Constraining gravity theories through the scalar induced gravitational waves from primordial black hole Poisson fluctuations

Theodoros Papanikolaou

National Observatory of Athens

[T. Papanikolaou, V. Vennin, D. Langlois, JCAP 2103 (2021) 053, 2010.11573]
[T. Papanikolaou, C. Tzerefos, S. Basilakos, E.N. Saridakis, 2112.15059]

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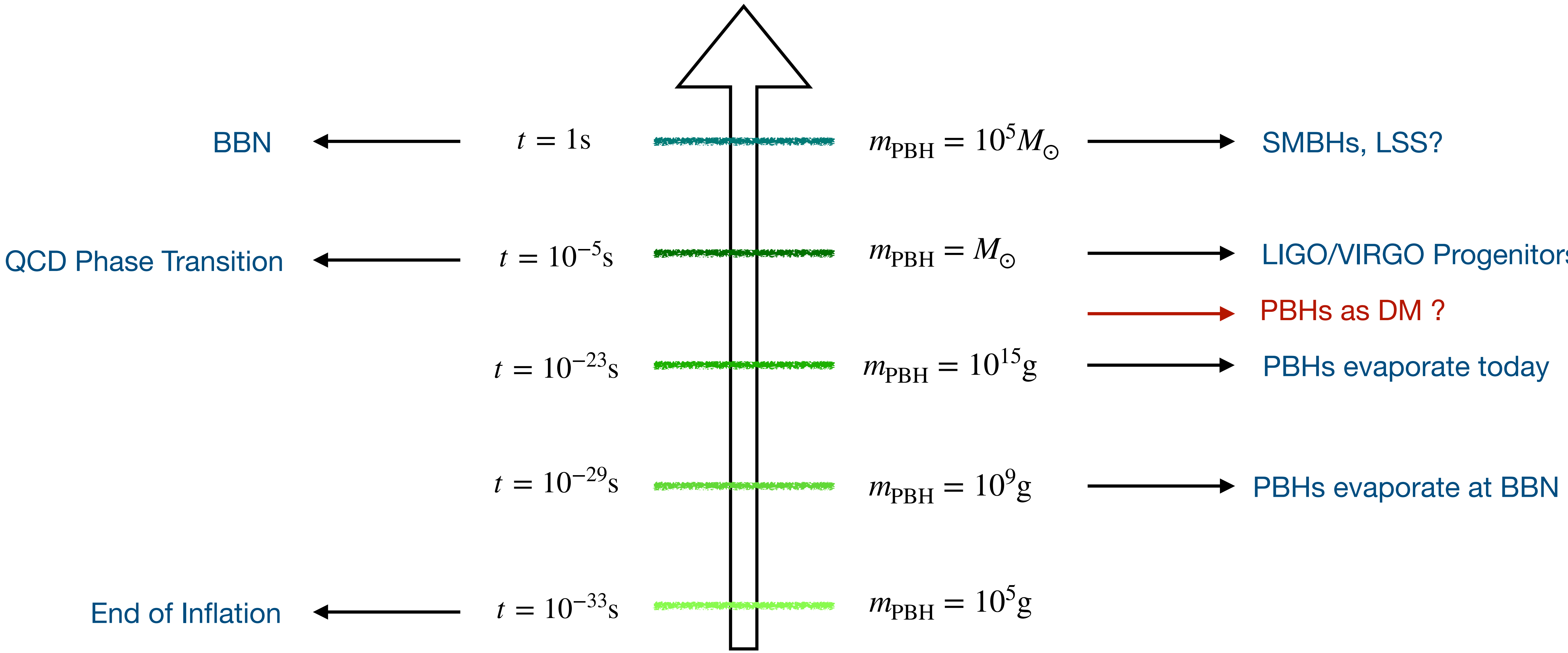
Contents

Introduction

- Primordial Black Holes (PBHs) are formed out of the **collapse of enhanced energy density perturbations** upon horizon reentry of the typical size of the collapsing overdensity region. This happens when $\delta > \delta_c(w \equiv p/\rho)$ [Carr - 1975].

$$m_{\mathrm{PBH}} = \gamma M_{\mathrm{H}} \propto H^{-1} \text{ where } \gamma \sim \mathrm{O}(1)$$



See for reviews in [Carr et al.- 2020, Sasaki et al - 2018, Clesse et al. - 2017]

PBHs and GWs

- 1) **Primordial induced GWs** generated through second order gravitational effects: $\mathscr{L}^{(3)}_{\Phi,h} \ni h\Phi^2$, [Bugaev - 2009, Kohri & Terada - 2018].

  GWs PBHs

- 2) **Relic Hawking radiated gravitons** from PBH evaporation [Anantua et al. - 2008, Dong et al. - 2015].

- 3) **GWs** emitted **by PBH mergers** [Eroshenko - 2016, Raidal et al. - 2017].

- 4) **GWs induced** at second order **by PBHs themselves** [Papanikolaou et al. - 2020].

Induced GWs from ultralight PBHs

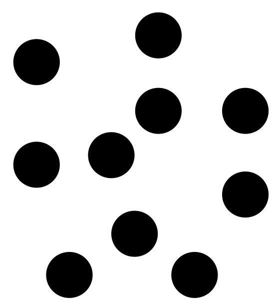
- We studied ultralight PBHs with $m_{\mathrm{PBH}} < 10^9\mathrm{g}$ which have formed in the very early universe and have evaporated by BBN.

- These ultralight PBHs can change drastically the standard cosmological scenario by giving rise to **an early matter dominated era (eMD)** and **driving the reheating process** through their evaporation [Zagorac et al. - 2019, Martin et al. - 2019, Inomata et al. - 2020].

- However, these very small PBHs **leave no direct observational imprint**, apart from possible Planckian relics, since they Hawking evaporate even before Big Bang Nucleosynthesis (BBN).

- A possible way to constrain them is to study the emitted secondary GWs induced from a gas of PBHs which can propagate until today and leave an **indirect imprint** of the PBH past experience.

The PBH Matter Field



$\longrightarrow$ **Poisson Statistics** [Desjacques & Riotto - 2018, Ali-Haimoud - 2018]

Same mass [Dizgah, Franciolini & Riotto - 2019]

$$P_{\delta_{\rm PBH}}(k) \equiv \langle |\delta_k^{\rm PBH}|^2 \rangle = \frac{4\pi}{3}\left(\frac{\bar{r}}{a}\right)^3 = \frac{4\pi}{3k_{\rm UV}^3}, \;\; \text{where} \;\; k < k_{\rm UV} = \frac{a}{\bar{r}}$$

$\rho_{\rm PBH}$ **is inhomogeneous**

$\rho_{\rm tot}$ is **homogeneous**

$\Big\}$ $\delta_{\rm PBH}$ can be seen as an **isocurvature perturbation.**

$\Omega_{\rm PBH} = \rho_{\rm PBH}/\rho_{\rm tot} \propto a^{-3}/a^{-4} \propto a \Rightarrow$ **the isocurvature perturbation,** $\delta_{\rm PBH}$ **will convert during the PBHD era to a curvature perturbation** $\zeta_{\rm PBH}$**, associated to a PBH gravitational potential** Φ**.**

$$\mathcal{P}_\Phi(k) = \frac{2}{3\pi}\left(\frac{k}{k_{\rm UV}}\right)^3\left(5 + \frac{4}{9}\frac{k^2}{k_{\rm d}^2}\right)^{-2}$$

Basics of Scalar Induced Gravitational Waves

- Choosing as the gauge for the GW frame the Newtonian gauge, the metric is written as

$$\mathrm{d}s^2 = a^2(\eta)\left\{-(1+2\Phi)\mathrm{d}\eta^2 + \left[(1-2\Phi)\delta_{ij} + \frac{h_{ij}}{2}\right]\mathrm{d}x^i\mathrm{d}x^j\right\}$$

- The equation of motion for the Fourier modes, $h_{\vec{k}}$, read as:

$$h^{s,''}_{\vec{k}} + 2\mathcal{H}h^{s,'}_{\vec{k}} + k^2 h^s_{\vec{k}} = 4S^s_{\vec{k}}$$

- The source term, $S_{\vec{k}}$ can be recast as:

$$S^s_{\vec{k}} = \int \frac{\mathrm{d}^3\vec{q}}{(2\pi)^{3/2}} e^s_{ij}(\vec{k})q_i q_j \left[2\Phi_{\vec{q}}\Phi_{\vec{k}-\vec{q}} + \frac{4}{3(1+w)}(\mathcal{H}^{-1}\Phi'_{\vec{q}} + \Phi_{\vec{q}})(\mathcal{H}^{-1}\Phi'_{\vec{k}-\vec{q}} + \Phi_{\vec{k}-\vec{q}})\right]$$

- At the end, the energy density of GWs can be recast as [M. Maggiore - 2000]:

$$\rho_{\mathrm{GW}}(\eta, \vec{x}) = \frac{M^2_{\mathrm{Pl}}}{8}\overline{\left(\partial_t h_{\alpha\beta}\partial_t h^{\alpha\beta} + \partial_i h_{\alpha\beta}\partial_i h^{\alpha\beta}\right)}.$$

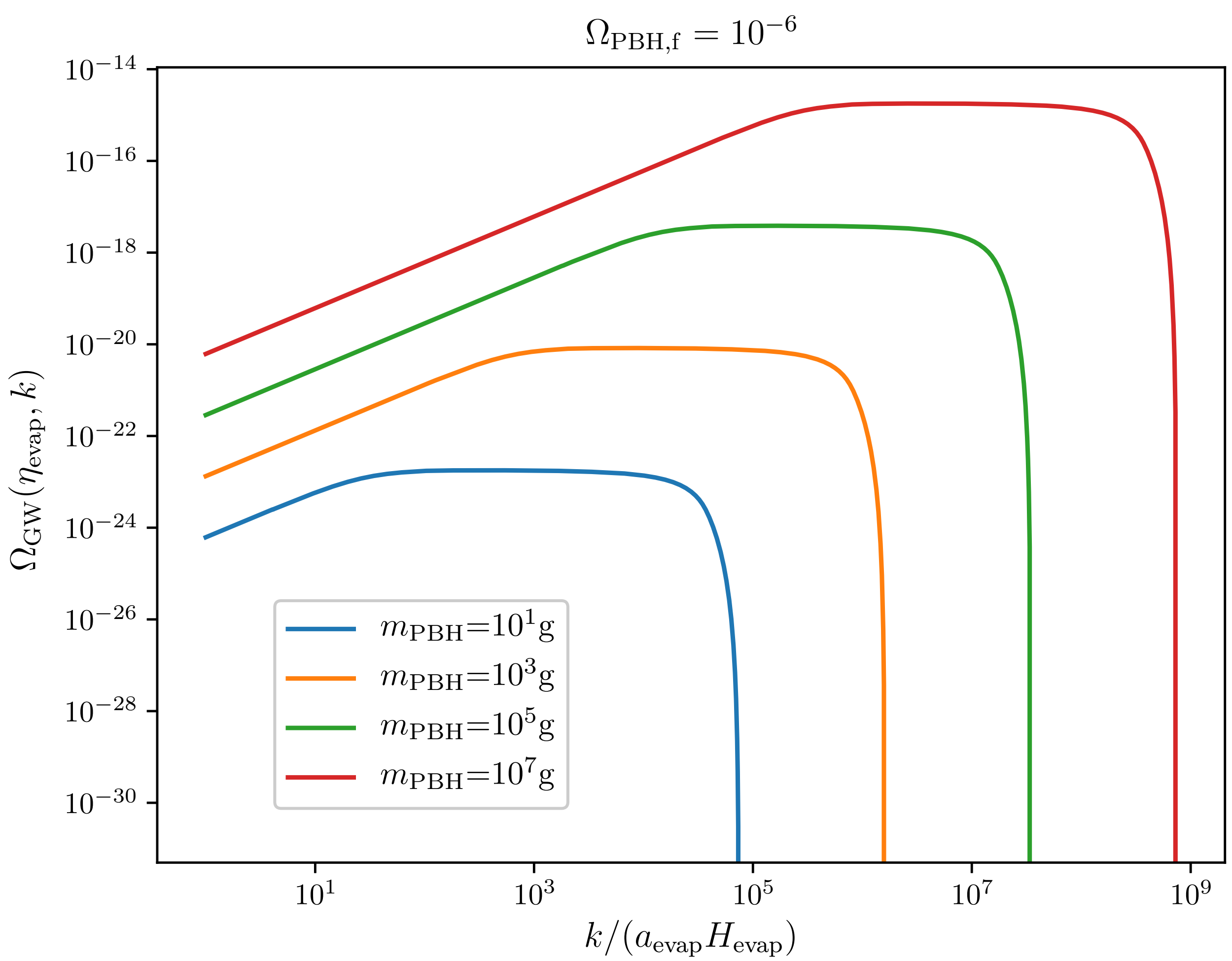
Kinetic Energy (KE) Gradient Energy (GE)

The Gravitational Wave Spectrum

- The spectral abundance, $\Omega_{\rm GW}(\eta, k)$ of GWs can be written as:

$$\Omega_{\rm GW}(\eta, k) \equiv \frac{1}{\rho_{\rm tot}} \frac{\mathrm{d}\rho_{\rm GW}}{\mathrm{d}\ln k} = \frac{1}{24}\left(\frac{k}{a(\eta)H(\eta)}\right)^2 \mathscr{P}_h(\eta, k)$$

$$\text{with} \quad \mathscr{P}_h(\eta, k) \propto \int \mathrm{d}k_1 \int \mathrm{d}k_2 \left(\int f(k_1, k_2, \eta)\mathrm{d}\eta\right)^2 \mathscr{P}_\Phi(k_1)\mathscr{P}_\Phi(k_2)\,.$$

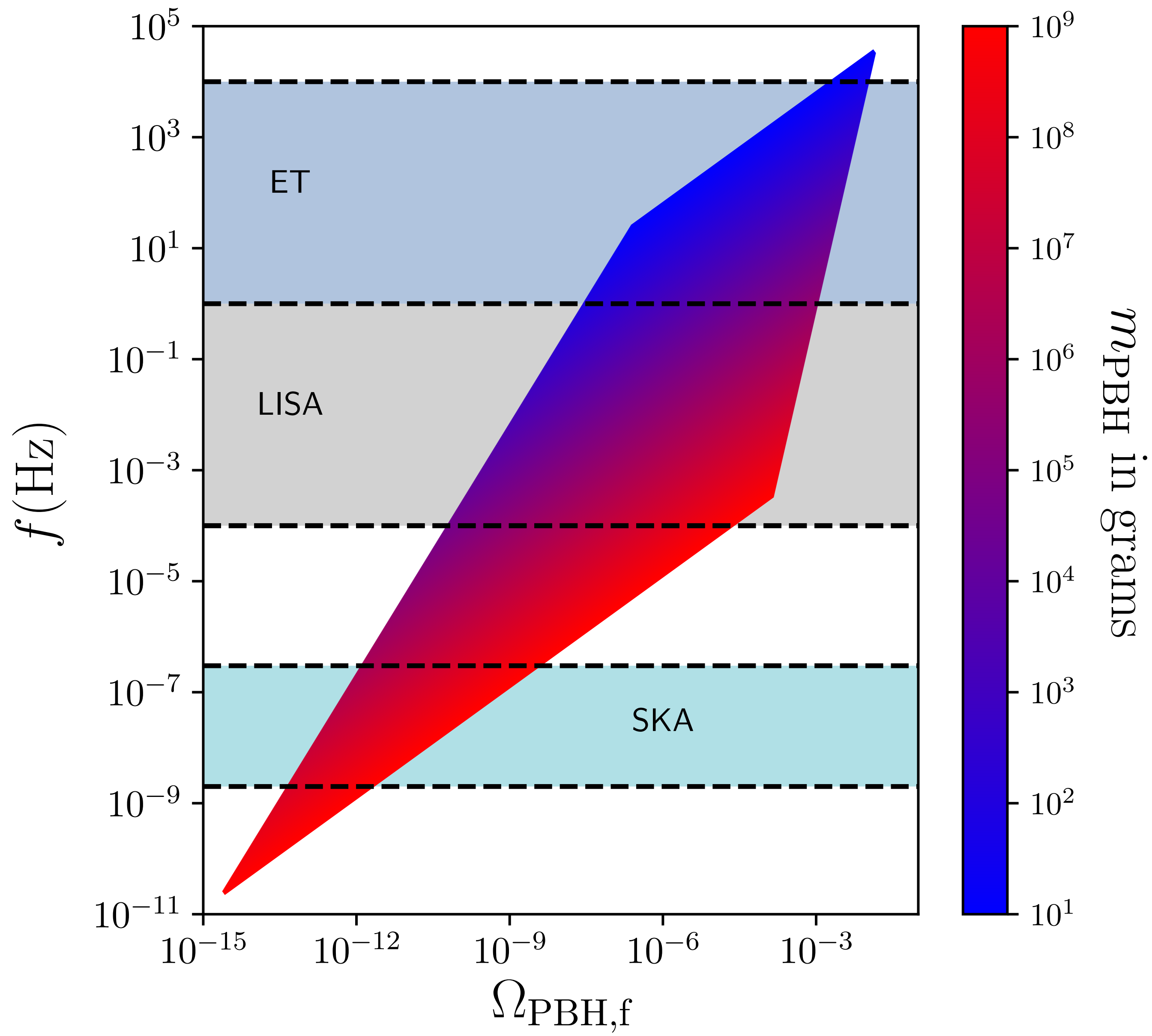


$$\Omega_{\rm GW}(\eta_{\rm evap}, k) \propto \left(\frac{m_{\rm PBH}}{10^9\mathrm{g}}\right)^{4/3} \Omega_{\rm PBH,f}^{16/3} \times \begin{cases} \frac{k}{k_{\rm d}} & \text{for} \quad k \ll \mathscr{H}_{\rm d} \\ 8 & \text{for} \quad k \gg \mathscr{H}_{\rm d} \end{cases}$$

$$\boxed{\Omega_{\rm GW,tot}(\eta_{\rm evap}) \leq 1 \Rightarrow \Omega_{\rm PBH,f} \leq 10^{-4}\left(\frac{10^9\mathrm{g}}{m_{\rm PBH}}\right)^{1/4}}$$

[Papanikolaou et al. - 2020]

GW Frequency



[Papanikolaou et al. - 2020]

- **GWs induced by a dominating gas of PBHs might still be detectable** in the future with gravitational-waves experiments.

The case of $f(R)$ gravity

- Compared to GR, within $f(R)$ gravity the gravitational action reads as:

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,R \Longrightarrow S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,f(R)\,.$$

$$\Big\downarrow$$

$$FR_{\mu\nu} - \frac{1}{2}g_{\mu\nu}f + (g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu)F = 8\pi G T^{\mathrm{m}}_{\mu\nu} \quad \text{with} \quad F(R) \equiv \frac{\mathrm{d}f(R)}{\mathrm{d}R}$$

- Treating again the PBH energy density fluctuations as isocurvature perturbations converting to curvature perturbations during the PBH dominated era one can straightforwardly show that

$$\mathscr{P}_\Phi(k) \equiv \frac{k^3}{2\pi^2}P_\Phi(k) = \frac{2}{3\pi}\left(\frac{k}{k_{\mathrm{UV}}}\right)^3\left[5 + \frac{2}{3}\left(\frac{k}{\mathscr{H}}\right)^2 F\left(\frac{1+3\frac{k^2}{a^2}\frac{F_{,R}}{F}}{1+2\frac{k^2}{a^2}\frac{F_{,R}}{F}}\right)\xi(a)\right]^{-2},$$

$$\text{where } \xi(a(t)) \equiv \frac{\delta_{\mathrm{PBH}}(a(t))}{\delta_{\mathrm{PBH}}(a_{\mathrm{f}})}$$

The tensor perturbations

- In $f(R)$ gravity, one is met with the existence of a new propagating degree of freedom, namely the scalaron field $\phi_{\rm sc} \equiv F(R)$ which follows a wave equation:

$$\Box F(R) = m_{\rm sc}^2 F(R), \quad \text{with} \quad m_{\rm sc}^2 \equiv \frac{1}{3}\left(\frac{F}{F_{,R}} - R\right)$$

- The equation of motion for the tensor perturbations is now modified and can be recast as:

$$h^{s,''}_{\vec{k}} + 2\mathcal{H} h^{s,'}_{\vec{k}} + (k^2 - \lambda m_{\rm sc}^2) h^s_{\vec{k}} = 4 S^s_{\vec{k}},$$

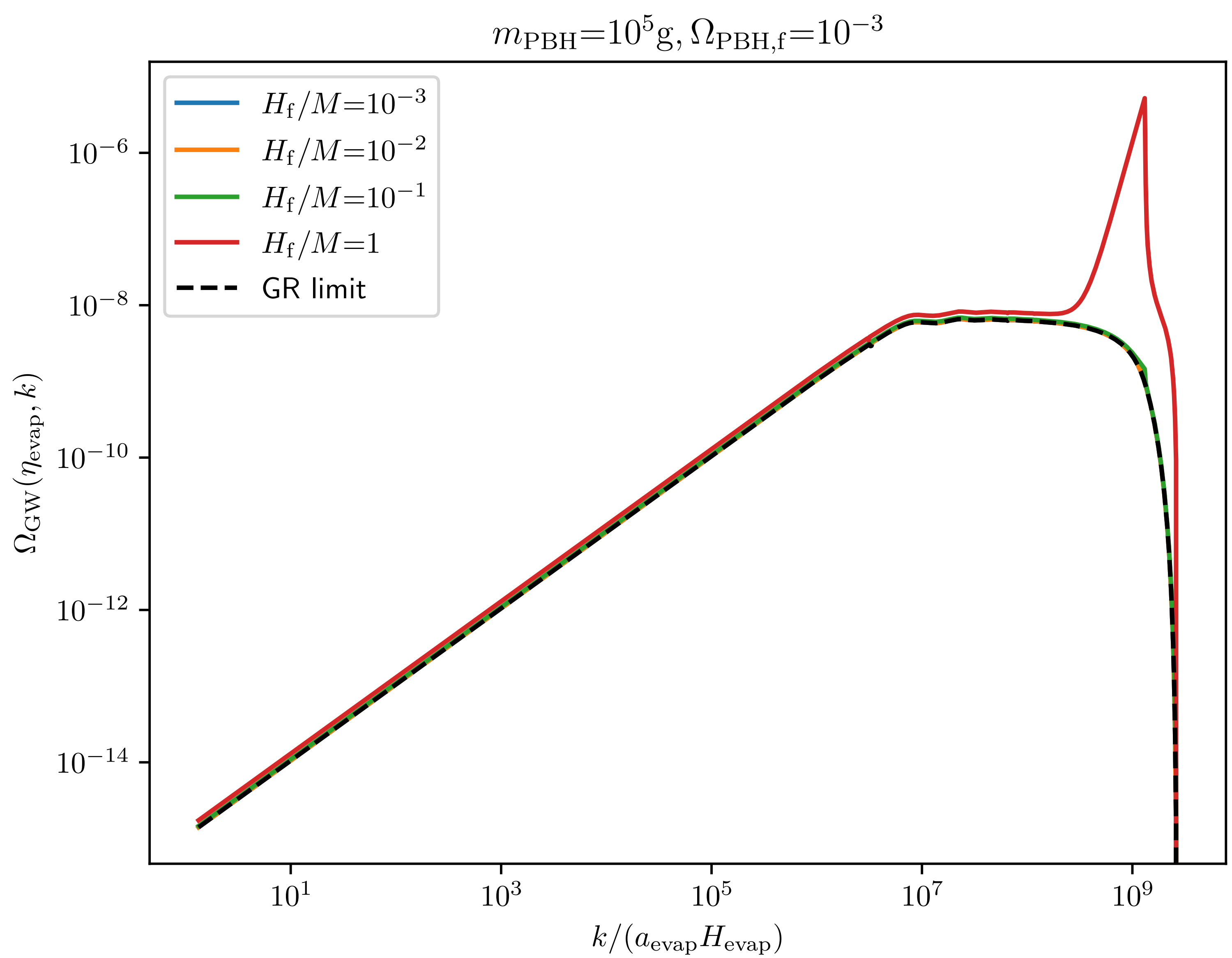
where $\lambda = 0$ when $s = (+), (\times)$ and $\lambda = 1$ when $s = (\mathrm{sc})$.

- The respective polarisation tensors read as

$$e^{(+)}_{ij}(\vec{k}) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad e^{(\times)}_{ij}(\vec{k}) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad e^{(\mathrm{sc})}_{ij}(\vec{k}) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

The case of Starobinsky gravity

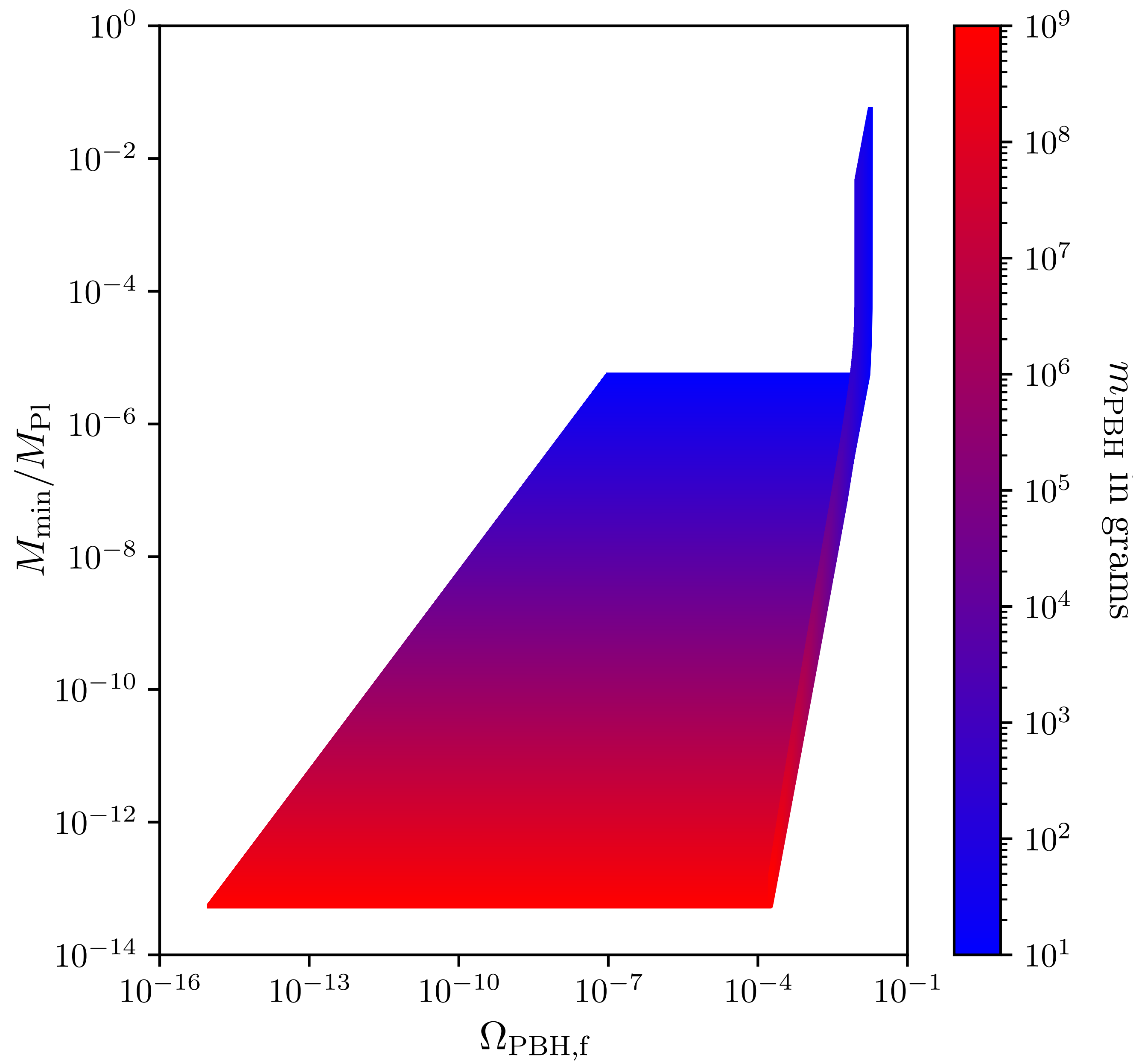
- We make here a case study considering a Starobinsky gravity model with a non-fixed mass scale parameter M: $f(R) = R + \dfrac{1}{6M^2}R^2$.



[Papanikolaou et al. - 2021]

Constraints on the parameters of the gravity theory

$$\boxed{\Omega_{\mathrm{GW,tot}}(\eta_{\mathrm{evap}}) \leq 1}$$



$$\boxed{\Omega_{\mathrm{PBH,f}} \leq 10^{-4}\left(\frac{10^{9}\mathrm{g}}{m_{\mathrm{PBH}}}\right)^{1/4} \xi_{\mathrm{d}}^{21/32}(M, \Omega_{\mathrm{PBH,f}})}$$

$$\Updownarrow$$

$$\Omega_{\mathrm{PBH,f}} < 10^{-3} \Rightarrow M_{\mathrm{min}} = H_{\mathrm{f}} = \frac{4\pi M_{\mathrm{Pl}}^{2}}{m_{\mathrm{PBH}}}$$

$$\Omega_{\mathrm{PBH,f}} > 10^{-3} \Rightarrow 5 \times 10^{-14} < M_{\mathrm{min}}/M_{\mathrm{Pl}} < 0.1$$

[Papanikolaou et al. - 2021]

Conclusions

- We studied the scalar induced GWs during a transient period of PBH domination in the early universe.

- By requiring that GWs do not lead to a back-reaction problem at evaporation time, i.e. $\Omega_{\mathrm{GW,tot}}(\eta_{\mathrm{evap}}) < 1$, we derive a **model-independent** upper bound on the initial abundance of PBHs.

- We applied our formalism in the case of $f(R)$ gravity making a **case study** for Starobisky gravity extracting at the end constraints on $\Omega_{\mathrm{PBH,f}}$ and vice-versa on the mass scale M of the Starobinsky model.

- This novel probe of SIGWs can be studied within the context of any gravity theory being used as an extra tool to **detect possible deviations from GR** and **break the observational degeneracy between different gravity theories.**

Thank you for your attention!

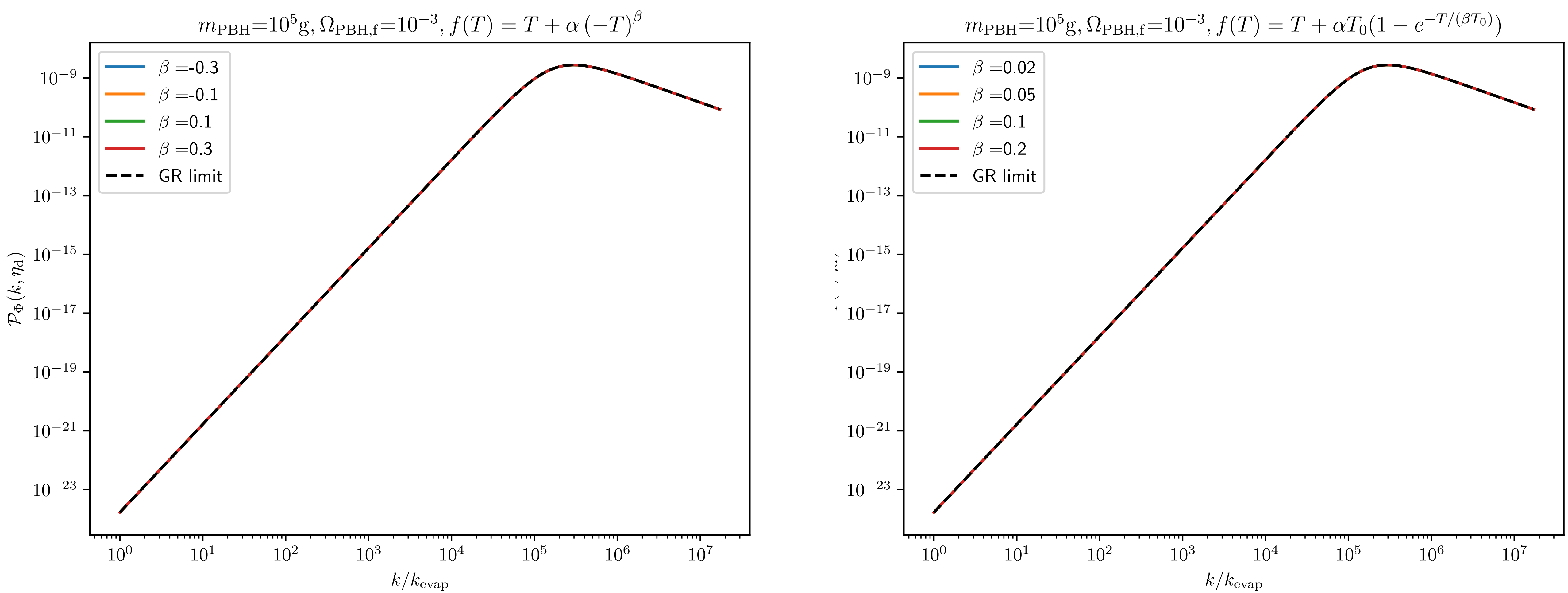
Appendix

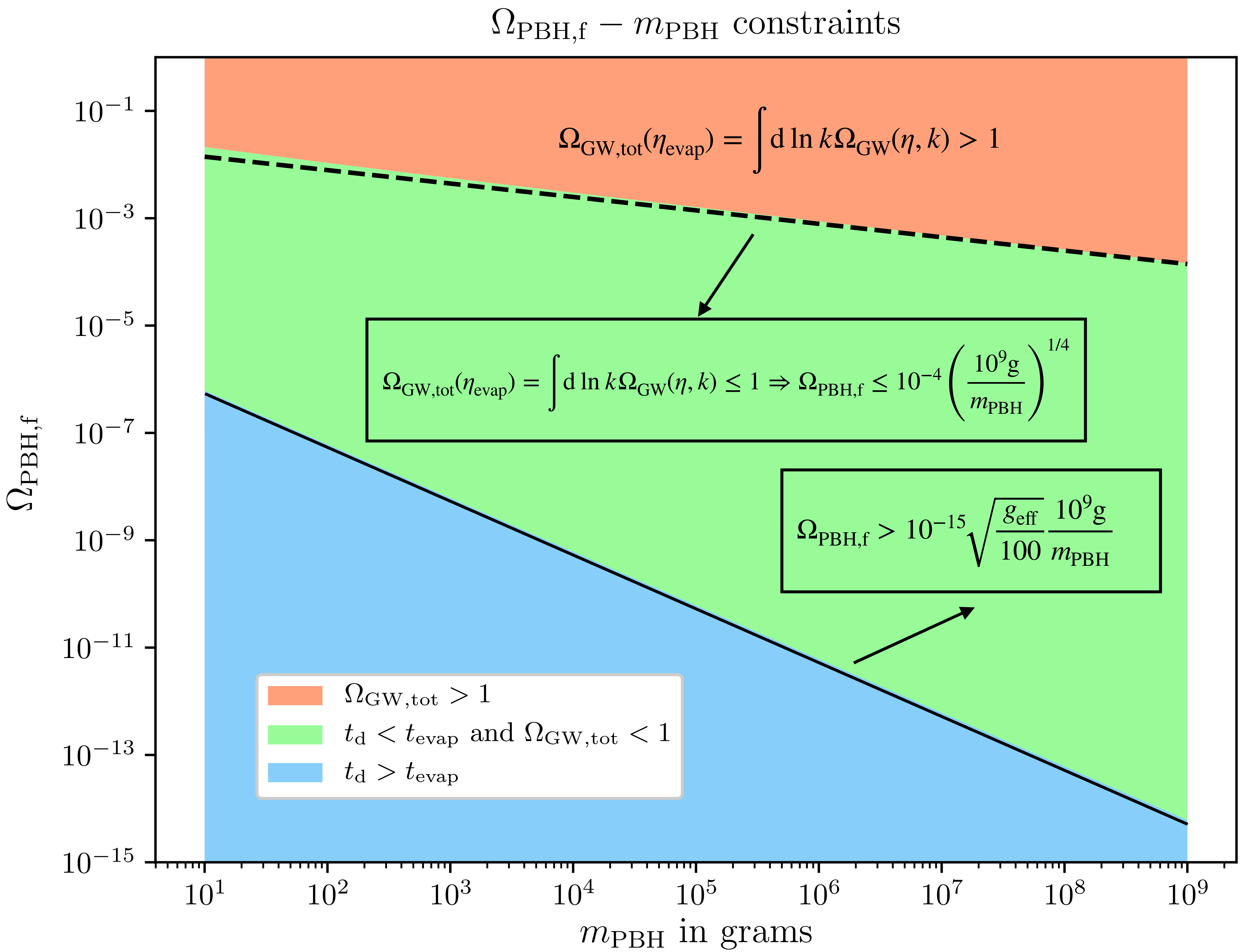
The case of $f(T)$ gravity

- In $f(T)$ gravity, the gravitational action reads as:

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,f(T)\,.$$

- Accounting for **a) source** and **b) propagation** effects we find a GW signal with no significant deviations from GR.

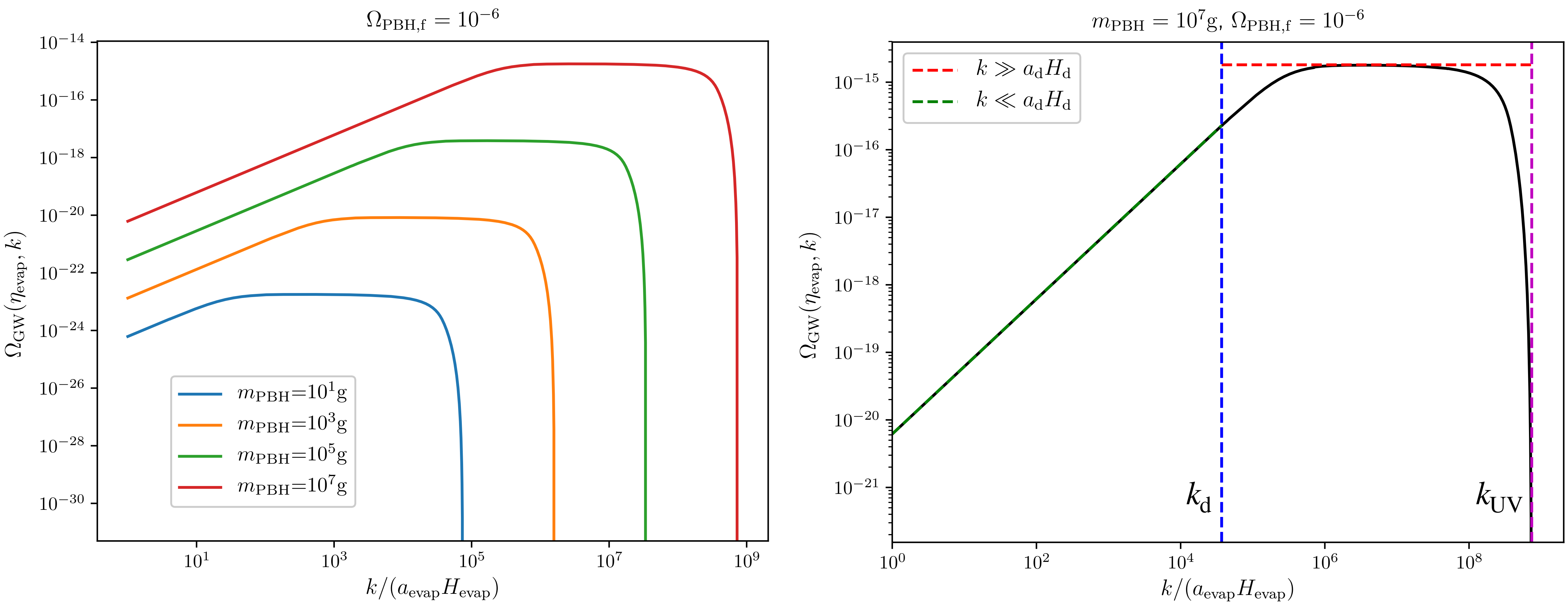




$$\rho_{\rm f}^{1/4} < \rho_{\rm inf}^{1/4} < 10^{16}{\rm GeV} \Rightarrow m_{\rm PBH} > 10{\rm g}$$

$$\rho_{\rm evap}^{1/4} > \rho_{\rm BBN}^{1/4} \simeq 1{\rm MeV} \Rightarrow m_{\rm PBH} < 10^9{\rm g}$$

The Gravitational Wave Spectrum



$$\Omega_{\rm GW}(\eta_{\rm evap}, k) \simeq 10^{19} \left(\frac{g_{\rm eff}}{100}\right)^{-2/3} \left(\frac{m_{\rm PBH}}{10^9 {\rm g}}\right)^{4/3} \Omega_{\rm PBH,f}^{16/3} \times \begin{cases} \frac{k}{k_{\rm d}} & \text{for} \quad k \ll \mathscr{H}_{\rm d} \\ 8 & \text{for} \quad k \gg \mathscr{H}_{\rm d} \end{cases}$$

- One identifies a broken power law for the GW spectrum. Two scales enter in the problem, $k_{\rm d} = \mathscr{H}_{\rm d}$ and $k_{\rm UV} = a_{\rm f} H_{\rm f} \Omega_{\rm PBH,f}^{1/3}$.
- For some regions in the parameter space $(\Omega_{\rm PBH,f}, m_{\rm PBH})$, we find that $\Omega_{\rm GW,tot}(\eta_{\rm evap}) > 1$!

Basics of Scalar Induced Gravitational Waves

- The solution of the equation of motion of tensor modes can be written analytically by the use of the Green function, which satisfies the homogeneous equation,

$$h^s_{\vec{k}}(\eta) = \frac{4}{a(\eta)} \int_{\eta_\mathrm{d}}^{\eta} \mathrm{d}\bar{\eta} G_{\vec{k}}(\eta, \bar{\eta}) a(\bar{\eta}) S^s_{\vec{k}}(\bar{\eta}) \,.$$

- The gravitational potential, $\Phi_{\vec{k}}$, present in the source term, $S^s_{\vec{k}}$, in the absence of anisotropic stress, satisfies the following equation,

$$\Phi''_{\vec{k}} + \frac{6(1+w)}{1+3w}\frac{1}{\eta}\Phi'_{\vec{k}} + wk^2\Phi_{\vec{k}} = 0.$$

- The solution of the above equation can be written analytically as.

$$\Phi_{\vec{k}}(\eta) = \frac{1}{y^\lambda}\left(C_1(k)J_\lambda(y) + C_2(k)Y_\lambda(y)\right) \text{ where } \lambda = \frac{1}{2}\frac{5+3\mathrm{w}}{1+3\mathrm{w}} \text{ and } \mathrm{y} = \sqrt{\mathrm{w}}\mathrm{k}\eta$$

- If there is a dominant mode, which is the case when $w = 0$ or $1/3$ then $\Phi_{\vec{k}} = \phi_{\vec{k}} T_\Phi(k\eta)$. In the case of a PBH era ($w = 0$), $T_\Phi(k\eta) = \text{constant}.$

The PBH Gravitational Potential Power Spectrum

- We assume that PBHs form in the radiation era, $\rho_{\rm PBH} \ll \rho_{\rm tot}$. Thus, PBH formation can be regarded as a transition of a fraction of radiation into dust matter.

- Given the random spatial distribution of PBHs, $\rho_{\rm PBH}$ **is inhomogeneous** while $\rho_{\rm tot}$ is homogeneous. Thus, the $\delta_{\rm PBH}$ can be viewed as **an isocurvature perturbation.**

- If $\Omega_{\rm PBH,f}$ is sufficiently large then PBHs will dominate the energy budget of the universe since $\Omega_{\rm PBH} = \rho_{\rm PBH}/\rho_{\rm tot} \propto a^{-3}/a^{-4} \propto a$. Consequently, in the subsequent PBH domination era, **the isocurvature perturbation will convert to a curvature perturbation associated to a PBH gravitational potential Φ.**

- By treating separately the sub and super-horizon scales, the PBH gravitational potential power spectrum reads as

$$\mathscr{P}_\Phi(k) = \frac{2}{3\pi}\left(\frac{k}{k_{\rm UV}}\right)^3\left(5+\frac{4}{9}\frac{k^2}{\mathscr{H}_{\rm d}^2}\right)^{-2} = \begin{cases} \propto k^3 \text{ for } k \ll \mathscr{H}_{\rm d} \\ \propto \frac{1}{k} \text{ for } k \gg \mathscr{H}_{\rm d} \end{cases}$$

The isocurvature perturbation

- The uniform energy density curvature perturbations ζ_{PBH} and ζ_{r}:

$$\zeta_{\mathrm{r}} = -\Phi + \frac{1}{4}\delta_{\mathrm{r}}, \quad \zeta_{\mathrm{PBH}} = -\Phi + \frac{1}{3}\delta_{\mathrm{PBH}}$$

- The isocurvature perturbation

$$S = 3\left(\zeta_{\mathrm{PBH}} - \zeta_{\mathrm{r}}\right) = \delta_{\mathrm{PBH}} - \frac{3}{4}\delta_{\mathrm{r}}$$

- The comoving curvature perturbation $\mathscr{R}$

$$\mathscr{R} = \frac{2}{3}\frac{\Phi'/\mathscr{H} + \Phi}{1+w} + \Phi \simeq -\zeta \text{ for k} \ll \mathscr{H}$$

- When $w = 0$, $\Phi = \text{constant}$ and $\Phi' = 0$. Thus, for $k \ll \mathscr{H}$, $\mathscr{R} = -\zeta = \frac{5}{3}\Phi$.
- For $k \ll \mathscr{H}$, $\zeta = \zeta_{\mathrm{PBH}} = \zeta_{\mathrm{r}} + S/3 \simeq S/3 \simeq \delta_{PBH}(t_{\mathrm{f}})/3$. Thus, $\Phi = \delta_{\mathrm{PBH}}(t_{\mathrm{f}})/5$ for $k \ll \mathscr{H}$.
- For sub-horizon scales, i.e. $k \gg \mathscr{H}$,

$$\frac{d^2\delta_{\mathrm{PBH}}}{ds^2} + \frac{2+3s}{2s(s+1)}\frac{d\delta_{\mathrm{PBH}}}{ds} - \frac{3}{2s(s+1)}\delta_{\mathrm{PBH}} = 0 \Rightarrow \delta_{\mathrm{PBH}} = \frac{2+3s}{2+3s_{\mathrm{f}}}\delta_{\mathrm{PBH}}(t_{\mathrm{f}}) \text{ with s} = \mathrm{a}/\mathrm{a}_{\mathrm{d}}$$

- Knowing that $\delta_{\mathrm{PBH}} = -\frac{2}{3}\left(\frac{k}{aH}\right)^2\Phi$, one gets that $\Phi = -\frac{9}{4}\left(\frac{\mathscr{H}_{\mathrm{d}}}{k}\right)^2\delta_{\mathrm{PBH}}(t_{\mathrm{f}}) \text{ for k} \gg \mathscr{H}$.

The isocurvature perturbation in $f(R)$ gravity

- The uniform energy density curvature perturbations $\zeta_{\rm PBH}$ and $\zeta_{\rm r}$:

$$\zeta_{\rm r} = -\Phi + \frac{1}{4}\delta_{\rm r}, \quad \zeta_{\rm PBH} = -\Phi + \frac{1}{3}\delta_{\rm PBH}, \quad \zeta_{\rm f(R)} = -\Phi + \frac{1}{3(1+w_{\rm f(R)})}\delta_{\rm f(R)}$$

- The isocurvature perturbation can be recast as:

$$S = 3\left(\zeta_{\rm PBH} - \zeta_{\rm r}\right) = \delta_{\rm PBH} - \frac{3}{4}\delta_{\rm r}$$

- The total curvature perturbation can be written as:

$$\zeta = -\Phi + \frac{\delta_{\rm tot}}{3(1+w_{\rm tot})} = \frac{\frac{4}{3}\bar{\rho}_{\rm r}\zeta_{\rm r} + \bar{\rho}_{\rm PBH}\zeta_{\rm PBH} + (1+w_{\rm f(R)})\bar{\rho}_{\rm f(R)}\zeta_{\rm f(R)}}{\frac{4}{3}\bar{\rho}_{\rm r} + \bar{\rho}_{\rm PBH} + (1+w_{\rm f(R)})\bar{\rho}_{\rm f(R)}} \xrightarrow{w_{\rm f(R)}\simeq -1} \frac{\frac{4}{3}\bar{\rho}_{\rm r}\zeta_{\rm r} + \bar{\rho}_{\rm PBH}\zeta_{\rm PBH}}{\frac{4}{3}\bar{\rho}_{\rm r} + \bar{\rho}_{\rm PBH}}$$

- The matter growth equation becomes

$$\frac{{\rm d}^2\delta_{\rm PBH}}{{\rm d}s^2} + \frac{2+3s}{2s(s+1)}\frac{{\rm d}\delta_{\rm PBH}}{{\rm d}s} - \frac{3}{2s(s+1)}\frac{1}{F}\frac{1+4\frac{k^2}{a^2}\frac{F_{,\rm R}}{F}}{1+3\frac{k^2}{a^2}\frac{F_{,\rm R}}{F}}\delta_{\rm PBH} = 0 \text{ with } {\rm s = a/a_d}\,.$$

- In sub-horizon scales, ie. $k \gg \mathscr{H}$: $\delta_{\rm PBH} = -\frac{2}{3}\left(\frac{k}{\mathscr{H}}\right)^2 \frac{F\left(1+3\frac{k^2}{a^2}\frac{F_{,R}}{F}\right)}{1+2\frac{k^2}{a^2}\frac{F_{,R}}{F}}\Phi\,.$

The Gravitational Wave Spectrum

- The spectral abundance then of GWs, $\Omega_{\rm GW}(\eta, k)$, after a straightforward calculation reads as:

$$\Omega_{\rm GW}(\eta, k) = \frac{4}{75\pi^2}\left(\frac{k}{aH}\right)^2\left(\frac{k}{k_{\rm UV}}\right)^6 \mathcal{F}(y, \Omega_{\rm PBH,f}), \text{ where}$$

$$\mathcal{F}(y, \Omega_{\rm PBH,f}) = \int_0^{\Lambda} \mathrm{d}v \int_{|1-v|}^{\min(\Lambda, 1+v)} \mathrm{d}u \left[\frac{4v^2 - (1+v^2-u^2)^2}{4\left(3+\frac{4}{15}y^2u^2\right)\left(3+\frac{4}{15}y^2v^2\right)}\right]^2 uv$$

In the above expressions, $y = k/\mathcal{H}_{\rm d}$ and $\Lambda = \dfrac{k_{\rm UV}}{k} = y^{-1}\Omega_{\rm PBH,f}^{-2/3}$.

$$\mathcal{F}(y, \Omega_{\rm PBH,f})\begin{cases} \dfrac{1125\sqrt{5}\pi}{256y^7} \text{ for y} \ll 1 \text{ and } \Omega_{\rm PBH,f} \ll 1 \\ \dfrac{50625\pi^2}{2048y^8} \text{ for y} \gg 1 \end{cases}$$

Conclusions

- **A word of caution** should be raised here. From the point of view of the gravitational potential Φ, or from the point of view of the curvature perturbation ζ, all scalar fluctuations incorporated in the calculation lie in the perturbative regime. However, in the PBH era $\delta_{\rm PBH} \sim a$ and can take values larger than one while Φ or ζ is constant and smaller than one.

- The status of these scales is unclear: the growth of δ above one may signal the onset of PBH clustering, which might result in the enhancement of the power spectrum above the Poissonian value, which might in turn be responsible for an even larger signal than the one we have computed. In this sense, the bounds we have derived could be conservative only, although a more thorough investigation of the virialisation dynamics at small scales would be required.