Generalized uncertainty principle or curved momentum space?

• Fabian Wagner, University of Szczecin •

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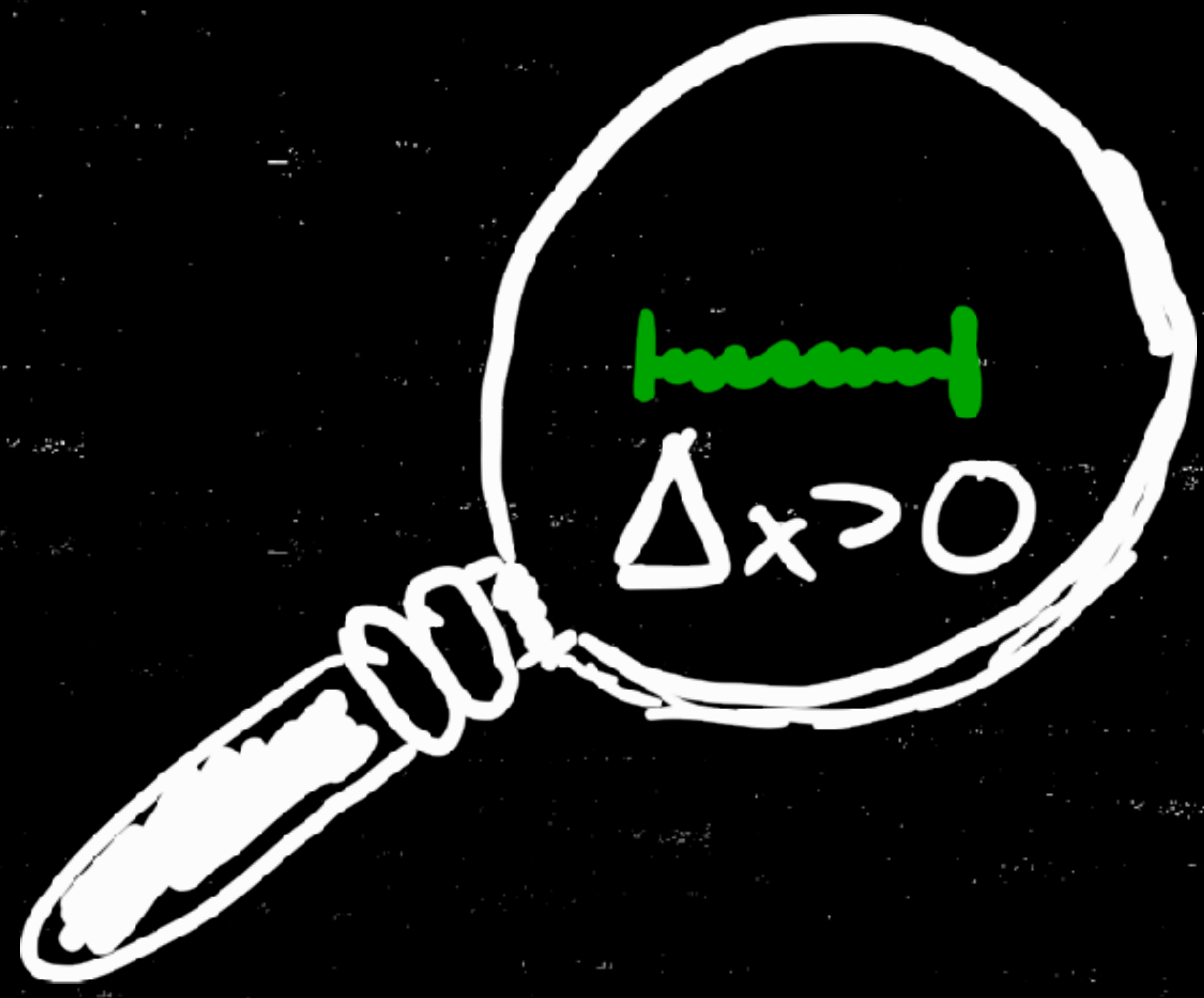
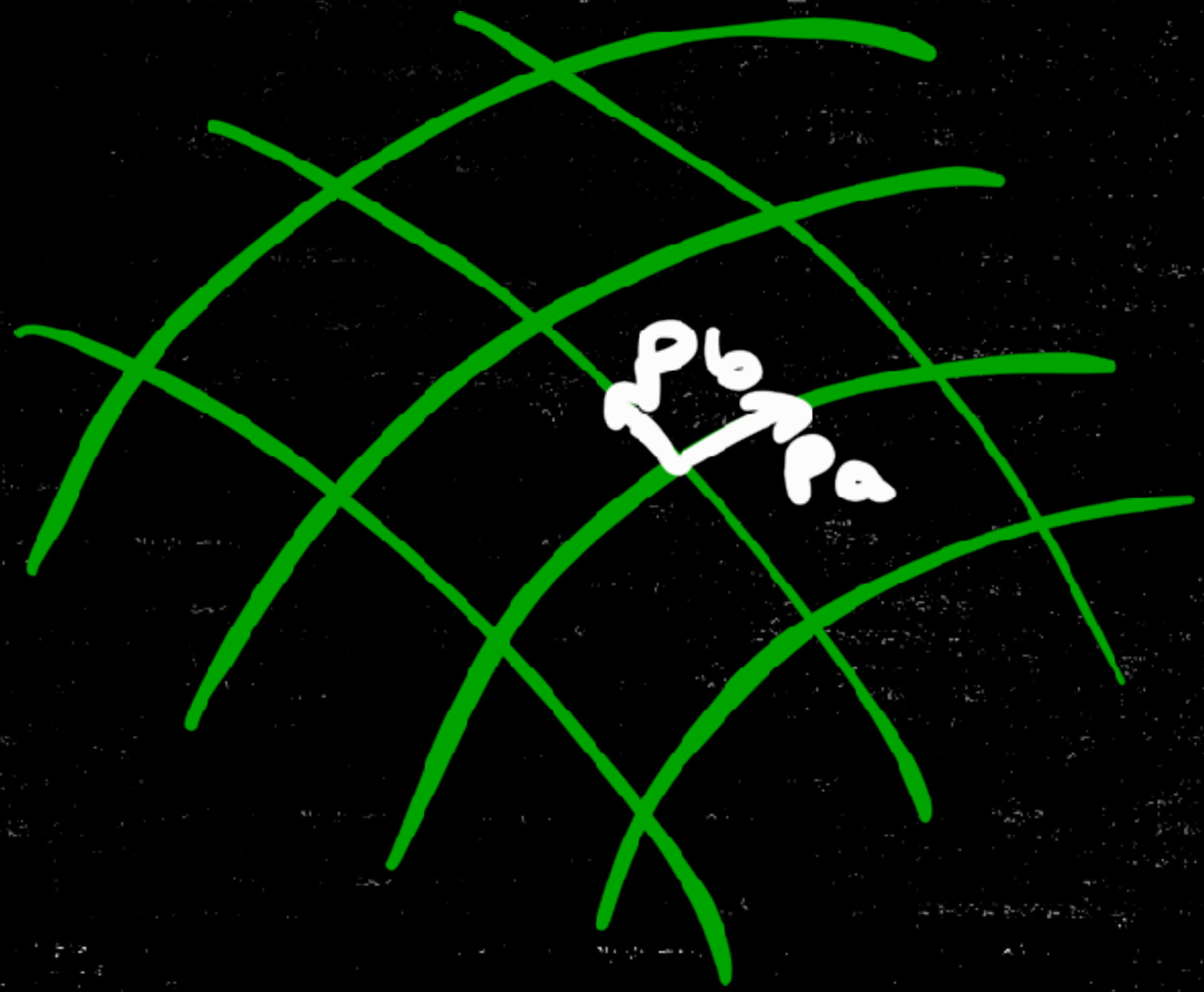
GUP

=

nontrivial mom. space





noncommutativity of space

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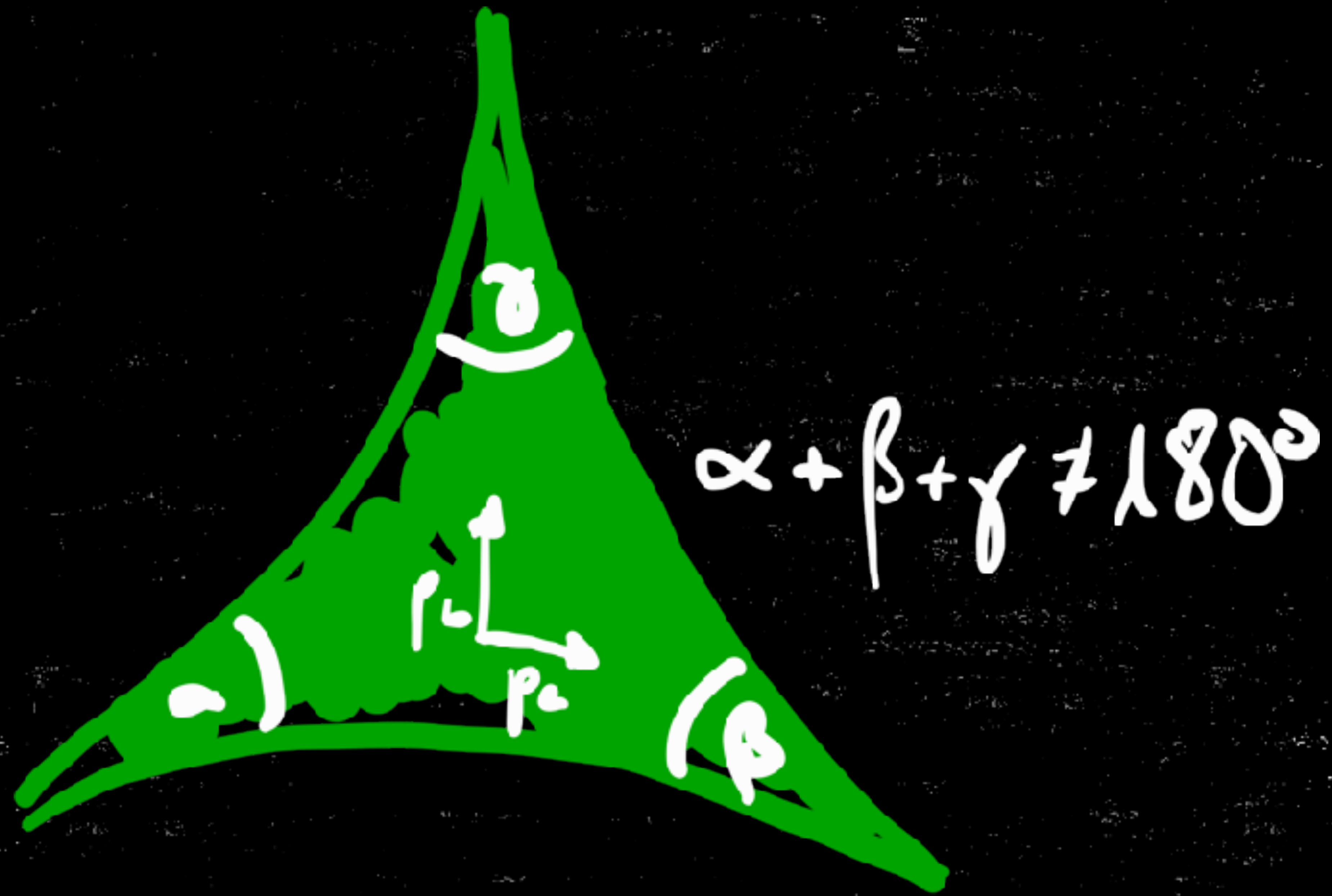
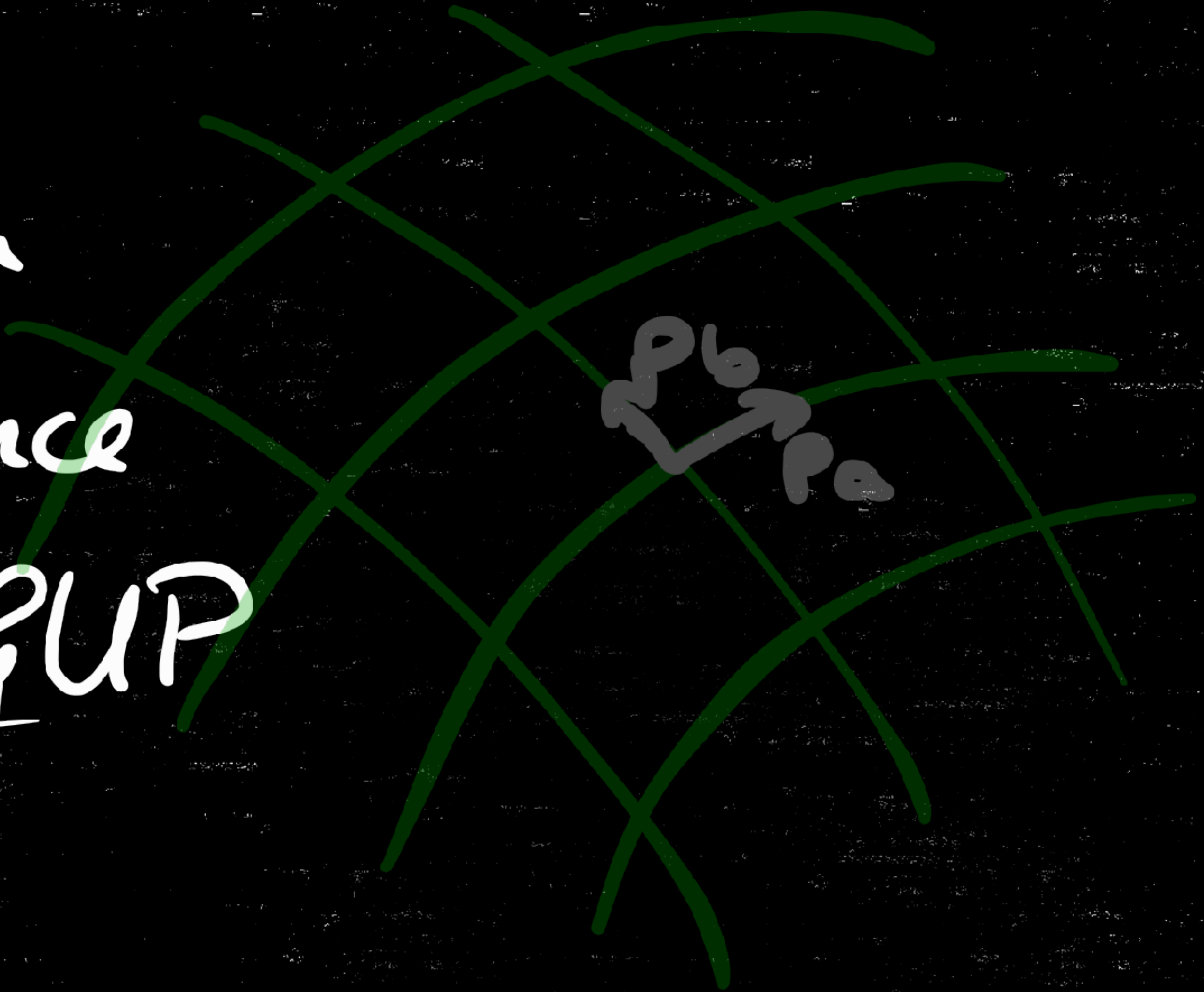
curvature in mom. space



Outline

1. Introduction
2. Correspondence
3. Quadratic GUP
4. Discussion

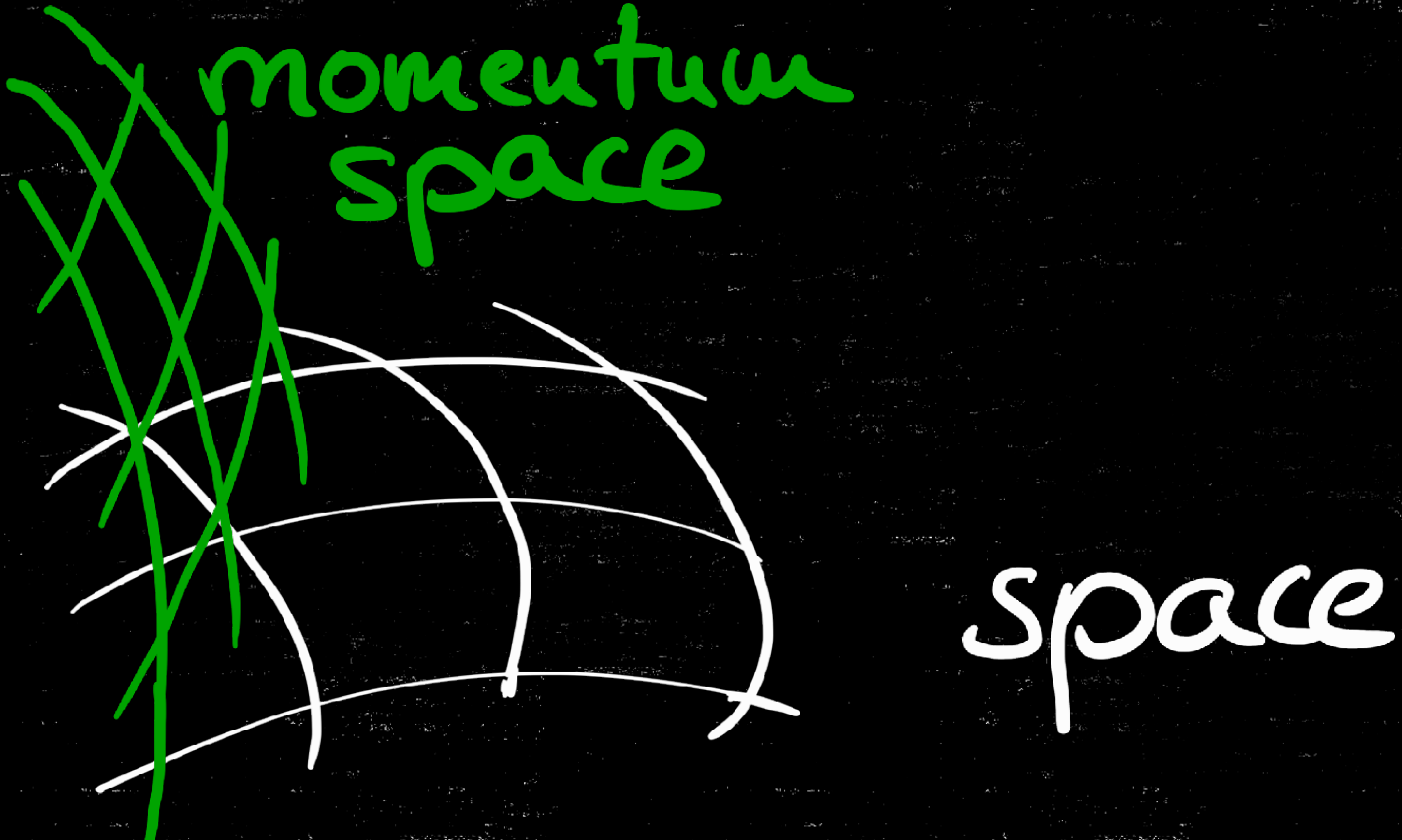


Introduction

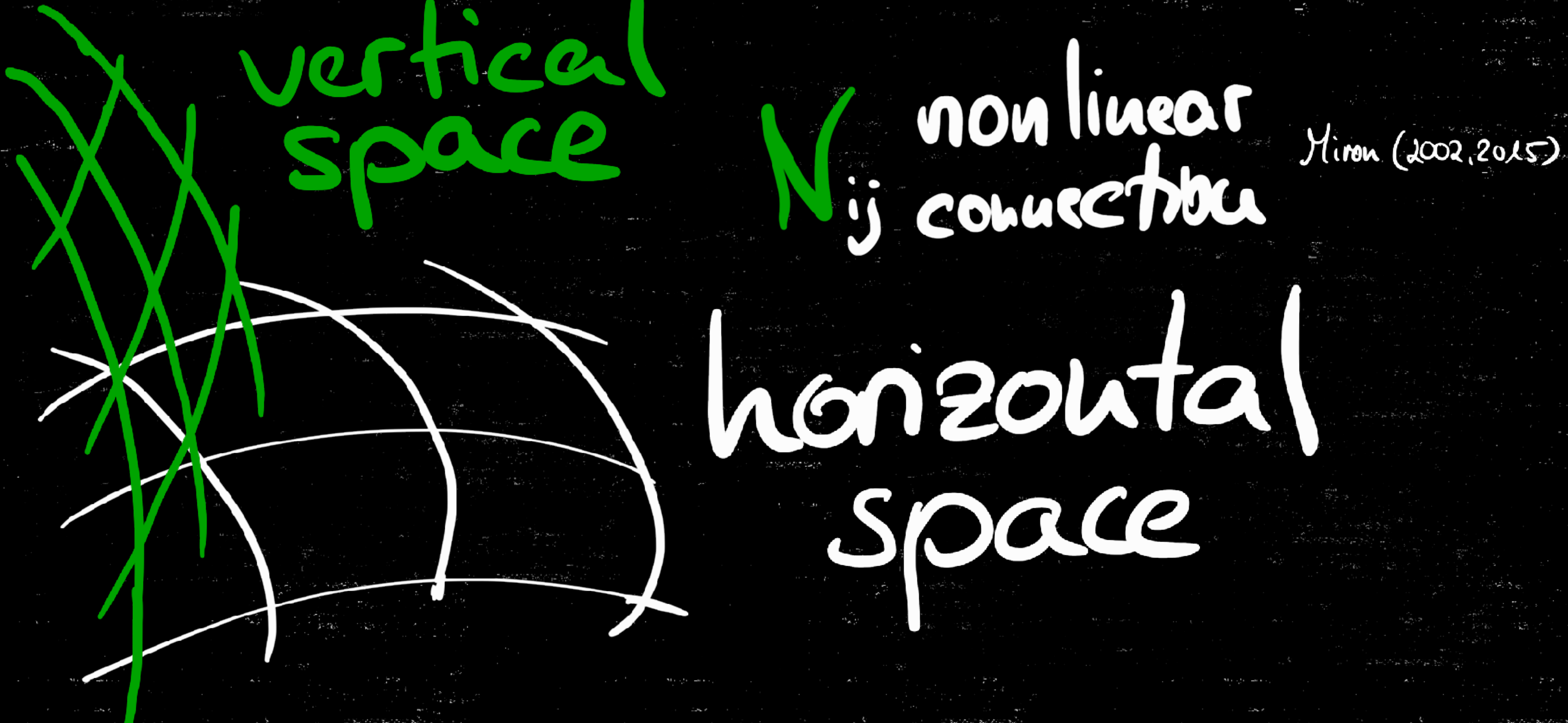
Curved momentum space

space

Curved momentum space



Curved momentum space



Curved momentum space

$$H = \frac{1}{2m} g^{ij}(p) p_i p_j + V[x^i, g_{ij}(p)]$$

$$= \frac{1}{2m} \delta^{ab} e^i_a e^j_b p_i p_j + V[(e^{-1})^a_i x^i, \delta_{ab}]$$

Curved momentum space

$$H = \frac{1}{2m} g^{ij}(p) p_i p_j$$

or

mom. space covariant

$$H = \frac{1}{2m} \sigma_p^2(p) \, ?$$

Curved momentum space

$$H = \frac{1}{2m} g^{ij}(p)\, p_i p_j$$

or

mom. space covariant

$$\cancel{H = \frac{1}{2m}\sigma_p^2(p)}\;?$$

Curved momentum space

$$g_{ij} = g_{ij}(p)$$

$$\xrightarrow{\text{coordinate transformation}}$$

$$g_{ij} = g_{ij}(p, x)$$

Curved momentum space

$$g_{ij} = g_{ij}(p)$$

$$\xrightarrow{\text{coordinate transformation}}$$

$$g_{ij} \cancel{= g_{ij}(p, x)}$$

Curved momentum space

$$g_{ij} = g_{ij}(p) \Longrightarrow N_{ij} = 0$$

curved mom. space

$\Longleftrightarrow$

curved pos. space

(deWitt 1952)

Minimum length

- measurement length scale > $l_0 \sim l_p$
- possible implementation:

$$\sigma_x \sigma_p \geq \hbar/2 \left(1 + \frac{\sigma_p^2 l_0^2}{\hbar^2}\right)$$

Minimum length

- measurement length scale $> l_0 \sim l_p$
- Possible implementation:

$$\sigma_x \sigma_p \geq \frac{\hbar}{2}\left(1 + \frac{\sigma_p^2 l_0^2}{\hbar^2}\right)$$

GUP

d-Dimensional GUP

$$[\hat{x}^a, \hat{x}^b] = 0 \qquad [\hat{p}_a, \hat{p}_b] = 0$$

$$[\hat{x}^a, \hat{p}_b] = i\hbar\,\delta^a_b$$

d-Dimensional GUP

$$[\hat{x}^a, \hat{x}^b] = i\hbar\theta^{ab}(x,p) \qquad [\hat{p}_a, \hat{p}_b] = 0$$

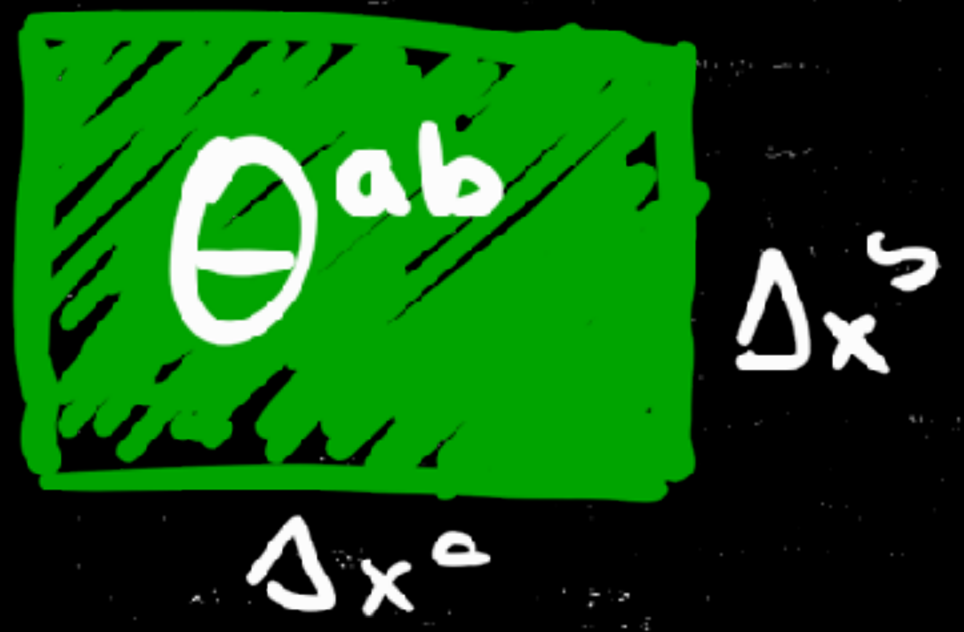
$$[\hat{x}^a, \hat{p}_b] = i\hbar f^a_b(p)$$

Noncommutative space

$$\Delta x^a \Delta x^b \geq \frac{\hbar}{2} |\langle \Theta^{ab} \rangle| \neq 0$$

$\Longrightarrow$ cannot measure more precisely

than



Can we distinguish between them?

Transformation

$$\hat{P}_i = (e^{-1})^a{}_i \, \hat{P}_a$$

$$\hat{X}^i = e^i{}_a \, \hat{X}^a$$

Assumptions

$$[\hat{x}^a, \hat{x}^b] = i\hbar\theta^{ab}(x,p) \qquad [\hat{p}_a, \hat{p}_b] = 0$$

$$[\hat{x}^a, \hat{p}_b] = i\hbar f^a_b(p)$$

Assumptions

$$[\hat{X}^i, \hat{X}^j] = 0 \qquad [\hat{P}_i, \hat{P}_j] = 0$$

$$[X^i, \hat{P}_j] = i\hbar\delta^i_j$$

overconstrained system

$\Longrightarrow$ consistent!

e^i_a determined explicitly from f^a_b

Hamiltonian

$$\hat{H} = \frac{1}{2m}\delta^{ab}\hat{P}_a\hat{P}_b + V(\hat{x}^a, \delta_{ab})$$

$$= \frac{1}{2m}\delta^{ab} e^i_a e^j_b \hat{P}_i \hat{P}_j + V[(e^{-1})^a_i X^i, \delta_{ab}]$$

$\Longrightarrow$ nontrivial mom. space

Quadratic GUP

Parametrization

$$f^a_b = \delta^a_b \left[1 + \beta\left(\frac{l_p \hat{p}}{\hbar}\right)^2\right] + \beta' \hat{p}^a \hat{p}_b$$

$$\Theta^{ab} = (2\beta - \beta')\left(\frac{l_p}{\hbar}\right)^2 \hat{J}^{ab}$$

Curvature

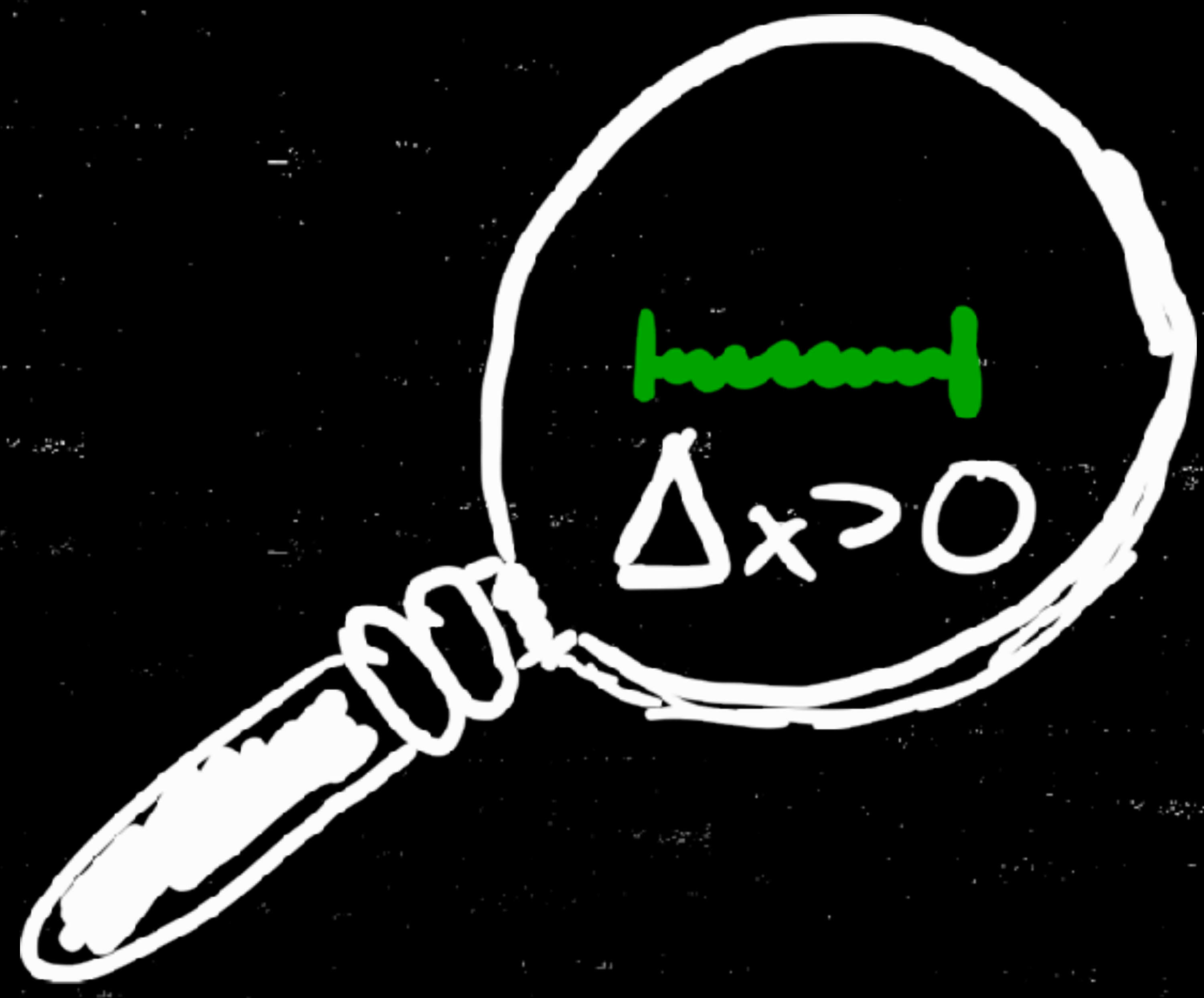
- maximally sym. space

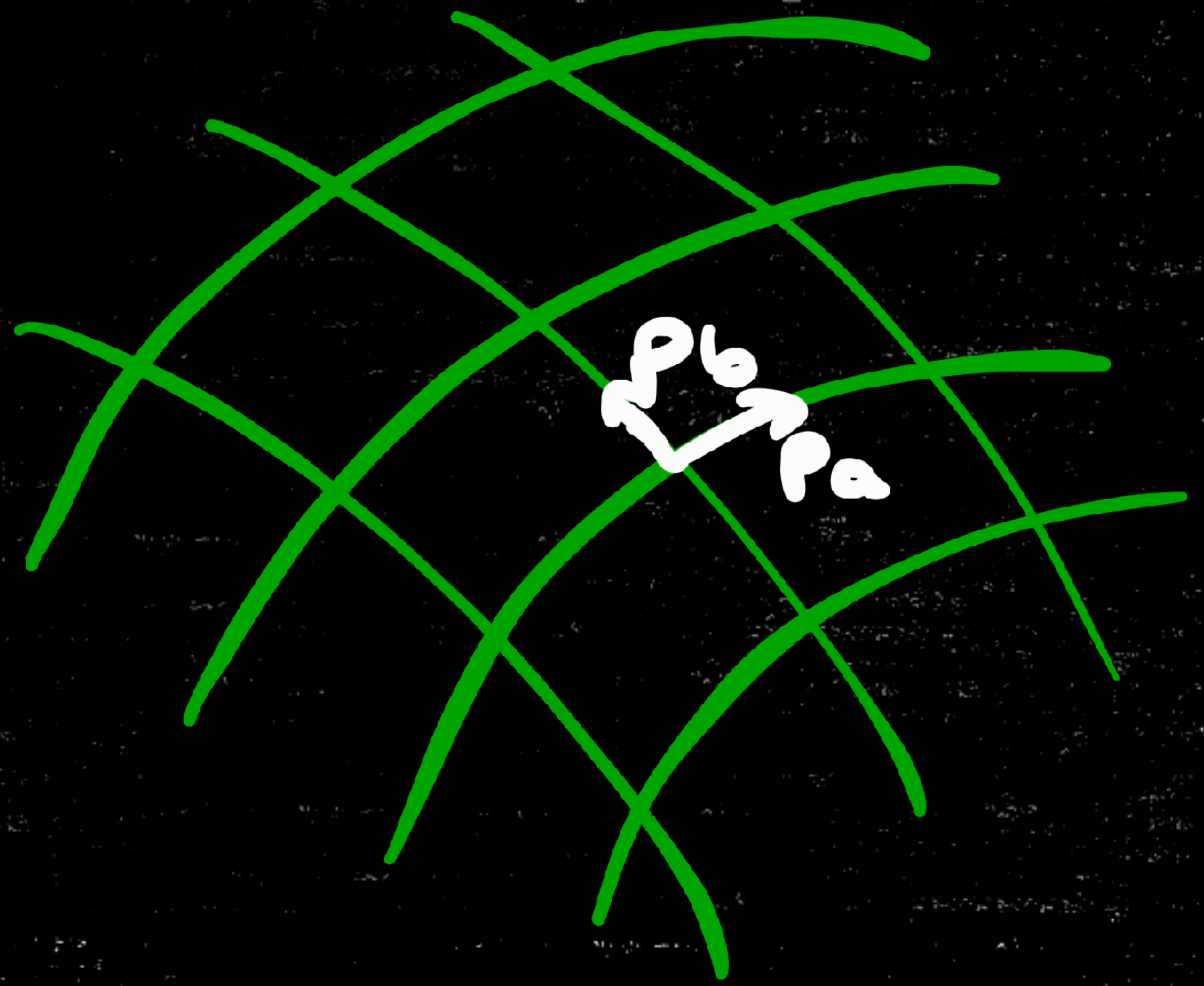
$$S = 2d(d+1)(2\beta - \beta')\left(\frac{l_P}{r}\right)^2$$

$$< 10^{27}\left(l_P/r\right)^2$$

Conclusion

GUP = nontrivial mom. space





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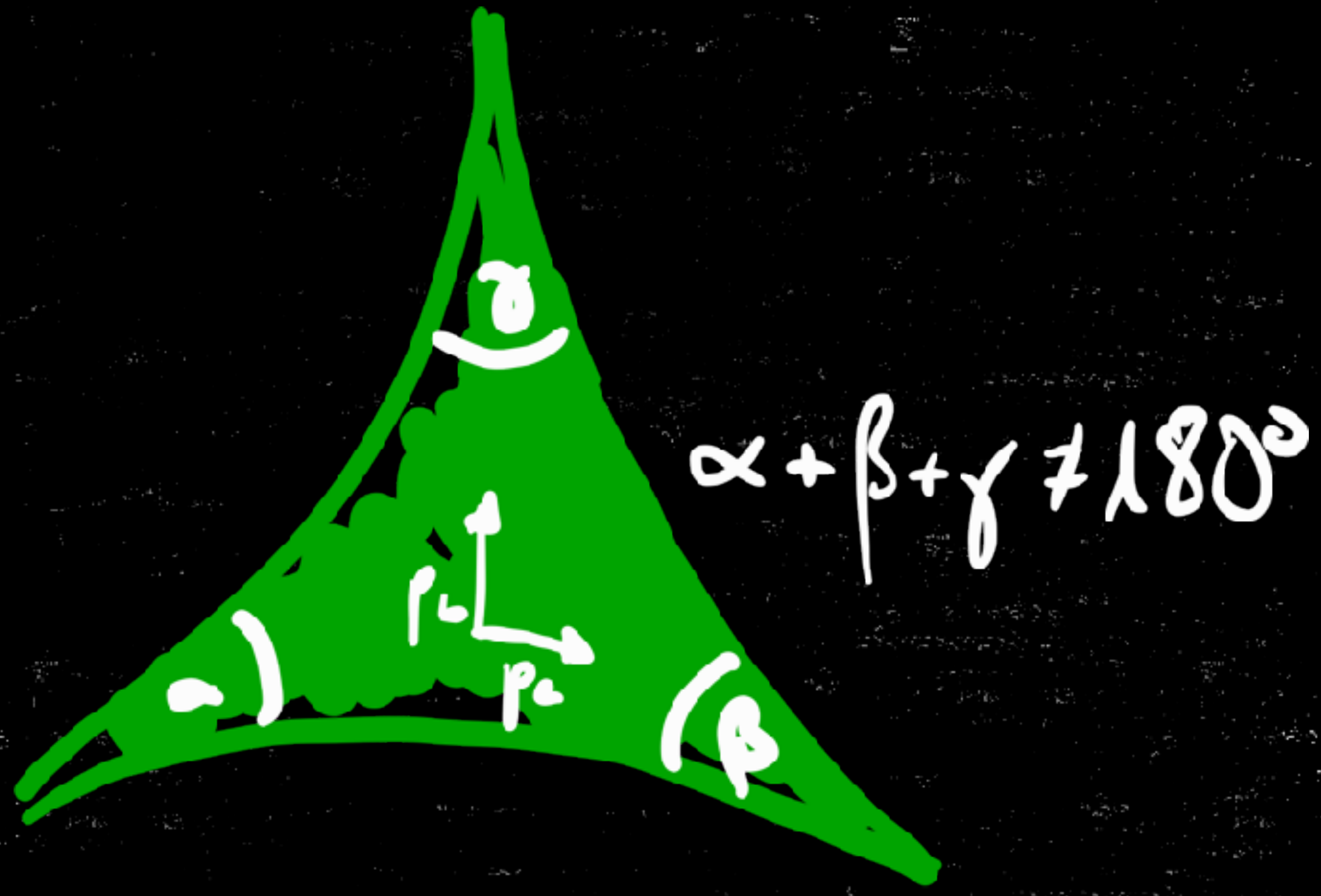

Metric

$$g^{ij} = \delta^{ij}\left[1 + (\beta' - \beta)^2\left(\frac{l_p \hat{p}}{t}\right)^2\right] + 2\beta\left(\frac{l_p}{t}\right)^2 \hat{p}^i \hat{p}^j$$