Cosmological models evolving through the Big Bang

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Quantum gravity phenomenology in the multi-messenger approach
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Gravitational singularities and determinism

> A singularity can be regarded as a place where there is a breakdown of the classical concept of spacetime as a manifold with a pseudo-Reimannian metric. Because all known laws of physics are formulated on a classical spacetime background, they will all break down at a singularity. This is a great crisis for physics because it means that **one cannot predict the future**. One does not know what will come out of a singularity.

S. W. Hawking, "Breakdown of Predictability in Gravitational Collapse", Phys. Rev. D14, 246 (1976)

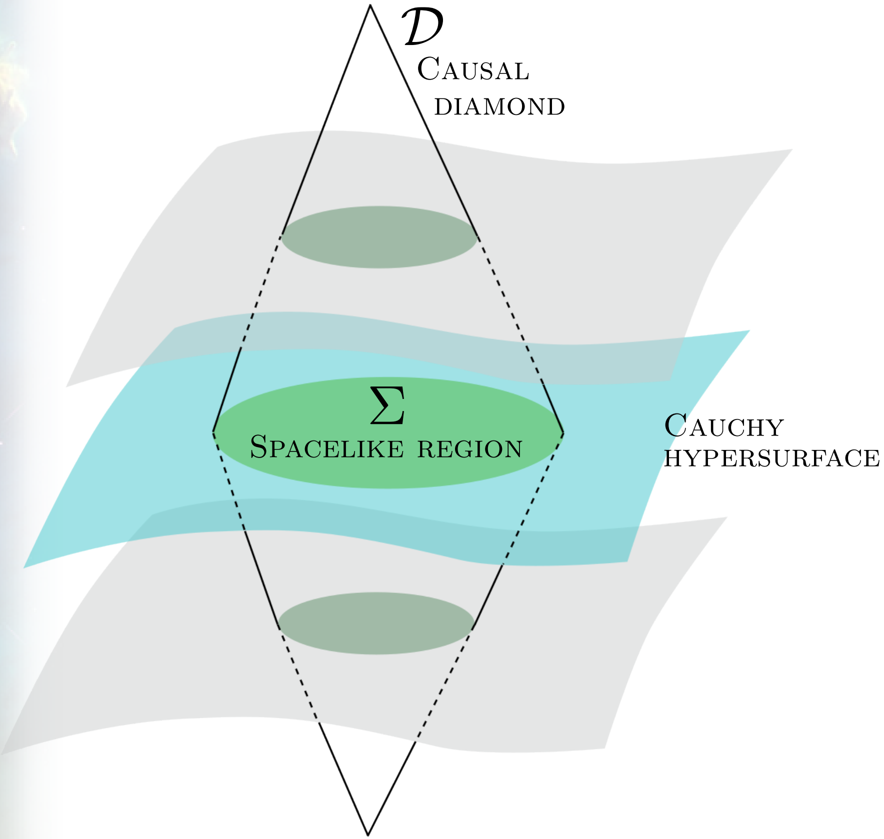
Gravitational singularities and determinism

Gravitational singularities are regions of spacetime where geometry or other fundamental physical structures become meaningless, and this happens in a **coordinate-independent way**

- the volume goes to zero
- some eigenvalues of the energy-momentum tensor diverge
- some curvature invariants diverge
- the geodesic equations are singular

But does this imply that dynamics is not well-defined?
Is this enough to give up on classical determinism?

Classical determinism in General Relativity



GR:

- Infinite number of DOFs
- Einstein's equations are a system of hyperbolic PDEs

Determinism:

Given all field values within Σ, it is possible to predict uniquely their values anywhere within $\mathcal{D}$

Classical determinism in General Relativity

Homogeneous cosmologies:

- Infinite → finite number of DOFs
- PDEs → ODEs

Determinism:

Picard–Lindelöf theorem of existence and uniqueness
under a certain set of conditions for the ODEs, an initial-value problem has a unique solution

GR is a gauge Hamiltonian system: **not all degrees of freedom are physical**, and determinism fails only if there is no way to evolve uniquely all physical DOFs

The Bianchi-IX universe

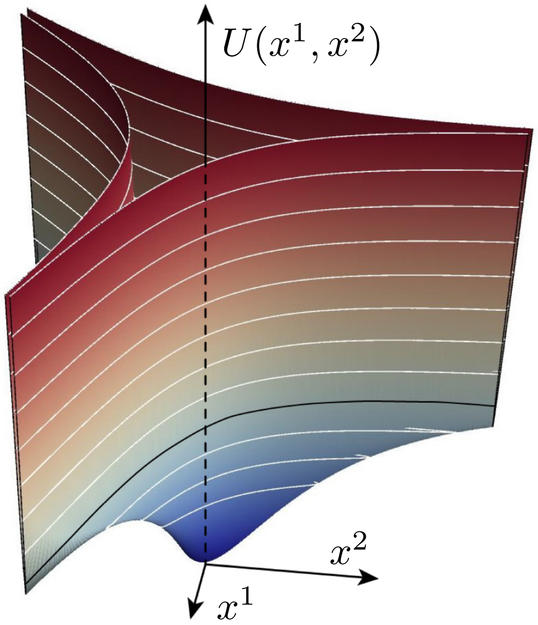
The BIX universe is a spatially **homogeneous** but not necessairly isotropic universe with a 3-sphere topology

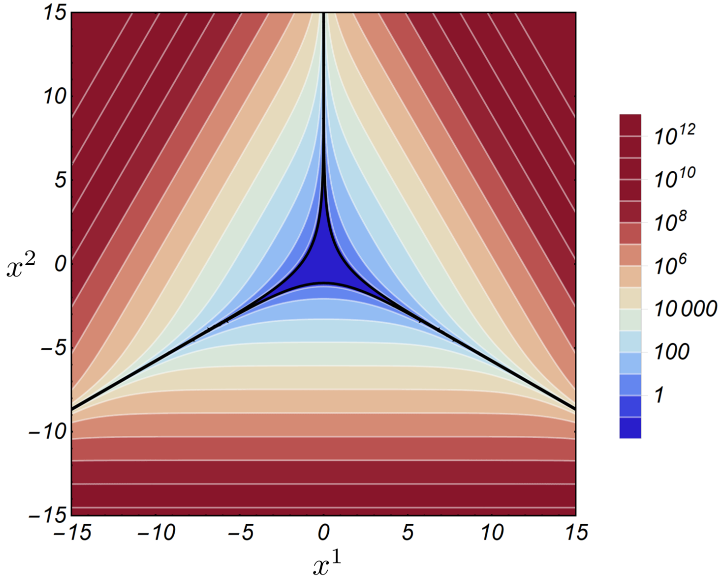
$$\mathcal{H} = \frac{1}{2}\left(-k_0^2 + k_1^2 + k_2^2\right) + \frac{1}{2} e^{\frac{2x^0}{\sqrt{3}}} U(x^1, x^2)$$

- x^0 **scale** variable (function of volume v)
  Big Bang singularity $x^0 \to -\infty$ ($v \to 0$) reached in a **finite amount of proper time**

- x^1, x^2 **shape** variables (measure large-scale anisotropy)
  when $x^1 = x^2 = 0$ → round 3-sphere metric

The Shape potential



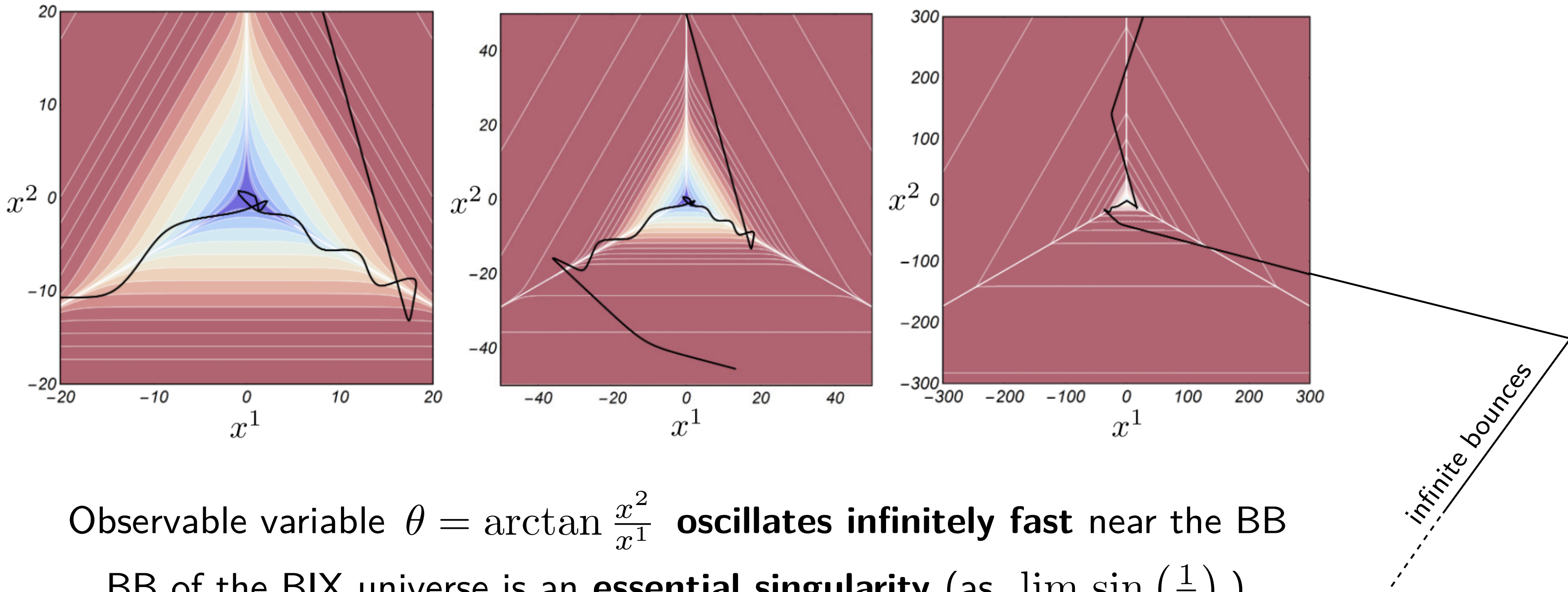


kinetic energy $\simeq$ potential energy $\rightarrow$ **bouces against the potential walls** (Taub transitions)

kinetic energy $\gg$ potential energy $\rightarrow$ **straight uniform motion** (Kasner epoch)

Essential singularity at the BB

Infinite bounces in a finite amount of proper time → eternal chaotic behaviour



Observable variable $\theta = \arctan \frac{x^2}{x^1}$ **oscillates infinitely fast** near the BB

BB of the BIX universe is an **essential singularity** (as $\lim_{x \to 0} \sin\left(\frac{1}{x}\right)$)

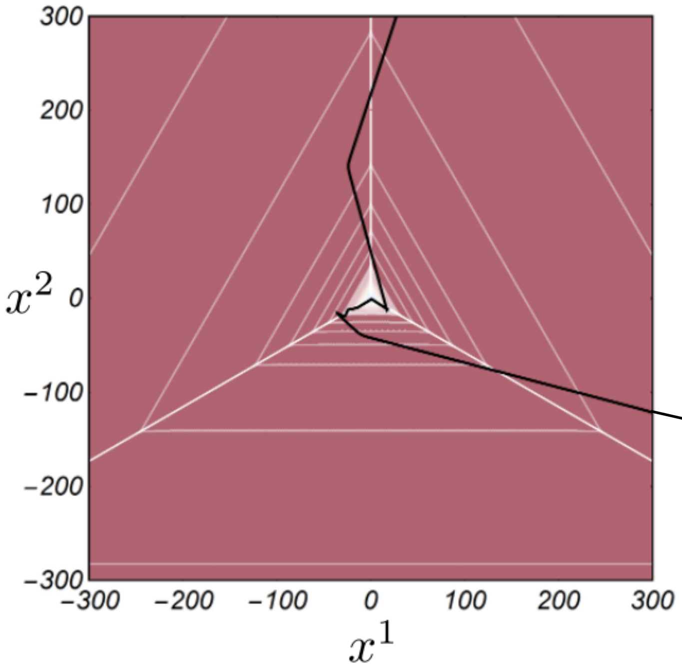
Quiescence

Adding an **homogeneous scalar field**
whose potential does not grow too fast

$$\mathcal{H} = \frac{1}{2}\left(-k_0^2 + k_1^2 + k_2^2 + \pi_\phi^2\right) + \frac{1}{2}e^{\frac{2x^0}{\sqrt{3}}}U(x^1, x^2) + e^{\sqrt{3}x^0}V(\phi)$$

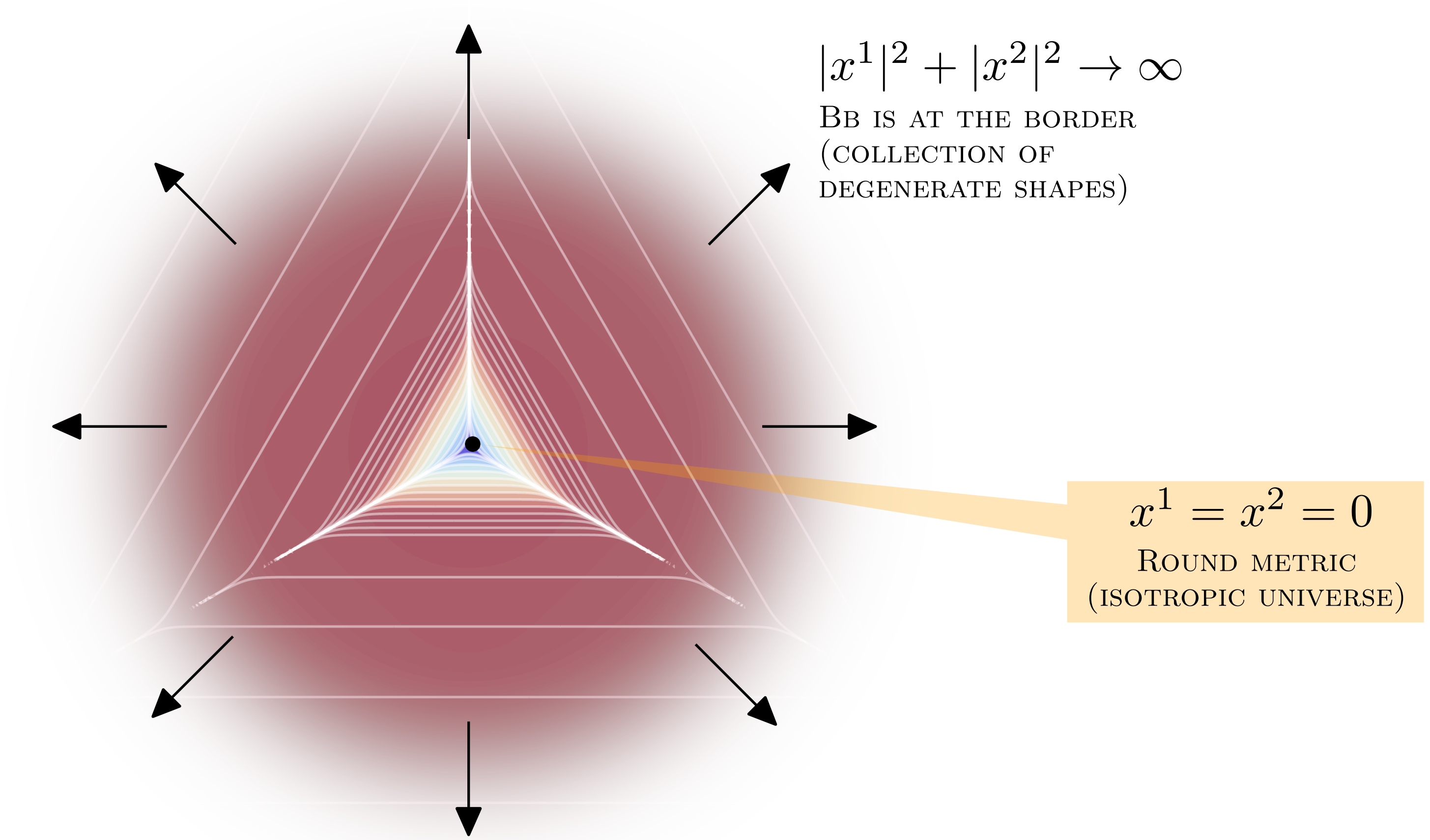
After a finite number of bounces, the **bounces will stop**, and the solution will settle on a **single straight-line (Kasner) solution**

Quiescence



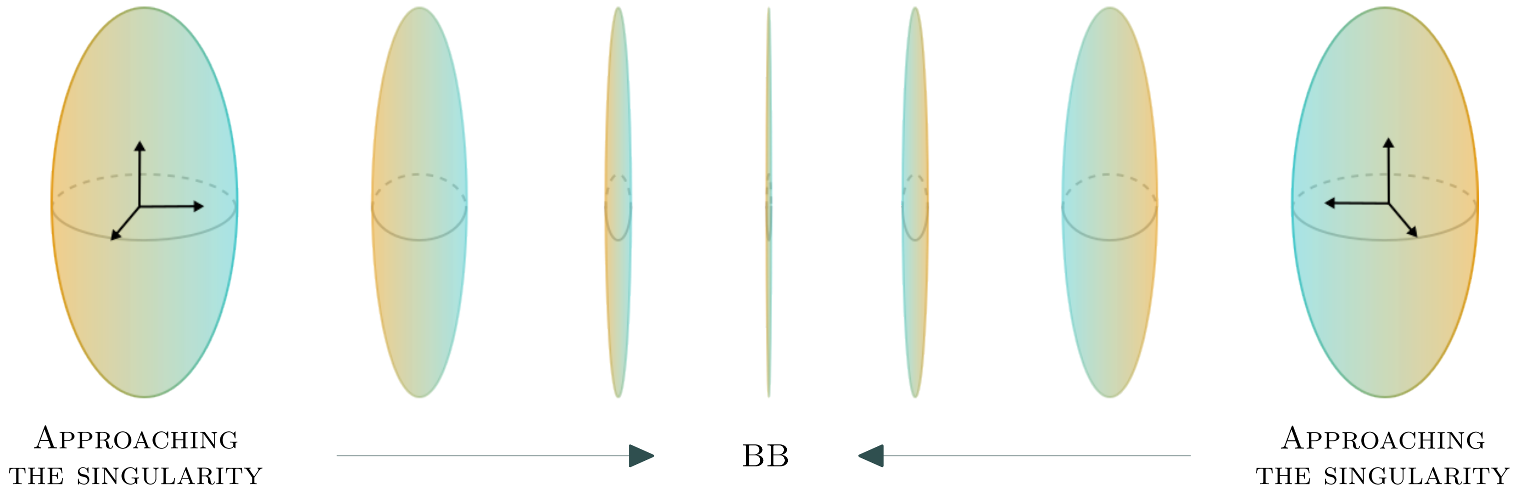
After the last bouce there is an endless Kasner epoch all the way to the singularity

Shape-plane topology

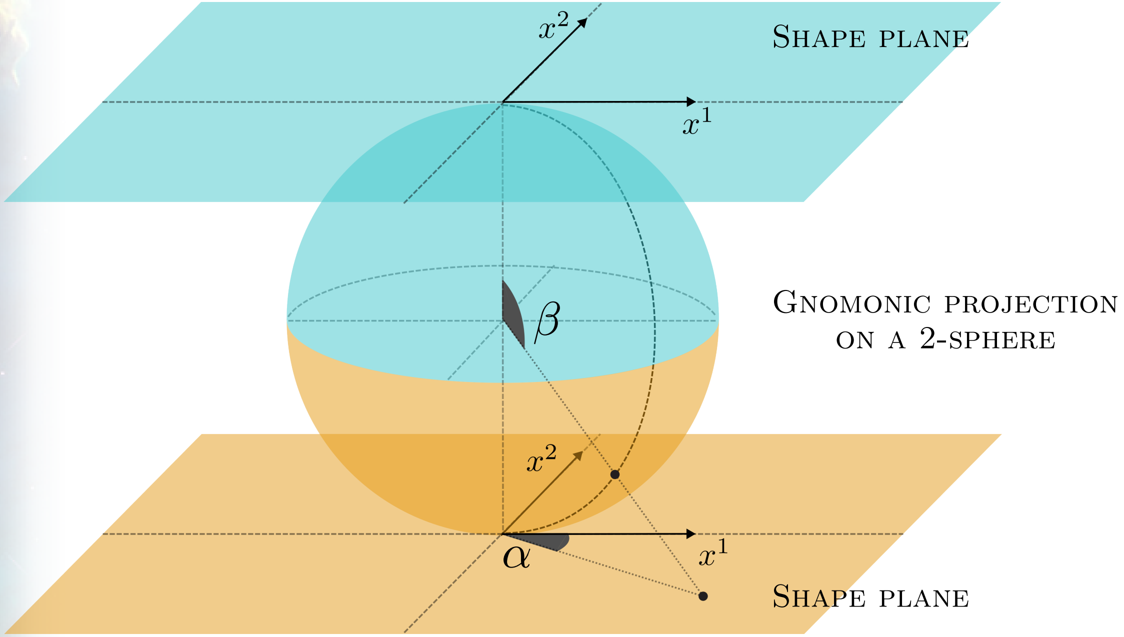


Phase-space compactification

GR is blind under changes of space orientation
→ two shape planes with different orientations



Phase-space compactification

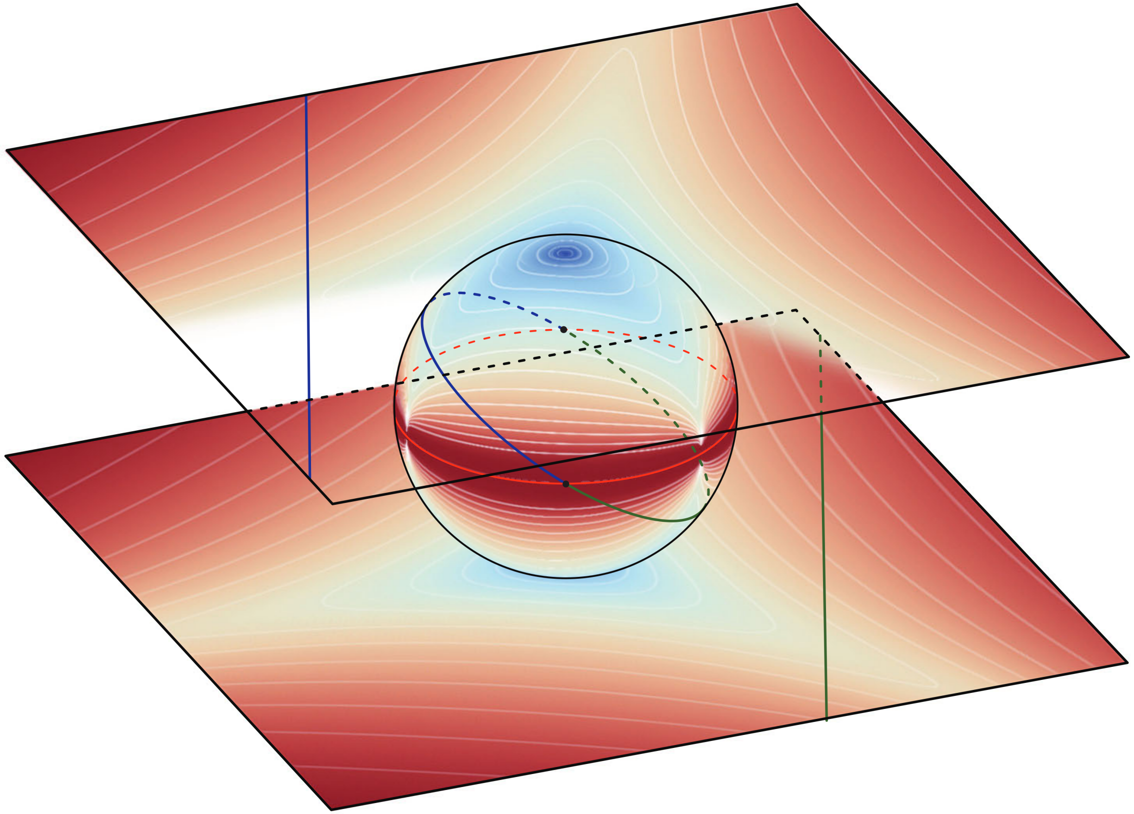


- Considering two shape planes with opposite orientations
- Compactifing phase-spaces via gnomonic projection on a 2-sphere
- Each shape plane is projected into an hemisphere

$$0 \leq \beta < \frac{\pi}{2},\ \frac{\pi}{2} < \beta \leq \pi$$

Phase-space compactification

- Kasner solutions are projected into great circles on the 2-sphere
- BB singularity $\beta \to \pm\frac{\pi}{2}$



Continuation through the BB

Compactifing all physical DOFs y_i, the EOMs satisfy the Picard-Lindelöf Theorem

$$\frac{dy_i}{d\beta} = f_i(y)$$

$f_i(y)$ are differentiable functions (a stronger property than the Lipschitz-continuity required by the theorem)

Each BIX solution on one side of the singularity is associated with **one and only one** BIX solution on the other side of the singularity

Extension of the model to gauge-theories

Is the orientation change **physical**? To detect an orientation change we need **matter fields**

GR minimally coupled with:

- U(1)-gauge fields
- SU(2)-gauge fields
- a 1-component SU(3)-gauge field

$\longrightarrow$

- Quiescence is preserved
- EOMs are still well-behaved at the singularity
- Gauge-fields do not flip their orientation

EOMs, describing **gravity minimally coupled with stiff-matter and with YM fields**, satify a theorem of existence and uniqueness at the BB singularity of a BIX universe: each solution passes through the singularity **without loss of informations**

YM-fields detect the orientation change

Future perspectives and some references

- Generalization to a generic SU(3)-YM field
- Adding fermionic fields
- Including inhomogeneities as perturbations about homogeneous terms

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