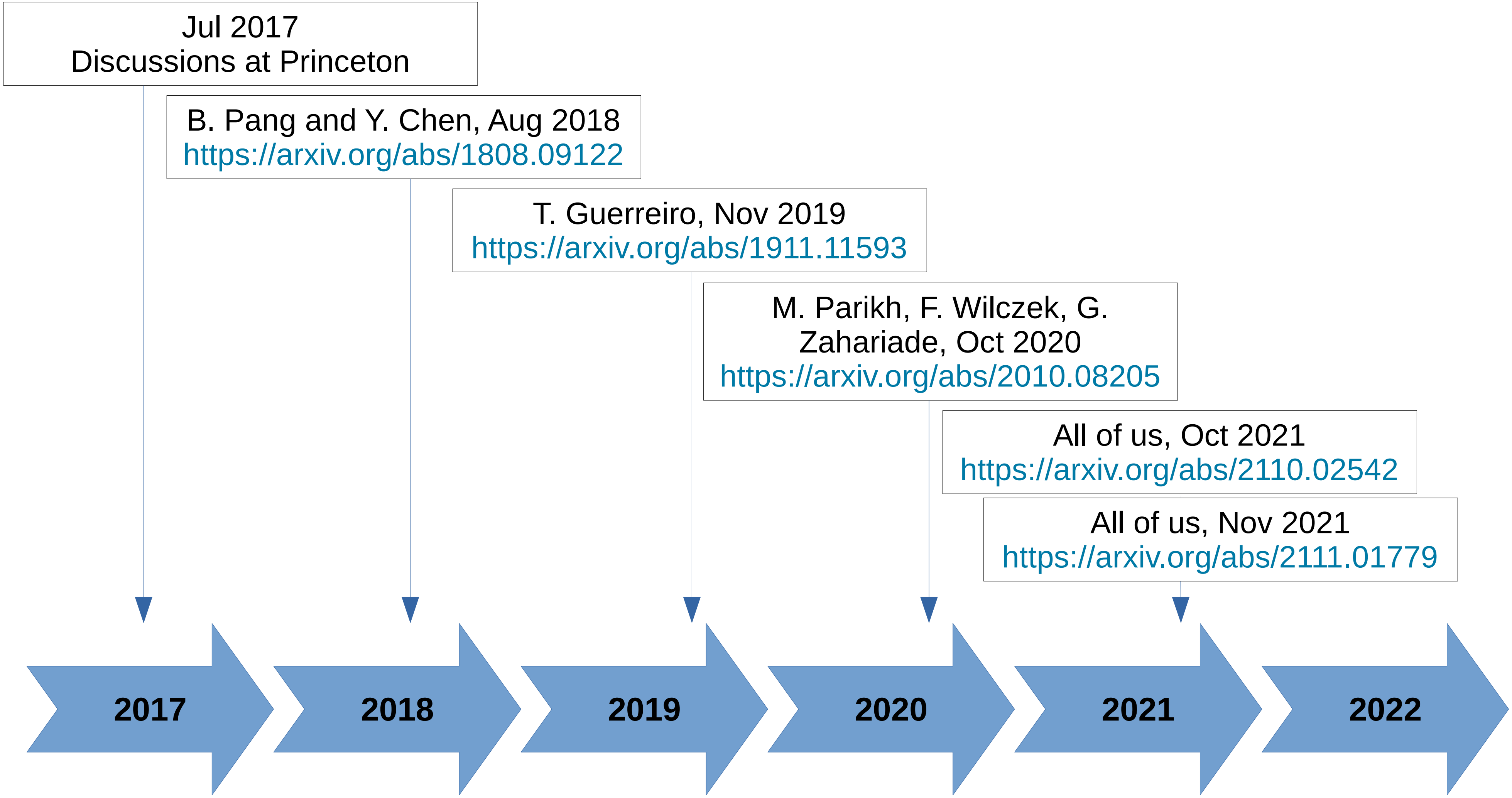
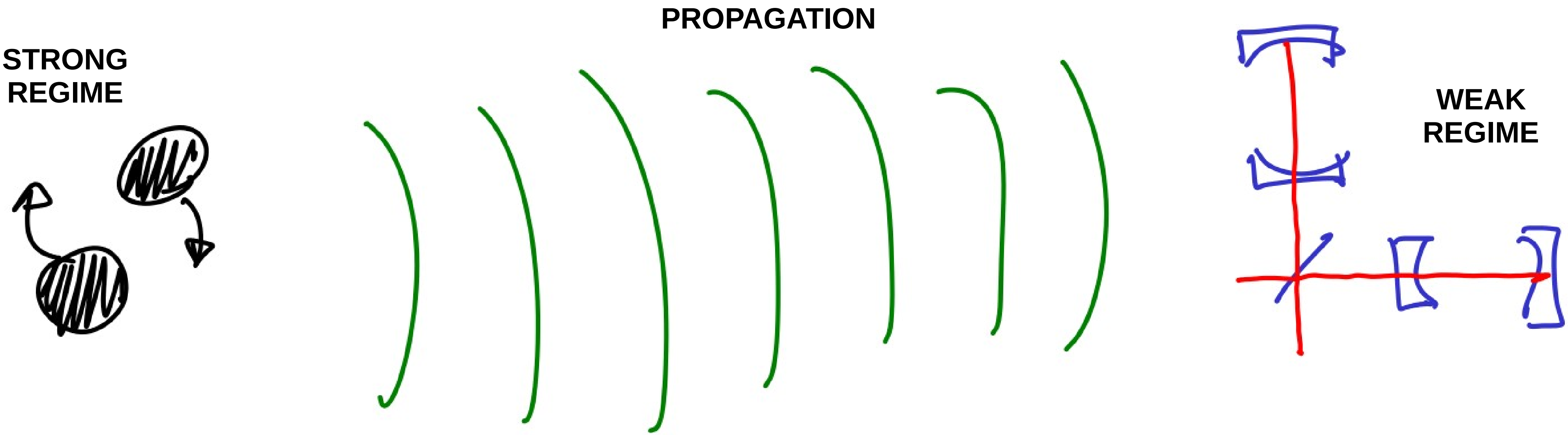

Jul 2017
Discussions at Princeton
B. Pang and Y. Chen, Aug 2018
https://arxiv.org/abs/1808.09122
T. Guerreiro, Nov 2019
https://arxiv.org/abs/1911.11593
M. Parikh, F. Wilczek, G. Zahariade, Oct 2020
https://arxiv.org/abs/2010.08205
All of us, Oct 2021
https://arxiv.org/abs/2110.02542
All of us, Nov 2021
https://arxiv.org/abs/2111.01779
2017
2018
2019
2020
2021
2022

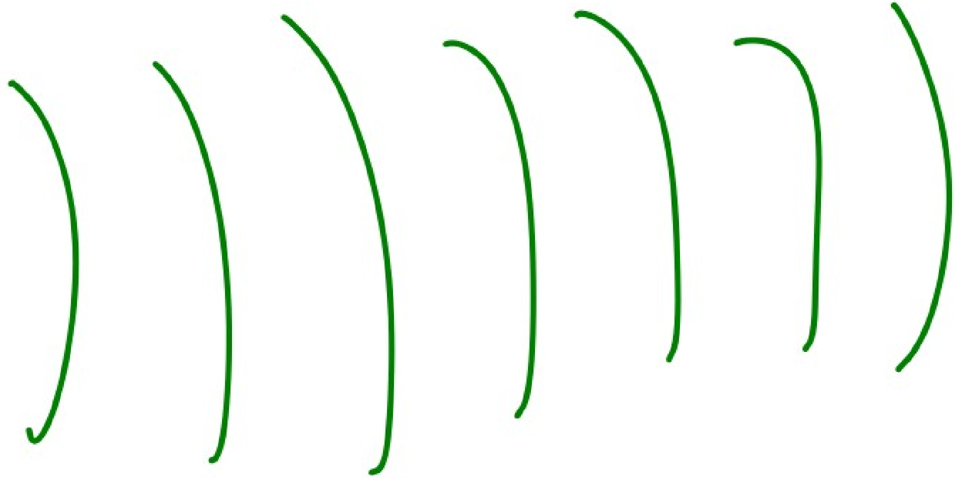
Setup

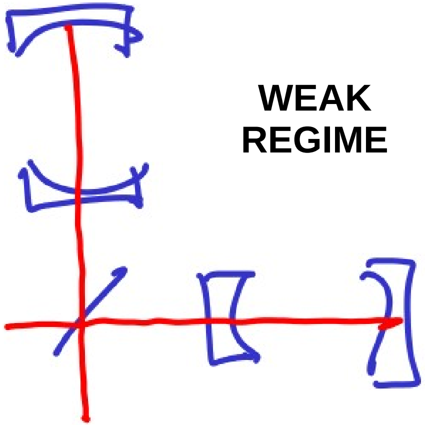


Setup

PROPAGATION

?





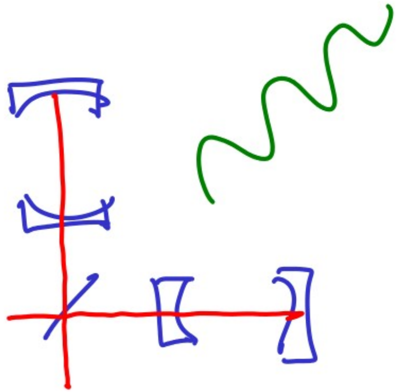
WEAK
REGIME

What is the
effect here?
Is it measurable?

IF

QUANTUM STATES WITH <u>NO</u>
<u>CLASSICAL ANALOG</u>

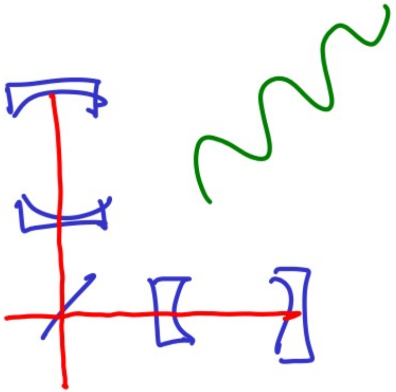
Procedure



$$|\Psi\rangle = |\psi\rangle_{EM} \otimes |\psi\rangle_{GW}$$

1) Hilbert space

Procedure

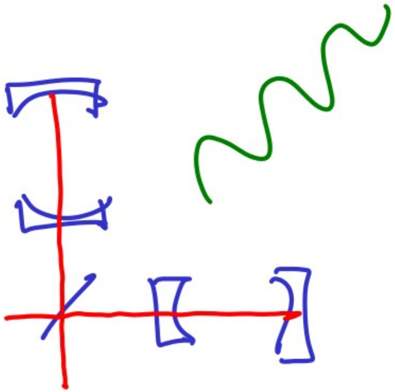


$$|\psi\rangle = |\psi\rangle_{EM} \otimes |\psi\rangle_{GW}$$

1) Hilbert space

2) Hamiltonian $\hat{H} = \hat{H}_0 + \hat{H}_{INT}$

Procedure



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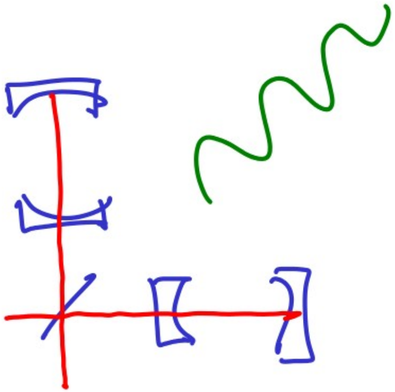

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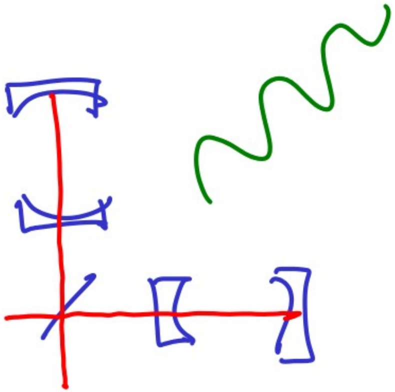

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5) Pick a (hypothetical!) state

$$|\psi\rangle_{GW}$$

(weak) gravity setup: canonical quantization of GW

$$h^{TT}_{ij}(t,\vec{x}) = \int \frac{d^3k}{(2\pi)^{3/2}} \, \epsilon^{\lambda}_{ij}(\vec{k}) \, h_{\lambda}(t,\vec{k}) \, e^{i\vec{k}\cdot\vec{x}}$$

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$$h_\lambda(t,\vec{k}) \rightarrow \hat{b}^{\lambda}_{\vec{k}}$$

$$h^*_\lambda(t,\vec{k}) \rightarrow \hat{b}^{\dagger\lambda}_{\vec{k}}$$

$$\left[\hat{b}^{\lambda}_{\vec{k}}\ \hat{b}^{\dagger\lambda'}_{\vec{k}'}\right] = \delta_{\lambda\lambda'}\,\delta^{(3)}(\vec{k}-\vec{k}')$$

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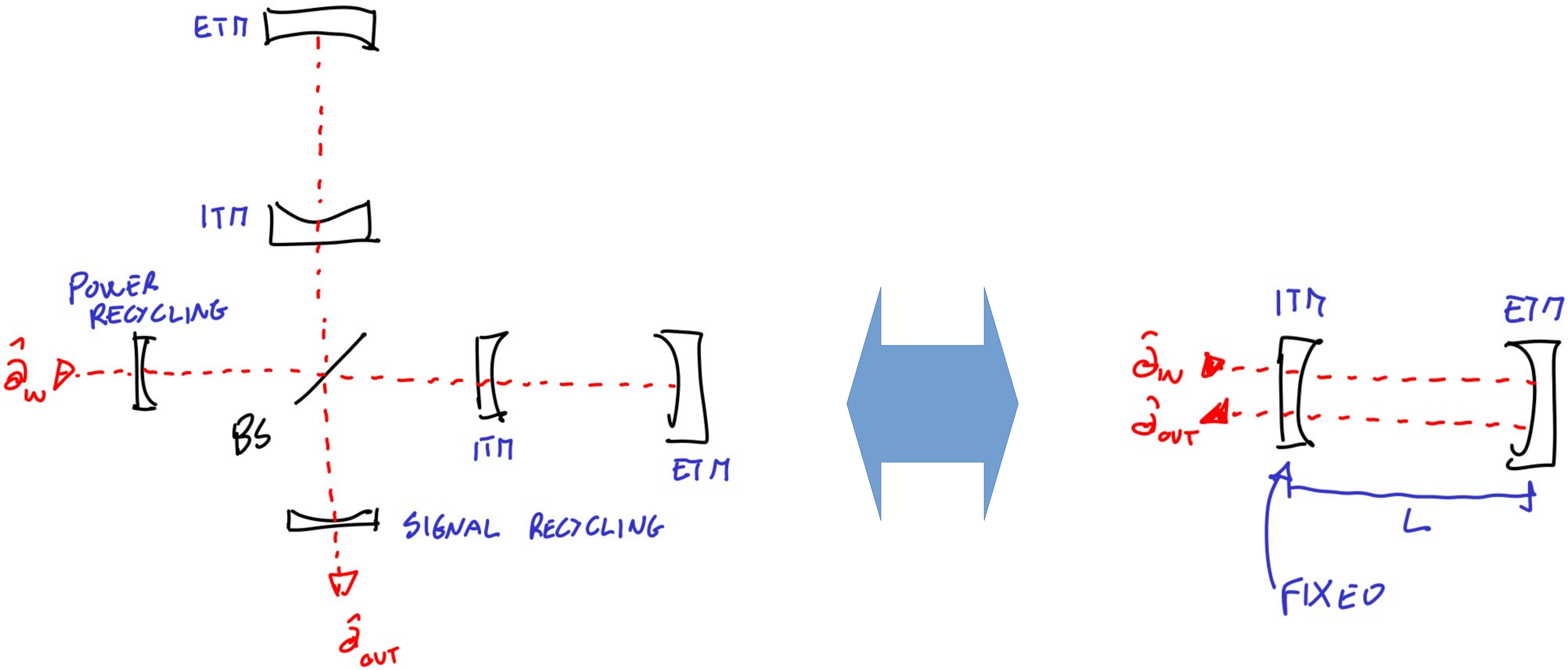
$$\hat{h}_{ij}(t,\vec{x}) = \int \frac{d^3k}{(2\pi)^{3/2}} \left(\sqrt{\frac{8\pi G}{k}}\, \epsilon^{\lambda}_{ij}(\vec{k})\, \hat{b}^{\lambda}_{\vec{k}}\, e^{i(\vec{k}\cdot\vec{x}-\Omega t)} + h.c.\right)$$

“Quantum LIGO”



Alessandra Buonanno and Yanbei Chen, Phys. Rev. D 67, 062002 – Published 26 March 2003

The Hamiltonian

$$\hat{H} = \hat{H}_0 + \hat{H}_{INT}$$

$$\hat{H}_0 = \omega \hat{a}^\dagger \hat{a} + \sum_{\vec{k}} \Omega_k \hat{b}^\dagger_{\vec{k}} \hat{b}_{\vec{k}}$$

$$\hat{H}_{INT} = -\hat{a}^\dagger \hat{a} \sum_{\vec{k}} \underbrace{g_k}_{\text{SMALL COUPLING}} \Omega_k \left(\hat{b}_{\vec{k}} + \hat{b}^\dagger_{\vec{k}}\right)$$

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The time evolution operator

For each GW mode k (index not shown):

$$\hat{U}(t) = e^{-i\omega \hat{a}^{\dagger}\hat{a} t}\, e^{-i\Omega \hat{b}^{\dagger}\hat{b} t}\, e^{g \hat{a}^{\dagger}\hat{a}\left[\gamma(t)\hat{b}^{\dagger} - \gamma^{*}(t)\hat{b}\right]}\, e^{i B(t) (\hat{a}^{\dagger}\hat{a})^{2}}$$

$$\gamma(t) = 1 - e^{-it} \qquad\qquad B(t) = g^{2}(t - \sin t)$$

The time evolution operator

For each GW mode k (index not shown):

$$\hat{U}(t) = e^{-i\omega \hat{a}^\dagger \hat{a} t}\, e^{-i\Omega \hat{b}^\dagger \hat{b} t}\, e^{q \hat{a}^\dagger \hat{a} \left[\gamma(t) \hat{b}^\dagger - \gamma^*(t) \hat{b}\right]}\, e^{i B(t) (\hat{a}^\dagger \hat{a})^2}$$

$$\gamma(t) = 1 - e^{-it} \qquad B(t) = q^2 (t - \sin t)$$

$$|\psi(t)\rangle = e^{-i\Omega \hat{b}^\dagger \hat{b} t} |\psi\rangle$$

The time evolution operator

For each GW mode k (index not shown):

$$\hat{U}(t) = e^{-i\omega \hat{a}^\dagger \hat{a} t}\, e^{-i\Omega \hat{b}^\dagger \hat{b} t}\, e^{g \hat{a}^\dagger \hat{a} \left[\eta(t) \hat{b}^\dagger - \eta^*(t) \hat{b}\right]}\, e^{i B(t) (\hat{a}^\dagger \hat{a})^2}$$

$$\eta(t) = 1 - e^{-it} \qquad\qquad B(t) = g^2 (t - \sin t)$$

$$|\psi(t)\rangle = e^{-i\Omega \hat{b}^\dagger \hat{b} t} |\psi\rangle$$

$$\boxed{\hat{U}(t) |\psi(t)\rangle = e^{g \hat{a}^\dagger \hat{a} \left[\eta(t) \hat{b}^\dagger - \eta^*(t) \hat{b}\right]} |\psi(t)\rangle}$$

Electric field

$$\hat{\mathcal{E}} = \sqrt{\frac{\omega}{V}}\,\frac{\hat{a}^\dagger + \hat{a}}{\sqrt{2}} \quad \Rightarrow \quad \mathcal{E}(t) = \langle \psi | \, \hat{U}^\dagger \, \hat{\mathcal{E}} \, \hat{U} \, | \psi \rangle$$

Electric field

$$\hat{\mathcal{E}} = \sqrt{\frac{\omega}{V}}\,\frac{\hat{a}^{\dagger} + \hat{a}}{\sqrt{2}} \quad \Longrightarrow \quad \mathcal{E}(t) = \langle\psi|\,\hat{U}^{\dagger}\,\hat{\mathcal{E}}\,\hat{U}\,|\psi\rangle$$

$$\mathcal{E}(t) = \sqrt{\frac{\omega}{V}}\left[\frac{\prod_{\vec{k}} \langle\psi|\,\hat{D}(q_k\,\gamma(t))\,\hat{a}\,|\psi\rangle}{\sqrt{2}} + h.c.\right]$$

$$|\alpha\rangle_{GW} = \hat{D}(\alpha)|0\rangle_{GW}$$

Displacement operator acting on GW state

coherent

Electromagnetic state

$|\psi\rangle = |\psi\rangle_{EM} \otimes |\psi\rangle_{GW}$

$|\psi_{EM}\rangle = |\alpha\rangle$

coherent

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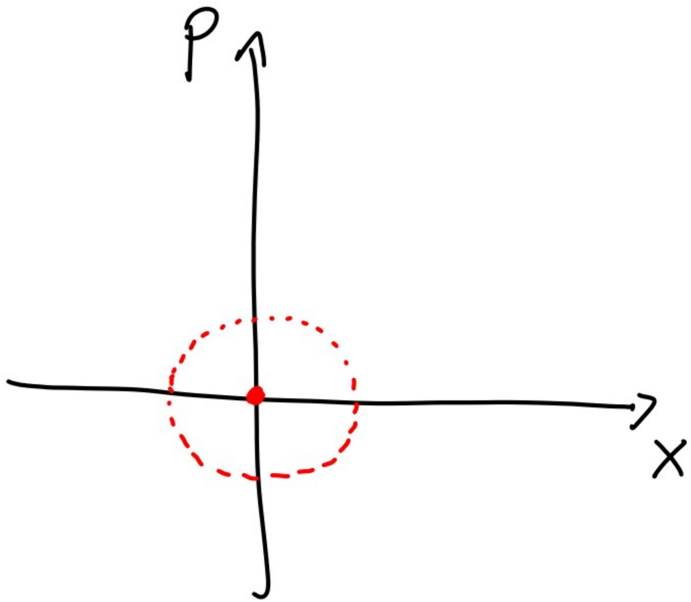
$$\mathcal{E}(t) = \sqrt{\frac{\omega}{V}} \left[\frac{\prod_{\vec{k}} \langle\psi| \hat{D}(q_k \gamma(t))\, \hat{a}\, |\psi\rangle}{\sqrt{2}} + h.c. \right]$$

$$\mathcal{E}(t) = \sqrt{\frac{\omega}{V}} \left[\frac{\alpha \prod_{\vec{k}} \langle\psi_{GW}| \hat{D}(q_k \gamma(t)) |\psi_{GW}\rangle}{\sqrt{2}} + h.c. \right]$$

Now let’s pick (quantum) gravity states

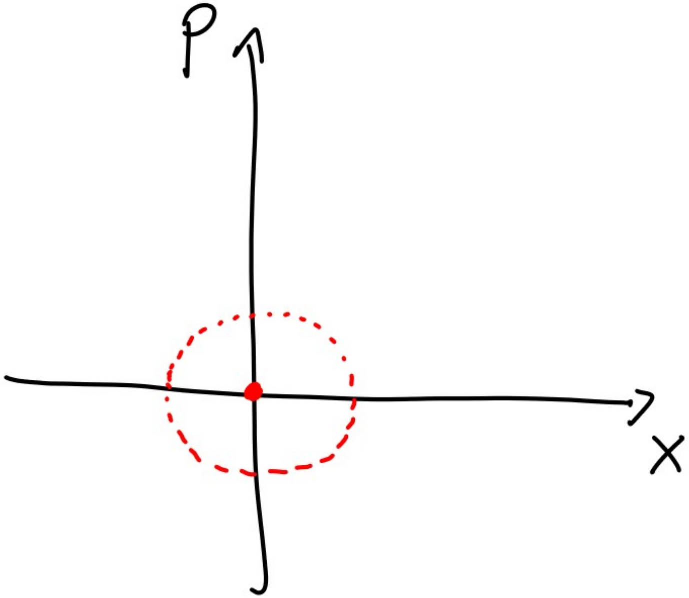
$$|\psi\rangle = |\psi\rangle_{EM} \otimes |\psi\rangle_{GW}$$

Vacuum



$$|0_{\vec{k}}(t)\rangle = e^{-i\hat{b}_k^{\dagger}\hat{b}_k \Omega_k t}|0_k\rangle$$

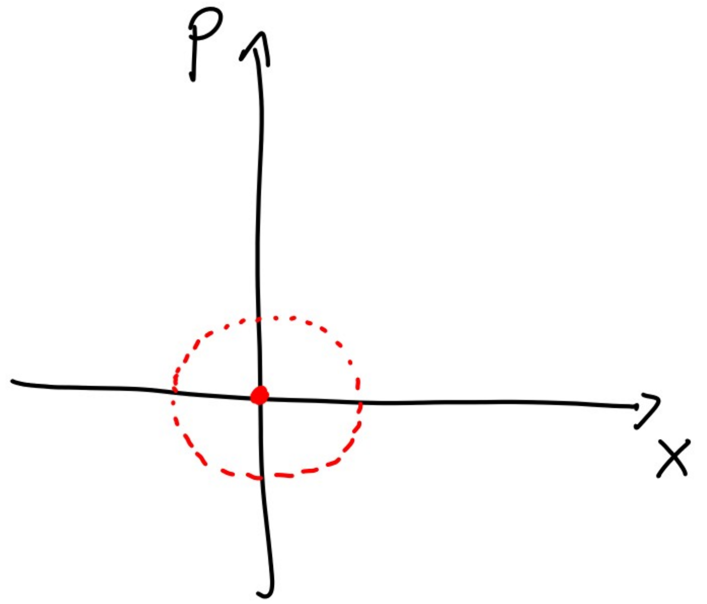
Vacuum



$$\mathcal{E}(t) \sim \prod_{\vec{k}} e^{-\frac{1}{2} q_k^2} \sim 1$$

$$|0_{\vec{k}}(t)\rangle = e^{-i b_{\vec{k}}^{\dagger} b_{\vec{k}} \Omega_k t} |0_k\rangle$$

Vacuum



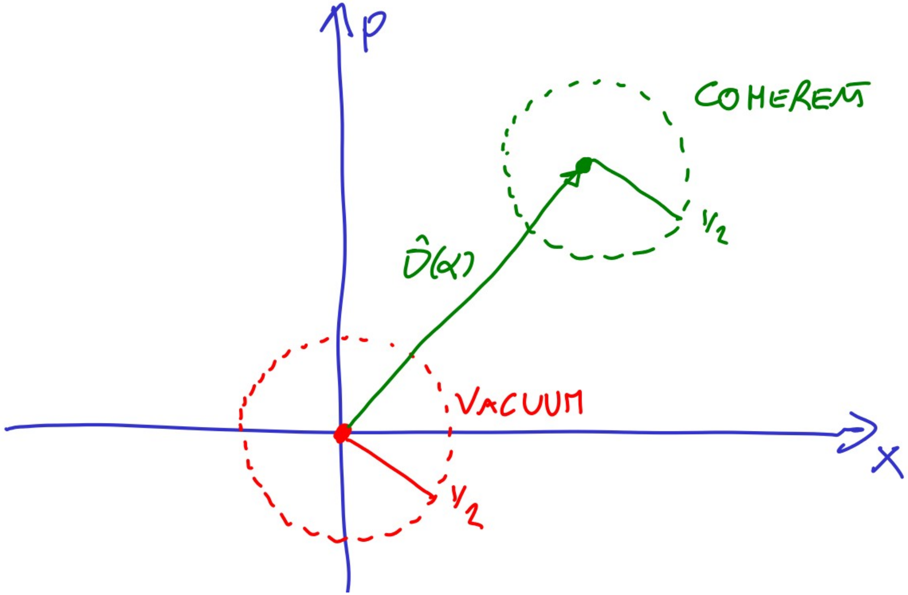
$$\mathcal{E}(t) \sim \prod_{\vec{k}} e^{-\frac{1}{2} q_k^2} \sim 1$$

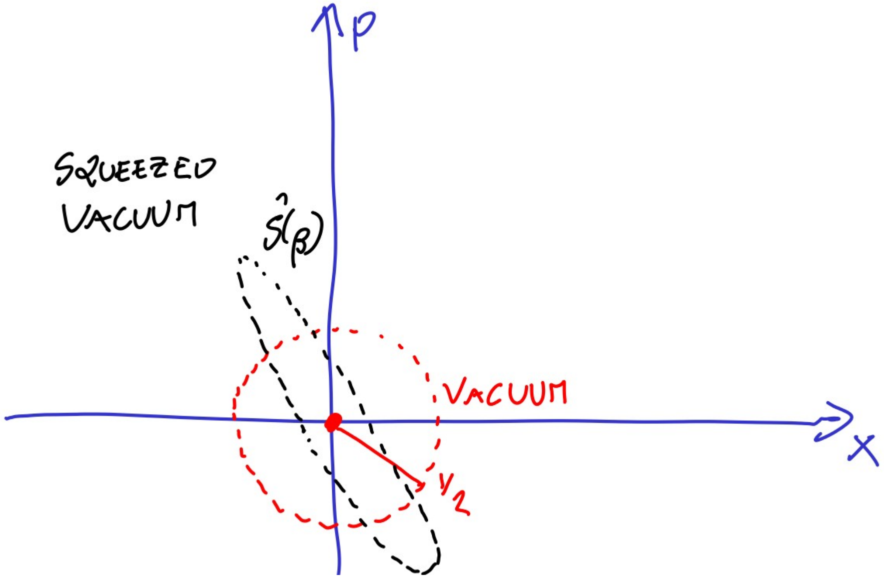
Interesting effect if we re-include the term we neglected:

$$\hat{U}(t) = \dots e^{i B(t) (\hat{a}^\dagger \hat{a})^2} \dots$$

$$|0_{\vec{k}}(t)\rangle = e^{-i \hat{b}_k^\dagger \hat{b}_k \Omega_k t} |0_k\rangle$$

Digression: quantum squeezing

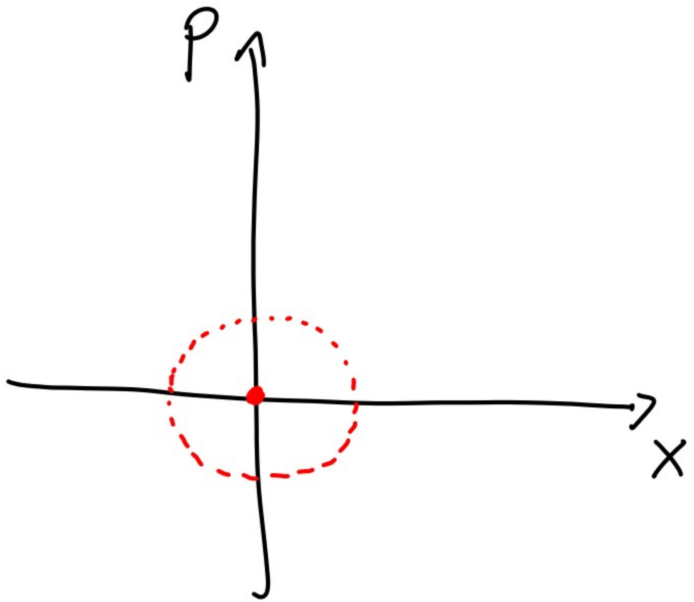




$$\hat{D}(\alpha) = e^{\alpha \hat{a}^\dagger - \alpha^* \hat{a}}$$

$$\hat{S}(\beta) = e^{\frac{1}{2}\beta^* \hat{a}^2 - \frac{1}{2}\beta (\hat{a}^\dagger)^2}$$

Vacuum



$$\mathcal{E}(t) \sim \prod_{\vec{k}} e^{-\frac{1}{2} q_k^2} \sim 1$$

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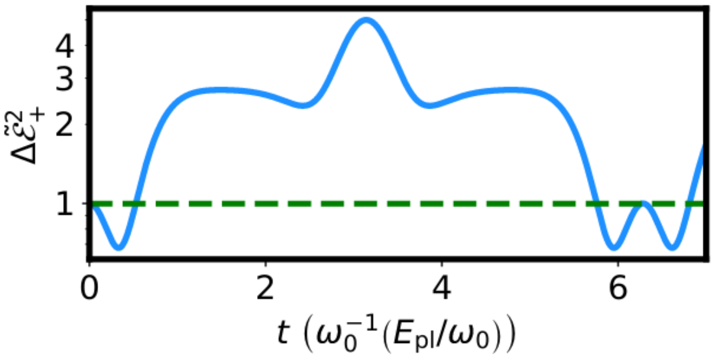

$$|0_{\vec{k}}(t)\rangle = e^{-i\hat{b}_k^\dagger \hat{b}_k \Omega_k t} |0_k\rangle$$

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Single mode coherent

$$|\psi_{GW}(t)\rangle = \begin{cases} |h e^{i\Omega t}\rangle & \Omega = |\vec{k}|^2 \\ |0\rangle & \text{OTHERWISE} \end{cases}$$

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$$\mathcal{E}(t) \propto e^{i\Delta\phi}$$

CLASSICAL SIGNAL

$$\Delta\phi \sim \frac{\omega}{\Omega} f \sin \Omega t$$

Thermal

1) Treat gravity as a thermal bath
2) Describe it as an ensemble of **mixed states**
3) Write down the **density matrix** for the full Hilbert space
4) Trace out gravity

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$$\rho_{i\neq j} \sim e^{-\Gamma t}$$

$$\Gamma \sim k_B T \left(\frac{N\omega}{E_{Pl}}\right)^2$$

GRAVITATIONAL-INDUCED DECOHERENCE

Squeezed vacuum

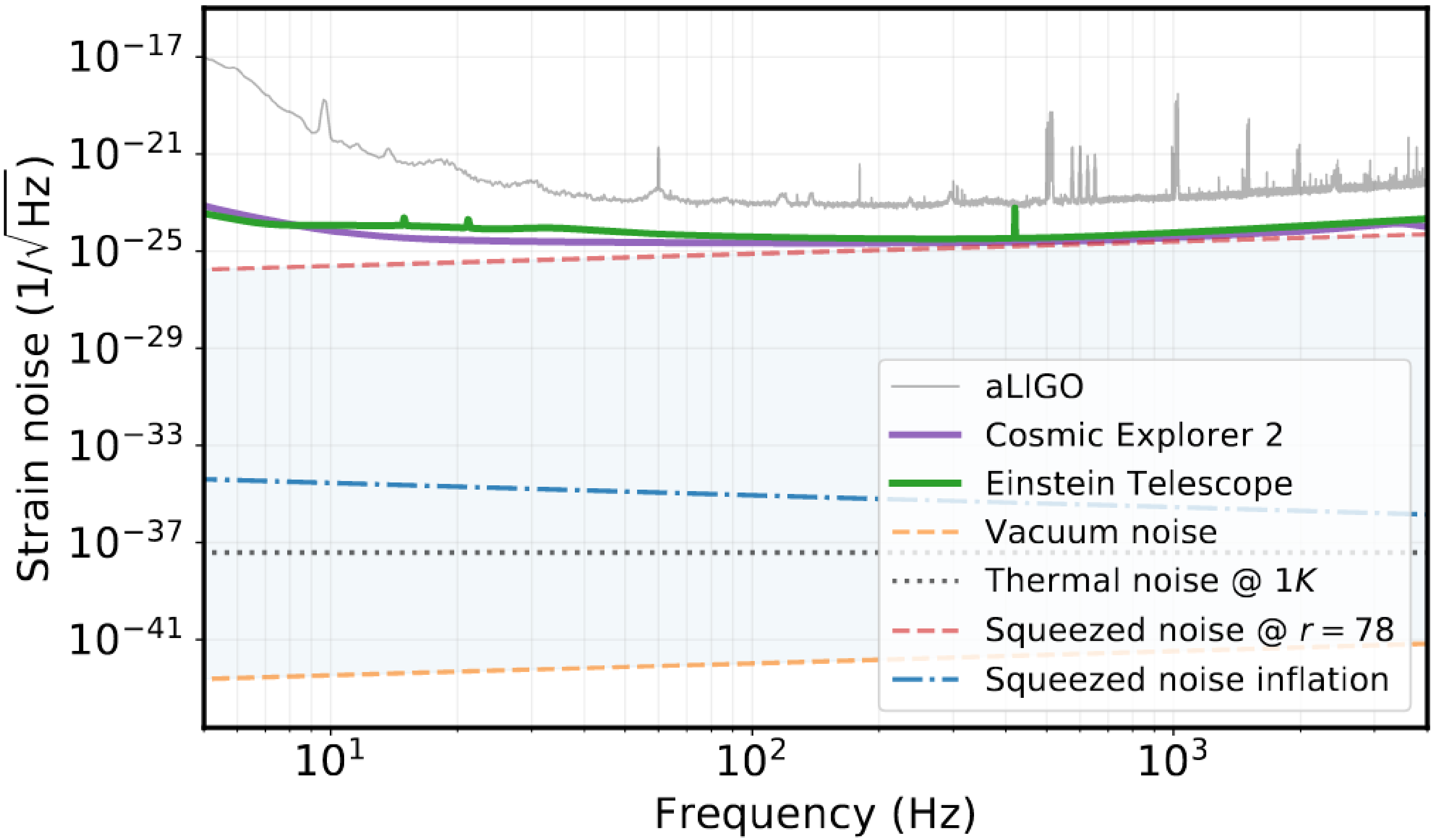
$$|\beta\rangle_{GW} = \hat{S}(\beta)|0\rangle_{GW}$$

Squeezed vacuum

$$|\beta\rangle_{GW} = \hat{S}(\beta)\,|0\rangle_{GW}$$

$$\mathcal{E}(t) \sim 2\alpha\left[1 - 8q^2 e^{|\beta|} \sin^4\left(\frac{\Omega t}{2}\right)\right]$$

EFFECT ON NOISE

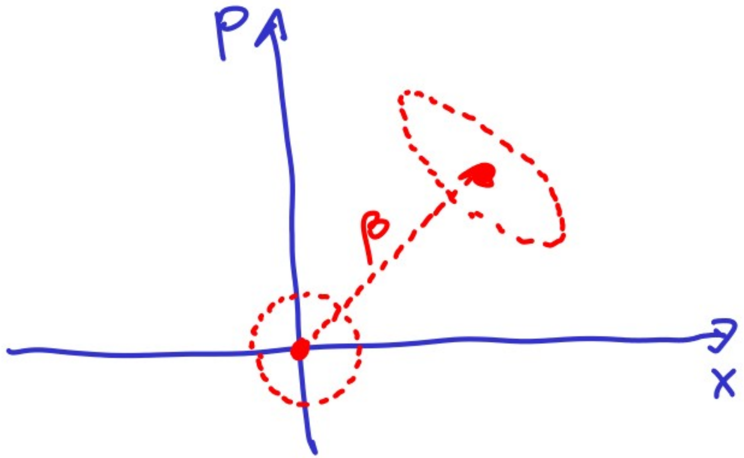


Coradeschi, F.; Frassino, A.M.; Guerreiro, T.; West, J.R.; Schioppa, E.J. Can We Detect the Quantum Nature of Weak Gravitational Fields? Universe 2021, 7, 414. https://doi.org/10.3390/universe7110414

Squeezed coherent

$\boxed{\text{SQUEEZED - COHERENT GRAVITY}}$: $|\psi\rangle_{GW} = \hat{S}(r, \phi_0)\,\hat{D}(\beta)\,|0\rangle_{GW}$

$$\hat{S}(r, \phi_0) = e^{\frac{1}{2} r \left(\hat{a}^2 e^{-2i\phi_0} - (\hat{a}^\dagger)^2 e^{2i\phi_0} \right)}$$



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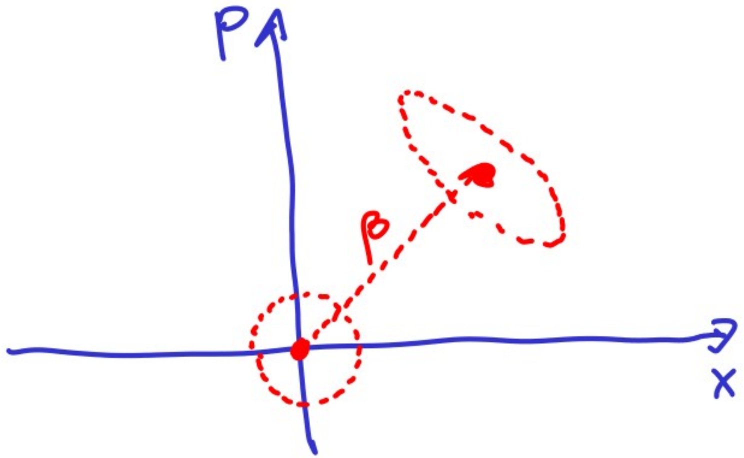
$$\Delta\phi = 2|h|g\left[\sin\Omega t \cosh r + \sin(2\phi_0 - \Omega t)\sinh r - \sin 2\phi_0 \sinh r\right]$$

1) $r=0 \rightarrow \Delta\phi = 2g|h|\sin\Omega t$

2) $\phi_0 = 0 \rightarrow \Delta\phi = 4|h|g\,e^{-r}\sin\Omega t$

3) $\phi_0 = \frac{\pi}{2} \rightarrow \Delta\phi = 4|h|g\,e^{r}\sin\Omega t$



Summary and conclusions

- We work in the weak regime and consider hypothetical quantum states of gravity
- We derive the effects they could have on a LIGO-like experiments
- We find an **exponential effect on the signal**

Many open questions, among which:

- Are these states realized? ($\rightarrow$ strong regime)
- Are these stated propagated? (…)
- In which circumstances can these effect be of order 1 ?