Testing gravitational theories and QG proposals with GWs

Mairi Sakellariadou

KING'S College LONDON

King's College London

COST CA18108 Third Annual Conference - Napoli (Italy)

LIGO

LISA

Outline

- Tests of General Relativity and modified gravity

  Consistency tests / parametrised tests / other model-independent tests

  *LVC (including **MS**) (2019)*

  *LISA Collaboration (including **MS**) (2022)*

- Measuring the propagation speed of GWs with LISA

  *Baker et al (including **MS**) (2022)*

- Constraining extra dimensions with LIGO/Virgo data

  *LVC (including **MS**) (2018)*

- Constraining Quantum Gravity candidates with GWs

  *Calcagni, Kuroyanagi, Marsat, **MS**, Tamanini, Tasinato (2019)*

- **Consistency tests of GR** *Assuming that GR is correct compare GR waveform with GW data*

- **Consistency tests of GR** *Assuming that GR is correct compare GR waveform with GW data*

○ **Residual tests** *Pick up arbitrary departures of the theory from the data*

Bayesian inference algorithm: Estimate parameters that maximise likelihood $P(d|h)$

If GR correct description of GWs $\Longrightarrow$ data consistent with waveform predicted by GR

$\Longrightarrow$ Residual, constructed by subtracting best-fit waveform from data, would be consistent with background noise

GW data often contaminated by non-stationary and non-Gaussian background

- Such tests have been applied to the existing GW events by the LIGO/Virgo collaboration

Challenge: - LISA will always have a background of sources that are on during the entire observation period

- **Consistency tests of GR** *Assuming that GR is correct compare GR waveform with GW data*

- **Residual tests** *Pick up arbitrary departures of the theory from the data*

Bayesian inference algorithm: Estimate parameters that maximise likelihood $P(d|h)$

If GR correct description of GWs $\Longrightarrow$ data consistent with waveform predicted by GR

$\Longrightarrow$ Residual, constructed by subtracting best-fit waveform from data, would be consistent with background noise

GW data often contaminated by non-stationary and non-Gaussian background

- Such tests have been applied to the existing GW events by the LIGO/Virgo collaboration

Challenge: - LISA will always have a background of sources that are on during the entire observation period

- **Inspiral-merger-ringdown tests** *Check if 90% credible contours corresponding to different ways of measuring parameters overlap with each other*

Compare parameters determined using inspiral phase of signal only to those using the late-time merger-ringdown

If GR correct description of GWs $\Longrightarrow$ parameters determined from the two phases consistent with each other

Challenge: - Better understanding of mass and spin of the remnant in mergers of intermediate mass ratio systems and systems with non-negligible eccentricity and double-spin precession (crucial for LISA binary sources)

- No simple mapping to convert theory-agnostic IMR test to bounds on specific theories

- **Consistency tests of GR** *Assuming that GR is correct compare GR waveform with GW data*

- **Residual tests** *Pick up arbitrary departures of the theory from the data*

Bayesian inference algorithm: Estimate parameters that maximise likelihood $P(d|h)$

If GR correct description of GWs $\Longrightarrow$ data consistent with waveform predicted by GR

$\Longrightarrow$ Residual, constructed by subtracting best-fit waveform from data, would be consistent with background noise

GW data often contaminated by non-stationary and non-Gaussian background

- Such tetsts have been applied to existing GW events by the LIGO/Virgo collaboration

Challenge:- LISA will always have a background of sources that are on during the entire observation period

- **Inspiral-merger-ringdown tests** *Check if 90% credible contours corresponding to different ways of measuring parameters overlap with each other*

Compare parameters determined using inspiral phase of signal only to those using the late-time merger-ringdown

If GR correct description of GWs $\Longrightarrow$ parameters determined from the two phases consistent with each other

Challenge: - Better understanding of mass and spin of the remnant in mergers of intermediate mass ratio systems and systems with non-negligible eccentricity and double-spin precession (crucial for LISA binary sources)

- No simple mapping to convert theory-agnostic IMR test to bounds on specific theories

- **Multipolar test** *Look for consistency of the source parameters (mass, spin angular momenta of the companion BHs) determined independently from the quadrupole and higher order modes*

Higher order modes make appreciable contribution to GWs from BBHs with large mass ratios and misaligned spins

Challenge: Mapping between generic higher-mode deviations from GR and specific modified theories

- **Parametrised tests** *Model-independent tests that introduce generic parameters capturing non-GR effects*

- **Parametrised tests** *Model-independent tests that introduce generic parameters capturing non-GR effects*

- **Inspiral tests** *to verify the PN structure of the waveform phase*

Decompose the Fourier-domain waveform model into a frequency-dependent amplitude and a frequency-dependent phase and write the phase as $\Psi_{\rm GR}(f) = \sum_{n=0}^{7} \alpha_n v(f)^{-5+n}$

Orbital velocity $v(f) = (\pi m f)^{1/3}$

In GR these PN coefficients are known functions of the binary

Treat coefficients α_n as independent and find their best-fit values by comparing this template waveform with the data
Check the consistency of the measured masses from each of these coefficients

Challenge: - Extend the mapping between generic deviations and specific theories
- Determine how to handle parametrically modifications of GR that do not admit simple PN expansion in inspiral

Note: Multiband observations of stellar-mass BBHs are important in constraining modified gravity via parametrised tests
Multibanding allows to combine information from early inspiral dynamics using LISA with late inspiral, merger and ringdown observations using 3rd generation ground-based detectors

$$\tilde{h}(f) = \tilde{A}_{\rm GR}(f)\,[1 + \alpha_{\rm ppE}\, v(f)^a]\, e^{i\Psi_{\rm GR}(f) + i\beta_{\rm ppE}\, v(f)^b}$$

Real numbers that determine the type of deviation

Parametrised post-Einsteinian constants

- **Parametrised tests** *Model-independent tests that introduce generic parameters capturing non-GR effects*

- **Inspiral tests** *to verify the PN structure of the waveform phase*

Decompose the Fourier-domain waveform model into a frequency-dependent amplitude and a frequency-dependent phase and write the phase as $\Psi_{\rm GR}(f) = \sum_{n=0}^{7} \alpha_n v(f)^{-5+n}$

Orbital velocity $v(f) = (\pi m f)^{1/3}$

In GR these PN coefficients are known functions of the binary

Treat coefficients α_n as independent and find their best-fit values by comparing this template waveform with the data. Check the consistency of the measured masses from each of these coefficients

Challenge: - Extend the mapping between generic deviations and specific theories
- Determine how to handle parametrically modifications of GR that do not admit simple PN expansion in inspiral

Note: Multiband observations of stellar-mass BBHs are important in constraining modified gravity via parametrised tests
Multibanding allows to combine information from early inspiral dynamics using LISA with late inspiral, merger and ringdown observations using 3rd generation ground-based detectors

- **Ringdown tests**

Attempt to construct parametrised ringdown waveforms based on perturbative treatments of the ringdown in GR

Challenge: - Modelling of rotating BHs beyond GR (NR and GW observations indicate that spin of merger remnant is large)
- Calculation of beyond-GR QNMs (separability of master eqs for perturbations in GR is absent in modified theories
$\Rightarrow$ QNMs from numerical simulations. or, parametrise deviations in QNM frequencies and damping times

▪ Parametrised tests of GW

phenomenological parameters

$$E^2 = p^2c^2 + A_\alpha p^\alpha c^\alpha$$

massive gravity

$$A_0 > 0$$
$$m_g = A_0^{1/2}/c^2.$$

doubly special relativity

$$\alpha = 0, 0.5, 1, 1.5, 2.5, 3, 3.5, 4$$

multi-fractal ST

Horava-Lifshitz & extra dim

Frequency dependence of speed of GWs

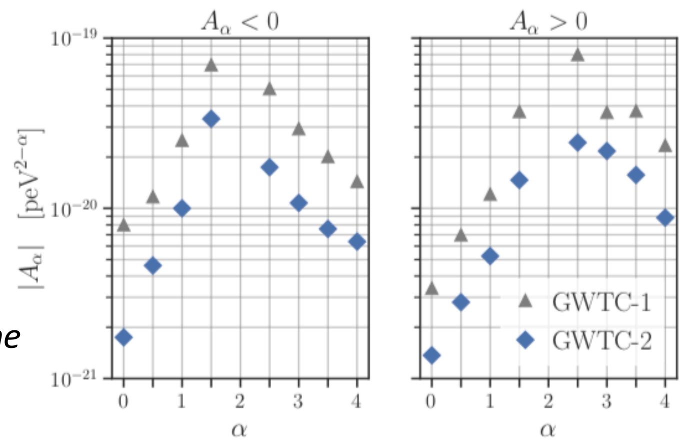
$$\tilde{h}(\nu) = A(\nu)e^{i\Phi(\nu)}$$

$$\boxed{\tilde{h}(\nu) = A(\nu)e^{i(\Phi(\nu)+\delta\Phi_\alpha(\nu))}}$$

90% credible upper bounds on the absolute value of the modified dispersion relation parameter A_α

$$\boxed{m_g \leq 1.76 \times 10^{-23}\mathrm{eV}/c^2}$$

1.8 times more stringent than the most recent Solar System bound



*LVC (including **MS**) (2019)*

- **Parametrised tests of GW**

phenomenological parameters

$$E^2 = p^2c^2 + A_\alpha p^\alpha c^\alpha$$

massive gravity

$A_0 > 0$

$m_g = A_0^{1/2}/c^2$.

doubly special relativity

$$\alpha = 0, 0.5, 1, 1.5, 2.5, 3, 3.5, 4$$

multi-fractal ST

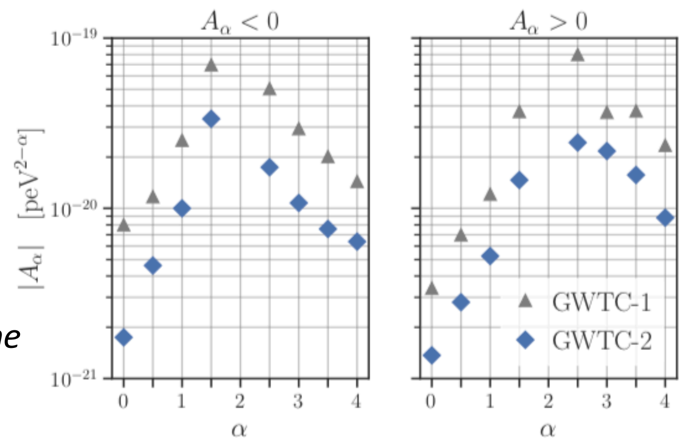
Horava-Lifshitz & extra dim

Frequency dependence of speed of GWs

$$\tilde{h}(\nu) = A(\nu)e^{i\Phi(\nu)}$$

$$\boxed{\tilde{h}(\nu) = A(\nu)e^{i(\Phi(\nu)+\delta\Phi_\alpha(\nu))}}$$

90% credible upper bounds on the absolute value of the modified dispersion relation parameter A_α



$$\boxed{m_g \leq 1.76 \times 10^{-23}\text{eV}/c^2}$$

1.8 times more stringent than the most recent Solar System bound

*LVC (including **MS**) (2019)*

Fourth order gravity nonminimally coupled to a massive scalar field

*Lambiase, **MS**, Stabile (2021)*

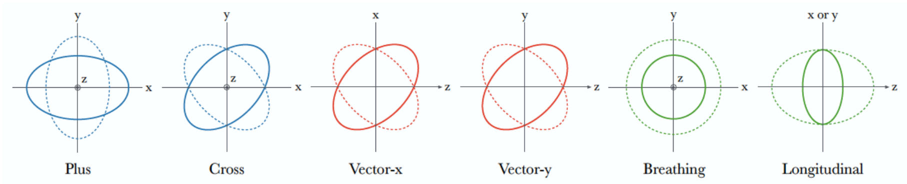
- **Other model-independent tests**

- **Other model-independent tests**

- **Polarisation tests** *Possibility of detecting additional polarisations not present in GR*

Alternative theories of gravity: scalar (S), vector (V), tensor (T) polarisations

$$\boxed{\Omega_{\mathrm{SVT-PL}}(f) = \sum_{\mathrm{p}} \beta_{IJ}^{(\mathrm{p})}(f)\Omega_{\mathrm{ref}}^{(\mathrm{p})}\left(\frac{f}{f_{\mathrm{ref}}}\right)^{\alpha_{\mathrm{p}}}} \qquad \mathrm{p} = \{\mathrm{T},\mathrm{V},\mathrm{S}\} \qquad \beta_{IJ}^{(\mathrm{p})}(f) = \gamma_{IJ}^{(\mathrm{p})}(f)/\gamma_{IJ}(f)$$



The current generation (number and orientation) of ground-based detectors is limited in its ability to sensitively determine the polarisation content of transient GW signals

Bayesian method to detect and characterise the polarisation of the SGWB

Callister et al (including MS) (2017)

There is no evidence of non-GR polarisations
The non-detection of scalar and vector polarised GWBs is consistent with predictions of GR

LVC (including MS) (2021)

- **Other model-independent tests**

- **Polarisation tests** *Possibility of detecting additional polarisations not present in GR*

Note: For frequencies larger than $6 \times 10^{-2}\,\mathrm{Hz}$, LISA sensitivity for scalar-longitudinal and vector polarisation modes can be 10 times larger compared to tensorial or scalar-transverse modes; in the lower frequency the sensitivity is the same

Challenge: - Idea of construction of null channels suggested for LIGO can be extended for LISA in the ppE framework
- Mapping of generic polarisation tests to specific modified theories (not yet been developed)

- **Other model-independent tests**

- **Polarisation tests** *Possibility of detecting additional polarisations not present in GR*

Note: For frequencies larger than $6\times10^{-2}\,\mathrm{Hz}$, LISA sensitivity for scalar-longitudinal and vector polarisation modes can be 10 times larger compared to tensorial or scalar-transverse modes; in the lower frequency the sensitivity is the same

Challenge: - Idea of construction of null channels suggested for LIGO can be extended for LISA in the ppE framework
- Mapping of generic polarisation tests to specific modified theories (not yet been developed)

- **Propagation tests**

GW propagation over cosmological distances

primes denote derivative wrt lna

$$h''_{ij} + \underbrace{\left(3 + \frac{H'}{H} + \nu\right)}_{\text{damping term}} h'_{ij} + \left[c_{\mathrm{T}}^2 \left(\frac{k}{aH}\right)^2 + \frac{\mu^2}{H^2}\right] h_{ij} = \frac{\Gamma}{H^2}\gamma_{ij}$$

Hubble friction

graviton speed

graviton mass

source term(e.g., in bimetric theories or from anisotropic stress)

can parametrise effective Planck mass evolution rate or the impact of extra dimensions

$\mathrm{GW170817}$
$c_T/c = 1$ of $\lesssim \mathcal{O}(10^{-15})$

LISA: massive BBH up to z~8 and their EM counterparts

Other model-independent tests

Polarisation tests

Possibility of detecting additional polarisations not present in GR

Note: For frequencies larger than $6\times10^{-2}\,\mathrm{Hz}$, LISA sensitivity for scalar-longitudinal and vector polarisation modes can be 10 times larger compared to tensorial or scalar-transverse modes; in the lower frequency the sensitivity is the same

Challenge: - Idea of construction of null channels suggested for LIGO can be extended for LISA in the ppE framework
- Mapping of generic polarisation tests to specific modified theories (not yet been developed)

Propagation tests

GW propagation over cosmological distances

primes denote derivative wrt lna

$$h''_{ij} + \underbrace{\left(3 + \frac{H'}{H} + \nu\right)}_{\text{damping term}} h'_{ij} + \left[c_T^2 \left(\frac{k}{aH}\right)^2 + \frac{\mu^2}{H^2}\right] h_{ij} = \frac{\Gamma}{H^2}\gamma_{ij}$$

Hubble friction

graviton mass

graviton speed

source term(e.g., in bimetric theories or from anisotropic stress)

can parametrise effective Planck mass evolution rate or the impact of extra dimensions

$$\boxed{\begin{array}{c}\text{GW170817}\\ c_T/c = 1 \text{ of } \lesssim \mathcal{O}(10^{-15})\end{array}}$$

LISA: massive BBH up to z~8 and their EM counterparts

The function that modifies the friction term can affect the **luminosity distance extracted** from detection of GWs from CBC

phenomenological model

$$\boxed{\frac{d_L^{\rm gw}(a)}{d_L^{\rm em}(a)} = \Xi_0 + a^n(1-\Xi_0)}$$

Belgacem et al (2018)

Construct simulated catalogues of LISA massive BBH with EM counterparts and use these mock catalogues to constrain modified gravity theories

*Belgacem et al (including **MS**) (2018)*

Measured to an accuracy between 4.4 % (worse case) and 1.1 %

- **Other model-independent tests**

  - **Stochastic GW background tests**

$$\Omega_{\rm GWB} = \Omega_\beta \left(\frac{f}{f_*}\right)^\beta$$

Challenge: Remove astrophysical background from stellar origin BH and neutron star mergers and find how much its residual affect tests of GR

log-likelihood for a single detector pair

$$\log p(\hat{C}_{ij}(f)|\boldsymbol{\theta}_{\rm GW}) = -\frac{1}{2}\sum_f \frac{\left[\hat{C}_{ij}(f) - \Omega_{\rm GW}(f, \boldsymbol{\theta}_{\rm GW})\right]^2}{\sigma_{ij}^2(f)} - \frac{1}{2}\sum_f \log\left[2\pi\sigma_{ij}^2(f)\right]$$

CBC Power Law: $\boldsymbol{\theta} = (\Omega_{2/3})$,

CBC + CS: $\boldsymbol{\theta} = (\Omega_{2/3}, \Omega_{\rm CS})$.

CBC + BPL: $\boldsymbol{\theta} = (\Omega_{2/3}, \Omega_*, f_*)$.

Model selection — To compare two models we use Bayes factors

Detector networks

- Hanford, Livinston, Virgo, O4 sensitivity, 1 year of run time
- Cosmic Explorers (CE) at Hanford and Livingston locations, Einstein Telescope (ET) at Virgo, 1 year of run time

- Current GW detectors are unable to separate astrophysical from cosmological sources
- Future GW detectors (CE, ET) may dig out cosmological signals, if one can subtract the *loud* astrophysical foreground

*Martinovic, Meyers, **MS**, Christensen (2021)*

Measuring the propagation speed of GWs with LISA

The propagation speed of GWs may vary as a function of the energy scale.

Measuring the propagation speed of GWs with LISA

The propagation speed of GWs may vary as a function of the energy scale.

Low energies: many theories spontaneously break Lorentz invariance through a time-dependent vacuum expectation value (essential for driving cosmic acceleration) of an additional field(s) $\Rightarrow$ tensor speed $c_T < 1$

Examples: *Horndeski theories and their extensions, DHOST (degenerate higher order scalar-tensor theories)*

If the UV completion of an extended gravity theory is required to be Lorentz invariant, then **the graviton speed becomes luminal at high energies**. The transition between non-luminal and luminal speed is likely to occur well before (or at most, around) the strong-coupling scale of the theory, which for Horndeski-like theories is typically $\Lambda \sim 260$ Hz.

Measuring the propagation speed of GWs with LISA

The propagation speed of GWs may vary as a function of the energy scale.

Low energies: many theories **spontaneously break Lorentz invariance** through a time-dependent vacuum expectation value (essential for driving cosmic acceleration) of an additional field(s) $\Rightarrow$ tensor speed $c_T < 1$
Examples: *Horndeski theories and their extensions, DHOST (degenerate higher order scalar-tensor theories)*
If the UV completion of an extended gravity theory is required to be Lorentz invariant, then **the graviton speed becomes luminal at high energies**. The transition between non-luminal and luminal speed is likely to occur well before (or at most, around) the strong-coupling scale of the theory, which for Horndeski-like theories is typically $\Lambda \sim 260$ Hz.

A frequency-dependent propagation speed can also arise in any scenario of gravity (typical for many QG theories) where the **spectral dimension of spacetime changes with the probed scale**. The ensuing dispersion relation features a non-trivial mixing between time and momentum and leads to a mixed redshift-frequency dependence of GW speed.
Also, a frequency dependent GW speed arises in **brane-world models** motivated by string theory.

Measuring the propagation speed of GWs with LISA

The propagation speed of GWs may vary as a function of the energy scale.

Low energies: many theories spontaneously break Lorentz invariance through a time-dependent vacuum expectation value (essential for driving cosmic acceleration) of an additional field(s) $\Rightarrow$ tensor speed $c_T < 1$

Examples: *Horndeski theories and their extensions, DHOST (degenerate higher order scalar-tensor theories)*

If the UV completion of an extended gravity theory is required to be Lorentz invariant, then **the graviton speed becomes luminal at high energies**. The transition between non-luminal and luminal speed is likely to occur well before (or at most, around) the strong-coupling scale of the theory, which for Horndeski-like theories is typically $\Lambda \sim 260$ Hz.

A frequency-dependent propagation speed can also arise in any scenario of gravity (typical for many QG theories) where the **spectral dimension of spacetime changes with the probed scale**. The ensuing dispersion relation features a non-trivial mixing between time and momentum and leads to a mixed redshift-frequency dependence of GW speed.

Also, a frequency dependent GW speed arises in **brane-world models** motivated by string theory.

A massive graviton (or the related bigravity) scenario can lead to a frequency-dependent GW velocity, with interesting and testable consequences for GW waveforms.

Measuring the propagation speed of GWs with LISA

Aim: *Develop a general theoretical and numerical toolkit for quantifying the perspective of LISA to measure a frequency-dependent* c_T *only through its effects on GW waveforms from merging massive BH binaries*

Measuring the propagation speed of GWs with LISA

Aim: *Develop a general theoretical and numerical toolkit for quantifying the perspective of LISA to measure a frequency-dependent c_T only through its effects on GW waveforms from merging massive BH binaries*

LVC: BNS GW170817 ⟹ $-3\times 10^{-15} \leq c_T - 1 \leq 7\times 10^{-16}$ (in $c = 1$ units)

One should construct a function for $c_T(f)$ which satisfies the LIGO-Virgo bounds whilst modifying the millihertz regime significantly

⟹ sharp transitions for $c_T(f)$ are needed in the frequency band between LISA and LIGO frequencies, to ensure consistency with the results from GW170817

Measuring the propagation speed of GWs with LISA

Aim: *Develop a general theoretical and numerical toolkit for quantifying the perspective of LISA to measure a frequency-dependent c_T only through its effects on GW waveforms from merging massive BH binaries*

LVC: BNS GW170817 $\Longrightarrow$ $-3\times 10^{-15} \leq c_T - 1 \leq 7\times 10^{-16}$ (in $c=1$ units)

One should construct a function for $c_T(f)$ which satisfies the LIGO-Virgo bounds whilst modifying the millihertz regime significantly

$\Longrightarrow$ sharp transitions for $c_T(f)$ are needed in the frequency band between LISA and LIGO frequencies, to ensure consistency with the results from GW170817

Framework:
Dynamics of GW at emission and detection is described by GR (possibly thanks to screening mechanisms)
Deviations from GR can occur during the propagation of GW through cosmological spacetime from source to observation

Measuring the propagation speed of GWs with LISA

Aim: *Develop a general theoretical and numerical toolkit for quantifying the perspective of LISA to measure a frequency-dependent* c_T *only through its effects on GW waveforms from merging massive BH binaries*

LVC: BNS GW170817 $\Longrightarrow$ $-3\times 10^{-15} \leq c_T - 1 \leq 7\times 10^{-16}$ (in $c = 1$ units)

Assume GW are massless and propagate freely through cosmological background from their source (inspiralling binary) to detection

Quadratic action for the linearised transverse-traceless GW modes

$$\boxed{S_T = \frac{M_{\rm Pl}^2}{8} \int dt\, d^3x\, a^3(t)\, \bar{\alpha} \left[\dot{h}_{ij}^2 - \frac{c_T^2(f)}{a^2(t)} (\vec{\nabla} h_{ij})^2\right]}$$

dimensionless normalisation constant

$$\bar{\alpha} = c_T^{-1}(f_s)$$

Assumption: in proximity of the source, modified gravity effects have no time to develop

the frequency of GW as emitted by an inspiralling binary process

Measuring the propagation speed of GWs with LISA

Aim: *Develop a general theoretical and numerical toolkit for quantifying the perspective of LISA to measure a frequency-dependent c_T only through its effects on GW waveforms from merging massive BH binaries*

LVC: BNS GW170817 ⇒ $-3\times 10^{-15} \leq c_T - 1 \leq 7\times 10^{-16}$ (in $c = 1$ units)

Assume GW are massless and propagate freely through cosmological background from their source (inspiralling binary) to detection

Quadratic action for the linearised transverse-traceless GW modes

$$\boxed{S_T \;=\; \frac{M_{\rm Pl}^2}{8}\int dt\, d^3x\; a^3(t)\,\bar{\alpha}\left[\dot{h}_{ij}^2 \;-\; \frac{c_T^2(f)}{a^2(t)}(\vec{\nabla}h_{ij})^2\right]}$$

dimensionless normalisation constant

$$\bar{\alpha} \;=\; c_T^{-1}(f_s)$$

Assumption: in proximity of the source, modified gravity effects have no time to develop

the frequency of GW as emitted by an inspiralling binary process

⇓

Lineralised evolution equation describing a free GW propagating through a cosmological spacetime with arbitrary speed $c_T(f)$

$$ds^2 \;=\; c_T(f)\,\bar{\alpha}\left[-c_T^2(f)\,dt^2 + a^2(t)\,d\vec{x}^2\right]$$

effective metric to describe propagation of GW

Ansätze for $c_T(f)$

- Polynomial ansatz

motivated from a perturbative expansion in powers of $(f/f_\star)$

small quantity

$$c_T(f) = 1 + \sum_n \beta_n \left(\frac{f}{f_*}\right)^n$$

positive or negative integer

fixed frequency scale controlling the onset of the deviations

set of parameters controlling deviations from GR

Ansätze for $c_T(f)$

LIGO bound implies:

$$|\beta_n| \lesssim 10^{-15-n}(f_*/\mathrm{Hz})^n$$

- Polynomial ansatz

motivated from a perturbative expansion in powers of $(f/f_\star)$

small quantity

$$c_T(f) = 1 + \sum_n \beta_n \left(\frac{f}{f_*}\right)^n$$

positive or negative integer

fixed frequency scale controlling the onset of the deviations

set of parameters controlling deviations from GR

Note: generalisation by considering a non-trivial function $c_T(z,f)$ of the redshift and the frequency

Expanding $c_T(f)$ up to quadratic order will prove sufficient to study the dominant corrections to the waveform detectable with LISA

Ansätze for $c_T(f)$

- Polynomial ansatz

motivated from a perturbative expansion in powers of $(f/f_\star)$

small quantity

$$c_T(f) = 1 + \sum_n \beta_n \left(\frac{f}{f_*}\right)^n$$

positive or negative integer

fixed frequency scale controlling the onset of the deviations

set of parameters controlling deviations from GR

LIGO bound implies:

$$|\beta_n| \lesssim 10^{-15-n}(f_*/\mathrm{Hz})^n$$

Note: generalisation by considering a non-trivial function $c_T(z, f)$ of the redshift and the frequency

Expanding $c_T(f)$ up to quadratic order will prove sufficient to study the dominant corrections to the waveform detectable with LISA

Positive power case: require $f_* > f$ everywhere in the LISA band, meaning that the deviation $c_T/c - 1$ will grow as the inspiral evolves (it implicitly requires that some termination mechanism switches off the deviations between the LISA band and the band of ground-based detectors, to maintain consistency with current bounds on c_T)

Negative power case: f_* should be outside the LISA frequency interval, so that $(f/f_*)^{-1}$ stays small in the LISA band

Ansätze for $c_T(f)$

- Polynomial ansatz

motivated from a perturbative expansion in powers of $(f/f_\star)$

$$c_T(f) = 1 + \sum_n \beta_n \left(\frac{f}{f_*}\right)^n$$

positive or negative integer

fixed frequency scale controlling the onset of the deviations

set of parameters controlling deviations from GR

Note: generalisation by considering a non-trivial function $c_T(z,f)$ of the redshift and the frequency

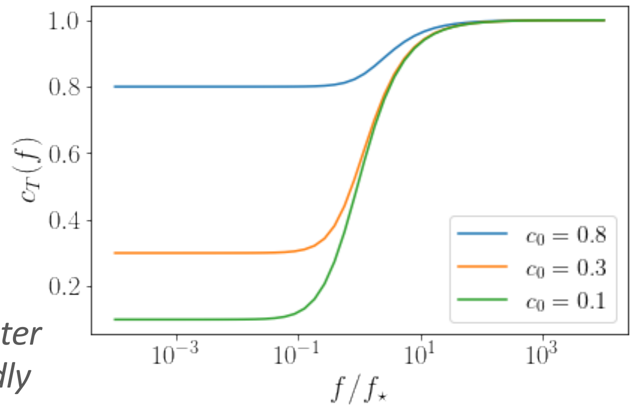
- EFT-inspired ansatz

Smooth transition in c_T *from some lower value to c, taking place inside or close to the LISA band*

$$c_T(f) = \left[1 + \frac{f_\star^2}{f^2} - \frac{f_\star^2}{f^2}\sqrt{1 + 2\left(1 - c_0^2\right)\frac{f^2}{f_\star^2}}\right]^{1/2}$$

low-frequency speed with $0 < c_0 \leq 1$

fiducial frequency, a free parameter around which c_T *changes rapidly*



The two parameters $f_\star$ and c_0 controlling the location and height of the transition, influence considerably GW waveforms and can be constrained with LISA. For MBH around $M_{\rm tot} \sim 10^5 M_\odot$ they can be constrained to a fractional error of order percent level or better, wrt their fiducial values

Theoretical motivation for the EFT ansatz

Suppose there exists a scalar theory valid up to a strong coupling scale Λ, with new physics entering at the scale $M \leq \Lambda$.

Assume a homogeneous scalar background $\phi_0(t)$ that spontaneously breaks Lorentz invariance, $\phi_0(t) = \alpha \Lambda t$, patametrised with a constant parameter α

The spontaneous breaking of Lorentz invariance typically leads to **a scalar speed different to that of light**

⇨ dispersion relation $$\omega^2 = k^2 - \alpha^2 \frac{\omega^2 M^2}{M^2 - \omega^2 + k^2}$$

⇨ scalar speed $$c_s^2(k) = 1 + \frac{k_\star^2}{k^2} - \frac{k_\star^2}{k^2}\sqrt{1 + 2\left(1 - c_0^2\right)\frac{k^2}{k_\star^2}}$$ with $$k_\star = \frac{M}{\sqrt{2}\,c_0} \quad ; \quad c_0^2 = \frac{1}{1+\alpha^2}$$

Rewrite tensor speed in terms of frequency $(f \equiv 2\pi\, k)$: $$c_T(f) = \left[1 + \frac{f_\star^2}{f^2} - \frac{f_\star^2}{f^2}\sqrt{1 + 2\left(1 - c_0^2\right)\frac{f^2}{f_\star^2}}\right]^{1/2}$$

Ansätze for $c_T(f)$

- **Polynomial ansatz**

$$c_T(f) = 1 + \sum_n \beta_n \left(\frac{f}{f_*}\right)^n$$

positive or negative integer

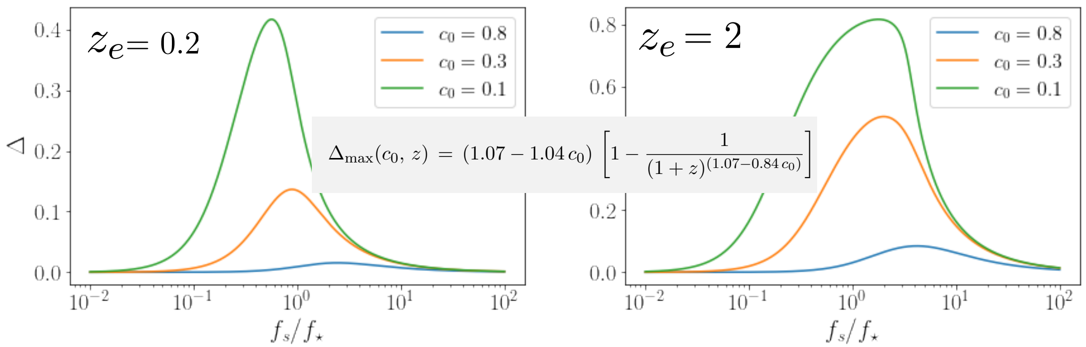
fixed frequency scale controlling the onset of the deviations

set of parameters controlling deviations from GR

Note: generalization by considering a non-trivial function $c_T(z, f)$ of the redshift and the frequency

- **EFT-inspired ansatz**

$$c_T(f) = \left[1 + \frac{f_\star^2}{f^2} - \frac{f_\star^2}{f^2}\sqrt{1 + 2\left(1 - c_0^2\right)\frac{f^2}{f_\star^2}}\right]^{1/2}$$



$$\Delta_{\max}(c_0, z) = (1.07 - 1.04\, c_0)\left[1 - \frac{1}{(1+z)^{(1.07 - 0.84\, c_0)}}\right]$$

dimensionless quantity

$$\Delta = \frac{f_s - (1+z)\, f_o(f_s, z)}{f_s} = 1 - \frac{c_T(f_o)}{c_T(f_s)}$$

that measures deviation from its standard relation $f_s = (1+z)\, f_o$ *connecting frequencies at emission and detection*

GW luminosity distance

$$\Delta = \frac{f_s - (1+z)\, f_o(f_s, z)}{f_s} = 1 - \frac{c_T(f_o)}{c_T(f_s)}$$

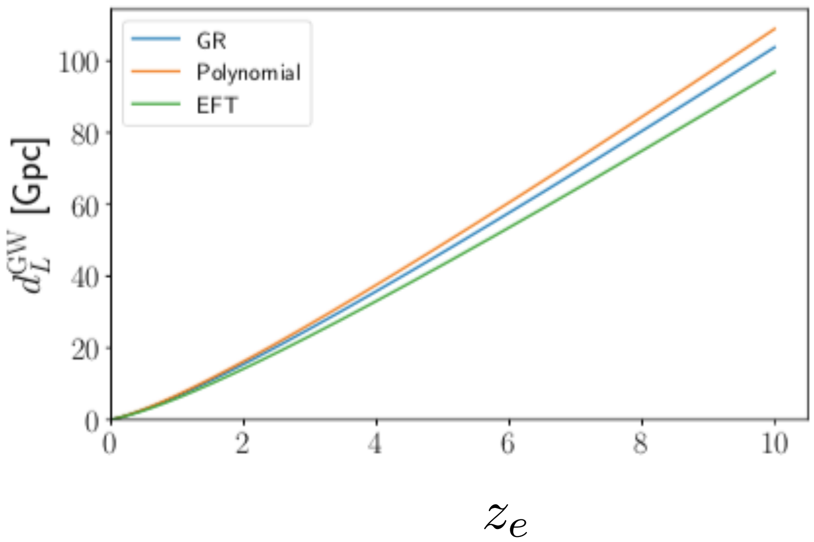
Consider $\Delta \neq 0$ (c_T non-constant function of frequency)

luminosity distance

$$\boxed{d_L^{\rm GW} = (1+z_e)\,(1-\Delta)^{-\frac{1}{2}}\, r_{\rm com}^{\rm GW}}$$

redshift of the source

$$r_{\rm com}^{\rm GW}(t) = \int_{t_e}^{t} \frac{c_T[f(t')]}{a(t')}\, dt'$$ *comoving distance*



Polynomial case: positive-power case with (exaggerated) values of $\beta_1 = \beta_2 = 10$ *at the frequency of* $(f_o/f_*) = 10^{-2}$

EFT case: with $c_0 = 0.9$ *at the frequency of* $(f_o/f_*) = 1$

The values of $d_L^{\rm GW}$ in the polynomial case are larger than in GR for positive values of β_1, β_2 (and vice-versa for negative β_1 , β_2)

For the EFT-inspired case , $d_L^{\rm GW}$ is suppressed with respect to GR behaviour

GW amplitude

The two helicities of GW waveform for the binary compact object inspiral in Fourier space:

$$h_+(f) = A(f)\frac{1+\cos^2\iota}{2}e^{i\Psi(f)}, \qquad h_\times(f) = iA(f)\cos\iota\ e^{i\Psi(f)}$$

redshifted GW amplitude

$$\boxed{A^{\rm GR}(f_z) = \sqrt{\frac{5\pi}{24}}\frac{\mathcal{M}_z^2}{(1+z)r_{\rm com}}(\pi\mathcal{M}_z f_z)^{-7/6}}$$

$$\mathcal{M}_z = (1+z)\mathcal{M}_s$$

$$\mathcal{M}_s = M_{\rm tot}\eta^{3/5}$$

chirp mass of binary at the source

$$\eta = m_1 m_2/M_{\rm tot}$$

reduced mass parameter

$$f_z = f_s/(1+z)$$

redshift frequency

GW amplitude

The two helicities of GW waveform for the binary compact object inspiral in Fourier space:

$$h_+(f) = A(f)\frac{1+\cos^2\iota}{2}e^{i\Psi(f)}, \qquad h_\times(f) = iA(f)\cos\iota\; e^{i\Psi(f)}$$

redshifted GW amplitude

$$\boxed{A^{\mathrm{GR}}(f_z) = \sqrt{\frac{5\pi}{24}}\frac{\mathcal{M}_z^2}{(1+z)r_{\mathrm{com}}}(\pi\mathcal{M}_z f_z)^{-7/6}}$$

$$\mathcal{M}_z = (1+z)\mathcal{M}_s$$

$$\mathcal{M}_s = M_{\mathrm{tot}}\eta^{3/5}$$

chirp mass of binary at the source

$$\eta = m_1 m_2/M_{\mathrm{tot}}$$

reduced mass parameter

$$f_z = f_s/(1+z)$$

redshift frequency

modified GW amplitude

$$\boxed{A^{\mathrm{MG}}(f_o) = \sqrt{\frac{5\pi}{24}}\frac{\mathcal{M}_o^2}{d_L^{\mathrm{GW}}}(\pi\mathcal{M}_o f_o)^{-\frac{7}{6}}\left[\frac{c_T(f_o)}{c_T(f_s)}\right]^{\frac{3}{2}}}$$

$$d_L^{\mathrm{GW}} = (1+z_e)\,(1-\Delta)^{-\frac{1}{2}}\,r_{\mathrm{com}}^{\mathrm{GW}}$$

$$f_o = f_z\frac{c_T(f_o)}{c_T(f_s)}$$

frequency at detection

$$\mathcal{M}_o = \mathcal{M}_z\frac{c_T(f_s)}{c_T(f_o)}$$

observed chirp mass

GW phase

The phase of GW during inspiral can be computed analytically using **PN expansion**

We focus on GW propagation effects, so **we do not consider modifications to the physics of the merging process at the source position** $\Longrightarrow$ **the rate of change of GW frequency in the source frame should match that of GR**

GW phase

The phase of GW during inspiral can be computed analytically using **PN expansion**

We focus on GW propagation effects, so **we do not consider modifications to the physics of the merging process at the source position** $\Longrightarrow$ **the rate of change of GW frequency in the source frame should match that of GR**

Consider **non-spinning binary systems on circular orbits**

$$\boxed{\frac{df_o}{dt_o} = (1-\Delta)^2 \left(\frac{1}{1+\frac{f_o}{1-\Delta}\frac{\partial \Delta}{\partial f_o}} \right) \frac{96}{5\pi\,\mathcal{M}_z^2}\, u^{\frac{11}{3}} \left[1 + \psi_1 u^{\frac{2}{3}} + \psi_{1.5} u + \psi_2 u^{\frac{4}{3}} + \psi_{2.5} u^{\frac{5}{3}} \right]}$$

red-shifted chirp mass

$u = \pi \mathcal{M}_s f_s = \pi \mathcal{M}_z f_z = \pi \mathcal{M}_o f_o$ *frame-independent*

$$\Delta = \frac{f_s - (1+z)\, f_o(f_s, z)}{f_s}$$

to parametrise deviations from GR

$\psi_k \;\; (k = 1, 1.5, 2, 2.5)$ *the PN phase parameters*

Note: - The 3PN term remains subdominant in all our calculations

GW phase

The phase of GW during inspiral can be computed analytically using **PN expansion**

We focus on GW propagation effects, so **we do not consider modifications to the physics of the merging process at the source position** $\Rightarrow$ **the rate of change of GW frequency in the source frame should match that of GR**

Consider **non-spinning binary systems on circular orbits**

$$\boxed{\frac{df_o}{dt_o} = (1-\Delta)^2 \left(\frac{1}{1+\frac{f_o}{1-\Delta}\frac{\partial \Delta}{\partial f_o}} \right) \frac{96}{5\pi \mathcal{M}_z^2}\, u^{\frac{11}{3}} \left[1 + \psi_1 u^{\frac{2}{3}} + \psi_{1.5} u + \psi_2 u^{\frac{4}{3}} + \psi_{2.5} u^{\frac{5}{3}} \right]}$$

red-shifted chirp mass

$u = \pi \mathcal{M}_s f_s = \pi \mathcal{M}_z f_z = \pi \mathcal{M}_o f_o$ *frame-independent*

$$\Delta = \frac{f_s - (1+z)\, f_o(f_s, z)}{f_s}$$

to parametrise deviations from GR

$\psi_k \;\; (k = 1, 1.5, 2, 2.5)$ *the PN phase parameters*

Note: - The 3PN term remains subdominant in all our calculations

time and phase at the end of inspiral

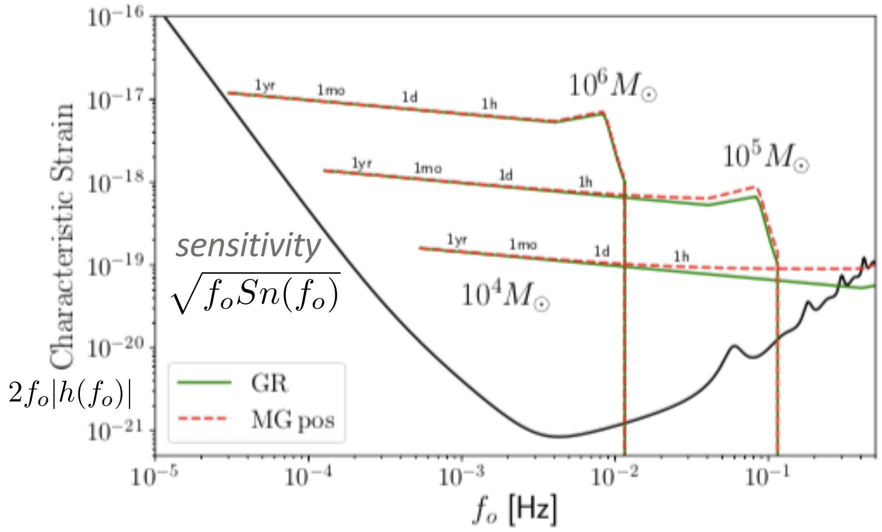
Integrate twice to find time to coalescence and then the GW phase

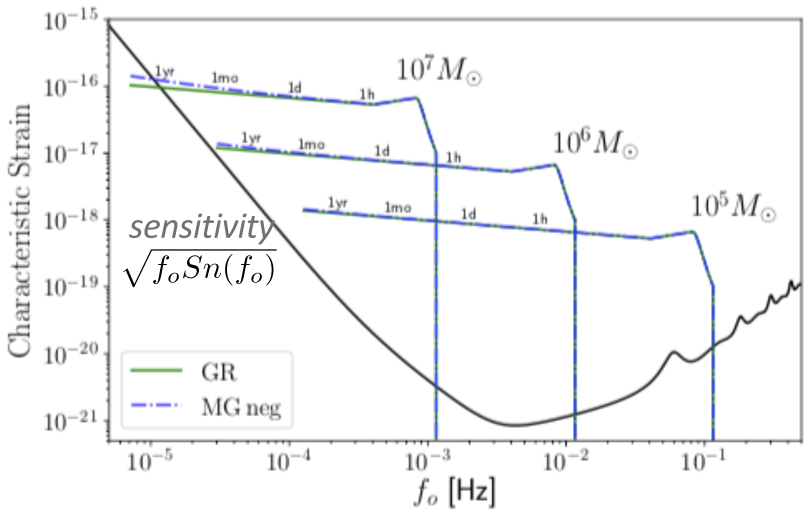
$$\Psi = 2\pi \int_{f_c}^{f_o} [t_o(\tilde{f}_o) - t_c]\, d\tilde{f}_o + 2\pi f_o t_c - \Psi_c - \frac{\pi}{4}$$

twice the frequency of inner-most stable circular orbit

Polynomial model:

Timeline on the amplitude: time before merger at Newtonian order





Positive-power polynomial with $\beta_1 = \beta_2 = 100$ *and* $f_* = 2$ Hz *for binaries with different total masses at* $z = 1$

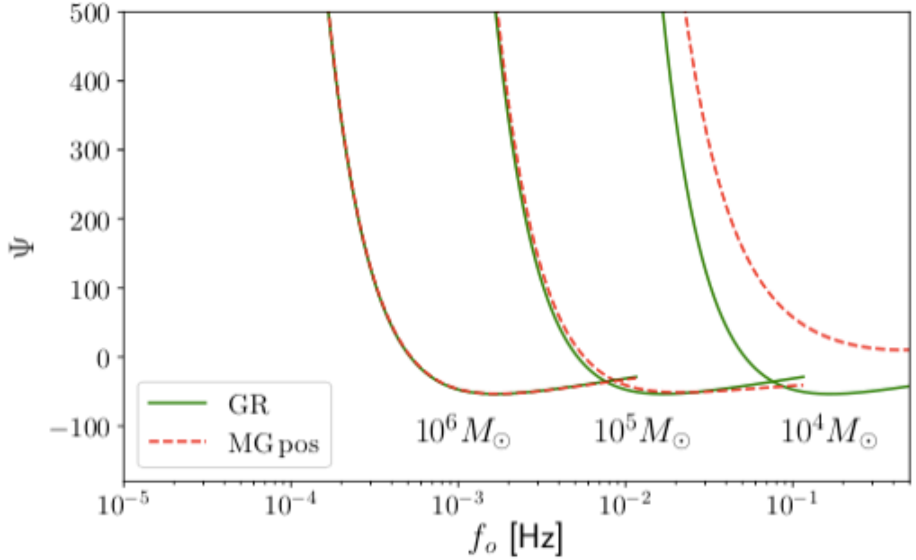
Negative-power polynomial with $\beta_1 = \beta_2 = 200$ *and* $f_* = 2 \times 10^{-7}$ Hz *for binaries with different total masses at* $z = 1$

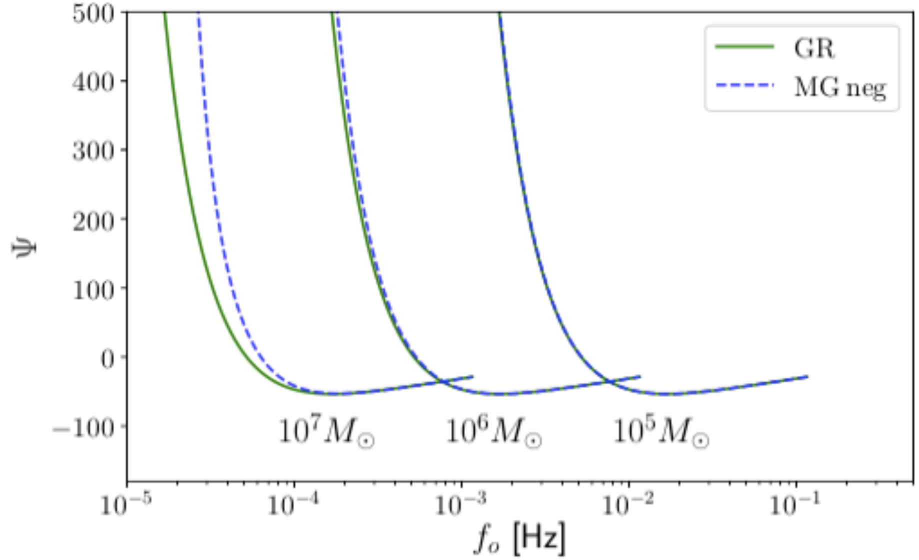
$$\boxed{c_T(f) = 1 + \sum_n \beta_n \left(\frac{f}{f_*}\right)^n}$$

The modified amplitudes deviate from their GR equivalents as f_o approaches f_*

Lighter systems are preferred for detecting beyond Einstein models described by the positive-power polynomial ansatz; heavier systems for the negative-power polynomial ansatz

Polynomial model:





Positive-power polynomial with $\beta_1 = \beta_2 = 20$ *and* $f_* = 2$ Hz *for binaries with different total masses at* $z = 1$

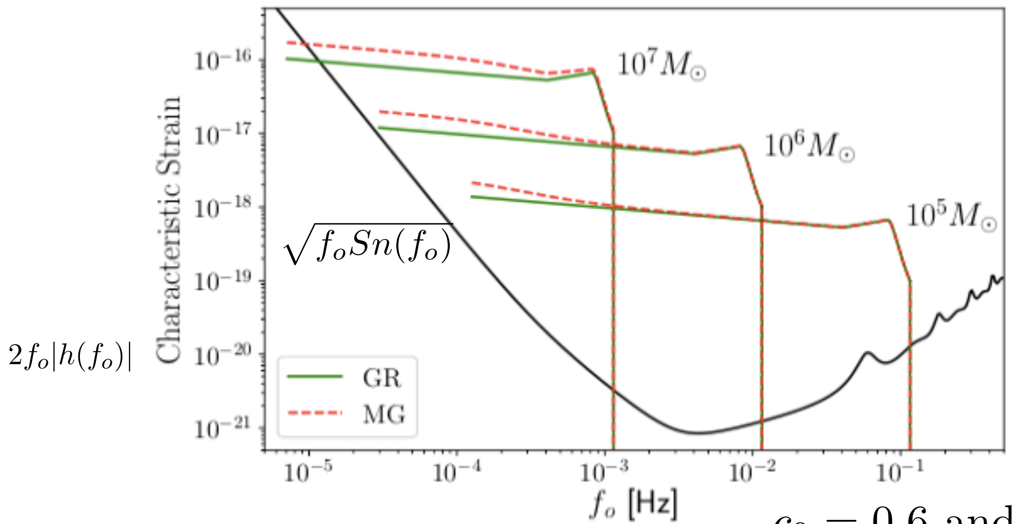
Negative-power polynomial with $\beta_1 = \beta_2 = 100$ *and* $f_* = 2 \times 10^{-7}$ Hz *for binaries with different total masses at* $z = 1$

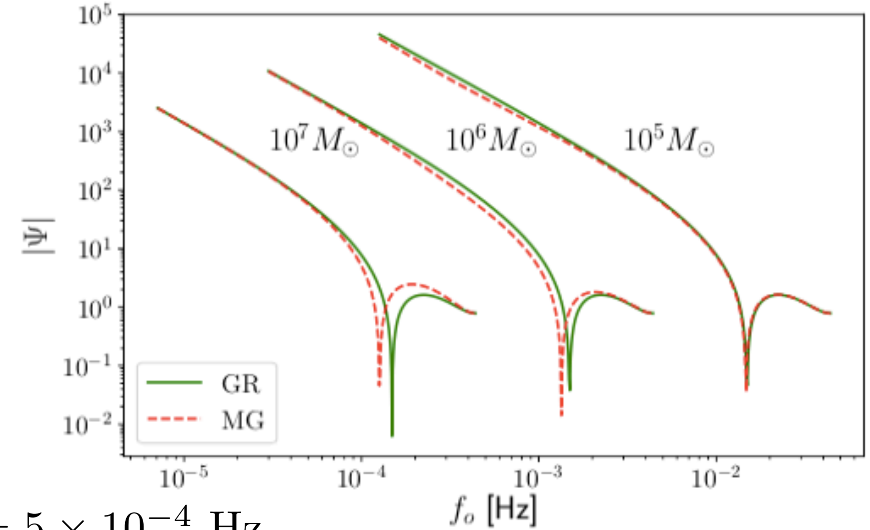
$$\boxed{c_T(f) = 1 + \sum_n \beta_n \left(\frac{f}{f_*}\right)^n}$$

Deviations of the modified phases from their GR correspondences as f_o approaches f_*

The deviations are larger for lighter binary systems in the positive-power case;
heavier binary systems in the negative-power case

EFT-inspired model:





$$c_0 = 0.6 \text{ and } f_* = 5 \times 10^{-4} \text{ Hz}$$

$$\boxed{c_T(f) = \left[1 + \frac{f_\star^2}{f^2} - \frac{f_\star^2}{f^2}\sqrt{1 + 2\left(1 - c_0^2\right)\frac{f^2}{f_\star^2}}\right]^{1/2}}$$

The characteristic strains and phases of the EFT ansatz case compared with GR for binaries with different total masses at $z = 1$

- The modified gravity effects start to become manifest when the observed frequency approaches f_* (the frequency that sets the position of the rapid growth of $c_T(f)$)
- The modified amplitudes show constant offsets from their GR equivalences at low frequencies much smaller than f_*
- The modified phase for the high mass system seems equivalent to GR values, but actually the deviation from GR of the phase does not vanish at low frequencies

Inspiral-Merger-Ringdown (IMR) extension

The inspiral waveform starts to become invalid above $f_c \sim 2 f_{\rm ISCO}$

GR: extended template waveforms that include IMR phases obtained from numerical studies of BBH mergers

Inspiral-Merger-Ringdown (IMR) extension

The inspiral waveform starts to become invalid above $f_c \sim 2 f_{\rm ISCO}$

GR: extended template waveforms that include IMR phases obtained from numerical studies of BBH mergers

- **Inspiral-only waveform** (*conservative case*)

  If considering departures from GR, we allow for the strong-field regime itself to be modified as well

  Post Newtonian expansion at 2.5PN order

Inspiral-Merger-Ringdown (IMR) extension

The inspiral waveform starts to become invalid above $f_c \sim 2 f_{\mathrm{ISCO}}$

GR: extended template waveforms that include IMR phases obtained from numerical studies of BBH mergers

- **Inspiral-only waveform** (*conservative case*)

  If considering departures from GR, we allow for the strong-field regime itself to be modified as well

  Post Newtonian expansion at 2.5PN order

- **Full IMR** (PhenomA) **waveform** (*optimistic case*)

  If we are confident that the strong-field regime is identical to GR (the screened case), and allow the continuation of GW propagation effects into the merger and ringdown regime
  (so, we use a modified PhenomA waveform which derives from GR simulations)

Inspiral-Merger-Ringdown (IMR) extension

The inspiral waveform starts to become invalid above $f_c \sim 2f_{\rm ISCO}$

GR: extended template waveforms that include IMR phases obtained from numerical studies of BBH mergers

Approximate analytic description (*we are not modifying the intrinsic strong-field dynamics of the source*)

Modified version of the frequency-domain PhenomA waveform *Ajith et al (2008)*

$$\boxed{A_{\rm ins}(f) = C\left(\frac{f}{f_{\rm merg}}\right)^{-\frac{7}{6}},} \quad \boxed{A_{\rm merg}(f) = C\left(\frac{f}{f_{\rm merg}}\right)^{-\frac{2}{3}},} \quad \boxed{A_{\rm ring}(f) = C\omega\mathcal{L}(f, f_{\rm ring}, \sigma)}$$

Lorentzian function in the ringdown phase

$$C^{\rm MG} = \sqrt{\frac{5}{24}}\pi^{-\frac{2}{3}}\frac{\mathcal{M}_o^{\frac{5}{6}}}{d_L^{\rm GM}}\left(\frac{c_T(f_o)}{c_T(f_s)}\right)^{\frac{3}{2}} f_{\rm merg}^{-\frac{7}{6}}; \qquad \omega = \frac{\pi\sigma}{2}\left(\frac{f_{\rm ring}}{f_{\rm merg}}\right)^{-2/3}$$

$$\mathcal{L}(f, f_{\rm ring}, \sigma) = \left(\frac{1}{2\pi}\right)\frac{\sigma}{(f - f_{\rm ring})^2 + \sigma^2/4}$$

$$f_k = (a_k\eta^2 + b_k\eta + c_k)/\pi M_{\rm tot}.$$

Ajith et al (2008)

$$\boxed{\Psi_{\rm merg}(f) = \bar{\Psi}_c + 2\pi\bar{t}_c f}$$

Phase during ringdown = 0 (as in ppE)

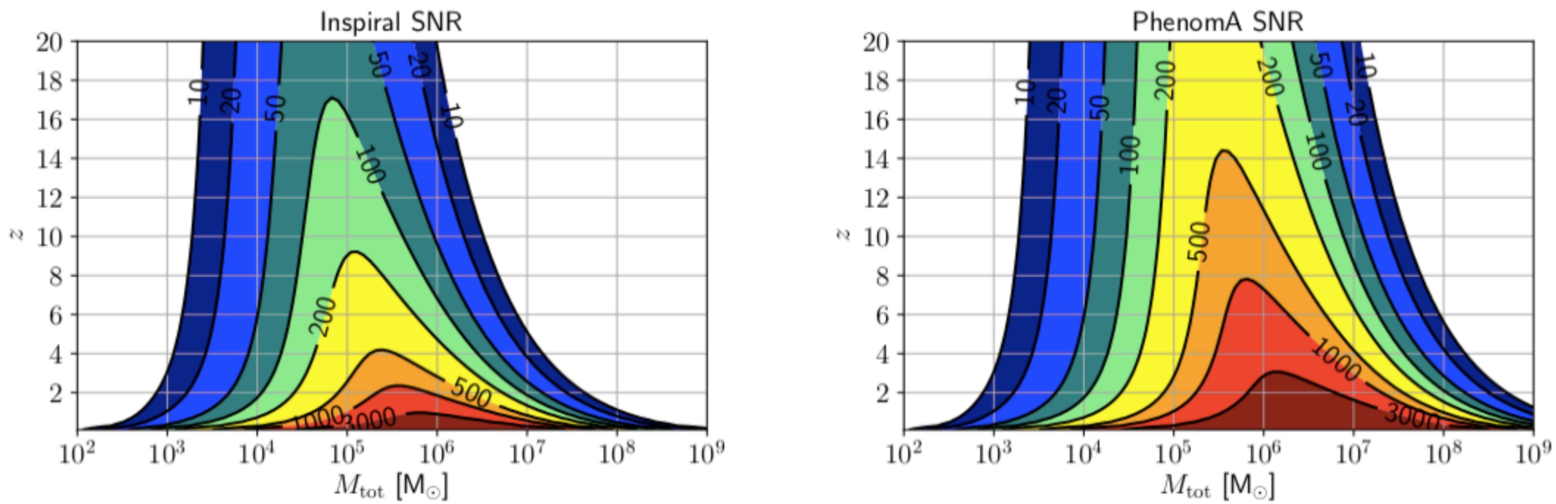
$$\bar{\Psi}_c = \Psi_{\rm ins}(f_{\rm merg}) - 2\pi\bar{t}_c f_{\rm merg} \qquad \bar{t}_c = \frac{1}{2\pi}\frac{d\Psi_{\rm ins}}{df}\bigg|_{f=f_{\rm merg}}.$$

<u>Question:</u> *How well a four-year LISA mission can constrain both the polynomial and EFT-inspired ansätze for* $c_T(f)$*, both with a single MBH merger and a population of MBH mergers*

Fisher matrix analysis to forecast constraints on five GR parameters and 2 modified gravity parameters

Naively one might expect that the best constraints will be obtained from systems with the highest total SNR

SNR contours for LISA detections within GR in terms of MBH total mass and redshift using only inspiral portion of the signal and full inspiral-merger-ringdown signal



Systems between 10^5 and 10^7 $M_\odot$ provide the highest SNR detections in both cases

These however are not necessarily the optimal systems for bounding $c_T(f)$, due to the **frequency-dependent nature of the corrections**. The constrains are controlled by **the SNR and the total mass of the system.**

Polynomial model

$$c_T(f) = 1 + \sum_n \beta_n \left(\frac{f}{f_*}\right)^n$$

- The presence of β_1 and β_2 weakens the constraints on the GR parameters

- The constraints on GR parameters are controlled by the SNR and the total mass of the system:

$\eta,\ z$ and Ψ_c tend to be better constrained when the SNR is higher

constraints on $\mathcal{M}_z$ and t_c are tighter for systems with lower masses

(signals from lower mass systems stay longer in the LISA band, so that more cycles are available for constraining the parameters)

Polynomial model

$$c_T(f) = 1 + \sum_n \beta_n \left(\frac{f}{f_*}\right)^n$$

Positive-power case: best constrained by systems with $M_{\rm tot} < 10^5 M_\odot$

Negative-power case: greatest deviation from GR for heaviest systems

β_2 is challenging to constraint (second order correction to $c_T(f)$

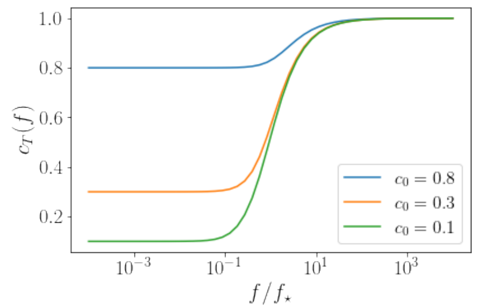
Including the merger and ringdown still tightens constraints on the GR parameters and due to the correlations between parameters, this leads to a mild improvement in β_1 and β_2

The constraints are quite sensitive to f_*

[EFT-inspired model]

The parameters f_* and c_0. control the location and height of the transition

$$c_T(f) = \left[1 + \frac{f_\star^2}{f^2} - \frac{f_\star^2}{f^2}\sqrt{1 + 2\left(1 - c_0^2\right)\frac{f^2}{f_\star^2}}\right]^{1/2}$$



The constraints on GR parameters are roughly as tight as the ones in the polynomial case

However, we now obtain tight constraints on c_0. and f_*

- Deviations from GR are strongest in the mid-inspiral phase, where both number of cycles and SNR accumulation are reasonable

- Both parameters play comparable roles in modifying the waveform (polynomial case: β_2 is significantly subdominant to β_1 In the LISA band)

Comments:

- The obtained constraints were derived from single event detections; we should consider an MBH population
  In most cases our method has robustness against realistic population models

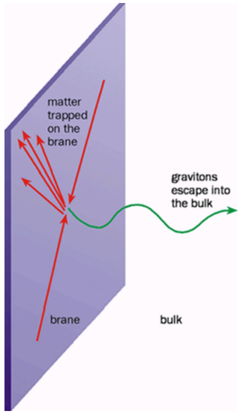
  ⟹ Tests of gravity at low frequency can be carried out with LISA in (almost) any scenario

- We focused exclusively on the frequency dependence of c_T ; as a result, the constraints are always tightest from low-redshift sources
  We have consequently considered a non-trivial function $c_T(z, f)$ of the redshift and the frequency, also including a non-trivial modification to the cosmological friction term

- Our method **does not rely on the presence of an electromagnetic counterpart**: for long-duration sources our analysis could be applied on-the-fly months op years before merger.

- We have a mapping between our beyond Einstein parameters and those of parametrised post-Einsteinian framework

Brane/String theory: Extra dimensions



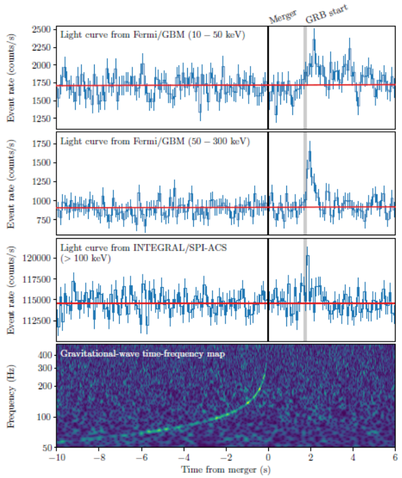
Constraints on the number of spacetime dimensions from GWs

Damping of the waveform due to gravitational leakage into extra dim

Deviation depends on the number of dimensions D and would result to a systematic **overestimation of the source** d_L^{EM} **inferred from GW data**

$$h \propto \frac{1}{d_L^{\mathrm{GW}}} = \frac{1}{d_L^{\mathrm{EM}}} \left[1 + \left(\frac{d_L^{\mathrm{EM}}}{R_c}\right)^n\right]^{-(D-4)/(2n)}$$

***Strain** measured in a GW interferometer* ⟷ ***Luminosity distance** measured for the optical counterpart of the standard siren*



- Consistency with GR in D=4 dim
- Some models (e.g. the Dvali-Gabadadze-Porrati (DGP) model) are ruled out

GRB 170817A and GW170817

GW event 1.7 s before γ-ray observation

BNS merger at 40 Mpc

Propagation of GWs in the context of Quantum Gravity

Long-range nonperturbative mechanism found in most QG candidates:
Dimensional flow (change of spacetime dimensionality)

ST distorted by QG effects characterised by ST measure ρ (how volume scales) and kinetic term K (modified dispersion relations)

Perturbed action for a small perturbation h over background

$$S = \frac{1}{2\ell_*^{2\Gamma}} \int d\varrho \sqrt{-g^{(0)}} \left[h_{\mu\nu} \mathcal{K} h^{\mu\nu} + O(h_{\mu\nu}^2) + \mathcal{J}^{\mu\nu} h_{\mu\nu} \right]$$

characteristic scale of geometry

scaling parameter

generic source term

$$\boxed{\Gamma(\ell) := \frac{d_{\rm H}(\ell)}{2} - \frac{d_{\rm H}^k(\ell)}{d_{\rm S}(\ell)}}$$

Propagation of GWs in the context of Quantum Gravity

Long-range nonperturbative mechanism found in most QG candidates:
Dimensional flow (change of spacetime dimensionality)

ST distorted by QG effects characterised by ST measure ρ (how volume scales) and kinetic term K (modified dispersion relations)

Perturbed action for a small perturbation h over background

$$S = \frac{1}{2\ell_*^{2\Gamma}} \int d\varrho \sqrt{-g^{(0)}} \left[h_{\mu\nu} \mathcal{K} h^{\mu\nu} + O(h_{\mu\nu}^2) + \mathcal{J}^{\mu\nu} h_{\mu\nu} \right]$$

$$\boxed{h \propto \int d\varrho\, \mathcal{J}\, G}$$

The GW amplitude is determined by the convolution of the source with the retarded Green function

$$\Downarrow$$

In radial coordinates, and in the local wave zone

$$\boxed{h(t,r) \sim f_h(t,r)\ (\ell_*/r)^{\Gamma}}$$

GW amplitude h is the product of a dimensionless function and a power-law distance behavior

depends on the source J and on the type of correlation function (advanced or retarded)

Propagation of GWs in the context of QG

QG corrections
are important

Scaling parameter

$$\boxed{\Gamma(\ell) := \frac{d_{\rm H}(\ell)}{2} - \frac{d_{\rm H}^k(\ell)}{d_{\rm S}(\ell)}}$$

	$\Gamma_{\rm UV}$	$\Gamma_{\rm meso} \gtrsim 1$
GFT/SF/LQG	$[-3, 0)$	yes
Causal dynamical triangulation	$-2/3$	
κ-Minkowski (other)	$[-1/2, 1]$	
Stelle gravity	0	
String theory (low-energy limit)	0	
Asymptotic safety	0	
Hořava–Lifshitz gravity	0	
κ-Minkowski bicross-product ∇^2	3/2	yes
κ-Minkowski relative-locality ∇^2	2	yes
Padmanabhan nonlocal model	2	yes

Contributions to GR
small but non-negligible

Scales at which QG corrections are important: UV regime

Intermediate scales where corrections to GR are small but not negligible: mesoscopic regime

$$\boxed{h(z) \sim f_h(z) \left[\frac{\ell_*}{d_L^{\rm EM}(z)}\right]^{\Gamma}}$$

Propagation of GWs in the context of QG

*The **strain** measured in a GW interferometer* ⟷ *The **luminosity distance** measured for the optical counterpart of the standard siren*

$$\boxed{h \propto \frac{1}{d_L^{\rm GW}}\,, \qquad d_L^{\rm GW} = d_L^{\rm EM}\left[1+\varepsilon\left(\frac{d_L^{\rm EM}}{\ell_*}\right)^{\gamma-1}\right], \qquad \gamma\neq 0,}$$

$$\varepsilon = \pm(\gamma-1)$$

If there is only one fundamental scale, $\ell_* = \mathcal{O}(\ell_{\rm Pl})$, the equation is exact and $\gamma = \Gamma_{\rm UV}$

If ℓ_* is a mesoscopic scale, then the equation is valid only near the IR regime and $\gamma = \Gamma_{\rm meso} \approx 1$

Propagation of GWs in the context of QG

*The **strain** measured in a GW interferometer* ⟷ *The **luminosity distance** measured for the optical counterpart of the standard siren*

$$\boxed{h \propto \frac{1}{d_L^{\mathrm{GW}}}, \qquad d_L^{\mathrm{GW}} = d_L^{\mathrm{EM}} \left[1 + \varepsilon \left(\frac{d_L^{\mathrm{EM}}}{\ell_*} \right)^{\gamma - 1} \right], \qquad \gamma \neq 0,}$$

$$\varepsilon = \pm(\gamma - 1)$$

Observations can place constraints on the parameters ℓ_* and γ in a model-independent way, by constraining the ratio $d_L^{\mathrm{GW}}(z)/d_L^{\mathrm{EM}}(z)$ as a function of the redshift of the source

Standard sirens: -- NS merger GW170817 (LIGO/Virgo & Fermi)
-- simulated z=2 supermassive BH merger within LISA detectability

Propagation of GWs in the context of QG

When $\gamma = \Gamma_{\mathrm{UV}}$ we cannot constrain the deep UV limit of QG, since $\ell_* = \mathcal{O}(\ell_{\mathrm{Pl}})$.
(*deviations from classical geometry occur at microscopic scales unobservable in astrophysics*)

The only theories that can be constrained in this way are those with $\Gamma_{\mathrm{meso}} > 1 > \Gamma_{\mathrm{UV}}$

$$0 < \Gamma_{\mathrm{meso}} - 1 < 0.02$$

Only GFT, SF or LQG *could* generate a signal detectable with standard sirens

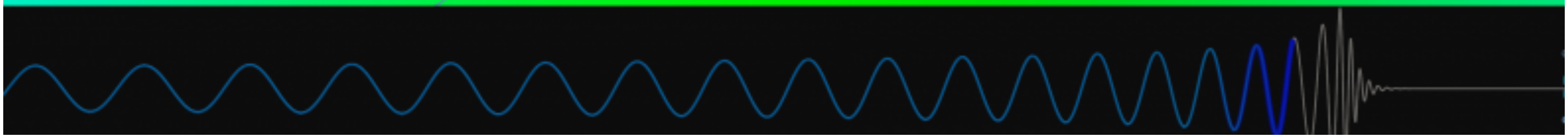
$$\Downarrow$$

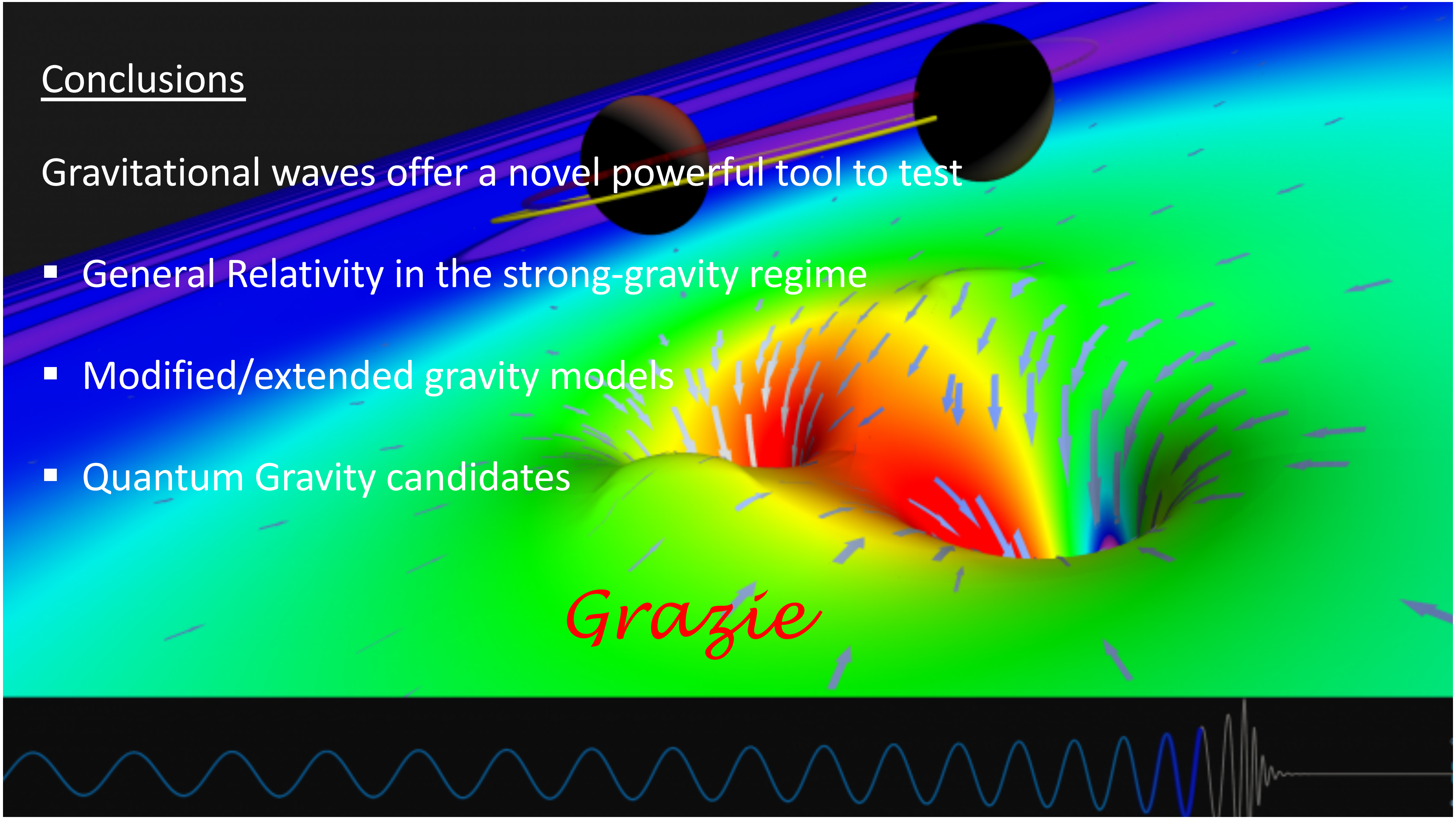
$$\boxed{\text{Look for realistic quantum states of geometry giving rise to such a signal}}$$

Conclusions

Gravitational waves offer a novel powerful tool to test

- General Relativity in the strong-gravity regime
- Modified/extended gravity models
- Quantum Gravity candidates




Conclusions
Gravitational waves offer a novel powerful tool to test
General Relativity in the strong-gravity regime
Modified/extended gravity models
Quantum Gravity candidates
Grazie