κ-FIELDS, κ-SYMMETRIES, AND κ-STATES

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WHAT I'M GOING TO TELL YOU ABOUT

- Free κ-deformed field theory, its continuous symmetries and conserved charges.
- κ-deformation of discrete symmetries and its phenomenological consequences.
- Multiparticle states in κ-deformed QFT.

- Based on works with M. Arzano, A. Bevilacqua, G. Rosati, J. Unger, W. Wislicki,
  - *κ*-deformed complex fields and discrete symmetries, *Phys.Rev.D* 103 (2021) 10, 106015 e-Print: 2011.09188 [hep-th]
  - κ-deformed complex scalar field: Conserved charges, symmetries, and their impact on physical observables, *Phys.Rev.D* 105 (2022) 10, 105004 e-Print: 2201.10191 [hep-th]
  - A group theoretic description of the *κ*-Poincaré Hopf algebra, e-Print: 2204.09394 [hep-th]
  - Multi-particle states with group-valued momenta: an identity crisis? To appear soon.

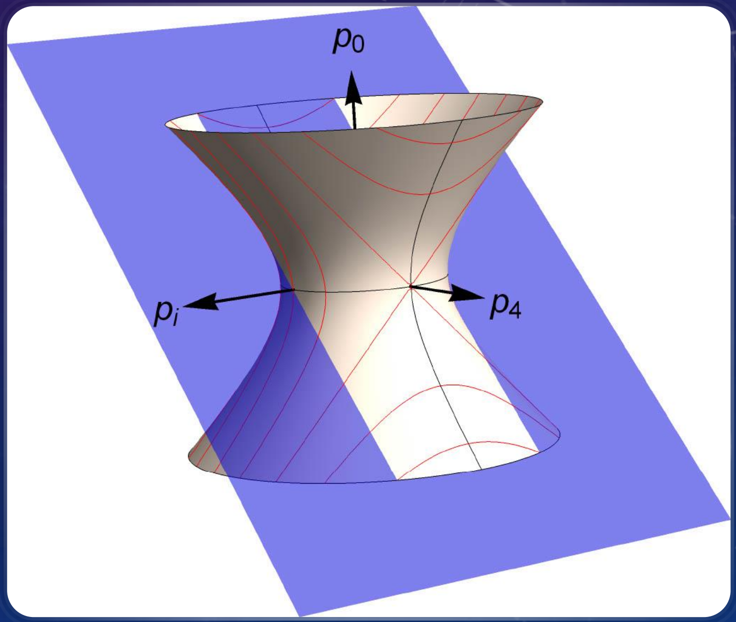
THE AN(3) ALGEBRA AND GROUP

- In κ-Poincaré the momentum space is a group manifold of the group AN(3), whose elements have the form

$$g = e^{ip_i X^i} e^{ip_0 X^0}$$

$$[X^0, X^i] = \frac{i}{\kappa} X^i\,, \quad [X^i, X^j] = 0\,; \quad i,j = 1, \ldots, 3$$

- The coefficients p_i , p_0 are interpreted as momenta.
- The group manifold is, geometrically, a half of de Sitter space.



FIELD THEORY

- In application to field theory, we regard the group element *g* as a plane wave, intertwining between curved momentum space and non-commutative (κ-Minkowski) spacetime. Instead of working in non-commutative spacetime we choose to use the standard commutative Minkowski space with star product, so that

$$g = e^{ip_i X^i} e^{ip_0 X^0} \to e^{ip_\mu x^\mu}$$

$$e^{ip_\mu x^\mu} \star e^{iq_\mu x^\mu} = e^{i(p\oplus q)_\mu x^\mu}$$

$$(p \oplus q)_0 = p_0 + q_0 \, , \quad (p \oplus q)_i = p_i + e^{-p_0/\kappa} \, q_0$$

$$S(p)_0 \equiv \ominus p_0 = -p_0 \, , \quad S(p)_i = \ominus p_i = -p_i e^{p_0/\kappa}$$

SCALAR FIELD AND ACTION

- The on-shell field

$$\phi(x) = \int \frac{d^3p}{\sqrt{2\omega_{\mathbf{p}}}}\, \zeta(p)\, a_{\mathbf{p}}\, e^{-i(\omega_{\mathbf{p}}t - \mathbf{px})} + \int \frac{d^3p}{\sqrt{2\omega_{\mathbf{p}}}}\, \zeta(p) b^\dagger_{\mathbf{p}}\, e^{-i(S(\omega_{\mathbf{p}})t - S(\mathbf{p})\mathbf{x})}$$

$$\phi^\dagger(x) = \int \frac{d^3p}{\sqrt{2\omega_{\mathbf{p}}}}\, \zeta(p)\, a^\dagger_{\mathbf{p}}\, e^{-i(S(\omega_{\mathbf{p}})t - S(\mathbf{p})\mathbf{x})} + \int \frac{d^3p}{\sqrt{2\omega_{\mathbf{p}}}}\, \zeta(p) b_{\mathbf{p}}\, e^{-i(\omega_{\mathbf{p}}t - \mathbf{p})\mathbf{x}}$$

- The action

$$\begin{aligned} S &= -\frac{1}{2}\int_{\mathbb{R}^4} d^4x\, \left[(\partial_\mu\phi)^\dagger \star \partial^\mu\phi + (\partial_\mu\phi) \star (\partial^\mu\phi)^\dagger + m^2(\phi^\dagger \star \phi + \phi \star \phi^\dagger)\right] \\ &= \frac{1}{2}\int \frac{d^3p}{2\omega_{\mathbf{p}}}\, \zeta(p)^2 \left(1 + \frac{|p_+|^3}{\kappa^3}\right)(p_\mu p^\mu + m^2)\, \left[a^\dagger_{\mathbf{p}} a_{\mathbf{p}} + b^\dagger_{\mathbf{p}} b_{\mathbf{p}}\right] \end{aligned}$$

SYMPLECTIC STRUCTURE AND POISSON BRACKET

- Having the action, we can compute the symplectic form

$$\Omega = -i \int d^3p \, \alpha(p) \left(\delta a^\dagger_{\mathbf{p}} \wedge \delta a_{\mathbf{p}} - \delta b_{\mathbf{p}} \wedge \delta b^\dagger_{\mathbf{p}} \right)$$

- Setting α =1 we have the Poisson brackets/commutators

$$[\hat{a}_{\mathbf{p}}, \hat{a}^\dagger_{\mathbf{q}}] = \delta^3(\mathbf{p} - \mathbf{q}),$$
$$[\hat{b}_{\mathbf{p}}, \hat{b}^\dagger_{\mathbf{q}}] = \delta^3(\mathbf{p} - \mathbf{q}).$$

CONTINUOUS SYMMETRIES CHARGES

- There is a straightforward construction of conserved Noether charges from the symplectic structure, called the "covariant phase space method". We use it to compute the charges associated with ten (κ) Poincaré symmetries.

$$\mathscr{P}_\mu = \int d^3p\, [-S(p)_\mu a^\dagger_{\mathbf{p}} a_{\mathbf{p}} + p_\mu b^\dagger_{\mathbf{p}} b_{\mathbf{p}}]$$

$$\mathscr{N}_i = -\frac{1}{2}\int d^3p \left\{ S(\omega_p)\left[\frac{\partial a^\dagger_{\mathbf{p}}}{\partial S(\mathbf{p})^i} a_{\mathbf{p}} - a^\dagger_{\mathbf{p}} \frac{\partial a_{\mathbf{p}}}{\partial S(\mathbf{p})^i}\right] + \omega_p \left[b_{\mathbf{p}} \frac{\partial b^\dagger_{\mathbf{p}}}{\partial \mathbf{p}^i} - \frac{\partial b_{\mathbf{p}}}{\partial \mathbf{p}^i} b^\dagger_{\mathbf{p}}\right]\right\}$$

$$\mathscr{M}_i = -\epsilon_i{}^{jk}\frac{1}{8}\int d^3q \left(S(\mathbf{q})_j \frac{\partial a^\dagger_{\mathbf{q}}}{\partial S(\mathbf{q})^k} a_{\mathbf{q}} - a^\dagger_{\mathbf{q}} S(\mathbf{q})_j \frac{\partial a_{\mathbf{q}}}{\partial S(\mathbf{q})^k} + b_{\mathbf{q}} \mathbf{q}_j \frac{\partial b^\dagger_{\mathbf{q}}}{\partial \mathbf{q}^k} - \mathbf{q}_j \frac{\partial b_{\mathbf{q}}}{\partial \mathbf{q}^k} b^\dagger_{\mathbf{q}} \right)$$

- The Poisson brackets of the charges form a representation of the standard Poincaré algebra.
- The charges are generators of the infinitesimal translations/boosts/rotations.

DISCRETE SYMMETRIES GENERATORS

- Of three discrete symmetries the most interesting is charge conjugation. The associated operators transforms a (one-) particle state into the (one-) antiparticle state.

$$\mathscr{C} = \int d^3p \, (b^\dagger_{\mathbf{p}} a_{\mathbf{p}} + a^\dagger_{\mathbf{p}} b_{\mathbf{p}})$$

$$\mathscr{C} a^\dagger |0\rangle = b^\dagger |0\rangle \qquad \mathscr{C} b^\dagger |0\rangle = a^\dagger |0\rangle$$

- It is remarkable, however that the charge generator **does not** commute with the boost generator

$$[\mathscr{N}_j, \mathscr{C}] \neq 0$$

- This leads to non-trivial CPT deformation (violation).

ASIDE: JOST—GREENBERG THEOREM

- There is a famous Jost—Greenberg theorem that states that there is a one-two-one correspondence between Poincaré symmetry and CPT symmetry (so that the violation of CPT inevitably leads to Poincaré symmetry violation, PIV).
- κ- Poincaré is an apparent counterexample to this theorem. The reason is that the Jost-Greenberg theorem makes use of some properties of Wightman functions that are no longer true in deformed case.

PHENOMENOLOGY

- Particles and anti-particles at rest have the same mass.
- But this changes when the (anti-) particle is boosted. If we boost them with rapidity ξ, they will have energy/momentum

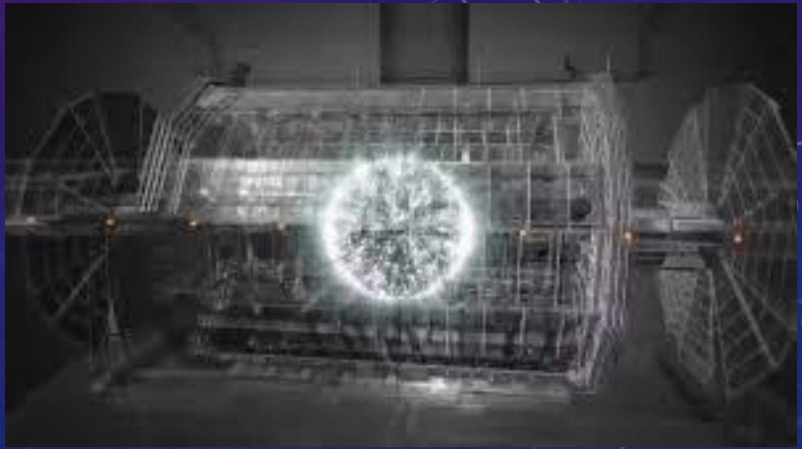
$$\mathscr{P}_1|M\cosh\xi, M\sinh\xi, 0, 0\rangle_a = -S(M\sinh\xi)|M\cosh\xi, M\sinh\xi, 0, 0\rangle_a$$
$$\mathscr{P}_0|M\cosh\xi, M\sinh\xi, 0, 0\rangle_a = -S(M\cosh\xi)|M\cosh\xi, M\sinh\xi, 0, 0\rangle_a$$

$$\mathscr{P}_1|M\cosh\xi, M\sinh\xi, 0, 0\rangle_b = M\sinh\xi|M\cosh\xi, M\sinh\xi, 0, 0\rangle_b$$
$$\mathscr{P}_0|M\cosh\xi, M\sinh\xi, 0, 0\rangle_b = M\cosh\xi|M\cosh\xi, M\sinh\xi, 0, 0\rangle_b$$

- The difference between energies is of order of p^2/κ

PHENOMENOLOGY

- This translates into different decay times of moving unstable particles and antiparticles.
- The most promising are muons, for which the current data makes it possible to constrain $\kappa \geq 10^{14}$ GeV for LHC, with a couple of orders of magnitude improvement in future collider. It might be further improved in some dedicated machines.
- We are also investigating the oscillations in Kaon—Anti Kaon systems, which might be detectable at LHCb.



TO DO FIELD THEORY

- Higher spins (1/2 seems straightforward, 1 might be problematic);
- Interactions;
- Deformed Standard Model;
- Loops.

THE MULTIPARTICLE STATES

PARTICLE EXCHANGE

- The reason why two particle state has to be symmetrized is that when we exchange the particles, the state cannot change since the particles are identical. This is a purely quantum effect. ***If two particles are indistinguishable swapping the factors in the tensor product describing their state should lead to another state which is indistinguishable from the original one.***
- Therefore, in (standard) QFT the two particle states is NOT

$$|k\rangle \otimes |l\rangle$$

- but rather

$$\frac{1}{\sqrt{2}}\left(|k\rangle \otimes |l\rangle + |l\rangle \otimes |k\rangle\right)$$

- Properties:
  1. It is an eigenstate of the momentum operator, with eigenvalue k+l=l+k;
  2. It is Lorentz-covariant;
  3. The two-particle state is composed by tensoring the on-shell one-particle states (in different orders).

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DEFORMED TWO-PARTICLE STATES

- We denote the one particle state by AN(3) group elements $|g\rangle$
- Tensoring them we obtain

$$|g\rangle \otimes |h\rangle$$

- One checks using the coproduct rule that the total momentum of such state is

$$\mathscr{P}\,|g\rangle \otimes |h\rangle = \mathscr{P}\,|gh\rangle = P(gh)$$

DEFORMED TWO-PARTICLE STATES

- If one just swaps the states

$$|g\rangle \otimes |h\rangle \to |h\rangle \otimes |g\rangle$$

- The new state is a momentum eigenstate with different momentum. NOT GOOD.
- Instead, we can try a different swap rule

$$|g\rangle \otimes |h\rangle \to |ghg^{-1}\rangle \otimes |g\rangle$$

- Obviously, both states have identical total momentum. It can be also checked that they transform identically under Lorentz transformation.

HOWEVER …

- There are infinitely many swaps one can construct, with the same total momentum and with correct Lorentz transformations

$$|g\rangle \otimes |h\rangle \to |ghg^{-1}\rangle \otimes |g\rangle \to |(gh)g(gh)^{-1}\rangle \otimes |ghg^{-1}\rangle \to \dots$$

- This makes it impossible to construct Fock space, because it seems ~~impossible~~ hard to make sense of the state

$$\psi = \frac{1}{\infty}\left(|g\rangle \otimes |h\rangle + |ghg^{-1}\rangle \otimes |g\rangle + |(gh)g(gh)^{-1}\rangle \otimes |ghg^{-1}\rangle + \dots\right)$$

MOREOVER (FOR BETTER OR WORSE)

- One can check that if $|g\rangle$, $|h\rangle$ are on-shell states with the same mass, the state $|ghg^{-1}\rangle$ is on-shell with the same mass iff the states $|g\rangle$, $|h\rangle$ can be simultaneously put to rest by Lorentz transformation. This means that if the swapped states are to describe the same particles as the original one, the state is to be perfectly stiff.
- Of course, all these problems disappear when deformation goes away.

SPECULATIONS

- It can be claimed (but yet not proved) that in the case of κ deformation (except, possibly, lightlike one) the Fock space construction is not possible. The emergence of Fock space is the just a low energy approximation.
- There are infinitely many multiparticle states, consisting of stiffly (topologically?) connected particles. The physical consequences of this state of affairs is not clear.
- If we take this message seriously, the semiclassical limit of QG cannot be an effective quantum field theory.