Lecture 3: The black hole that spins (2/2)

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ERGOSPHERE

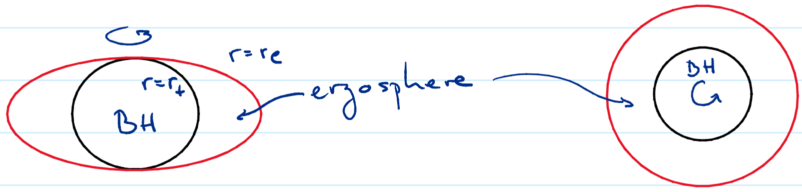
- The surface where $g_{tt} = 0$, ie, where $r^2 + a^2\cos^2\theta = 2Mr$ marks the static limit.

$r_e = M + \sqrt{M^2 - a^2\cos^2\theta} \geq r_H$

$r_e = r_H$ only at $\theta = 0, \pi$

The region $r_+ < r < r_e$ is called the ergosphere (or ergoregion)

from above



In the ergosphere ξ is spacelike : $\nexists$ static observers (relative to ∞)

Inside the ergosphere no observer can follow a trajectory parallel to $\xi = \partial/\partial t$, which would be static wrt infinity. So in the ergosphere no observer can remain at rest relative to infinity. They necessarily move along ϕ, since they are dragged by the rotation of the black hole.

Inside the ergosphere a particle that locally has positive energy (ie measured in its own reference frame) can have negative energy relative to observers at infinity.

Energy is conjugate to time, but a vector, like $\partial/\partial t$, which is timelike at ∞, can become spacelike in other regions, where $g_{tt} > 0$. Then in that region there can exist excitations that have -ve energy wrt ∞.
In Schw this happens only inside the horizon, but in Kerr this can happen outside the horizon, within the ergoregion w/ $r_H < r < r_e$

Penrose process and superradiance

The possibility of negative energies in a region outside the horizon allows a mechanism for extracting energy out of the black hole:

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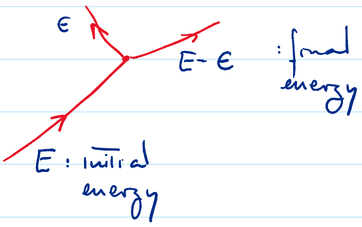
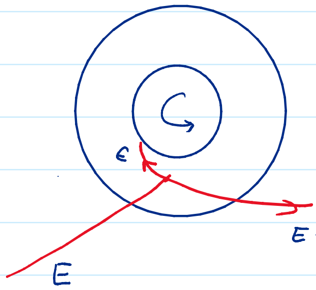
particle fission



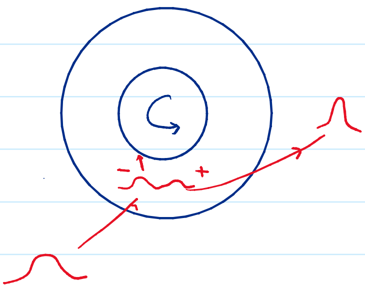
Fission in the ergosphere



May have $\epsilon < 0$

$E - \epsilon = E + |\epsilon| > E$: particle comes out w/ more energy than it came in!

If we use a wave field instead of particles, the phenomenon is called superradiance. The wave polarizes into +ve and -ve energy components



Flux out > Flux in if $0 < \omega \leq m\Omega_H$

$$\Phi = f(r,\theta)\, e^{-i\omega t + im\phi}$$

The wave extracts energy, but also angular momentum, and in fact it reduces the ratio J/M^2 : the bh spins down

SIDE REMARK

Superradiant amplification as stimulated emission of radiation

Superradiance can be interpreted as stimulated emission of radiation. It is related to spontaneous emission of radiation (Hawking radiation) by the usual relation between Einstein A and B coefficients, ie by detailed balance. Equivalently, if a mode (ω, ℓ, m) has a decay rate $\Gamma_{\ell m}(\omega)$, detailed balance requires that when there's an incident flux of the field $\mathcal{F}_{in}$, we have

$$\mathcal{F}_{in}\, \sigma_{\ell m}(\omega) + \Gamma_{\ell m}(\omega) = 0$$

where $\sigma_{\ell m}(\omega)$ is the absorption cross section of the mode.
Superradiant modes spontaneously decay, so $\Gamma_{\ell m}(\omega) > 0$ and so $\sigma_{\ell m}(\omega) < 0$.

One can prove using flux conservation that

$$\mathcal{F}_{out} - \mathcal{F}_{in} = C_\ell(\omega)\, \mathcal{F}_{in}\, \sigma_{\ell m}(\omega)$$

where $C_\ell(\omega) > 0$ is a universal coefficient (essentially a conversion factor between plane waves for the fluxes, and spherical waves for the absorption $\sigma_{\ell m}(\omega)$).

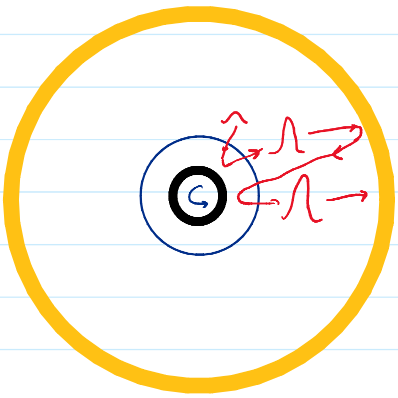
Then, since $\sigma_{\ell m}(\omega) < 0$, we have $\mathcal{F}_{out} > \mathcal{F}_{in}$: the outgoing wave is amplified.

For large occupation numbers, stimulated emission is a classical process. Spontaneous emission is always quantum.

See hep-th/0712.0791 for a microscopic interpretation of superradiance.

Superradiant instability (black hole bomb)

Imagine that outside black hole there is a barrier for wave propagation, eg field mass, or AdS potential



An initial perturbation can grow by bouncing back and forth between the ergoregion and the barrier, extracting energy and spin from the black hole and forming a long-lived cloud around the black hole (pseudo-hair)

Ultralight scalar fields can deplete spin from a black hole

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Perturbations of the Kerr black hole

Scalar wave equation: $\Phi = f(r,\theta)\, e^{-i\omega t + im\phi}$

$\Rightarrow$ PDE for $f(r,\theta)$: can separate into $R(r)\,\Theta(\theta)$ due to the existence of a hidden symmetry (a Killing tensor, and a conserved Carter constant).

$\Theta(\theta)$ are spheroidal harmonics, known numerically or approximately for small a.
Then can write down a radial equation for $R(r)$ and study the qnms of the scalar field.

Gravitational perturbations are much more complicated, a set of coupled equations, but, amazingly, it is possible to decouple them and obtain a single 2^{nd} order master equation for a gauge-invariant master variable $\Psi(r,\theta)$ from which all the metric perturbation can be recovered. This is the Teukolsky equation

- From the Teukolsky equation one can compute the spectrum of quasinormal frequencies, which are all stable:

$$\omega(M,J) - i\frac{1}{\tau(M,J)} \quad \left(= \frac{1}{M}\left(\omega(1, J/M^2) - \frac{i}{\tau(1, J/M^2)} \right) \right)$$

By measuring $\omega(M,J)$ and $\tau(M,J)$ for the slowest QNM we can determine M and J.

By measuring two QNMs, we can test the "Kerr hypothesis" (or "no-hair theorem"): all properties of the Kerr bh, and hence all QNMs, are fully determined by just knowing M and J.