Quantum gravitational decoherence from minimal length scale

Luciano Petruzziello

Università degli Studi di Salerno, Dipartimento di Fisica “E. R. Caianiello”
INFN Sezione di Napoli, Gruppo collegato di Salerno

L. P. and F. Illuminati, Nature Commun. **12**, 4449 (2021)

Outline

- Generalized Uncertainty Principle
- Lindblad-type master equation
- Experimental possibilities
- Conclusions

Generalized Uncertainty Principle (GUP)

Many theories of Quantum Gravity predict the existence of a minimal length scale that modifies the standard Heisenberg Uncertainty Principle (HUP)

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

In string theory, several works on superstring collisions* suggested that the presence of gravity should modify HUP according to ($c = 1$)

$$\Delta x \geq \frac{\hbar}{2\Delta p} \pm |\beta| \ell_p^2 \frac{\Delta p}{\hbar^2}$$

*D. Amati, M. Ciafaloni and G. Veneziano, Phys. Lett. B (1987); D. J. Gross and P. F. Mende, Phys. Lett. B (1987)

GUP

A similar outcome can be achieved also in the context of gedanken experiments on large* and micro† black holes

The previous uncertainty relation can also be cast in terms of a deformed commutator‡

$$[x,p] = i\hbar\left[1 \pm |\beta|\ell_p^2\frac{p^2}{\hbar^2}\right]$$

Typically, in some models of string theory the deformation parameter is assumed to be $|\beta| \sim \mathcal{O}(1)$

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*M. Maggiore, Phys. Lett. B (1993)

†F. Scardigli, Phys. Lett. B (1999)

‡A. Kempf, G. Mangano and R. B. Mann, Phys. Rev. D (1995)

What's new?

- Minimal uncertainty in position (for $\beta > 0$)

$$\Delta x_{\mathrm{min}} \propto \hbar\sqrt{\beta}$$

- Spatial non-commutativity in higher dimensions

$$\left[x_i, x_j\right] \propto \mathcal{O}\left(\beta^2 \ell_p^4 \frac{p^4}{\hbar^4}\right)$$

- Non-orthogonality of position eigenstates

$$\langle\psi_{\lambda'}|\psi_{\lambda}\rangle \propto \frac{\hbar\sqrt{|\beta|}}{(\lambda-\lambda')} \sin\left[\frac{(\lambda-\lambda')}{\hbar\sqrt{|\beta|}}\right]$$

What happens to standard Quantum Mechanics?

A similar modification of the uncertainty relations necessarily entails a change in the standard quantum mechanical formalism

$$\left[x_i, p_j\right] = i\hbar \left(\delta_{ij} \pm |\beta| \ell_p^2 \frac{\mathbf{p}^2}{\hbar^2} \delta_{ij} \pm 2|\beta| \ell_p^2 \frac{p_i p_j}{\hbar^2} \right)$$

$$x_i = x_i^{(0)} \qquad p_i = p_i^{(0)} \left[1 \pm |\beta| \ell_p^2 \frac{(\mathbf{p}^{(0)})^2}{\hbar^2} \right]$$

$$H = H_0 + H_\beta \qquad H_\beta = \pm 4 \frac{|\beta| m \, \ell_p^2}{\hbar^2} H_0^2 \left(1 \pm \frac{|\beta| m \, \ell_p^2}{\hbar^2} H_0 \right)$$

The nature of β

The parameter β has always been regarded as a constant of either positive or negative sign

But what if we embed the notion of foamy spacetime* in the above framework and relate spacetime fluctuations with the deformation parameter?

$$\beta = \sqrt{t_p}\,\chi(t) \qquad \langle \chi(t) \rangle = \bar{\beta} = \frac{1}{\sqrt{t_p}} \qquad \langle \chi(t)\,\chi(t') \rangle = \delta(t - t')$$

We can investigate the consequences of this ansatz

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*J. A. Wheeler, Phys. Rev. (1955); V. Vasileiou *et al.*, Nat. Phys. (2015); C. Rovelli and L. Smolin, Phys. Rev. D (1995)

Liouville-von Neumann equation

As a preliminary step, one can look at the Liouville-von Neumann equation and try to solve it in the presence of the stochastic β parameter

$$\partial_t \varrho = -\frac{i}{\hbar}\left[H_0 + H_\beta, \varrho\right]$$

In the interaction picture (i.e. $\tilde{\mathcal{O}} = e^{iH_0 t/\hbar}\, \mathcal{O}\, e^{-iH_0 t/\hbar}$), a formal solution is represented by

$$\partial_t \tilde{\varrho}(t) = -\frac{i}{\hbar}\left[\tilde{H}_\beta(t), \tilde{\varrho}(0)\right] - \frac{1}{\hbar^2}\int_0^t \left[\tilde{H}_\beta(t), \left[\tilde{H}_\beta(t'), \tilde{\varrho}(t)\right]\right] dt'$$

Lindblad master equation

By averaging over fluctuations we are left with

$$\partial_t \tilde{\rho}(t) = -\sigma \left[\tilde{H}_0^2(t), \left[\tilde{H}_0^2(t), \tilde{\rho}(t) \right] \right] \qquad \sigma = \frac{16\, m^2\, \ell_p^4\, t_p}{\hbar^6}$$

By going back in the Schrödinger picture we then have a Lindblad-type master equation*

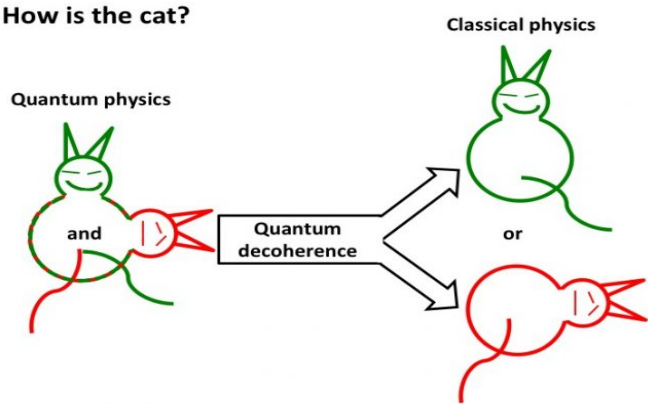
$$\partial_t \rho(t) = -\frac{i}{\hbar} \left[H_0, \rho(t) \right] - \sigma \left[H_0^2, \left[H_0^2, \rho(t) \right] \right]$$

This entails interesting consequences

*G. Lindblad, Commun. Math. Phys. (1976); V. Gorini, A. Kossakowski and E. Sudarshan, J. Math. Phys. (1976)

Quantum decoherence

The previous master equation is the starting point to analyze the quantum-to-classical transition, aka quantum decoherence



Momentum space solution

In momentum space, we can evaluate the density matrix elements as

$$\rho_{p,p'}(t) = \langle \mathbf{p} | \rho(t) | \mathbf{p}' \rangle$$

For a free system with $E(p) = p^2/2m$, the equation to solve is given by

$$\partial_t \rho_{p,p'} = \left[-\frac{i}{\hbar}\big(E(p) - E(p')\big) - \sigma \left(E^2(p) - E^2(p') \right)^2 \right] \rho_{p,p'}$$

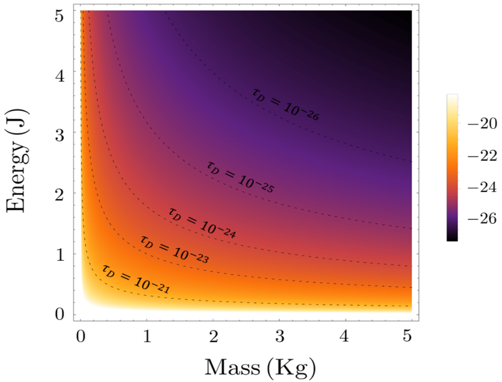
Provided $\Delta E^2 = E^2(p) - E^2(p')$, the solution is

$$\rho_{p,p'}(t) = \exp \left[-\frac{i\big(E(p) - E(p')\big)t}{\hbar} - \sigma \left(\Delta E^2 \right)^2 t \right] \rho_{p,p'}(0)$$

Decoherence time

From the previous equation, the decoherence time can be inferred

$$\tau_{\mathrm{D}} = \frac{1}{\sigma\,(\Delta E^2)^2} = \frac{\hbar^6}{16\, m^2\, \ell_{\mathrm{p}}^4\, t_{\mathrm{p}}\, (\Delta E^2)^2}\,.$$



Examples

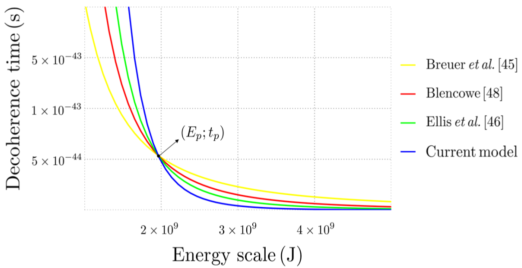
TABLE I: Values of τ_D for different physical systems

Physical system	**Energy scale (J)**	**Mass (Kg)**	**Value of τ_D (s)**
Slow car	2×10^3	900	2.749×10^{-42}
Thrown tennis ball	22.5	0.05	3.649×10^{-26}
Cosmic dust	3.125×10^{-1}	10^{-9}	2.452×10^{-3}
Benzene molecule	4.981×10^{-17}	1.296×10^{-25}	2.263×10^{92}
Neutron interferometer	4.053×10^{-21}	1.675×10^{-27}	3.086×10^{112}
Down quark at the quark epoch	3.134×10^{-14}	8.592×10^{-30}	3.283×10^{89}

Comparison

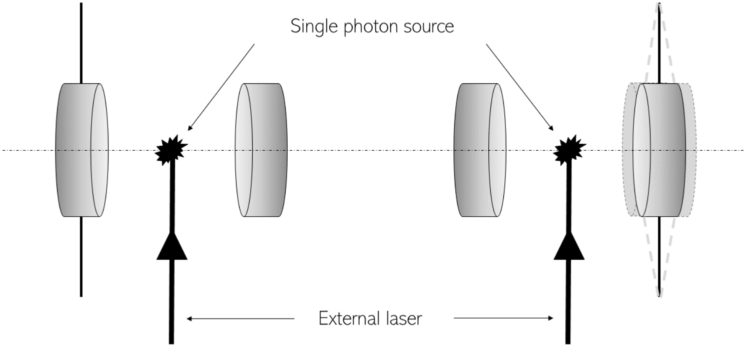
TABLE II: Comparison between different gravitational decoherence models

Reference	Physical source of decoherence	Decoherence time
Breuer, Goklu and Lammerzahl [38]	Perturbation around flat spacetime	$\frac{\hbar E_{\mathrm{p}}}{E^2}$
Ellis, Mohanty and Nanopoulos [39]	Classical gravity near a wormhole	$\frac{\hbar E_{\mathrm{p}}^3}{E^4}$
Blencowe [41]	Thermal background of gravitons	$\frac{1}{k_{\mathrm{B}}T}\frac{\hbar E_{\mathrm{p}}^2}{E^2}$
Anastopoulos and Hu [42]	Linearized gravity with thermal noise Θ	$\frac{1}{k_{\mathrm{B}}\Theta}\frac{\hbar E_{\mathrm{p}}^2}{E^2}$
Current model	Fluctuating minimal length and deformation parameter	$\frac{\hbar E_{\mathrm{p}}^5}{E^6}$



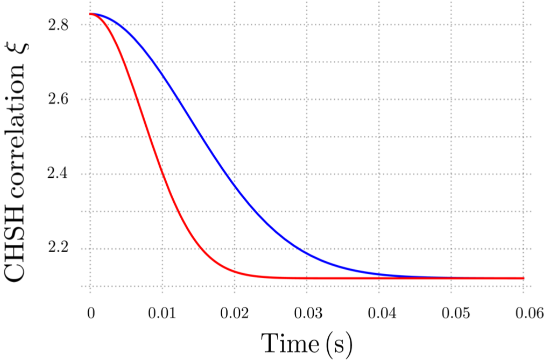
Experimental setup for the verification

Remarkably, there exists an experimental test capable of detecting the present quantum gravitational decoherence mechanism



The experimental evidence

The verification amounts to a CHSH experiment*



*C. Pfister, J. Kaniewski, M. Tomamichel, A. Mantri, R. Schmucker, N. McMahon, G. Milburn and S. Wehner, Nature Commun. (2016)

A tool to discriminate different decoherence mechanisms

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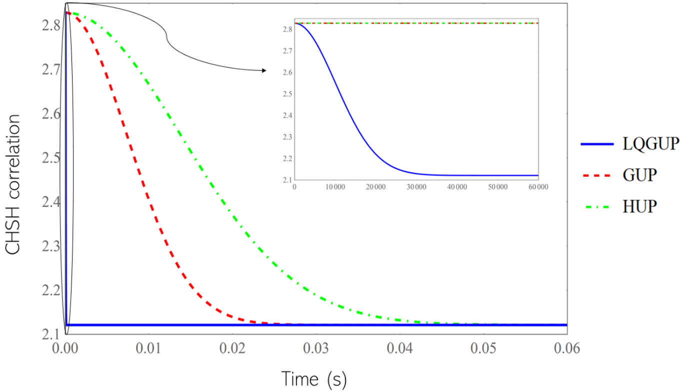
$$[x_i, p_j] = i\hbar\left[\delta_{ij} - \alpha\left(|\mathbf{p}|\delta_{ij} + \frac{p_i p_j}{|\mathbf{p}|}\right) + \alpha^2(\mathbf{p}^2\delta_{ij} + 3p_i p_j)\right]$$



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*E. Al-Nasrallah, S. Das, F. Illuminati, L. P. and E. C. Vagenas, arXiv:2110.10288 [gr-qc] (2021)

Conclusions and future perspectives

- The verification of the effect described so far would provide both a check of the existence of quantum gravity and a universal decoherence mechanism
- Energy localization can be accompanied by spatial localization if extended uncertainty principle is accounted for (work in progress)
- Higher-order generalizations of GUP can help to improve the development of the theory as well as the search for the experimental window where to investigate


THANK
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