Time delays,

choice of energy-momentum variables and relative locality **in doubly special relativity**

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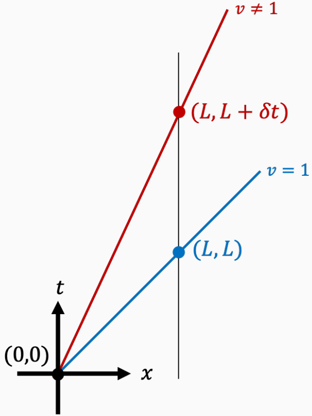
Introduction

Photon time delay can be defined as the difference between the temporal coordinates in the detection of a high-energy and a low-energy photon emitted simultaneously.

It is a necessary consequence of the LIV frameworks, in which a momentum-dependent velocity for photons stems inevitably from a modified energy-momentum relation.



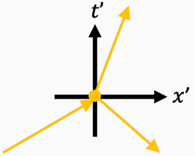
The existence of time delays in DSR is subtler due to the presence of additional ingredients apart from the modified energy-momentum relation, like a composition law of momenta, relative locality, non-trivial translations, and the choice of the energy-momentum variables.

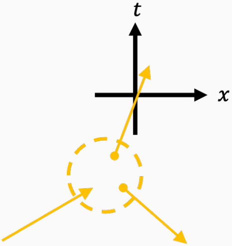
Some questions arise regarding time delays in DSR:

- Is there a consistent way to implement relative locality in a model of time delays?
- Are time delays a necessary consequence of DSR?
- Is the physics, in this case the time delay, different under a change of the energy-momentum variables?

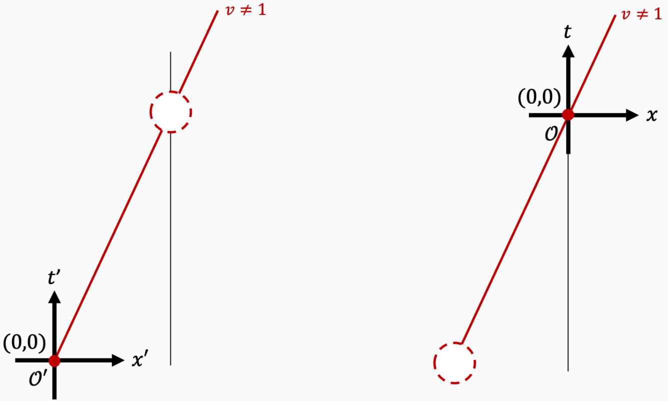
Model of time delay consistent with relative locality

As a consequence of a composition law of momenta, there exists only one observer which sees the worldlines of the particles participating in one interaction to meet at the same point, and it is the observer that sees the interaction in its origin of space-time coordinates.

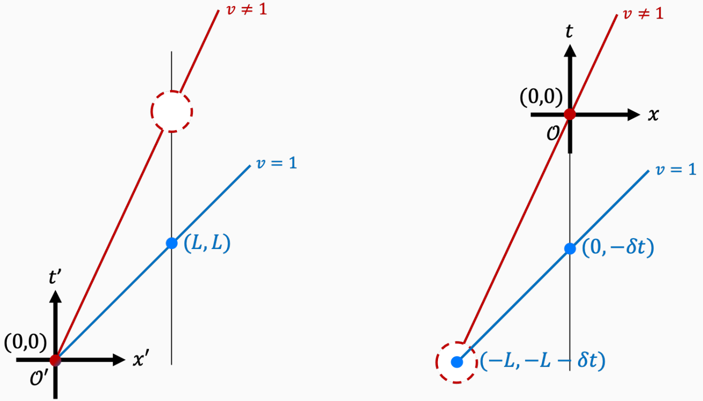




In order to consistently assign coordinates to the emission and detection of the high-energy photon one will need two observers. Let us call $\mathcal{O}'$ the one that is local to its emission, and $\mathcal{O}$ the one that is local to its detection.



One can neglect the effects of DSR in the propagation of the low-energy photon. Additionally, if one consider that its emission and detection are low-energy processes, both are seen as local for every observer.



If the source and detector are at relative rest, the observers $\mathcal{O}$ and $\mathcal{O}'$ are related by a translation. This translation can be described using the most general first-order deformation of the Poincaré algebra (in powers of a energy scale Λ) acting on the canonical one-particle phase space (x, t, Π, Ω),

$$
\begin{aligned}
E &= \Omega + \frac{a_1}{\Lambda}\Omega^2 + \frac{a_2}{\Lambda}\Pi^2 \,, \quad P = \Pi + \frac{a_3}{\Lambda}\Omega\,\Pi \,, \\
N &= x\,\Omega - t\,\Pi + \frac{a_4}{\Lambda}x\,\Omega^2 + \frac{a_5}{\Lambda}x\,\Pi^2 - \frac{a_6}{\Lambda}t\,\Omega\,\Pi \,,
\end{aligned} \tag{1}
$$

which includes, along the generators of space-time translations (E, P), the generator of the boosts N.

This transformations leave unchanged the Casimir

$$C = \Omega^2 - \Pi^2 + \frac{\alpha_1}{\Lambda}\,\Omega^3 + \frac{\alpha_2}{\Lambda}\,\Omega\,\Pi^2\,, \tag{2}$$

where the coefficients α_1 and α_2 are determined by $\{N, C\} = 0$.

From $C(\Pi, \Omega) = m^2$ one can obtain the energy-momentum relation for photons ($m = 0$),

$$\Omega = \Pi - \frac{(\alpha_1 + \alpha_2)}{2\,\Lambda}\,\Pi^2 = \Pi - \frac{(a_4 + a_5 - a_6)}{3\,\Lambda}\,\Pi^2\,, \tag{3}$$

from which one can obtain the velocity of the photons,

$$v = 1 - \frac{(\alpha_1 + \alpha_2)}{\Lambda}\,\Pi = 1 - \frac{2\,(a_4 + a_5 + a_6)}{3\,\Lambda}\,\Pi\,, \tag{4}$$

The translation connecting $\mathcal{O}$ and $\mathcal{O}'$ will be generated by (E, P) with parameters (ϵ_0, ϵ_1), still to be determined,

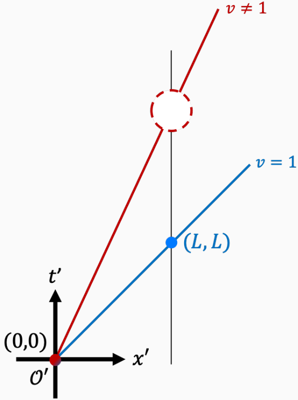
$$x' = x - \epsilon_1\{P, x\} + \epsilon_0\{E, x\}\,, \qquad t' = t - \epsilon_1\{P, t\} + \epsilon_0\{E, t\}\,. \tag{5}$$

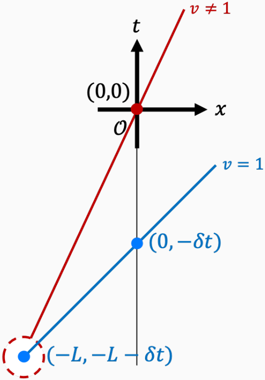
The trajectory of the high-energy photon must be $x = vt$ for $\mathcal{O}$ and $x' = vt'$ for $\mathcal{O}'$, respectively. This fixes a condition over the parameters of the translation,

$$\frac{\epsilon_0}{\epsilon_1} = 1 + \frac{2(a_4 + a_5 - a_6)}{3\Lambda}\,\Pi - \frac{2(a_1 + a_2 - a_3)}{\Lambda}\,\Pi\,. \tag{6}$$

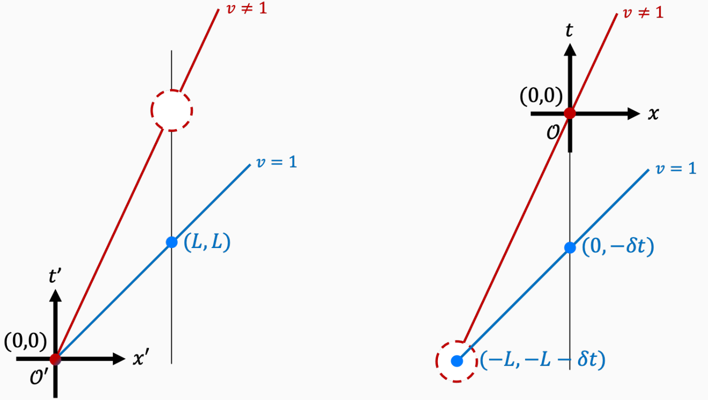
which has two contributions, one from the momentum-dependency of the velocity and another from the deformed space-time translations.

$\mathcal{O}'$ sees the detection of the low-energy photon as local and at some point of its trajectory ($x' = t'$) that we can call $(x', t') = (L, L)$, where L is what we usually call the "distance between source and detector" in SR.





$\mathcal{O}$ sees the detection of the low-energy photon as local and at the same spacial coordinate as the detection of the high-energy one, $x = 0$, but the temporal coordinate can be different, and its value is (minus) the time delay, $t = -\delta t$.



$(x', t') = (L, L)$ and $(x, t) = (0, -\delta t)$ are the coordinates that $\mathcal{O}'$ and $\mathcal{O}$ assign to the same event, the detection of the low-energy photon; then they should be related by the translation generated by (E, P) with parameters (ϵ_0, ϵ_1). Solving for δt one obtains the time delay,

$$\delta t \;=\; L \left[\frac{2(a_4 + a_5 - a_6)}{3\Lambda}\, \Pi - \frac{2(a_1 + a_2 - a_3)}{\Lambda}\, \Pi \right] , \tag{7}$$

which has two contributions, one from the momentum-dependency of the velocity and another from the deformed space-time translations. One will have cancellation of the two effects, and then no observable time delay, when

$$a_4 + a_5 - a_6 \;=\; 3(a_1 + a_2 - a_3)\,. \tag{8}$$

Let us notice that one can change the choice of energy momentum variables, e.g. from (Π, Ω) to some $(\bar{\Pi}, \bar{\Omega})$, through a canonical transformation,

$$\overline{\Omega} = \Omega + \frac{\delta_1}{\Lambda}\Omega^2 + \frac{\delta_2}{\Lambda}\Pi^2\,, \quad \overline{\Pi} = \Pi + \frac{\delta_3}{\Lambda}\Omega\,\Pi\,,$$
$$\bar{t} = t - \frac{2\delta_1}{\Lambda}t\Omega + \frac{\delta_3}{\Lambda}x\,\Pi\,, \quad \bar{x} = x + \frac{2\delta_2}{\Lambda}t\,\Pi - \frac{\delta_3}{\Lambda}x\,\Omega\,, \tag{9}$$

but one can check by explicit calculation that the value of the time delay is invariant under such transformation (freedom of *labeling*),

$$\delta\bar{t} = L\,\frac{2(\overline{a}_4 + \overline{a}_5 - \overline{a}_6 - 3\overline{a}_1 - 3\overline{a}_2 + 3\overline{a}_3)}{3\Lambda}\,\overline{\Pi}$$
$$= L\,\frac{2(a_4 + a_5 - a_6 - 3a_1 - 3a_2 + 3a_3)}{3\Lambda}\,\Pi = \delta t\,. \tag{10}$$

In fact, one can write the time delay in terms of the deformed algebra of (E, P, N) only, independently of its representation in the phase space,

$$\delta t = L \frac{2(w_1 + w_2 - w_3)}{3\Lambda} P, \tag{11}$$

where (w_1, w_2, w_3) are defined as

$$\{N, E\} = P + \frac{w_3}{\Lambda} EP, \qquad \{N, P\} = E + \frac{w_1}{\Lambda} E^2 + \frac{w_2}{\Lambda} P^2. \tag{12}$$

Let us highlight that a change of energy-momentum variables (Π, Ω) is different from changing the generators (E, P),

$$\overline{E} \,=\, E + \frac{\Delta_1}{\Lambda} E^2 + \frac{\Delta_2}{\Lambda} P^2 \,, \qquad \overline{P} \,=\, P + \frac{\Delta_3}{\Lambda} E\, P \,. \tag{13}$$

Because this last kind of change modifies the algebra of (E, P, N),

$$\{N, \overline{P}\} \,=\, \overline{E} + \frac{\overline{w}_1}{\Lambda} \overline{E}^2 + \frac{\overline{w}_2}{\Lambda} \overline{P}^2 \,, \qquad \{N, \overline{E}\} \,=\, \overline{P} + \frac{\overline{w}_3}{\Lambda} \overline{E}\, \overline{P} \,, \tag{14}$$

so it corresponds to different physics,

$$\delta\bar{t} \,=\, L\, \frac{2(\overline{w}_1 + \overline{w}_2 - \overline{w}_3)}{3\Lambda}\, \Pi \,=\, \delta t - L\, \frac{2(\Delta_1 + \Delta_2 - \Delta_3)}{\Lambda}\, \Pi \,. \tag{15}$$

Composition law and relative locality

One can consider, similarly to the deformation of the generators of space-time translations, a lineal deformation of the generators of the energy-momentum translations X and T,

$$X = x + \frac{b_1}{\Lambda} x \Omega + \frac{b_2}{\Lambda} t \Pi , \qquad T = t + \frac{b_3}{\Lambda} x \Pi + \frac{b_4}{\Lambda} t \Omega , \tag{16}$$

which along with the generator of the boosts N, close a Lie algebra,

$$\{T, X\} = \frac{1}{\Lambda} X , \qquad \{N, X\} = T , \qquad \{N, T\} = X + \frac{1}{\Lambda} N . \tag{17}$$

This determines the coefficients (b_1, b_2, b_3, b_4) in terms of the coefficients of the boost (a_4, a_5, a_6).

One can perform a translation in the energy-momentum space from $(\Pi, \Omega) \to (\Pi', \Omega')$, with parameters (π, ω),

$$\Pi' \;=\; \Pi + \pi\{X, \Pi\} - \omega\{T, \Pi\}\,, \qquad \Omega' \;=\; \Omega + \pi\{X, \Omega\} - \omega\{T, \Omega\}\,, \quad (18)$$

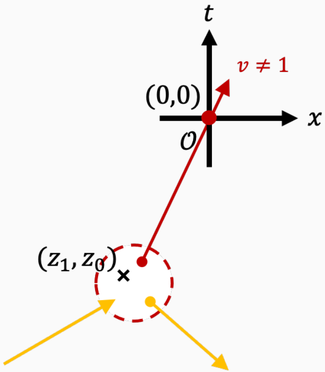
Identifying $q = (\Pi, \Omega)$, $p = (\pi, \omega)$ and $(p \oplus q) = (\Pi', \Omega')$, one can define a composition law of momenta,

$$\begin{aligned}(p \oplus q)_1 \;&\doteq\; p_1 + q_1 + \frac{\gamma_1}{\Lambda}\, p_0\, q_1 + \frac{\gamma_2}{\Lambda}\, p_1\, q_0\,,\\ (p \oplus q)_0 \;&\doteq\; p_0 + q_0 + \frac{\beta_1}{\Lambda}\, p_0\, q_0 + \frac{\beta_2}{\Lambda}\, p_1\, q_1\,. \end{aligned} \quad (19)$$

Using the composition law, one can define the total energy $\mathcal{E}$ and total momentum $\mathcal{P}$ of a certain interaction.

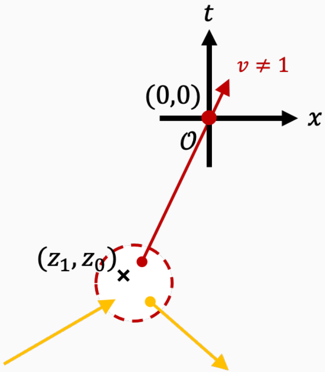
This is used in the action formulation of relative locality, to compute the end or starting points of the world-lines participating in an interaction.

$$
\begin{aligned}
t_i &= z_1 \frac{\partial \mathcal{P}}{\partial \Omega_i} - z_0 \frac{\partial \mathcal{E}}{\partial \Omega_i} , \\
x_i &= z_1 \frac{\partial \mathcal{P}}{\partial \Pi_i} - z_0 \frac{\partial \mathcal{E}}{\partial \Pi_i} .
\end{aligned} \tag{20}
$$



One can obtain the values of (z_1, z_0) assigned by $\mathcal{O}$ to the process of emission, from the coordinates that this observer assigns to the start of the worldline of the high-energy photon.

Knowing the vertex of the interaction, one can solve the ends or starting points of all the particles participating in the interaction.



Conclusions

- Is there a consistent way to implement relative locality in a model of time delays?

  Yes, because the time delay only depends on the energy of the high-energy photon, and not on all the details of each interaction.

- Are time delays a necessary consequence of DSR?

  No. Depending of the model of DSR, one will not have time delay; e.g. when the algebra (E, P, N) has $(w_1 + w_2 - w_3) = 0$.

- Is the physics, in this case the time delay, different under a change of the energy-momentum variables?

  The time delay does not change under a canonical transformation of the phase space (relabeling). On the contrary, changing the generators of space-time translations, effectively, changes the physics.

[1] J. M. Carmona et al. “Time delays, choice of energy-momentum variables and relative locality in doubly special relativity”. In: (July 2022). arXiv: *2207.03799 [gr-qc]*.

[2] Domenico Frattulillo. “Planck scale deformed realtivistic transformations in curved spacetime”. talk given at Corfu Summer Institute for the Second Annual Conference of COST CA18108, Greece. 2021. URL: *https://www.youtube.com/watch?v=twAMuqn5C-U&list=PLUX8Mk7mqLPKmlEE4w2qfYg4km__j0I1U&index=3*.

[3] Giovanni Amelino-Camelia et al. “The principle of relative locality”. In: *Phys. Rev.* D84 (2011), p. 084010. DOI: *10.1103/PhysRevD.84.084010*. arXiv: *1101.0931 [hep-th]*.