MODEL PARAMETERS IN THE CONTEXT OF LATE TIME COSMIC ACCELERATION IN MODIFIED THEORIES OF GRAVITY

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Topic Outlines

- Introduction
- Overview $f(Q, T)$ gravity
- Mathematical formalism
- Models with hybrid scale factor
- Analysis of the model
- Results and Discussion

Introduction

• Einstein's centennial theory of gravity accepts three formulations: general relativity based on curvature, teleparallel gravity based on torsion, and symmetric teleparallel gravity based on nonmetricity.

• General relativity assumes Levi-Civita connection and, hence, implies zero torsion and nonmetricity. GR has a well-researched alternative formulation known as teleparallel equivalent of general relativity.

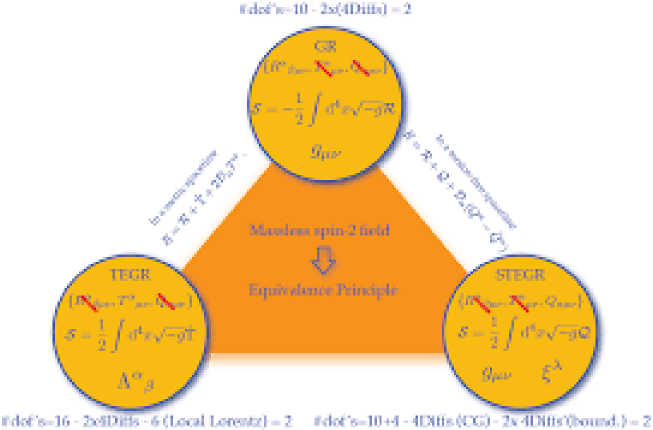
• GR and it's higher order derivative extensions have Symmetric teleparallel reformulation..

• The way of Einstein.

• The geometrical nature of gravity emerges from the universality dictated by the equivalence principle.

• Gravity and geometry have accompanied each other from the very conception of General Relativity (GR) brilliantly formulated by Einstein in terms of the spacetime curvature.

The tritiny of geometric models of gravity



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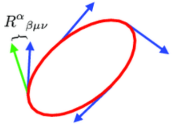
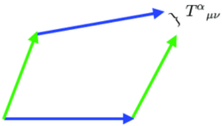
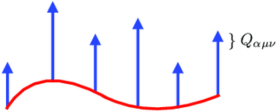
[1] J. B. Jiménez, L. Heisenberg, T. M. Koivisto, *Universe*, **5**, 173, (2019)



The rotation of a vector transported along a closed curve is given by the curvature: General Relativity.



The non-closure of parallelograms formed when two vectors are transported along each other is given by the torsion: Teleparallel Equivalent of General Relativity.



The variation of the length of a vector as it is transported is given by the non-metricity: Symmetric Teleparallel Equivalent of General Relativity. [2]

[2] J. B. Jiménez, L. Heisenberg, T. Koivisto, *Universe*, **5**, 173, (2019)

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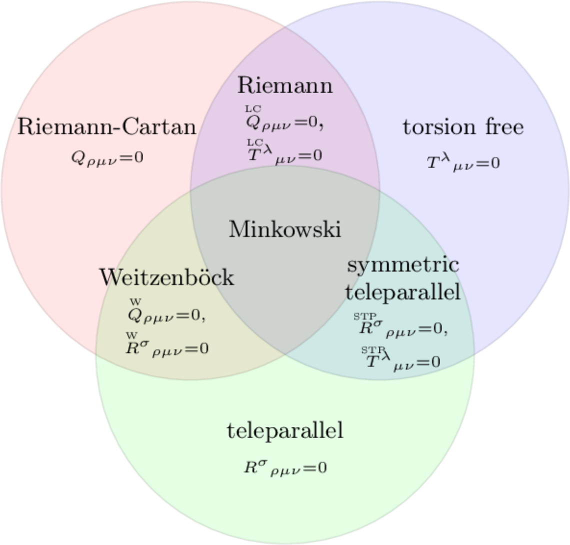

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[3] L. Järv, M. Rünkla, etal., *PHYSICAL REVIEW D*, **97**, 124025, (2018)

Overview $f(Q, T)$ gravity

• The cosmological model is a study of modified $f(Q, T)$ theory of gravity which was recently proposed by Xu et.al.

• We propose an extension of the symmetric teleparallel gravity in which the gravitational action L is given by an arbitrary function of the non-metricity Q and the trace of the matter-energy momentum tensor T. It represented as, $L = f(Q, T)$.

• The non-metricity tensor is defined as the covariant derivative of the metric tensor, which geometrically describes the variation in the length of a vector in parallel transit.

• We imposed a cosmological models which is the functional form of $f(Q, T)$ and the format of these functional forms are taken as,

$f(Q, T) = aQ^m + bT$

Mathematical formalism

The action of $f(Q,T)$ gravity given as,

$$S=\int\left(\frac{1}{16\pi}f(Q,T)+\mathcal{L}_m\right)d^4x\sqrt{-g}, \tag{1}$$

The non-metricity Q can be defined as,

$$Q\equiv-g^{\mu\nu}(L^k{}_{l\mu}L^l{}_{\nu k}-L^k{}_{lk}L^l{}_{\mu\nu}), \tag{2}$$

where

$$L^k{}_{l\gamma}\equiv-\frac{1}{2}g^{k\lambda}(\nabla_\gamma g_{l\lambda}+\nabla_l g_{\lambda\gamma}-\nabla_\lambda g_{l\gamma})$$

Varying the gravitational action (1), the field equation of $f(Q,T)$ gravity can be obtained as,

$$-\frac{2}{\sqrt{-g}}\nabla_k\left(f_Q\sqrt{-g}P^k_{\mu\nu}\right)-\frac{1}{2}fg_{\mu\nu}+f_T(T_{\mu\nu}+\Theta_{\mu\nu})-f_Q(P_{\mu kl}Q_\nu{}^{kl}-2Q^{kl}{}_\mu P_{kl\nu})=8\pi T_{\mu\nu}, \tag{3}$$

The super potential of the model can be expressed as,

$$P^k_{\mu\nu} = -\frac{1}{2}L^k_{\mu\nu} + \frac{1}{4}(Q^k - \tilde{Q}^k)g_{\mu\nu} - \frac{1}{4}\delta^k_{(\mu}Q_{\nu)}. \quad (4)$$

The trace of the energy momentum tensor and trace of the non-metricity tensor can be respectively denoted as,

$$\begin{aligned} T &= T_{\mu\nu}g^{\mu\nu} \\ Q_k &= Q_k{}^{\mu}{}_{\mu}, \tilde{Q}_k = Q^{\mu}{}_{k\mu} \end{aligned}$$

We consider the universe described by homogeneous, isotropic and spatially flat FLRW space-time as,

$$ds^2 = -N^2(t)dt^2 + a^2(t)(dx^2 + dy^2 + dz^2), \quad (5)$$

The dilation rate $\tilde{T} = \frac{\dot{N}(t)}{N(t)}$ and in the standard case $N = 1$, the non-metricity reduced to, $Q = 6H^2$.

$$\frac{f}{2} - 6F\frac{H^2}{N^2} = 8\pi\tilde{G}(\rho + p) \quad (6)$$

$$\frac{f}{2} - \frac{2}{N^2}\left[\left(\dot{F} - F\tilde{T}\right)H + F\left(\dot{H} + 3H^2\right)\right] = -8\pi p \quad (7)$$

Models with hybrid scale factor

$$\rho = \frac{1}{8\pi}\left[\frac{f}{2} - 6FH^2 - 2\frac{\tilde{G}}{1+\tilde{G}}(\dot{F}H + F\dot{H})\right] \tag{8}$$

$$p = -\frac{1}{8\pi}\left[\frac{f}{2} + 6FH^2 + 2(\dot{F}H + F\dot{H})\right] \tag{9}$$

Here by taking functional, $f(Q,T) = aQ^m + bT$, and $m = n+1$ then equation (8), (9) can be written as,

$$p = \frac{a\left(6H^2\right)^n\left[(12bn + 6b - 12n(3b+16\pi) - 6(3b+16\pi))H^2 - 4(3b+16\pi)(n+1)\dot{H} - \frac{2(3b+16\pi)n(n+1)}{3H}\right]}{b^2 - (3b+16\pi)^2} \tag{10}$$

$$\rho = \frac{a\left(6H^2\right)^n\left[(12n(3b+16\pi) + 6(3b+16\pi) - 12bn - 6b)H^2 - 4b(n+1)\dot{H} - \frac{2bn(n+1)}{3H}\right]}{b^2 - (3b+16\pi)^2} \tag{11}$$

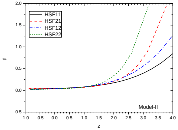
$$\omega = \frac{\left[(12bn + 6b - 12n(3b+16\pi) - 6(3b+16\pi))H^2 - 4(3b+16\pi)(n+1)\dot{H} - \frac{2(3b+16\pi)n(n+1)}{3H}\right]}{\left[(12n(3b+16\pi) + 6(3b+16\pi) - 12bn - 6b)H^2 - 4b(n+1)\dot{H} - \frac{2bn(n+1)}{3H}\right]} \tag{12}$$

Now we used the hybrid scale factor $a(t) = e^{\alpha t}t^{\beta}$. Then, $H = \alpha + \frac{\beta}{t}$. We have used this value in the above equation

Analysis of the graph

Table 1. EoS parameter at the present epoch as predicted by the HSF models within $f(Q,T)$ gravity theory.

Models	HSF11	HSF21	HSF12	HSF22
m	ω_0	ω_0	ω_0	ω_0
0.6	−0.847	−0.889	−0.798	−0.841
0.8	−0.796	−0.852	−0.730	−0.788
1.0	−0.745	−0.815	−0.663	−0.736
1.1	−0.719	−0.797	−0.629	−0.709
1.5	−0.617	−0.723	−0.494	−0.603
2	−0.489	−0.631	−0.325	−0.471



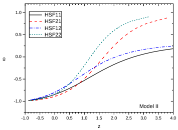


Figure: The evolutionary aspect of the energy density (left panel) and Equation of state parameter (right panel) vs redsfift. Here we have used the parameter space (a = -4.4, b = 0.01, m = 0.6).

- The EoS parameter value from observational sources, Supernova data[4]. $\omega = -1.084 \pm 0.063$, WMAP[5]. $\omega = -1.073^{+0.090}_{-0.089}$ favours ΛCDM.

[4] R. Amanullahet *al.*, *Astrophy. J.*, **712**, 716 (2010)

[5] G. Hinshaw *et al*, *Astrophys. J. Suppl. Ser.*, **19**, 208 (2013)

Results and Discussion

• The behaviour of energy density and EoS parameter mostly depends on the value of the model parameters 'a' and 'b', and that of scale factor parameters α and β. It is worthy to mention here that the late time cosmic acceleration behaviour can be assessed in dark energy and $f(Q, T)$ gravity cosmological models by constraining the parameter of the scale factor, $\alpha > 0$ and $0 < \beta < \frac{1}{3}$.

• This is essential in getting accelerating cosmological models without invoking any dark energy components to the matter field.

• It is found from our investigations that, the model parameter 'a' does not affect the evolutionary aspect of the EoS parameter. However, the parameter 'b' affects marginally the behaviour of ω.

• Another parameter 'm' that appears in the model $f(Q, T)$ which has a substantial effect on the EoS parameter in the sense that, with an increase in 'm', the value of ω at the present epoch increases.

- Besides the above, we also have used some recent constructed hybrid scale factors to investigate the cosmic dynamics.

- HSF models coincide with the concordant ΛCDM value $\omega =$ -1 at late time of cosmic evolution which is in conformity with recent observations.

- The model parameters are chosen suitably so as to provide a physically acceptable energy density.

- In view of this, these models may be useful in the context of the search for geometrical alternatives to dark energy.

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Thank you