Entanglement of superpositions of quantum gravity states

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Outline

1. The quest of Quantum Gravity

2. GFT Spin networks

3. Entanglement in QG

5. Superposition of Spin network states

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1. The quest of Quantum Gravity

$G_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu}$

General relativity that describes Gravity and large scale physics.

Quantum field theory: describes the other three forces governing the universe.

Black hole Physics:

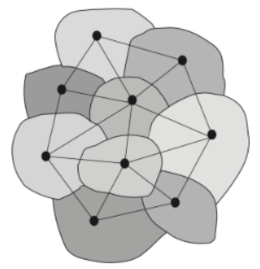
- are solution to Einstein equation —> Classical objects.
- Evaporate and can be described by quantum states.

Find the states that capture the geometry of spacetime at the quantum level.

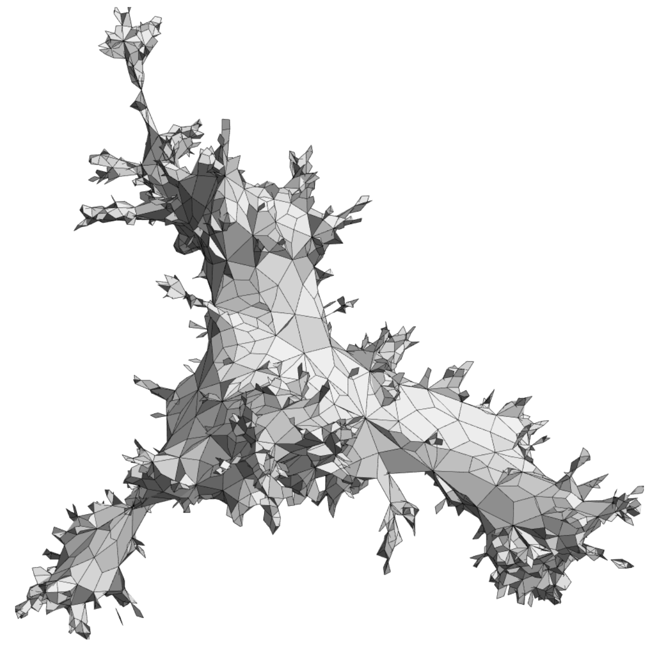
How to reconcile the lessons from **QM** and those of **GR**?
—> **Quantum gravity**

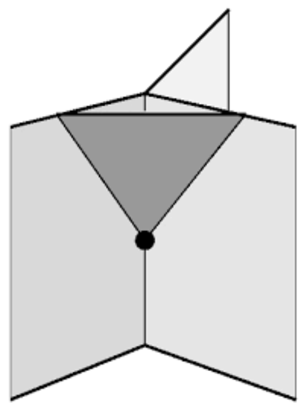
And many other un anserwerd question …

❖Some models apply quantization schemes directly to the full spacetime geometry (GR):

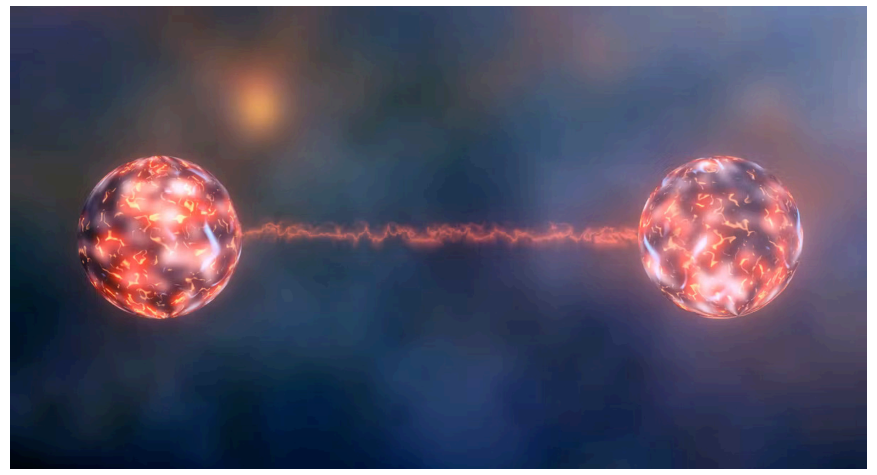


- Canonical loop quantum gravity.
- Path integral formulations of quantum GR and their modern evolution mostly based on lattice structures (e.g. causal dynamical triangulations).
- Group field theories.



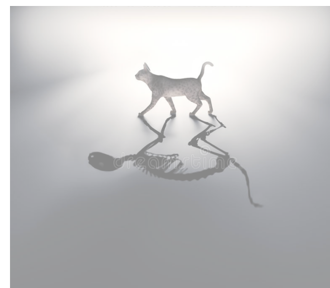


Entanglement and emergence of space time!



❖There are several motivation behind using the notion of entanglement in the QG setting:

- In background independent approach: it can deliver more insight on defining distance in terms of correlations/ entanglement between regions of quantum space.
- Defines a procedure to glue discrete geometric surfaces together (recover continuum).
- Calculations relating geometric quantities to entanglement measures: LQG, ADS/CFT.. —> Black hole physics and holography: **opens windows to QG phenomenology!**



2. GFT Spin network states

◉ Group Field Theories are QFT formulations of the dynamics of fundamental building blocks of space:

A) are endowed with an algebraic data,
B) can be interpreted as quantized tetrahedra,
C) can be glued together to form an extended dimensional triangulations.

A) The elementary portion of space is interpreting as a quantum (d–1)-simplex dual to a d-valent vertex, whose edges are decorated by elements of a group G (SU(2)).

- A **single** atom of space is given by the single vertex WF:

$$\psi(\vec{g}) = \sum_{\vec{j}\vec{n}\iota} \psi^{\vec{j}\iota}_{\vec{n}} S_{\vec{j}\vec{n}\iota}(\vec{g}),$$

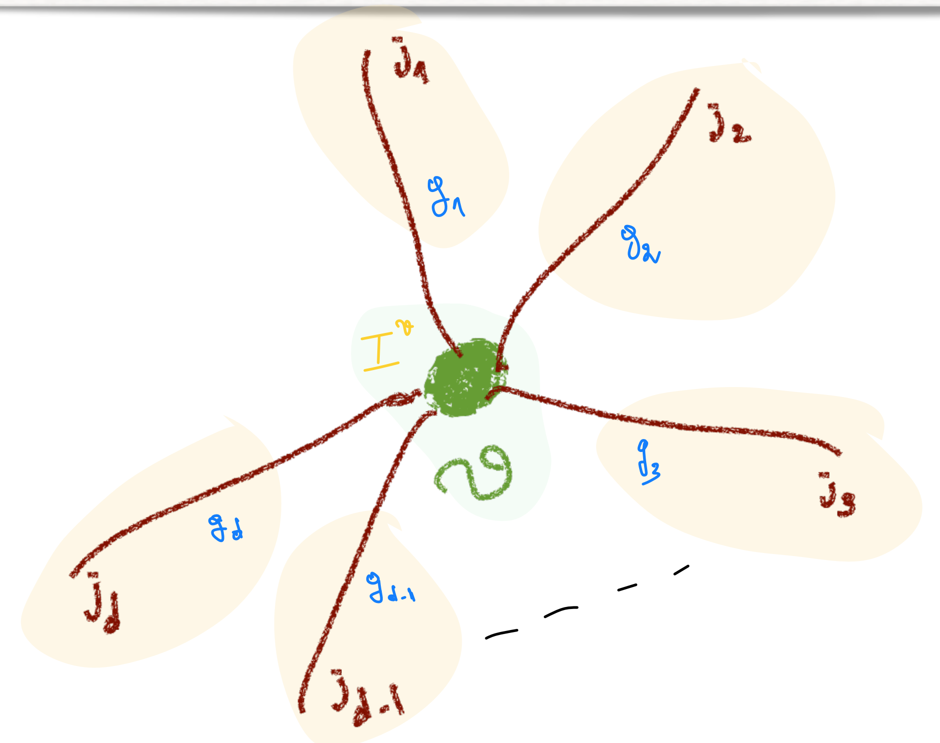
- Living on: $\mathscr{H} = L^2\left(G^d/G\right)$.

Spin network basis: $S_{\vec{j}\vec{n}\iota}(\vec{g}) = \sum_{\vec{m}} I^{\vec{j}\iota}_{\vec{m}} \prod_i \sqrt{d_{j_i}} D^{j_i}_{m_i n_i}(g_i),$

- **To each edge** $e \in \Gamma$: associate a $\mathrm{SU}(2)$ spin representation labeled by a half-integer $j_e \in \mathbb{N}/2$.

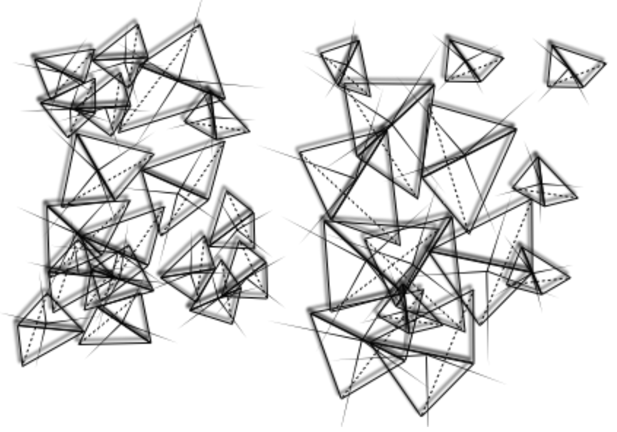
- **To each vertex** v we attach an intertwiner $\mathscr{I}_v$: $\mathrm{SU}(2)$-invariant map between the representation spaces V^{j_e} associated to all the edges e meeting at the vertex v:

$$\mathscr{I}_v : \bigotimes_{e \text{ ingoing}} V_{j_e} \to \bigotimes_{e \text{ outgoing}} V_{j_e}.$$

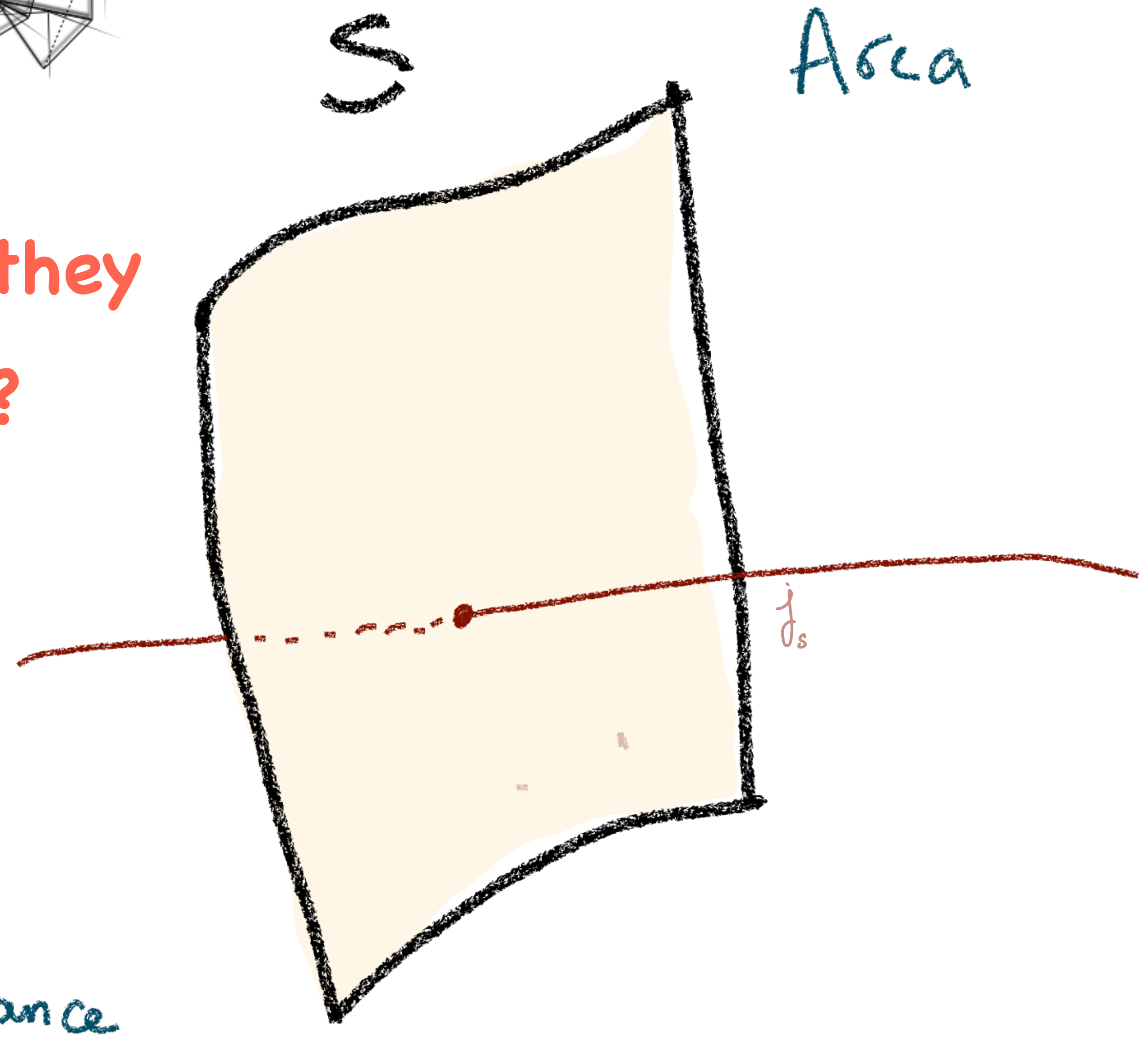


N distinguishable vertices, the corresponding GFT Hilbert space is nothing than the tensor product of all the single-vertex Hilbert spaces:

$$L^2\left(G^{d\times V}/G^V\right) := \underbrace{\mathscr{H}\otimes\ldots\otimes\mathscr{H}}_{N},$$



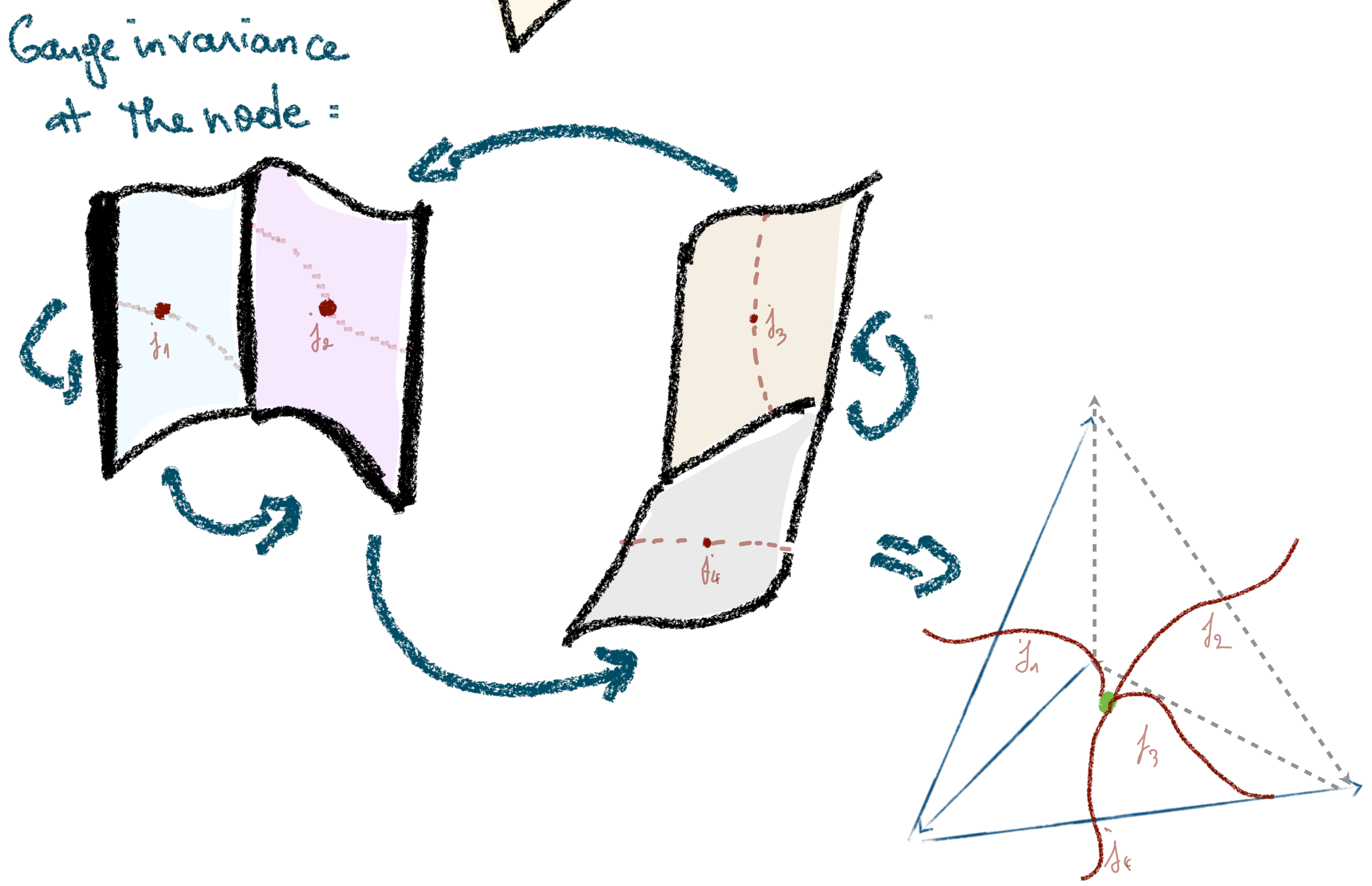
Important results of LQG is that area and volume have discrete spectrum



But why are they so special?

Spin network graphs are understood as collections of individual vertices (representing building blocks of space, where the "particles" picture thereof is described by the GFT Fock space).

B) They can be interpreted as quantized tetrahedra:



C) Can be glued together to form an extended dimensional triangulations. How?

3. Entanglement in QG

Entanglement measures

Entanglement measure is an entity that aims to quantify the quantum correlations within a bipartite system. It should satisfy certain axioms.

—> **Von Neumann Entropy**

QG states entanglement: Quantum information techniques.

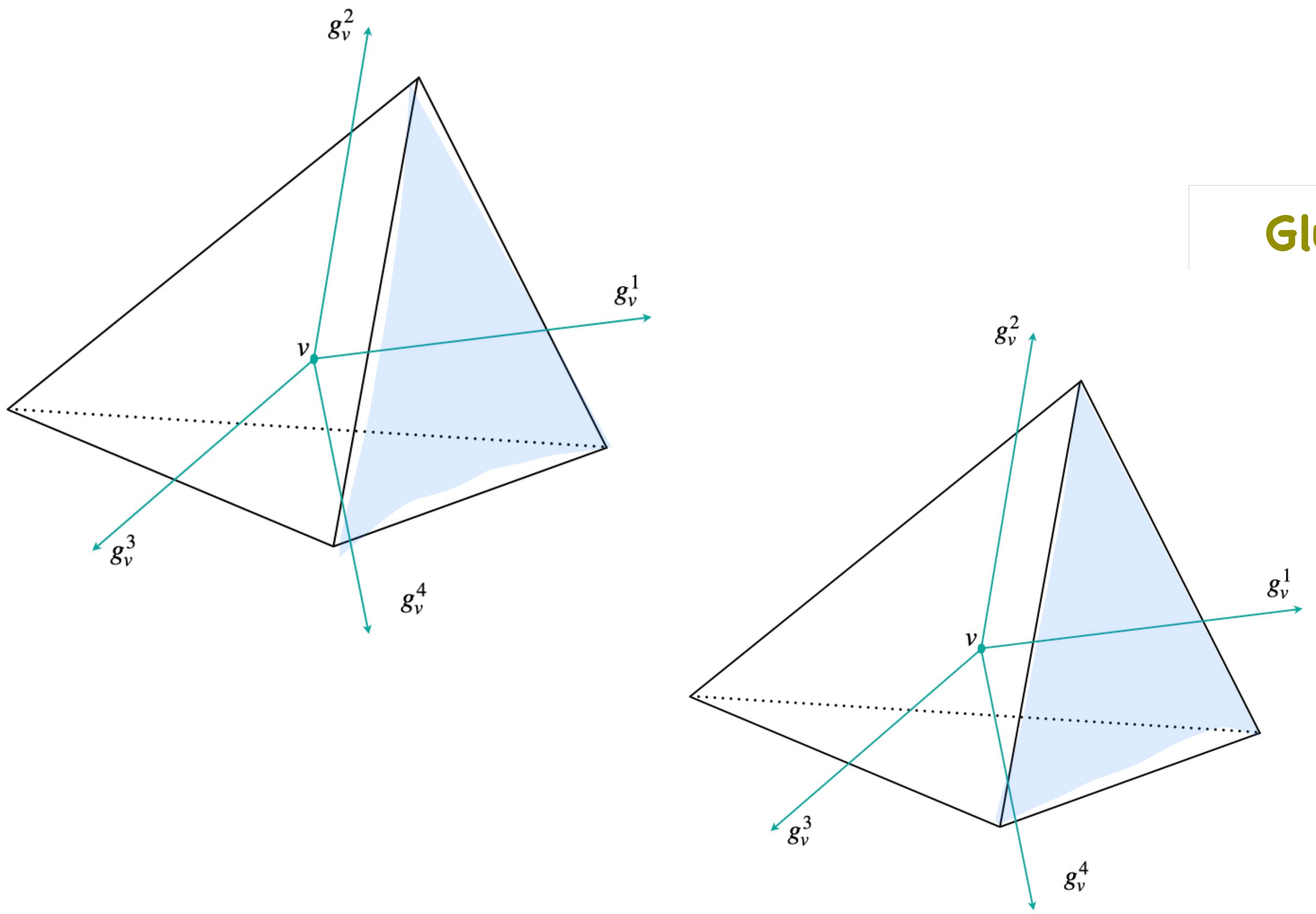
- A bipartite state is called entangled if it is not a product state: $|\psi\rangle \neq \phi_A \phi_B$.
- The entanglement between A and B as the Von Neumann entropy of the reduced density matrices:

$$\rho = |\psi\rangle\langle\psi| \,, \quad \rho_A = \mathrm{Tr}_B\, \rho, \quad \rho_B = \mathrm{Tr}_A\, \rho,$$
$$S(A \mid B) = -\,\mathrm{Tr}\, \rho_A \ln \rho_A = -\,\mathrm{Tr}\, \rho_B \ln \rho_B \,.$$

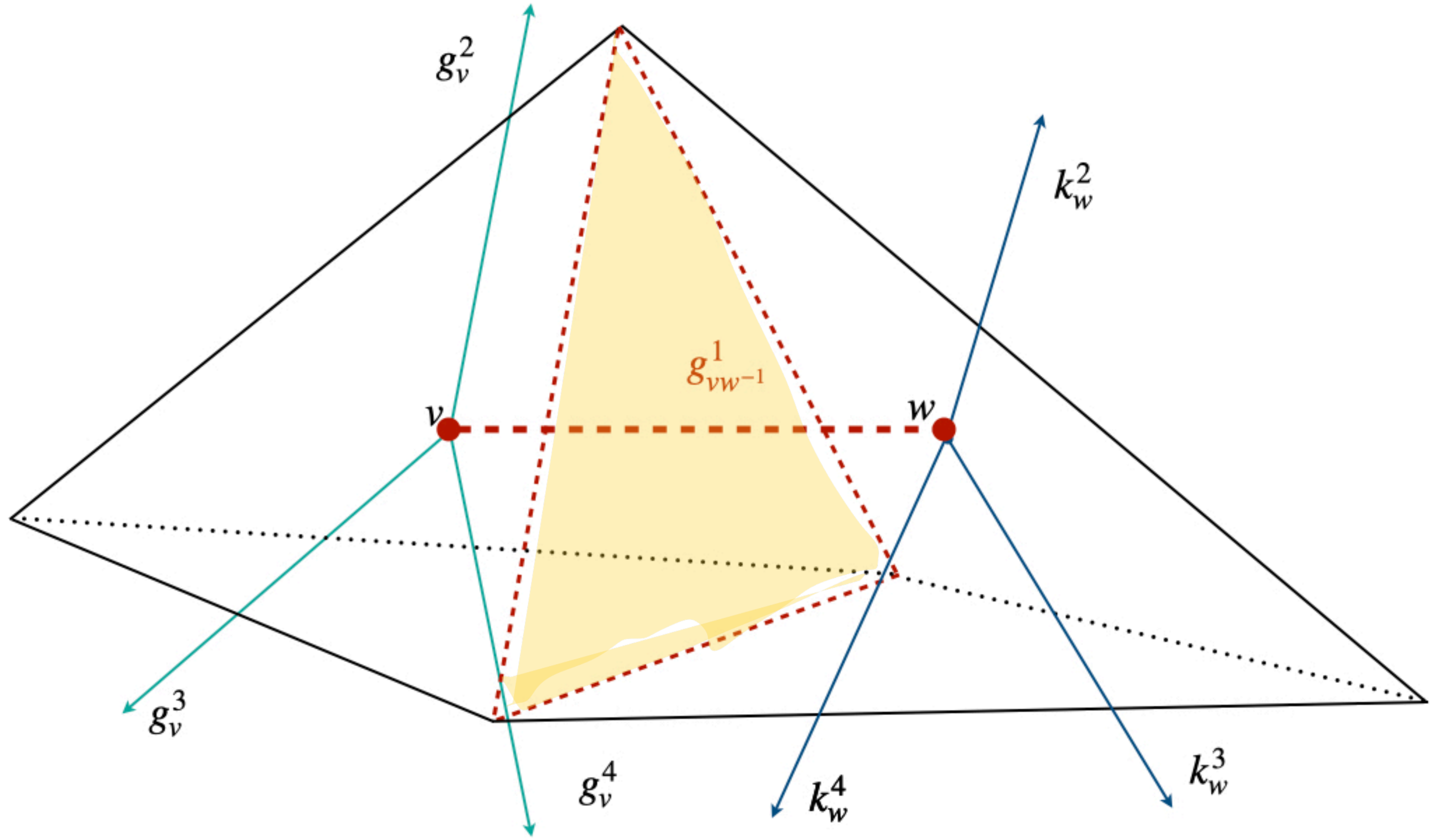
But how do we know if Spin network states are entangled ??

Entanglement in GFT

How do we entangle such states?



Gluing procedure



Gluing link-state:

$$|l^i_{vw}\rangle := \bigoplus_j \frac{1}{\sqrt{d_j}} \sum_n (1)^{j+m} |jn\rangle \otimes |j-n\rangle$$

Link dual to a face.
Fusing links is equivalent to identifying common faces of tetrahedra.

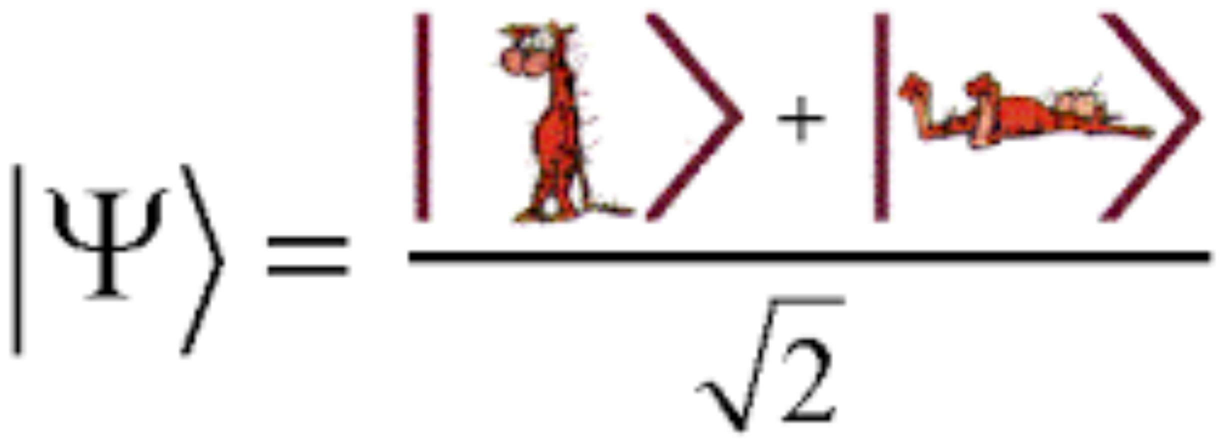
$$S_A(\rho) = S_B(\rho) = \log(2j+1)\,.$$

dim of V^j

What do we learn from this procedure?

- One can detect a Pattern of connectivity within the entangled network. (Adjacency matrix).
- The emergence of spacetime from something more fundamental than spatio-temporal entities.

However, this is the simplest case of single states of spin network graphs!



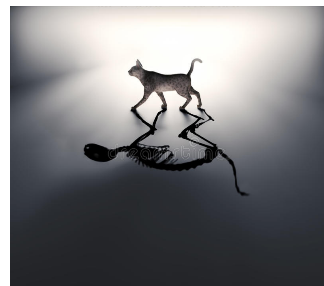
5. Superposition of Spin network graphs

$$\mathcal{H}_{GFT} = \bigoplus_{V=0}^{\infty} \mathcal{H}_V$$

A generic quantum state at the kinematical level of the theory is not an entanglement graph state, but a quantum **superposition** thereof.

$$|\psi_A\rangle = \sum_{\gamma} \alpha_{\gamma_i} |\psi_i\rangle \quad |\psi_B\rangle = \sum_{\delta} \beta_{\delta_j} |\psi_j\rangle$$

Interference of quantum geometries



Same number of particles (vertices)

$$S(\rho_A) = \sum_{\{\gamma_i\},\{p_{[\gamma_i,\delta_j]}\}} \left(\omega\left(\gamma_i, p_{[ij]}\right) \log\left[\omega\left(\gamma_i, p_{[ij]}\right)\right] + \sum_{j_{\gamma_i}} \omega\left(\gamma_i, p_{[ij]}\right) S\left(j_{\gamma_i}\right) \right)$$

$\omega\left(\gamma_i, p_{[ij]}\right)$: the probability of finding a given state with v vertices with a definite spins $j_{\gamma_i}, j_{[\gamma_i,\delta_j]}$

The entropy= the sum of the classical **Shannon entropy** of the probability distribution $\omega\left(\gamma_i, p_{[ij]}\right)$ and the **average entropy** at fixed spin of all possible combinatorial patterns governing the system.

- **Coherent interference:** superposition of graphs with different number of quanta → sum over all possible entanglement entropies.

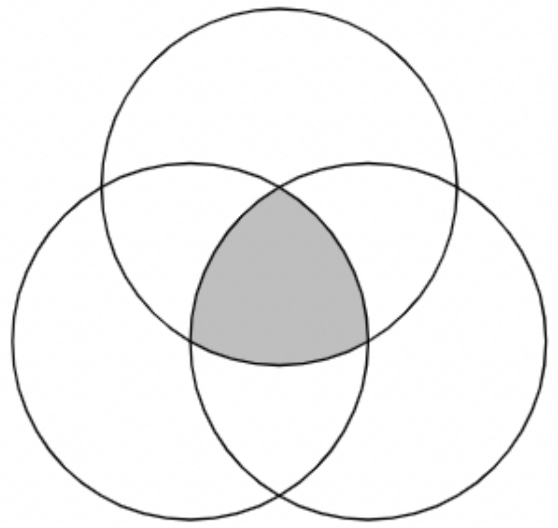
Different number of particles

$$S(\rho_A) = \sum_{\{\gamma_i\},\{p_{[\gamma_i,\delta_j]}\}} \omega\left(\gamma_i, p_{[ij]}\right) \log\left[\omega\left(\gamma_i, p_{[ij]}\right)\right] + \sum_{\{\gamma_i\},\{p_{[\gamma_i,\delta_j]}\}} \left(\omega\left(\gamma_i, p_{[ij]}\right) \log\left[\omega\left(\gamma_i, p_{[ij]}\right)\right] + \sum_{j_{\gamma_i}} \omega\left(\gamma_i, p_{[ij]}\right) S\left(j_{\gamma_i}\right) \right) + \mathrm{I}\left(\gamma_1 : \gamma_2 : \cdots : \gamma_{\mathcal{N}_A}\right)$$

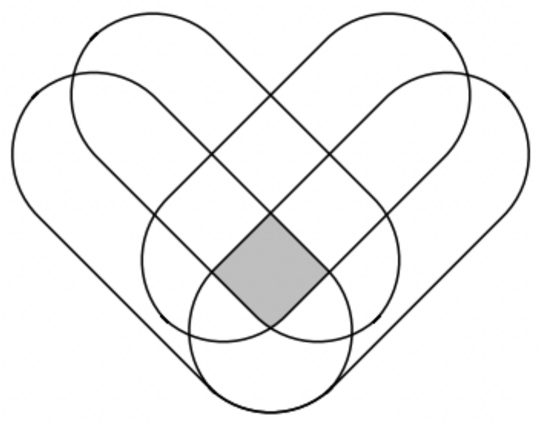
Usual entropy of the connected links reflecting the entanglement of the superposition of graphs.

+

Information interaction: an alternating signed sum of all the marginal entropies of the combinatorial states.



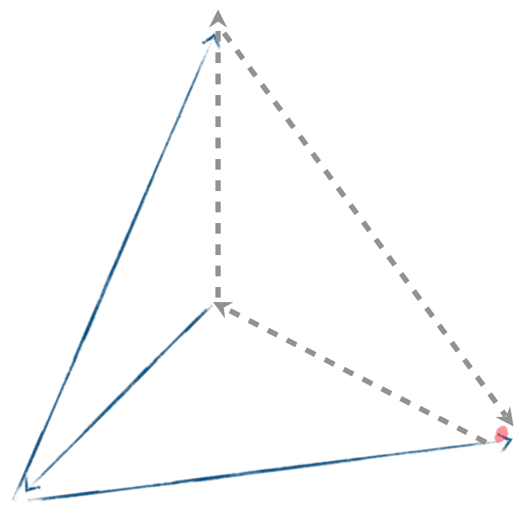
$$I(X;Y;Z) = +[S(X)+S(Y)+S(Z)] - [S(X,Y)+S(X,Z)+S(Y,Z)] + S(X,Y,Z)$$



I(A : B : C : D)

- **Incoherent interference:** superposition of graphs with the same number of quanta but different combinatorial patterns →emergence of interaction information → towards a definition of *relative distance measures*.

Conclusion and Outlook

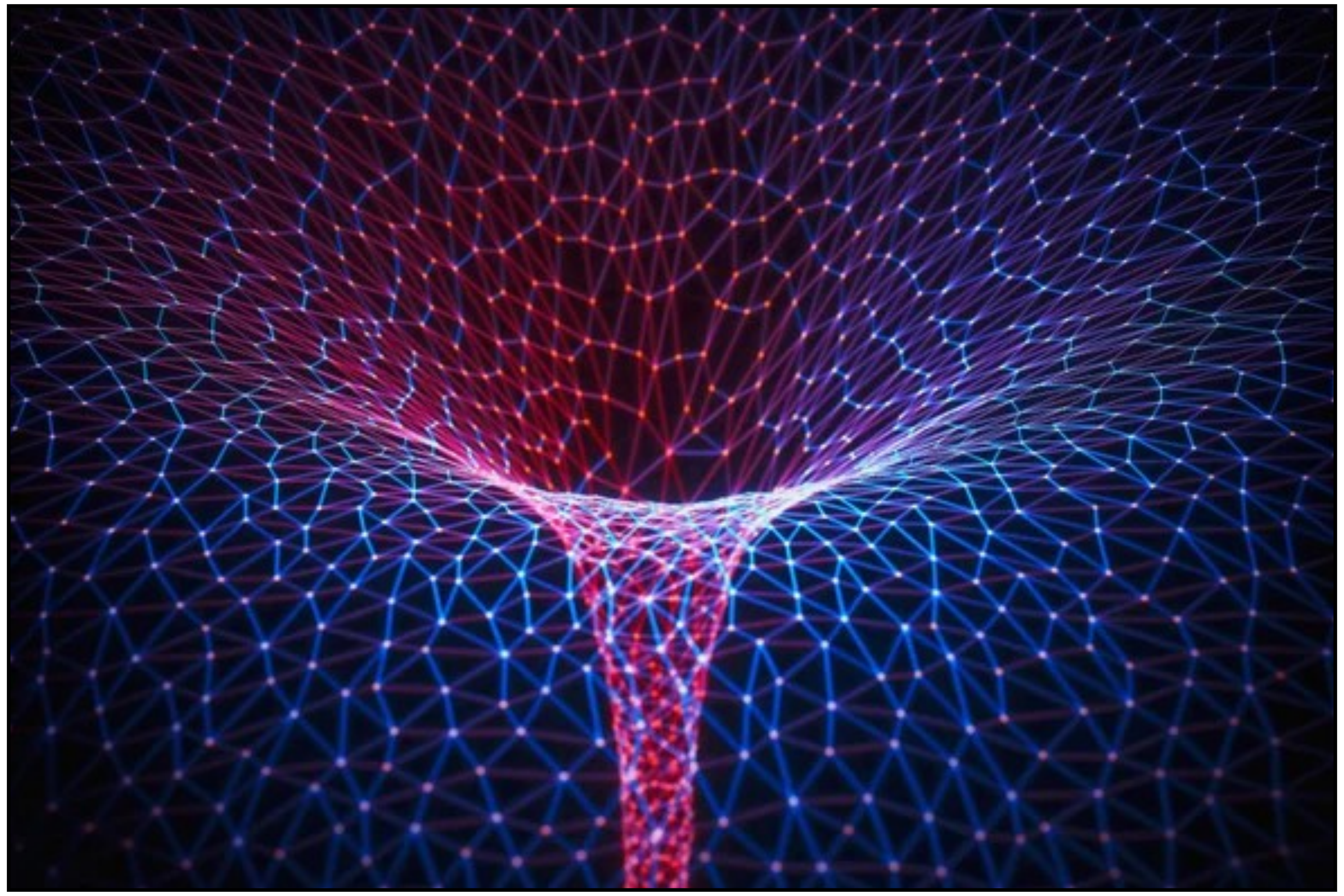


- Spacetime is not fundamental in quantum gravity models. It is a rather emergent entity: GFT formalism and quanta of spacetime.
- Spin network graphs and their characterising entanglement patterns.
- Entanglement plays a crucial role in the gluing process. Generic case, reveals more features to be analysed. Analysis of the interaction entropy already at the kinematical level.

Entanglement entropy: Entanglement entropy (probability of the connected components) of different combinatorial patterns + that of the overlap between these patterns (interaction entropy).

Outlook

- How can we define an appropriate semi classical limit? What about the dynamics of the theory?
- Can we define an appropriate Curvature operator?
- Fock space and second quantization? (Diffeomorphism, crossroad with condensed matter theory ..)
- And more other issues ..



Thank you very much for the attention!