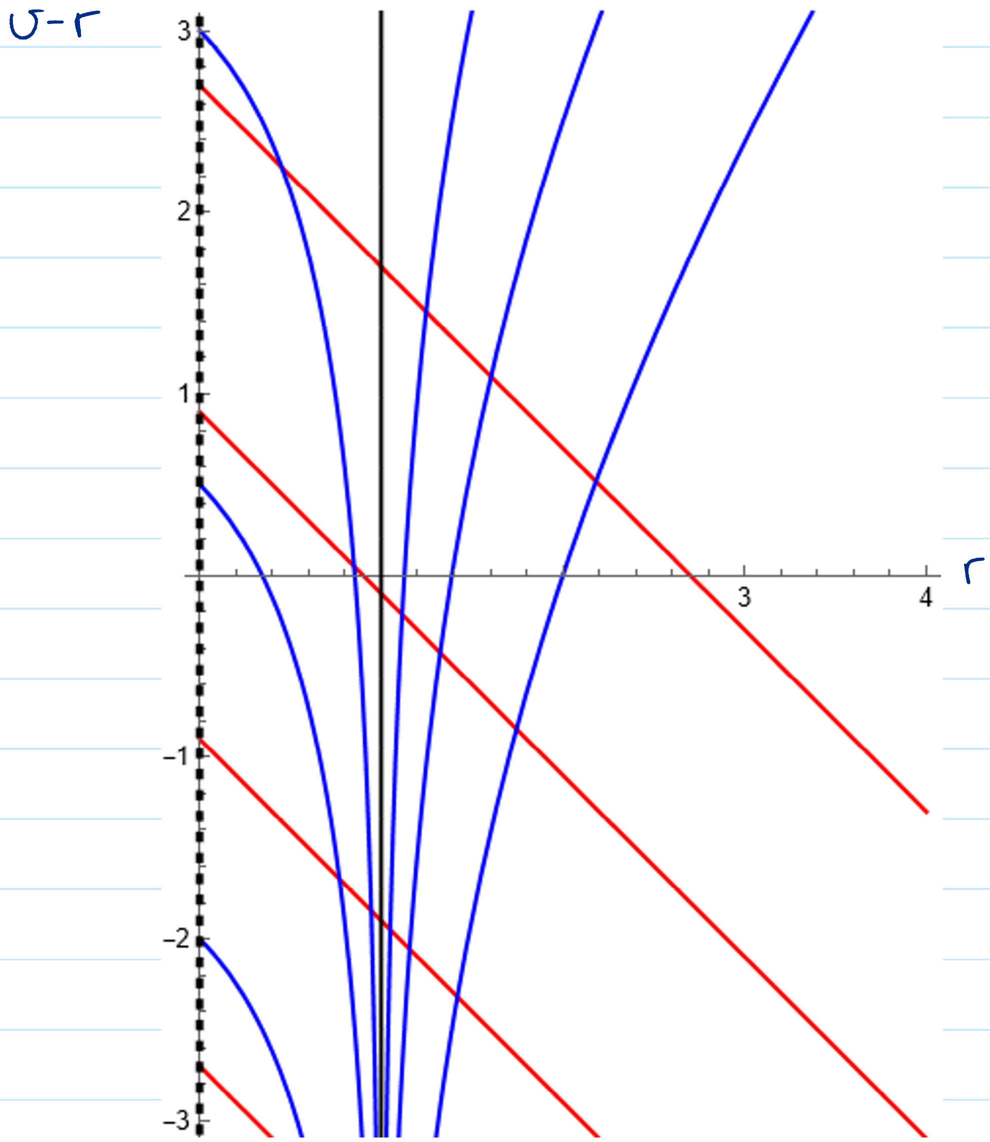
Lecture 1: The black hole as a tale of light and darkness (2/2)

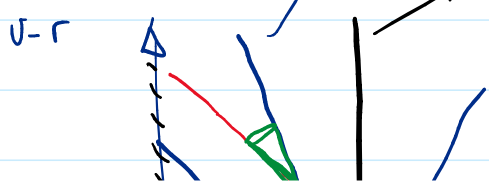
miércoles, 31 de agosto de 2022 15:56

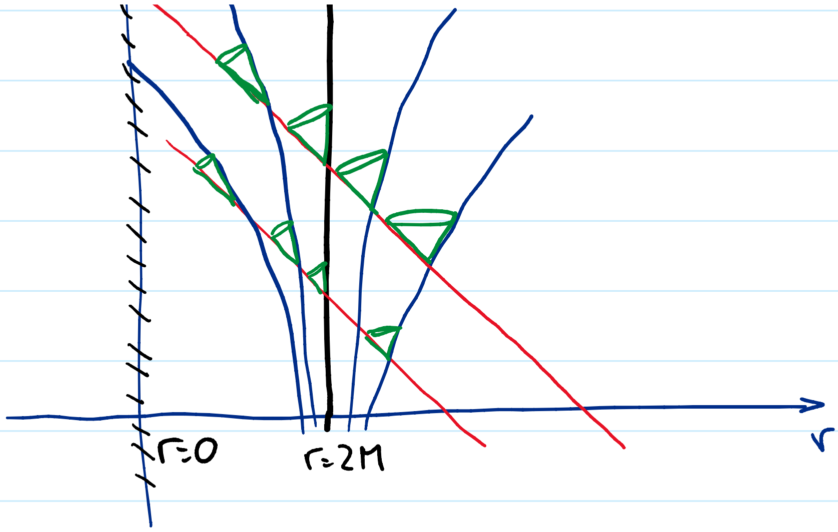


Light cones tip over

"outgoing ray" does not escape!

"outgoing ray" stays at fixed position





Since we (massive objects) move inside light cones,
Then inside $r<2M$ we cannot escape to $r>2M$

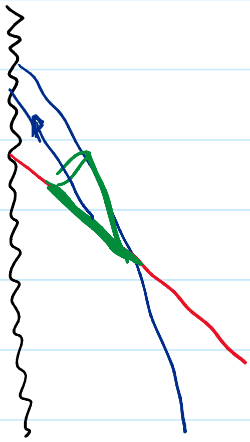
- $\underline{r=2M}$: "frozen light rays" : **Horizon**

  one per point in the sphere:

  "horizon is generated by null rays"

  Null hypersurface $S^2 \times \mathbb{R}$

- $\underline{r=0}$ is a **singularity** $\left(R_{ijke}R^{ijke} \to \infty \text{ at } r=0\right)$



No light rays emanate from the singularity: it cannot be seen!

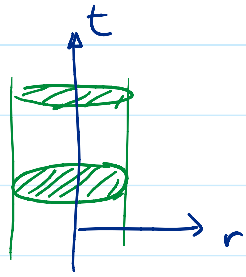
It is experienced as an event in the future.

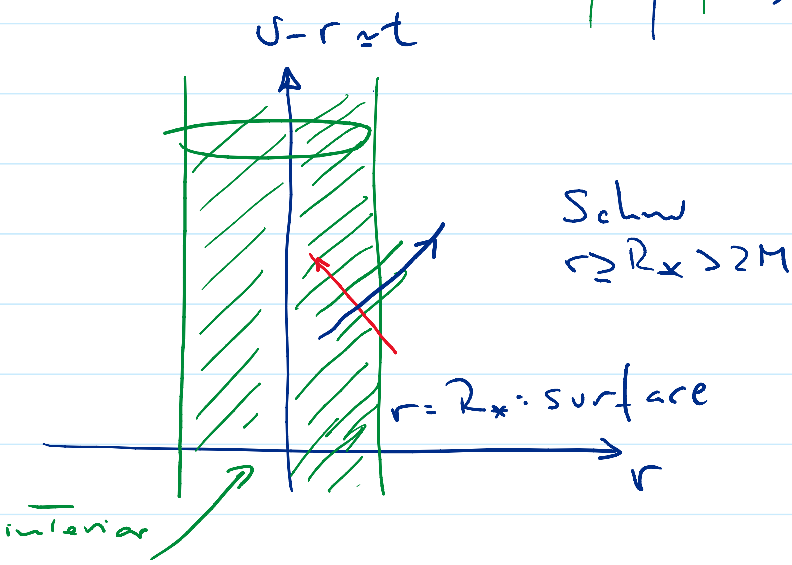
Singularity is not a point in space, but a moment in time

All particle trajectories in $r<2M$ end at $r=0$ in finite proper time.

• Collapsing star

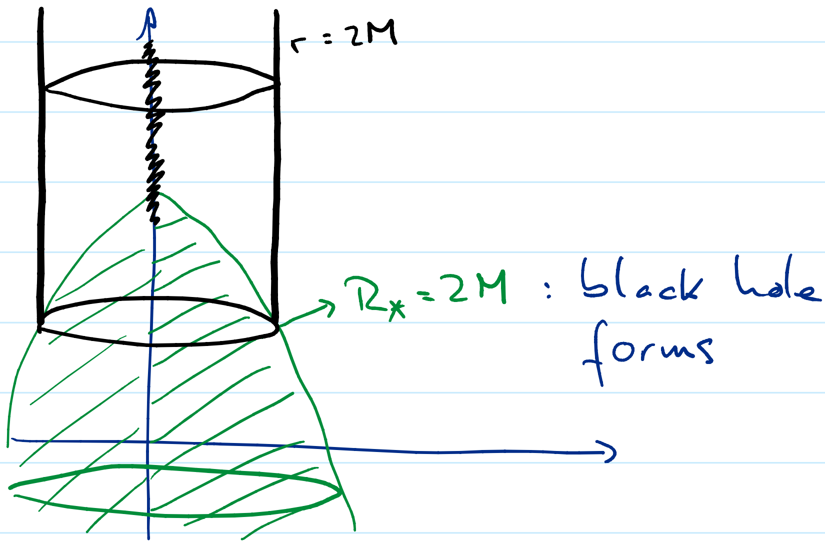
First, non-collapsing star



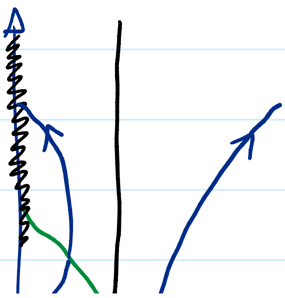


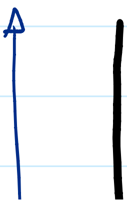
$r = 0$ is simply origin of coords

Now collapse: R_* decreases with time



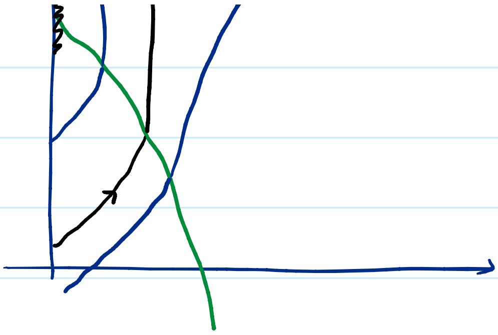
outgoing light rays

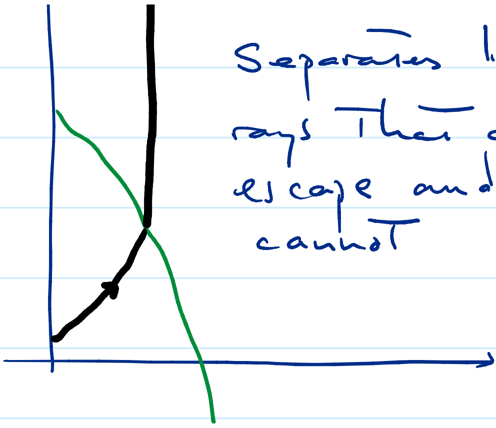




Event horizon

separates light





Separates light rays that can escape and cannot

[See supplements for more on the event horizon, the apparent horizon, and Penrose's singularity theorem]

• Non-radial null geodesics: The light ring

By conservation of angular momentum, they'll be on a plane. By symmetry, can consider $\theta = \pi/2$

$$ds^2\Big|_{\theta=\pi/2} = -\left(1-\frac{2M}{r}\right)dt^2 + \frac{dr^2}{1-\frac{2M}{r}} + r^2 d\phi^2$$

Spacetime is static and spherically symmetric

$\Rightarrow$ energy E and angular momentum L of geodesic are conserved: (constants of motion)

$$E = \left(1-\frac{2M}{r}\right)\frac{dt}{d\lambda}$$

$$L = r^2\frac{d\phi}{d\lambda}$$

(λ: affine parameter along null geodesic)

Then from $ds^2 = 0$ we have

Then from $ds^2 = 0$ we have

$$\left(\frac{dr}{d\lambda}\right)^2 + \left(1 - \frac{2M}{r}\right)\frac{L^2}{r^2} = E^2$$

like $\frac{1}{2}\dot{r}^2 + V = \mathcal{E}$

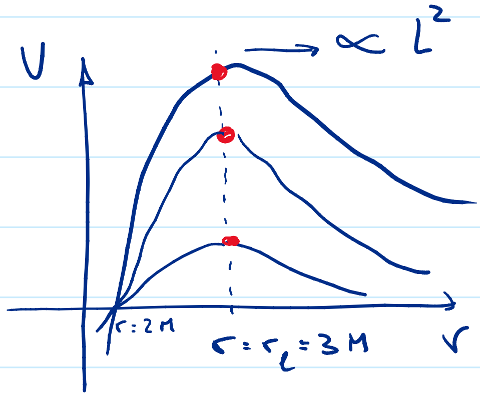
so

$$V = \frac{1}{2}\left(1 - \frac{2M}{r}\right)\frac{L^2}{r^2}$$

$$= \frac{L^2}{2r^2} - \frac{ML^2}{r^3}$$

↓ centrifugal barrier

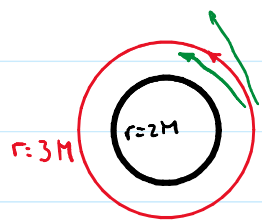
↳ GR stronger attraction



$$V' = 0 \Rightarrow r = 3M$$

Unstable circular light ray orbits at $r = r_\ell = 3M$

Light ring



It is easy to compute that a circular light ray has $\frac{L}{E} = 3\sqrt{3}M$ and takes time $\Delta t = 6\pi\sqrt{3}M$ to loop the circle