Hubble tension and quantum gravity effects

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based on

M. Lulli, A. Marciano & X. Shan, arXiv:2112.01490v2

M. Lulli, A. Marciano & L. Visinelli, Work in progress…

COST CA1808 Third Annual Conference — July 13-15, Naples

Plan of the seminar

Biased introduction to Quantum Gravity

The Ricci flow from the RG perspective

The Ricci flow and the Stochastic Quantization

Thermal time and conformal transformation

The Hamiltonian version of the Ricci flow and its interpretation

Ricci renormalization group flow of Λ

QG macroscopic effect: a way out from Hubble tension?

Searching over 100 years…

Summing over quantum histories

Imposition of the constraints at the quantum level vs fluctuations

The fate of the symmetries

Spontaneous vs dynamical symmetry breaking

Frisch, Rumpf…

Emergence of conformal symmetry in the UV

Finite gravity vs Convergence to zero of all coupling constants

Stelle, Toumbolis, Modesto…

Searching over 100 years…

Conformal anomaly and dimensional transmutation

Emergence of scale and new degrees of freedom

Renormalization group flow

Searching for a non-trivial fixed point and control of UV behaviour

Ricci flow

Developing a geometric intuition on the RG flow

$$S = \alpha' \int_{\mathcal{M}} d^2\sigma \sqrt{h} h^{ab}(\sigma) g_{ij}(X) \frac{\partial X^i}{\partial \sigma^a} \frac{\partial X^j}{\partial \sigma^b}$$

The case of the non-linear sigma models

$$\frac{\partial g_{ij}}{\partial \lambda} = -\alpha' R_{ij} - \frac{\alpha'^2}{2} R_{iklm} R_j{}^{klm} + \cdots$$

$$\frac{\partial}{\partial \lambda} g_{\mu\nu} = -2R_{\mu\nu}$$

Hamilton

Complementing with randomness

Ricci flow and the variational principle

$$\frac{\partial}{\partial\lambda} g_{\mu\nu} = -2R_{\mu\nu}$$
$$= -2\left[R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R\right] - g_{\mu\nu} R$$
$$= -2\frac{\delta S_G}{\delta g^{\mu\nu}} - g_{\mu\nu} R.$$

substitute with noise

Langevin equation and random noise

$$\frac{\partial}{\partial\lambda}\phi_A\left(x^\mu,\lambda\right) = -\frac{\delta S\left[\phi\right]}{\delta\phi_A} + \eta\left(x^\mu,\lambda\right)$$

Parisi & Wu

Stochastic quantization

Describe approach to equilibrium

$$\frac{\partial}{\partial\lambda}\phi_A\left(x^\mu,\lambda\right)=-\frac{\delta S\left[\phi\right]}{\delta\phi_A}+\eta_A\left(x^\mu,\lambda\right)$$

Additive noise associated

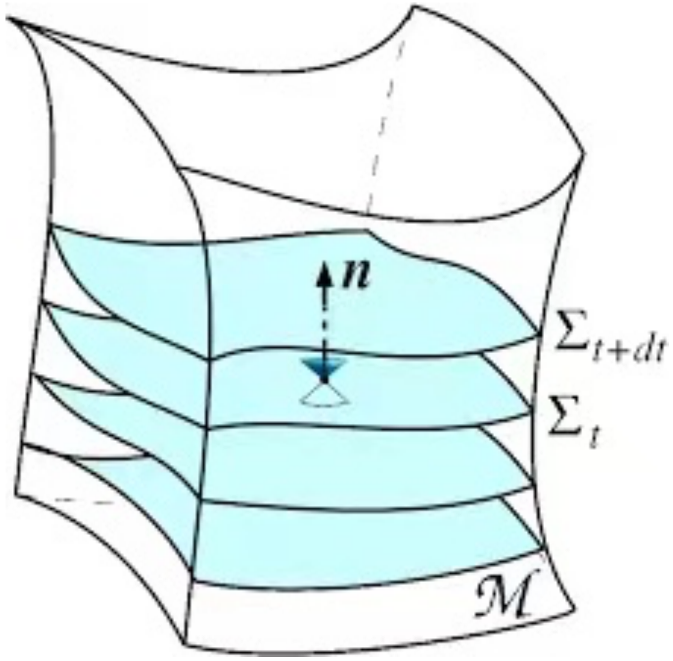
$$\langle\eta_A\left(x,\lambda\right)\eta_B\left(x',\lambda'\right)\rangle \;=\; \alpha_\eta\delta_{AB}\delta\left(x-x'\right)\delta\left(\lambda-\lambda'\right)$$

Compute gauge invariant quantities without fixing the gauge

Thermal time and conformal transformation

$$\varepsilon^2(\lambda) = \exp\left[2\int_{\tau_0}^{\tau} \mathrm{d}\bar{\tau}\varphi(\bar{\tau},\lambda)\right]$$

The norm of the metric is not constant



$$\lambda \to \tau$$

From having introduced a projective term in the connection

$$\frac{\mathrm{d}g_{\mu\nu}(\tau,\lambda)}{\mathrm{d}\tau} = -2\varphi(\tau,\lambda)g_{\mu\nu}(\tau,\lambda)$$

Projective connection

$$\bar{\Gamma}^{\gamma}_{\alpha\beta} = \Gamma^{\gamma}_{\alpha\beta} + \mathcal{C}^{\gamma}_{\alpha\beta}$$

$$\mathcal{C}^{\gamma}_{\alpha\beta} = \lambda_1 \delta^{\gamma}_{\alpha} u_{\beta} + \lambda_2 u_{\alpha} \delta^{\gamma}_{\beta} + \lambda_3 w_{\alpha\beta} u^{\gamma} + \lambda_4 u_{\alpha} u_{\beta} u^{\gamma}$$

$$\sqrt{-g}\bar{R} = \sqrt{-g}R + \sqrt{-g} g^{\beta\delta} \left(\mathcal{C}^{\mu}_{\beta\delta} \mathcal{C}^{\alpha}_{\mu\alpha} - \mathcal{C}^{\mu}_{\beta\alpha} \mathcal{C}^{\alpha}_{\mu\delta} \right)$$

Cosmological term induced in the action

$$\sqrt{-g} g^{\beta\delta} \left(\mathcal{C}^{\mu}_{\beta\delta} \mathcal{C}^{\alpha}_{\mu\alpha} - \mathcal{C}^{\mu}_{\beta\alpha} \mathcal{C}^{\alpha}_{\mu\delta} \right) = \sqrt{-g} \left[\left(\lambda_2^2 + \lambda_3^2 \right) (D-1) \, u_{\mu} u^{\mu} \right]$$

Breakdown of the conformal symmetry

$$h_{\mu\nu} = h^{\perp}_{\mu\nu} + \partial_\mu a^{\perp}_\nu + \partial_\nu a^{\perp}_\mu + \left(\partial_\mu \partial_\nu - \frac{1}{4}\eta_{\mu\nu}\Box\right) a + \frac{1}{4}\eta_{\mu\nu}\varphi$$

$$\Phi = \varphi - \bar{\Box} a$$

Einstein-Hilbert expanded on dS or AdS backgrounds

$$\mathcal{S}^{(2)}_{\rm EH} = \frac{c^3}{16\pi G}\int d^4x\sqrt{-\bar{g}}\left[\frac{1}{4}h^{\mu\nu}_{\perp}\left(\bar{\Box} - \frac{\bar{R}}{6}\right)h^{\perp}_{\mu\nu} - \frac{3}{32}\Phi\left(\bar{\Box} + \frac{\bar{R}}{3}\right)\Phi\right]$$

Residual gauge transformation: conformal Killing vector and disappearance of the ghost

$$h^{\perp}_{\mu\nu} \to h^{\perp}_{\mu\nu} + \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu\,, \qquad \Phi \to \Phi + 2\nabla^\mu k_\mu$$

Spectral dimension & Heat flow equation

$$\partial_s K(x,y;s) + \Delta_x K(x,y;s) = 0$$

$$d_s \equiv -2\frac{\partial \mathrm{Tr} K}{\partial \log s}$$

Schwinger time as the Wick rotation of the thermal time

$$D_F(x,y) = \int_0^\infty ds\, e^{-s\epsilon}\, e^{-\imath s m^2} K_\eta(x,y;-\imath s)$$

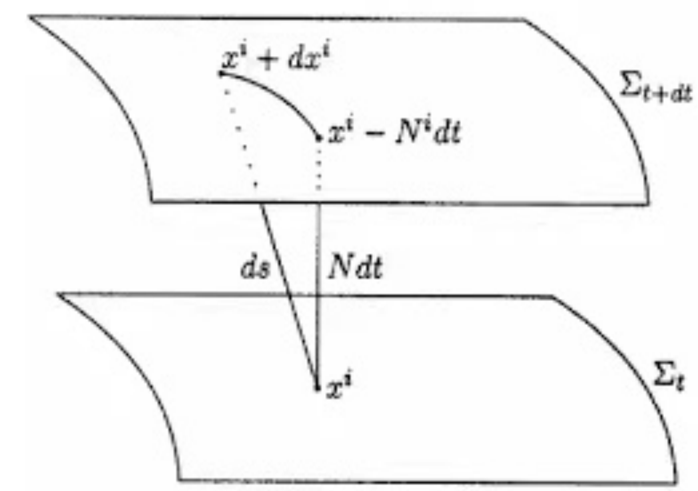
$$\frac{dg_{\mu\nu}}{ds} = -2R_{\mu\nu}$$

Ricci flow precisely as a heat equation of either Riemannian or pseudo-Riemannian space

ADM decomposition

$$ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu$$
$$= -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

$$n_\mu = (-N, 0)\,, \qquad n^\mu = \left(\frac{1}{N}, -\frac{N^i}{N}\right)$$



$$K_{ij} = -\nabla_{(j} n_{i)} =$$
$$= \frac{1}{2N}\left(-\partial_t h_{ij} + {}^{(3)}\nabla_{(i} N_{j)} + {}^{(3)}\nabla_{(j} N_{i)}\right)$$

$$\bar{\nabla}_{(\alpha} n_{\beta)} = \nabla_{(\alpha} n_{\beta)} - \mathcal{C}^{\gamma}_{(\alpha\beta)} n_\gamma$$
$$= \nabla_{(\alpha} n_{\beta)} - (\lambda_1 + \lambda_2)\, n_\alpha n_\beta$$
$$+ \varepsilon(\lambda)\left[\lambda_3 w_{\alpha\beta} + \lambda_4 n_\alpha n_\beta\right]$$

Hamiltonian analysis of the Ricci flow

$$\eta_{\mu\nu} = \eta \, g_{\mu\nu}$$

Multiplicative choice of the noise source entails additivity in the Hamiltonian ADM picture

$$\frac{\partial N}{\partial \lambda} = -\frac{N}{2}\left[\frac{\mathcal{H}}{\sqrt{h}} + \eta\right]$$

“00”

$$\frac{\partial N^k}{\partial \lambda} = \frac{N\mathcal{H}^k}{\sqrt{h}}$$

“0i”

$$\frac{\partial h_{ij}}{\partial \lambda} = \frac{1}{N}\mathcal{L}_m\left[\mathcal{H}, h_{ij}\right] + \left[\mathcal{H}, \left[\mathcal{H}, h_{ij}\right]\right] + \frac{h_{ij}\mathcal{H}}{2\sqrt{h}} - h_{ij}\eta$$

“ij”

Physical interpretation

Thermal time and time de-parametrization

Measurement problem and collapse of the wavefunction

“00”

Navier-Stokes at equilibrium

$$r_c^{3/2}\partial^k T_{ki} = \partial_t v_i - \zeta\partial^2 v_i + \partial_i P + v^k \partial_k v_i = 0$$

Turbulence away from equilibrium

$$r_c^{3/2}\partial_k T^{ki} = \frac{1}{N}\frac{\partial N^i}{\partial\lambda}$$

“0i”

$$\frac{\partial\nu}{\partial\lambda} = -4\pi e^{-\nu} r^2 \left[2\left(\frac{\partial^2\nu}{\partial r^2} + \frac{1}{r}\frac{\partial\nu}{\partial r}\right) + \frac{1}{2}\left(\frac{\partial\nu}{\partial r}\right)^2 + \frac{2}{r^2}\left(1 - e^{-\nu}\right)\right] + \eta$$

Kardar-Parisi-Zhang Equation

“ij”

Ricci RG flow of Λ

$$ds^2 = -N^2 dt^2 + a^2(t)\left[\frac{(dr)^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right]$$

FLRW background

$$S = 6\int d^4x N a^3 R + \int d^4x N a^3 (D-1)\lambda_2^2 \epsilon(\lambda) \qquad R = 6\left(\frac{\ddot{a}}{a} + \left(\frac{\dot{a}^2}{a^2}\right) + \frac{k}{a^2}\right)$$

$$\frac{\partial a}{\partial s} = -\frac{2\imath}{N^2}\left(a\dot{H} + 3aH^2 + \varepsilon N^2 \lambda_2^2\right) + a\eta\,,$$

$$\frac{\partial N}{\partial s} = -2\imath\left(\frac{3}{2N}(\dot{H} + H^2) + \frac{1}{16}N\left(\Lambda_0 + 8\lambda_2^2\right)\right) - N\eta\,,$$

$$\frac{\partial \lambda_2}{\partial s} = \imath\left(-2\varepsilon - \imath\eta\right)\lambda_2\,,$$

Ricci flow equations

Hubble tension: a macroscopic QG effect?

$$\langle \lambda_2^k(s) \rangle = \exp\left[\left(i(-2\varepsilon) + \frac{\Lambda_0}{2}\right)s\right]\langle \lambda_2^k(0) \rangle$$

Thermal time oriented as the proper time implies mild increase of Λ

Cosmological measurements $67.4 \pm 1.4 \quad (\mathrm{km/s})/\mathrm{Mpc}$

Astronomical measurements $74.03 \pm 1.42 \quad (\mathrm{km/s})/\mathrm{Mpc}$

Outlooks

RG flow for matter fields with gravitational back-reaction

Binary systems and growth of instabilities

Inflationary scenario from conformal symmetry breaking

Gravitational collapse of the wave-functions and its dynamics

Emergent gravity and topological phase

Thank you!

Grazie! **谢谢**