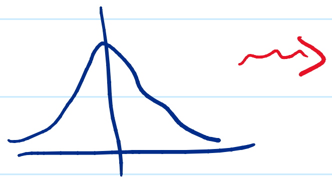
Lecture 2: The black hole that vibrates (2/2)

miércoles, 31 de agosto de 2022 20:33

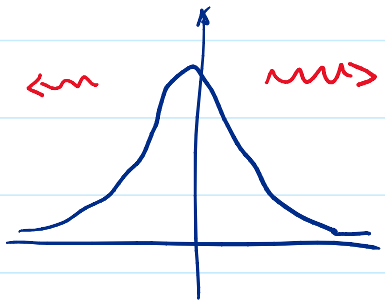
Quasinormal vibrations of the black hole

We are interested now in the "proper" vibrations of the black hole, without any incident waves, only outgoing waves



$$\Psi \sim e^{-i\omega(t - r_*)} \quad \text{as} \quad r_* \to \infty$$

No waves can come from the horizon, so the wave must be ingoing as $r_* \to -\infty$



$$\Psi \sim e^{-i\omega(t + r_*)} \quad \text{as} \quad r_* \to -\infty$$

These boundary conditions define the quasinormal mode spectrum.

We have a bdry-value problem for the ODE for Ψ, and it will admit solutions only for a discrete set of complex $\omega = \mathrm{Re}\,\omega - \frac{i}{\tau} = 2\pi f - \frac{i}{\tau}$

If $\tau > 0$ then stable

The amplitude after one oscillation decreases by a factor $\exp\left(\frac{1}{f\tau}\right)$

The least-damped modes are the most important.
For Schw this is the fundamental $\ell=2$ mode

$$\omega = (0.37 - 0.089i)\frac{c^3}{GM}$$

Since Schw has only one scale, M, we necessarily have

$$\omega \sim \frac{1}{M}$$

In observational units, using that $M_\odot \simeq 5\,\mu s$

$$\frac{1}{M_\odot} = 200\ Hz$$

$$\omega = 2\pi f - i/\tau$$

$$f = 1.21\,\frac{10M_\odot}{M}\ kHz$$

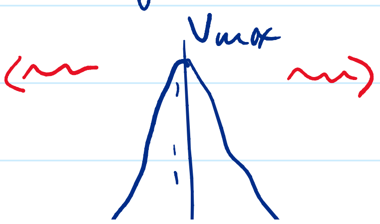
(choose $10M_\odot$ because of LV observations)

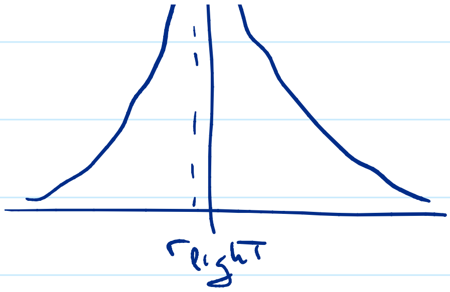
$$\tau = 0.55\,\frac{M}{10M_\odot}\ ms$$

$$\exp(-1/f\tau) \simeq 0.22$$

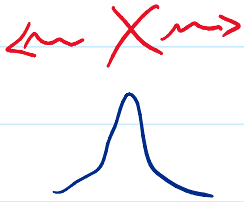
- Let us now gain some intuition about QNMs

They must match an ingoing wave and an outgoing wave

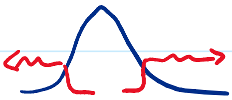




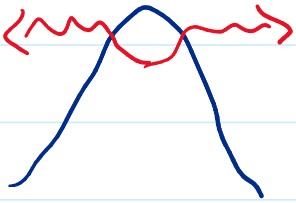
Can't do this if $\omega^2 \gg V_{max}$



For $\omega^2 \ll V_{max}$ the damping will be very large

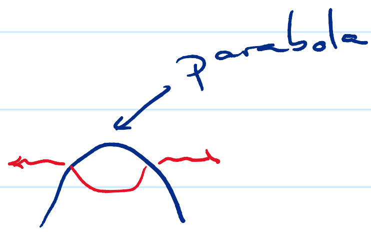


Optimal near the peak



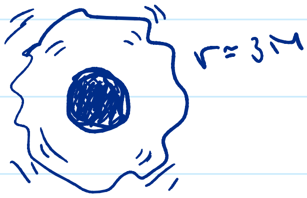
So:

- QNMs (least damped, dominant ones) are localized <u>near the light ring</u>
- May be computed in WKB approx:

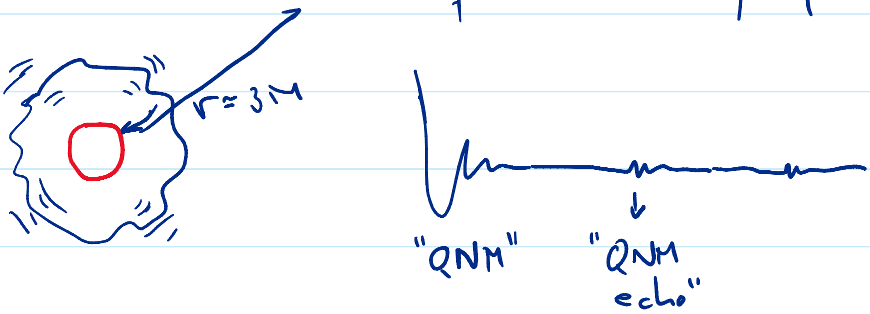


- Have higher frequency for higher ℓ (expected)
- Proper vibrations of the black hole should not be thought of as vibrations of the horizon (although it does vibrate): most of the vibration of the

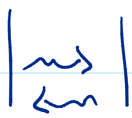
thought of as vibrating of the horizon (although it does vibrate): most of the vibration of the geometry occurs at a distance from the horizon (if too close, the vibration is quickly absorbed)



This is important: an exotic compact object (ECO) that is not a black hole (mimicker or impostor) will vibrate near the light ring, but may produce echoes if it is not perfectly absorbing.



QNMs and circular light rays

Normal modes can be regarded as stationary oscillations that travel bouncing around the system 

Similarly for bh, the circular photon ring, is a stationary trajectory for waves around the black hole. But it is unstable, so it has a characteristic decay time.

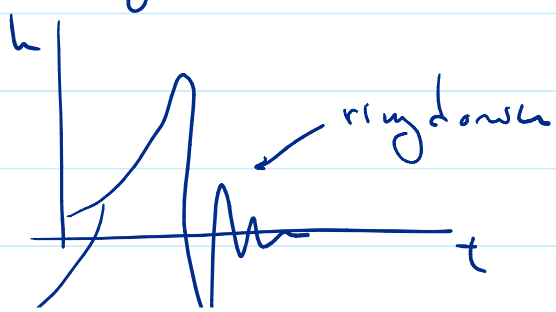
The frequency of a light ray in the photon ring gives ~ f of QNM

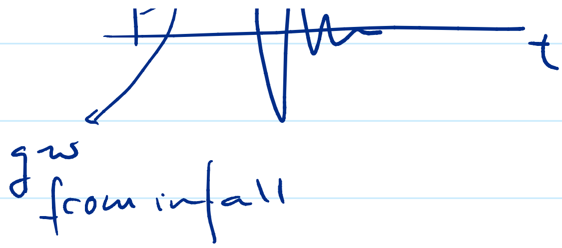
Orbit decay time ~ τ of QNM
(Lyapunov)

- QNM of Schwarzschild carry info about its mass
  Measuring f and τ of a single QNM provides two measurements of M
  (more interesting w/ rotation)

- In order to excite QNMs, one must "tickle" the region around the photon ring of the bh

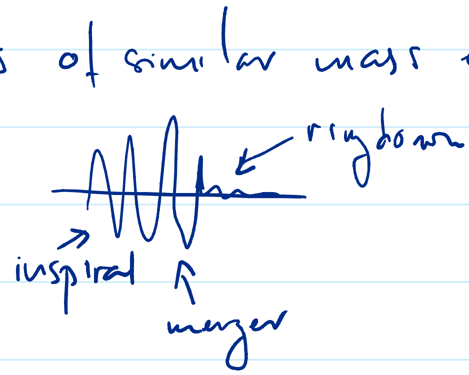
  - Accretion disk stops at $r = 6M$ (ISCO for Schw) so it doesn't excite much the QNMs
  - Infalling particles do excite the QNMs





- EMR inspirals excite little the qnms (EMR plunge would, but it's short and small)

- BH-BH mergers of similar mass excite QNMs the most



- Other properties of QNMs
  - unlike normal modes, they're not a complete basis of functions. There are "late-time tails" in the radiation that cannot be represented by QNMs
  - QNMs do not provide good initial data:

    if $\omega = 2\pi f - i/\tau$ w/ $\tau > 0$

    then $e^{-i\omega(t - r_*)}$ at constant t diverges at $r_* \to \infty$

- QNMs are also very important in AdS/CFT
  → relaxation of dual thermal plasma