Phenomenological signatures of two-body decays in deformed relativity

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Outline

- Finsler geometry as a tool for preserving relativistic principles at the Planck scale
- Novel Finite Lorentz-Finsler transformation between arbitrary frames
- Related composition law
- Lessons and phenomenological opportunities
- Summary

Motivation

- MDRs provide an effective way of describing the propagation of particles in a quantum spacetime

[Amelino-Camelia, Liv. Rev. Rel. (2013)].

$$m^2 = E^2 - p^2 + \frac{1}{E_{QG}} \left[\alpha E^3 + \beta E^2 p + \gamma E p^2 + \lambda p^3\right] + \dots$$

- Where one expects $$E_{QG} = \mathcal{O}(E_P) \quad , \quad E_P \approx 1.22 \times 10^{19}\,\mathrm{GeV}$$

- Forbidden decays of particles can be affected in a LIV scenario [Albert *et al.* (HAWK Collaboration), PRL (2020)]
  Depends on the conservation law of energy-momentum in decays $1 \to 2 + 3$

- Important constraints are given by the analysis of massless particles by searching for a time delay from astrophysical sources: LIV - $E_{QG} \geq 0.58 \times 10^{19}\,\mathrm{GeV}$ [Acciari *et al.* (MAGIC Collaboration), PRL (2020)]
  LIV+DSR [Levy *et al.* (PoS ICRC2021) arXiv:2108.03992]

Geometry as a tool

- The presence of a Planck scale departures of relativistic equations, like the MDR, does not necessarily imply in a violation of the relativity principle

- The DSR approach is responsible for showing that it is possible to deform rather than break Lorentz symmetry at the Planck scale

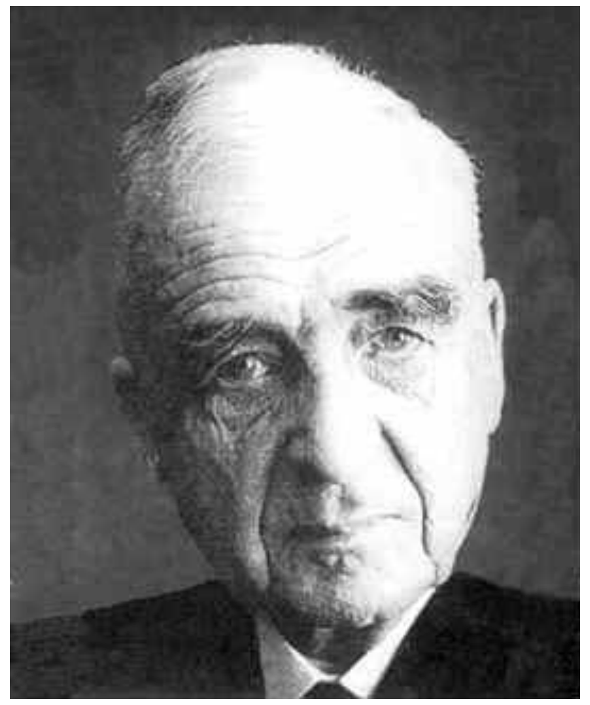
- The principles behind relativistic theories, like the relativity principle, equivalence principle and clock postulate, can be conveniently described by geometrical means

• Relativity principle	• Symmetries
• Equivalence principle	• Geodesic trajectory
• Dispersion Relation	• Norm of 4-momentum
• Clock postulate	• Proper time is the arc-length

Finsler geometry

- Convenient geometry

- We adopt the use of Finsler geometry of spacetime



$$S[x,p,\lambda]_H = \int d\mu[\dot{x}^a p_a - \lambda\,(H(x,p) - m^2)]$$

$$S[x] = m\int F(x,\dot{x})d\mu$$

[Girelli, Liberati, Sindoni, PRD (2007)]

- Geodesics are the extremizing trajectories of the action
- Killing vectors give DSR symmetries
- MDR is a norm calculated with the Finsler metric
  [Amelino-Camelia *et al.*, PRD (2014)],
  [Letizia, Liberati, PRD (2017)]
  [**Lobo**, Loret, Nettel, PRD (2017)],

- Action = Arc-length => Finsler function

$$\boxed{g_{\mu\nu}(x,\dot{x}) = \frac{1}{2}\frac{\partial^2 F^2}{\partial\dot{x}^\mu \partial\dot{x}^\nu}}$$

- **κ-Poincaré**

$$\boxed{H(p) = p_0^2 - |\vec{p}|^2 - \ell p_0 |\vec{p}|^2 = m^2}$$

$$\boxed{F(\dot{x}) = \sqrt{\eta(\dot{x},\dot{x})} + \frac{\ell m}{2} \frac{\dot{x}^0 \delta_{ij} \dot{x}^i \dot{x}^j}{\eta(\dot{x},\dot{x})}}$$

- Definition of momenta

[**IPL**, Pfeifer, PRD (2021)]

$$p_\mu = \frac{\partial F}{\partial \dot{x}^\mu} \qquad \begin{cases} p_0(v) = \gamma m - \frac{\ell}{2} m^2 (\gamma^2 - 1)(2\gamma^2 - 1) \\ p_i(v) = -\gamma v_i m + \ell m^2 \gamma^4 v_i \end{cases}$$

$$\gamma = 1/\sqrt{1 - v^2}$$

- Deformed Lorentz Transformation from the rest frame to the lab frame

$$H(p(v)) = H(p(0)) = m^2$$

!

- Can we find a more general transformation connecting arbitrary frames that move relative to each other with speed v?

- General Lorentz symmetry

$$\begin{cases} \tilde{p}_0 = \gamma(p_0 - vp_1) + \ell\left[Ap_0p_1 + Bp_1^2 - \frac{1}{2}p_0^2(\gamma^2-1)(2\gamma^2-1)\right] \\ \tilde{p}_1 = \gamma(p_1 - vp_0) + \ell\left(p_0^2 v\gamma^4 + Fp_0p_1 + Gp_1^2\right) \end{cases}$$

$$H(p) = H(\tilde{p})$$

- Finite deformed Lorentz transformation

$$\boxed{\Lambda(v,p) = \begin{cases} \tilde{p}_0 = \gamma(p_0 - vp_1) + \frac{\ell}{2}\left[p_1^2\gamma(2\gamma^3-\gamma-1) - p_0^2(\gamma^2-1)(2\gamma^2-1)\right] \\ \tilde{p}_1 = \gamma(p_1 - vp_0) + \ell\, v\left[p_0^2\gamma^4 - \frac{p_1^2}{2}\gamma(2\gamma^3-2\gamma-1)\right] \end{cases}}$$

- Usual k-Poincaré finite boost transformation [Gubitosi, Mercati, CQG (2013)]

$$\begin{cases} \tilde{p}_0 = p_0\cosh(\xi) + p_1\sinh(\xi) - \ell\sinh^2\left(\frac{\xi}{2}\right)\left[\left(p_0^2+p_1^2\right)\cosh(\xi) + p_0(p_0 + 2p_1\sinh(\xi))\right] \\ \tilde{p}_1 = p_0\sinh(\xi) + p_1\cosh(\xi) - \ell\sinh\left(\frac{\xi}{2}\right)\left[p_0^2\cosh\left(\frac{\xi}{2}\right) + \left(p_0^2+p_1^2\right)\cosh\left(\frac{3\xi}{2}\right) + 2p_0p_1\sinh\left(\frac{3\xi}{2}\right)\right] \end{cases}$$

- Map between rapidity and spacetime velocity

$$\xi(v) = \operatorname{arcsinh}(v\gamma) + \ell[-v^3\gamma^3p_0 + p_1(\gamma^3-1)]$$

Modified composition law

- General composition law

$$\begin{cases} (p \oplus q)_0 = p_0 + q_0 + \ell(\alpha p_0 q_0 + \beta p_1 q_1 + \omega p_0 q_1 + \eta p_1 q_0) \\ (p \oplus q)_1 = p_1 + q_1 + \ell(\delta p_1 q_0 + \epsilon p_0 q_1 + \lambda p_1 q_1 + \mu p_0 q_0) \end{cases}$$

- Covariance condition

$$\Lambda(v; ((p \oplus q)_0, (p \oplus q)_1)) = \Lambda(\mathfrak{v}_{v;q_0,q_1}; (p_0, p_1)) \oplus \Lambda(\mathfrak{v}_{v;p_0,p_1}; (q_0, q_1))$$

Back-reaction $\quad \mathfrak{v}_{v;q_0,q_1} = v + \ell(Hq_0 + Jq_1)$

$$\begin{cases} (p \oplus q)_0 = p_0 + q_0 + \ell p_1 q_1 \\ (p \oplus q)_1 = p_1 + q_1 \end{cases}$$

$$\mathfrak{v}_{v;k_0,k_1} = v + \ell\left[\left(\frac{1}{\gamma} - \gamma\right) k_1 - v\gamma k_0\right]$$

$$\begin{cases} (p \oplus q)_0 = p_0 + q_0 \\ (p \oplus q)_1 = p_1 + q_1 - \ell p_0 q_1 \end{cases}$$

$$\mathfrak{v}_{v;q_0,q_1} = v - \ell\left[v\frac{(\gamma^2 - 1)}{\gamma} q_0 + \left(\gamma - \frac{1}{\gamma^2}\right) q_1\right]$$

$$\mathfrak{v}_{v;p_0,p_1} = v - \ell\left[v\gamma p_0 + \left(\gamma - \frac{1}{\gamma}\right) p_1\right]$$

Phenomenological opportunities: Time dilation

- Deformed Lorentz transformation

$$p_0(v) \approx \gamma m - \ell m^2 \gamma^4$$

$$\gamma \approx (p_0/m)\left(1 + \ell p_0^3/m^2\right)$$

$$\boxed{\Delta t = \gamma \Delta\tau \approx (p_0/m)\Delta\tau \left(1 + \ell p_0^3/m^2\right)}$$

- Clock postulate

$$\Delta\tau = \int_{t_A}^{t_B} F(x, \dot{x})\, dt \approx \gamma^{-1}\Delta t\,(1 + \ell m \gamma^3/2)$$

$$\boxed{\Delta t \approx \gamma \Delta\tau (1 - \ell m \gamma^3/2)}$$

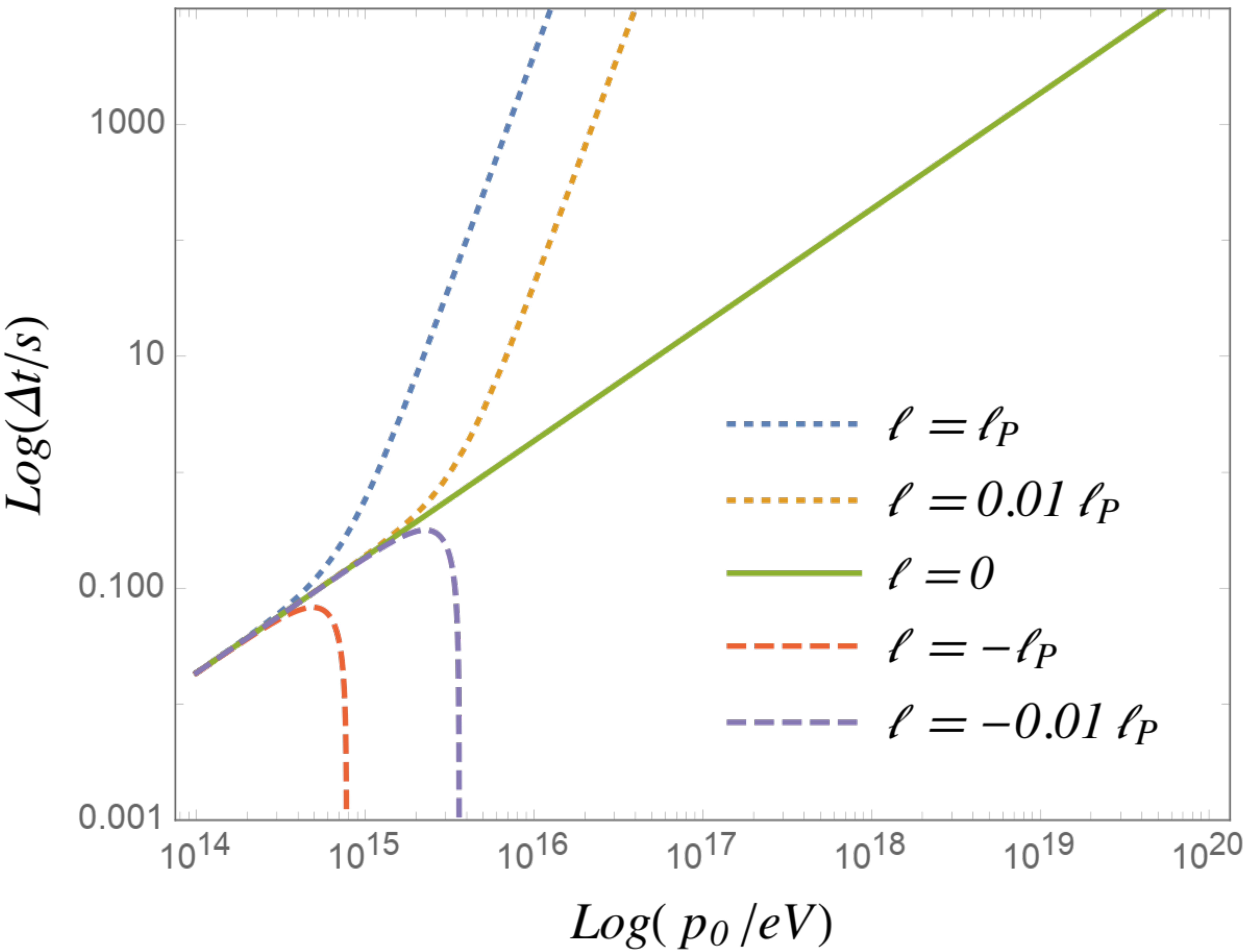
- Clock postulate + Deformed Lorentz transformation

$$\boxed{\Delta t \approx (p_0/m)\Delta\tau (1 + \ell p_0^3/2m^2)}$$

<u>Dimensionless correction of the order</u>

$$\ell m \gamma^3$$

$$\ell p_0^3/m^2$$



- Lifetime of pion π (m = 140 MeV)

Threshold effects

- We analyzed the two-body decay in this scenario $M \to m_1 + m_2$
- We particularly analyzed the case $\pi \to \mu + \nu$
- We calculated the threshold lab frame energies in the lab frame
- We calculated the maxima and minima energies in the center of momentum frame and then boosted them to the lab frame

The results are the same, as a verification of the relativity principle in DSR

<u>Results</u>

$$E^{(\pm)}_{\mu,\nu} = (E^{(\pm)}_{\mu,\nu})_{SR}\big(1 + \mathcal{O}(\ell E_\pi)\big)$$

There is no amplification in threshold energies analyses in DSR

Lessons from DSR

- The time dilation presents a correction of the order $\ell E^3/m^2$

- Modifications in the time dilation affect the distance particles propagate, which is an input in cosmic rays analyses

- This is a manifestation of the relativistic nature of this approach, therefore it should be valid for other powers of corrections

$$E^2 - p^2 + (\ell E)^n E^2 = m^2 \quad \Rightarrow \quad \Delta t \approx \Delta t_{SR}[1 + (\ell E)^n E^2/m^2]$$

[**IPL**, Pfeifer, PRD (2021)]

Threshold effects do not contribute significantly

- Verified in previous cases, but reconfirmed using our finite transformations

[Amelino-Camelia, Liv. Rev. Rel. (2013)]

LIV?

1. Unmodified composition law
   One frame is preferred and usual threshold effects of LIV are present [Albert *et al.* (HAWK Collaboration), PRL (2020)]

2. Lorentz transformations:
   2.1 Standard ones do not carry new measurable contributions
   2.2 Other kind of modified Lorentz transformation could carry measurable contributions
   [Abreu et al. (Pierre Auger) PoS IRC2021 (2021)]

$$E^2 - p^2 = m^2 + (\ell E)^n E^2 = m_{LIV} \quad \longrightarrow \quad E/m_{LIV} \approx (E/m)(1 - (\ell E)^n E^2/2m^2) = \gamma_{LIV}$$

$$\longrightarrow \quad \Delta t = \gamma_{LIV} \Delta \tau$$

3. Clock postulate
   3.1 If valid = measurable contribution
   3.2 If not valid = no measurable contribution

Conclusion

- From the Finsler function inspired by the bicrossproduct basis of κ-Poincaré, we identified a deformed Lorentz transformation that connects the rest frame of a particle to the lab frame

- We extended this transformation in $1+1$D to connect arbitrary frames and verified its compatibility with the boost generator of the κ-Poincaré algebra

- We constructed its relativistic-compatible composition law and back-reaction rules

- As results we found that the DSR effects only impacts the time dilation, without threshold effects in two-body decays

- Possible impact in addressing the muon puzzle, that corresponds to an excess in the measured number of muons showers initiated by UHECRs compared to cosmic predictions

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- As prospects, we shall proceed into further investigations considering cosmic rays thanks to the network provided by the COST Action CA18108

Thank you!

Grazie!

Clock postulate

The proper time an observer, or massive particle, experiences between events A and B along a time-like curve (her worldline) in a Finsler spacetime $(\mathscr{M}, F)$ is the length of this curve between events A and B:

$$\Delta\tau_{AB} \doteq \int_{t_A}^{t_B} F(x, \dot{x})\, dt$$

- **κ-Poincaré Bicrossproduct basis**

$$\boxed{H(p) = p_0^2 - |\overrightarrow{p}|^2 - \ell p_0 |\overrightarrow{p}|^2 = m^2}$$

- **Finsler function**

$$\boxed{F(\dot{x}) = \sqrt{\eta(\dot{x}, \dot{x})} + \frac{\ell m}{2} \frac{\dot{x}^0 \delta_{ij} \dot{x}^i \dot{x}^j}{\eta(\dot{x}, \dot{x})}}$$

- **Dilated lifetime**

$$\boxed{\Delta t_F \approx \gamma \Delta t \left[1 - \frac{\ell m}{2} \gamma^3\right]}$$

- Clock postulate + Deformed Lorentz Transformation

- For $p_0/m \gg 1$

$$\Delta t = \underbrace{\Delta t_{SR}}_{\frac{p_0}{m}\Delta\tau}\left(1 + \frac{\ell}{2}\frac{p_0^3}{m^2}\right)$$

MOTIVATION

- But now, if we want to complete the DSR approach and **diversify** the analysis, by considering for instance, data from cosmic rays, we need to find transformations between **arbitrary frames** and its corresponding **composition law**

Composition law

- To preserve interaction vertices under deformed Lorentz transformations, it's necessary to compensate the non-linearity of the transformation with a deformed composition law

$$\Lambda(p \oplus q) = \Lambda(p) \oplus \Lambda(q)$$

- General composition law

$$\begin{cases} (p \oplus q)_0 = p_0 + q_0 + \ell(\alpha p_0 q_0 + \beta p_1 q_1 + \omega p_0 q_1 + \eta p_1 q_0) \\ (p \oplus q)_1 = p_1 + q_1 + \ell(\delta p_1 q_0 + \epsilon p_0 q_1 + \lambda p_1 q_1 + \mu p_0 q_0) \end{cases}$$

- FOCUS ON THE MAIN ISSUES

- Back-reaction on boost parameter is needed to accomplish the relativistic condition [Majid, arXiv:hep-th/060413, (2006)]

$$\Lambda(v, p \oplus q) = \Lambda(v \triangleleft q, p) \oplus \Lambda(v \,\tilde{\triangleleft}\, p, q) \qquad \begin{cases} v \triangleleft q = v + \ell(H q_0 + J q_1) \\ v \,\tilde{\triangleleft}\, p = v + \ell(M p_0 + R p_1) \end{cases}$$

THRESHOLD EFFECTS (ALGORITHM OF THE PAPER)

- If "*" is the rest frame of the parent particle, "i" labels particles "1" and "2", P_L is the momentum of the parent particle and v connects the frame "*" and the lab [Gaisser, Cosmic Rays and Particle Physics, Cambridge (2016)]

- Special relativistic flat distribution $$\frac{dn_i}{dE_i} = \frac{1}{2\gamma v p_i^*} = \frac{M}{2p_i^* P_L}$$

- Derived from the upper and lower limits on the lab energy of the secondary "i" $$\gamma\left(E_i^* \pm v p_i^*\right)$$

- If we have a deformed Lorentz transformation, the limits change, but the difference is the same

$$\gamma(E_i^* \pm v p_i^*) + \frac{\ell}{2}\left[(p_i^*)^2\gamma(2\gamma^3 - \gamma - 1) - (E_i^*)^2(\gamma^2 - 1)(2\gamma^2 - 1)\right]$$

- Which shall give the same normalized, flat distribution $$\frac{dn_i}{dE_i} = \frac{1}{2\gamma v p_i^*} = ?$$

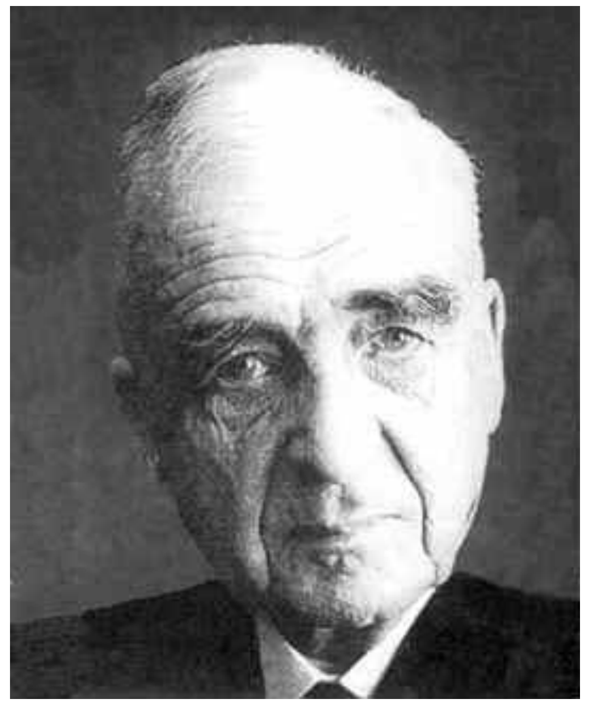
Clock postulate

- Time-based observable involving Massive Particles?

- Recently, the lifetime of fundamental particles has been identified as an observable that can be used to test deformed CPT symmetry inspired by QG close to the Planck scale [Arzano, Kowalski-Glikman, Wislicki, PLB (2019)]

- The lifetime of particles constitutes a natural definition of a "clock"

- We recently extended the clock postulate to the effective spacetime probed by fundamental particles [IPL, Pfeifer, PRD (2021)]



- **Finsler geometry** $g(x, \dot{x})$**:**

$$g_{\mu\nu} = \frac{1}{2}\frac{\partial^2 F^2}{\partial \dot{x}^\mu \partial \dot{x}^\nu}$$

- For a parity-invariant composition law, we can find relations between back-reaction parameters

General case in the forthcoming paper [IPL, Pfeifer, arXiv:2110.xxxxx]

$$J = -\frac{Hv\gamma}{1+\gamma} + \frac{1+\gamma-2\gamma^2}{\gamma}, \qquad M = -\frac{Rv\gamma}{\gamma-1} - v(1+2\gamma)$$

Invariant momentum conservation

$$\boxed{\begin{aligned}(p\oplus q)_0 &= p_0 + q_0 + \ell p_1 q_1\\ (p\oplus q)_1 &= p_1 + q_1\end{aligned}}$$

$$v \triangleleft k = v + \ell\left[\left(\frac{1}{\gamma} - \gamma\right)k_1 - v\gamma k_0\right]$$

- Infinitesimal form

$$v \triangleleft k = v(1 - \ell k_0)$$

Invariant energy conservation (coproduct)

$$\boxed{\begin{aligned}(p\oplus q)_0 &= p_0 + q_0\\ (p\oplus q)_1 &= p_1 + q_1 - \ell p_0 q_1\end{aligned}}$$

$$v \triangleleft q = v + \ell\left[v\frac{(\gamma^2-1)}{\gamma}q_0 - \frac{(3\gamma^3 - 2\gamma^2 - 2\gamma + 1)}{\gamma^2}q_1\right]$$

$$v \,\tilde{\triangleleft}\, p = v - \ell\left[v\gamma p_0 + \left(\gamma - \frac{1}{\gamma}\right)p_1\right]$$

- Infinitesimal form

$$v \triangleleft q = v$$
$$v \,\tilde{\triangleleft}\, p = v(1 - \ell p_0)$$

Coincides with [Gubitosi, Mercati, CQG (2013)]