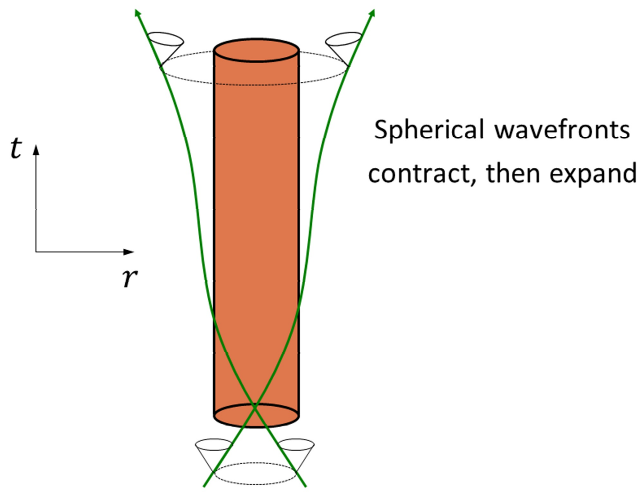
Lecture 1: Supplement I: Event horizon: simple illustrations

lunes, 22 de marzo de 2021 18:02

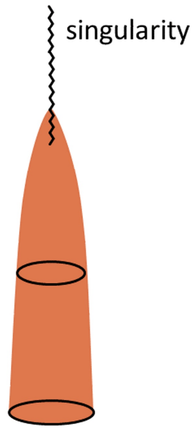
Star



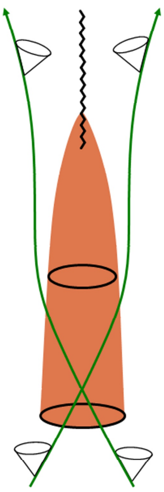
Consider the propagation of spherical lightfronts in the spacetime of a spherical star.

An initial spherical lightfront that is contracting will reach zero size inside the star, and then (since the geometry in the star interior is smooth), it will expand again -- feeling only some attraction from the star, which slightly delays its expansion

Collapsed Star

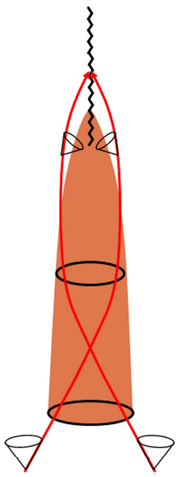


Now consider a star that has collapsed to form a singularity

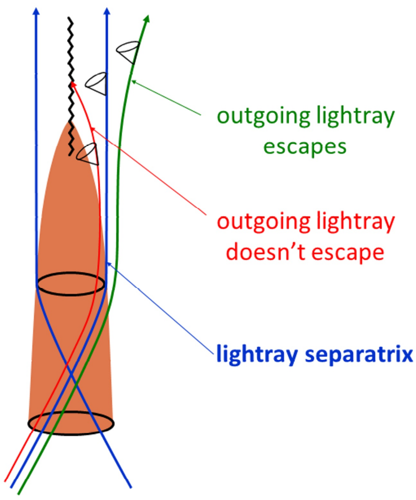


Lightfronts that begin their contraction early enough will contract to zero, then expand

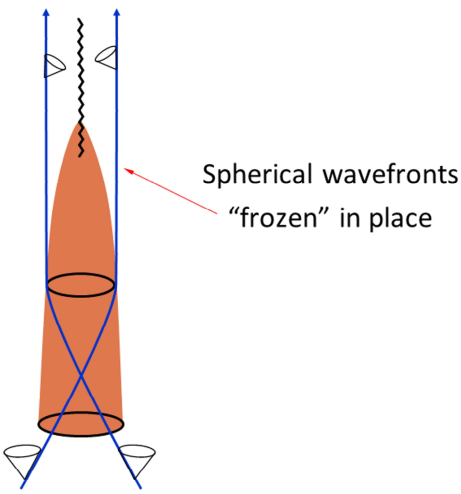
But there will also be later lightfronts that, when they try to expand, are dragged back so strongly



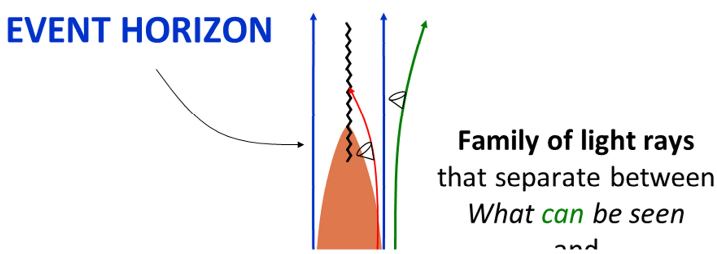
But there will also be later lightfronts that, when they try to expand, are dragged back so strongly that they collapse to the singularity and fail to escape away to infinity



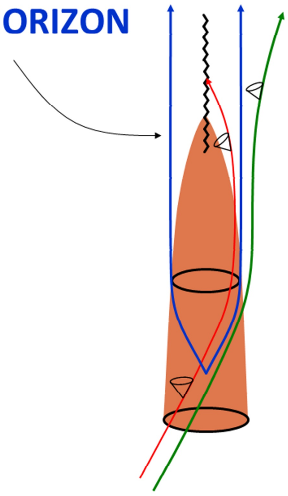
We have two classes of lightfronts: those that escape and those that don't. There will be a class of lightrays that separate the two



These separatrix light rays do not fall into the singularity, nor escape to infinity. They remain "frozen" at fixed radius



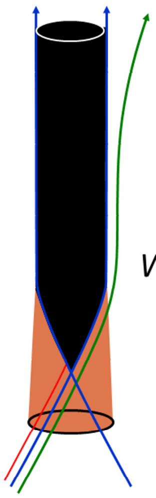
EVENT HORIZON



Family of light rays
that separate between
What can be seen
and
What cannot be seen
from outside

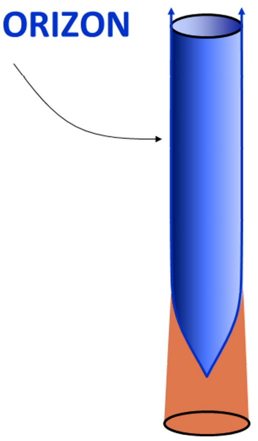
The **event horizon** is the surface traced (generated) by these frozen light rays. If an event occurs inside this surface, no light rays from it can escape to the outside: it cannot be seen by any external observers -- hence the name "event horizon"

BLACK HOLE



What cannot be seen
from outside
(asymptotic infty)

EVENT HORIZON

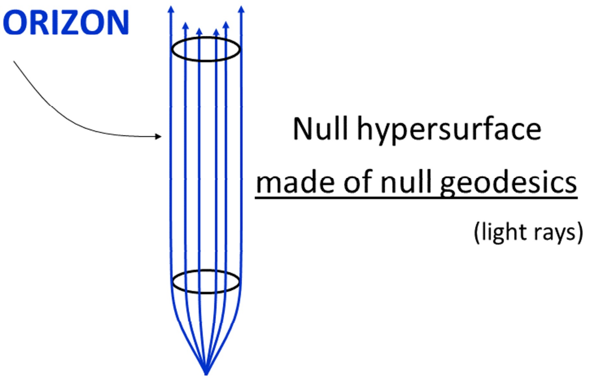


Null hypersurface

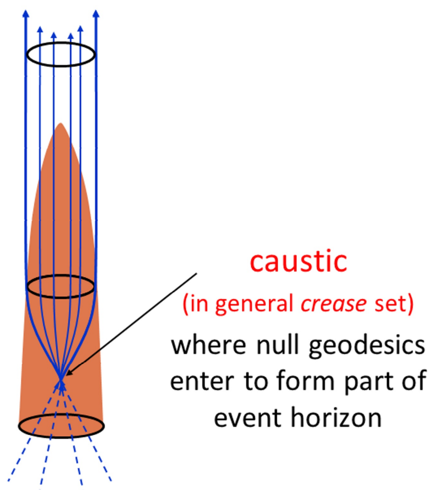
3-dimensional in
4-dimensional spacetime

The event horizon is a three-dimensional hypersurface in the four-dimensional spacetime. It is a null hypersurface. We often use the same name for its two-dimensional spatial sections -- in this case, spheres

EVENT HORIZON

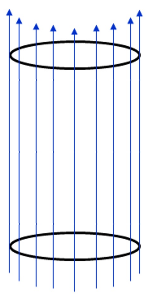


The event horizon is a family of null geodesics in spacetime



In the collapsing situation we are studying, the null geodesics only begin to form the event horizon at some instant -- before then, one can escape to infinity without problem. The points where the null geodesics begin to form part of the horizon are caustics (since light rays cross at them), or more generally, crease sets of the horizon. They are singular points of the surface, even though in general they are not singularities of the spacetime

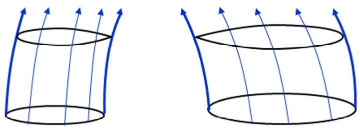
Event horizon
found by
tracing a family of light rays in a given spacetime



In general, in order to find an event horizon, we have to trace a family of light rays in a spacetime, which form a surface such that events inside the surface cannot send any light rays to infinity

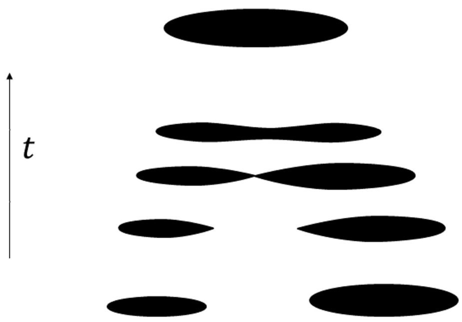
Event horizon of binary black hole fusion

Let us now consider the event horizon in the process where two black holes merge to form a single one



We begin with two cylindrical null surfaces corresponding to the event horizons of the initial black holes

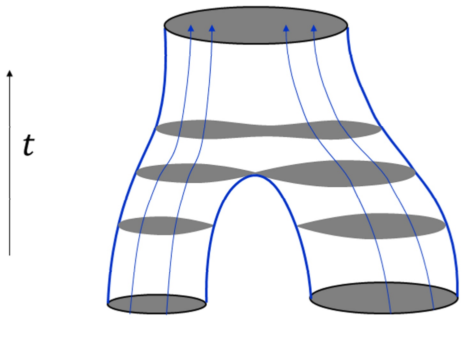
binary black hole fusion



Viewed in constant-time snapshots, we expect that the two (approximately) spherical black holes come together, and then fuse into a single one

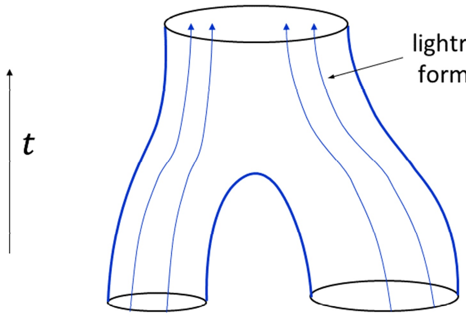
Event horizon of binary black hole fusion

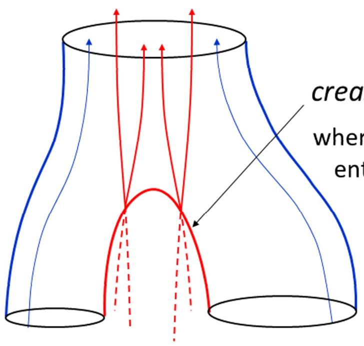
Continuously tracing out the merger, we find the shape of the surface of the black holes along the process



Event horizon of binary black hole fusion

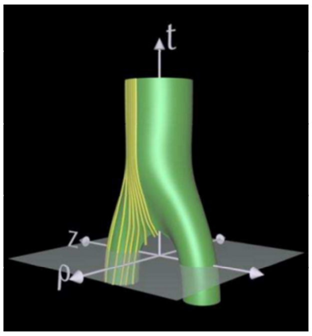
The event horizon has the shape of a pair of pants





The surface is not completely smooth, as there are points where new light rays are added to it. This happens at the crotch of the pants. The crease set is not a null surface, but a spacelike set

Event horizon of binary black hole fusion



head-on (axisymmetric)

equal masses

Cover of *Science,* November 10, 1995

Binary Black Hole ***Grand Challenge*** *Alliance* (Matzner et al)

An early numerical simulation of the simplest collision between two black holes was illustrated, in the cover of Science, by the event horizon and the light rays that form it

Hawking's black hole area theorem

Hawking proved that the total area of all the sections of the event horizon at a given instant can never decrease as time evolves.

The theorem makes two assumptions:

- Energy along null geodesics is non-negative (null energy condition)
- There are no naked singularities

Given these assumptions, one can prove that the null generators of the horizon can not contract -- that is, the area of a cross section of a pencil of null rays cannot decrease. Furthermore, it can also be proven that null generators can be added, but not removed, from the event horizon. Both of these effects can then only lead to an increase of the total horizon area, and never to a decrease.

Quantum effects can violate the null energy condition, and make the horizon of the black hole shrink. This is what happens during the evaporation of a black hole by emission of Hawking radiation