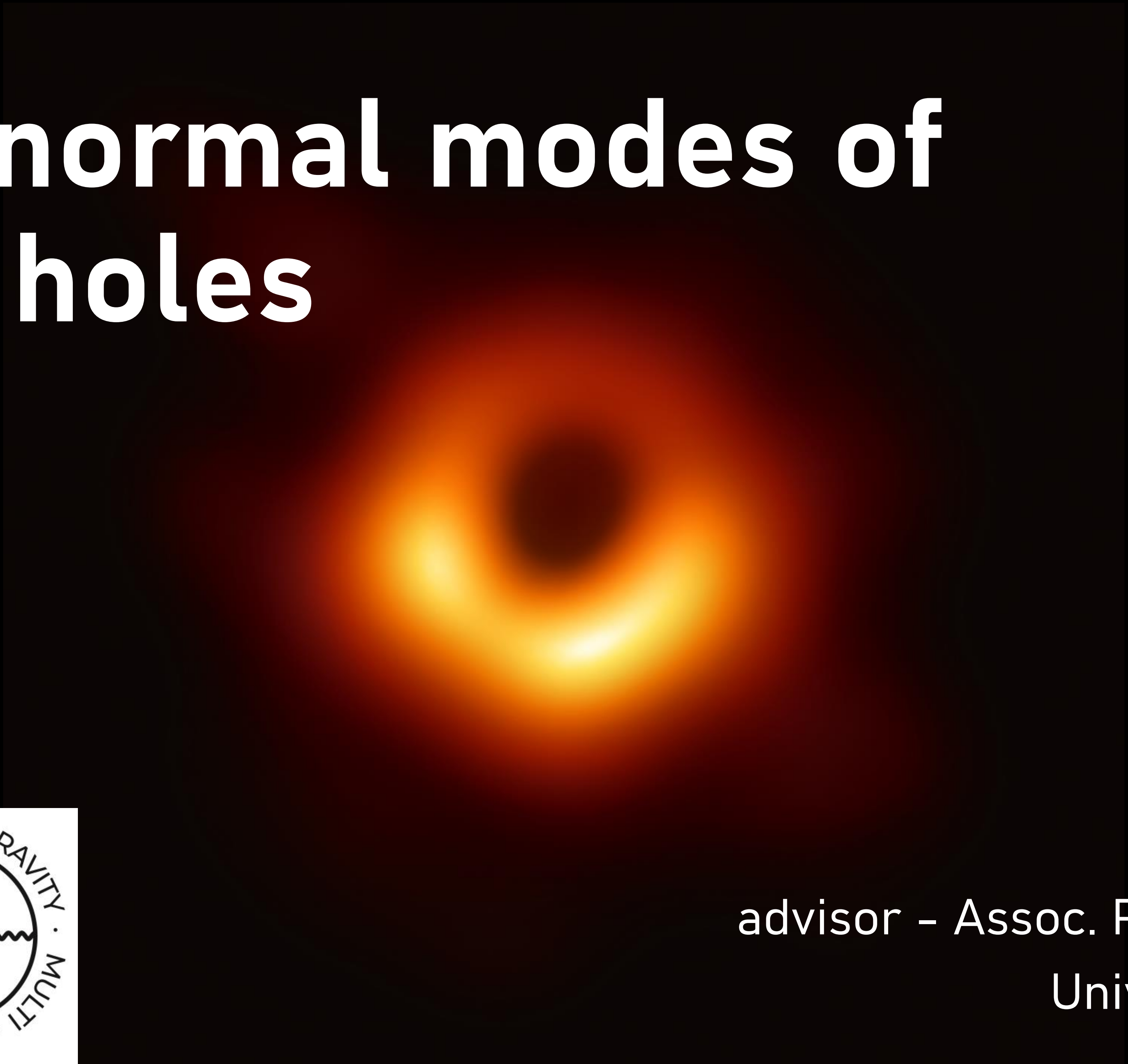
Quasinormal modes of black holes



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Recap

- **quasi**normal modes are eigenmodes of **damped** oscillations

- horizon **ingoing** waves $\psi \propto e^{-i\omega(t+r_*)}$
- infinity **outgoing** waves $\psi \propto e^{-i\omega(t-r_*)}$

- **damped** with time $\omega \in \mathbb{C}$, $\mathrm{Im}(\omega) < 0$
  if time dependance is $e^{-i\omega t}$

$$e^{-i\omega t} = e^{-i\omega_R t+\omega_I t}$$

Motivation

- **experimental** viability
- can be used to check **modified** theories
- **quantum** theory of gravity
    - eg. Motl&Neitzke motivate through a supposed connection to **LQG**

Perturbations by spin

- **BH perturbations**
  - SCALAR $\Box\phi = 0$
  - VECTOR (electromagnetic) $\nabla_\mu F^{\mu\nu} = 0$
  - TENSOR (gravitational) $R_{\mu\nu} = 0$

Perturbations by spin

$$\left(\frac{d^2}{dr_*^2} + \omega^2 - V(r)\right)\psi = 0$$

SCALAR $\Box\phi = 0$ $V_{s=0} = \left(1 - \frac{2M}{r}\right)\left(\frac{l(l+1)}{r^2} + \frac{2M}{r^3}\right)$

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TENSOR (gravitational) $\boldsymbol{R_{\mu\nu} = 0}$ $\boldsymbol{V_{s=2}^{ax} \neq V_{s=2}^{pol}}$

Effective potential for tensor perturbations

$$V_{s=0} = \left(1 - \frac{2M}{r}\right)\left(\frac{l(l+1)}{r^2} + \frac{2M}{r^3}\right)$$

$$V_{s=1}^{ax} = V_{s=1}^{pol} = \left(1 - \frac{2M}{r}\right)\frac{l(l+1)}{r^2}$$

$$V_{s=2}^{ax} = \left(1 - \frac{2M}{r}\right)\left(\frac{l(l+1)}{r^2} - \frac{6M}{r^3}\right)$$

$$V_{s=2}^{pol} = 2\left(1 - \frac{2M}{r}\right)\frac{n^2(n+1)r^3 + 3Mn^2r^2 + 9M^2nr + 9M^3}{r^3(nr+3M)^2}$$

$$n = \frac{1}{2}(l-1)(l+2)$$

Isospectrality of gravitational perturbations

$$V_{s=2}^{\substack{+pol \\ -ax}} = \pm\beta \frac{dg}{dr_*} + \beta^2 g^2 + \kappa g$$

$$\beta = 6M; \quad g = \frac{\left(1 - \frac{2M}{r}\right)}{r\big((l(l+1) - 2)r + 6M\big)}; \quad \kappa = (l(l+1) - 2)l(l+1)$$

Isospectrality of gravitational perturbations

$$\psi^{ax}(r,\omega) = A_1 \psi^{pol}(r,\omega) + A_2 \frac{d\psi^{pol}(r,\omega)}{dr_*}$$

$$\psi^{pol}(r,\omega) = A_3 \psi^{ax}(r,\omega) + A_4 \frac{d\psi^{ax}(r,\omega)}{dr_*}$$

- for **any solution which governs axial** perturbations, one can always **construct a solution which governs polar** perturbations **with the same quasinormal mode frequency** (and vice-versa)

Isospectrality of gravitational perturbations

$$(\kappa - 2i\omega\beta)\psi^{ax}(r,\omega) = (\kappa + 2\beta^2 g)\psi^{pol}(r,\omega) - 2\beta \frac{d\psi^{pol}(r,\omega)}{dr_*}$$

$$(\kappa + 2i\omega\beta)\psi^{pol}(r,\omega) = (\kappa + 2\beta^2 g)\psi^{ax}(r,\omega) + 2\beta \frac{d\psi^{ax}(r,\omega)}{dr_*}$$

- this implies that, not only are the quasinormal modes the same, but if you look at the scattering problem – the **reflection and transmission coeficciens are the same**

Isospectrality of gravitational perturbations

$$\left(\frac{d^2}{dr_*^2} + \omega^2 - V(r)\right)\psi = 0$$

$$V_s = \left(1 - \frac{2M}{r}\right)\left(\frac{l(l+1)}{r^2} + \frac{2M}{r^3}(1 - s^2)\right)$$

- for $s = 2$ we recover the **axial** gravitational potential

Thank you!

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