Lecture 4: The black hole that evaporates (1/2)

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Particle production in an external field. We intend to study the production of particles in a given background field, due to fluctuations of a quantum field.

Virtual quantum fluctuations of a field with mass m are described by

$$\langle \phi(x)\phi(0) \rangle \sim e^{-x\frac{mc}{\hbar}} . \tag{2.2.2}$$

This is the probability amplitude that a pair of quanta of the field spontaneously form separated by a distance x. In vacuum, these fluctuations do not materialize in the production of a real pair since that would violate energy conservation. But the energy required for this materialization can be provided if there is an external field to which the field couples.

Call the field strength (force per unit charge) F, and the coupling (charge) λ. Assuming the field is uniform, in order to materialize the pair of quanta we need

$$\lambda F x = 2mc^2 . \tag{2.2.3}$$

Then the probability for pair creation per unit volume and unit time is

$$\Gamma \sim |\langle \phi(x)\phi(0) \rangle|^2 \sim e^{-\frac{4m^2c^3}{\hbar\lambda F}} . \tag{2.2.4}$$

A proper calculation in quantum field theory involves a tunneling process (hence the exponential suppression) which can be evaluated in the WKB approximation, and indeed gives

$$\Gamma \sim A\, e^{-\gamma\frac{\pi m^2 c^3}{\hbar\lambda F}} \tag{2.2.5}$$

where A is the quantum one-loop determinant factor, which we will ignore in the following, and γ is a numerical factor of order one, which depends on the specific type of particle and its coupling to the field.

This is a process of pair creation by a background field, where the latter is a semiclassical, coherent state involving a large number of quanta in the background. The process is described in terms of a non-perturbative instanton bounce. It is different than the perturbative process of pair creation, *e.g.*, e^+e^- creation by photon-photon collision. The non-perturbative e^+e^- production in a background electric field was studied by Schwinger in a classic paper in 1950. In this case $\lambda = e$, $F = E$, and $\gamma = 1$. Schwinger's leading order result is then

$$\Gamma_{e^+e^-} \sim A\, e^{-\frac{\pi m^2 c^3}{\hbar e E}} . \tag{2.2.6}$$

The energy for the creation of the pair is provided by the background field, which as a result decays gradually.

A black hole creates a strong gravitational field, so we might also expect particle pairs to form near the horizon. In this case the coupling is the particle's mass, while for the force we take the surface gravity,

$$\lambda = m , \qquad F = \kappa \tag{2.2.7}$$

so

$$\Gamma \sim e^{-\gamma\frac{\pi m c^3}{\hbar\kappa}} . \tag{2.2.8}$$

Virtual pairs are entangled excitations, correlated excitations.

$\langle \phi(x)\, \phi(0) \rangle \sim e^{-x/\xi}$

where ξ is the correlation length.

Mass gives finite range to quantum correlations (eg Yukawa), so

$\xi = \frac{\hbar}{m c}$

Surface gravity: heuristic argument

$\frac{\text{Force}}{m} = \kappa$

$\kappa = \frac{1}{4M}$ for Schwarzschild

Observe that the exponent is proportional to m, *i.e.*, to the energy $E = mc^2$ of the particle. Thus we can write it as

$$\Gamma \sim e^{-E/T_H} \tag{2.2.9}$$

where

$$T_H = \frac{\hbar\kappa}{\gamma\pi c}\,. \tag{2.2.10}$$

This is a *thermal spectrum with temperature* $T_H \propto \kappa$. So, the black hole is expected to radiate like a blackbody. In contrast, the Schwinger production rate (2.2.6) is not thermal. The reason that it is thermal in the case of a black hole is that gravity couples to the particle's energy. It is very suggestive that the universal character of gravity appears to be related to a universal thermal behavior.

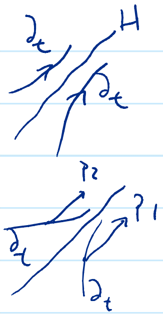
Some caveats about this heuristic argument:

- We have not pinned down the value of γ. For this we need Hawking's proper calculation, which yields $\gamma = 2$.
- The argument was made for massive particles. However, Hawking's result applies as well to massless quanta.
- The energy required by the creation of the pair is supplied by the black hole. The energy of the black hole decreases, since one of the members of the pair has negative energy (relative to asymptotic observers) and falls inside the black hole. In more detail, if the pair have four- momenta p_1 and p_2, four-momentum conservation requires that

$$p_1 + p_2 = 0\,. \tag{2.2.11}$$

If ∂_t is the generator of asymptotic time-translations, so that t is conjugate to the energy measured by asymptotic observers, then the energy of a particle with four-momentum p is

$$E = -p\cdot\partial_t\,. \tag{2.2.12}$$



Thus for the particle pair we must have $E_1 + E_2 = 0$. Now, if particle 1 is to escape to infinity, it must have $E_1 > 0$. If particle 2 goes inside the black hole, then in that region ∂_t is spacelike, so E_2 is actually not an energy but a component of momentum, which can be negative. Thus it is consistent to create the pair if one of the particles falls inside the black hole. In this case, the total mass of the black hole will decrease by an amount

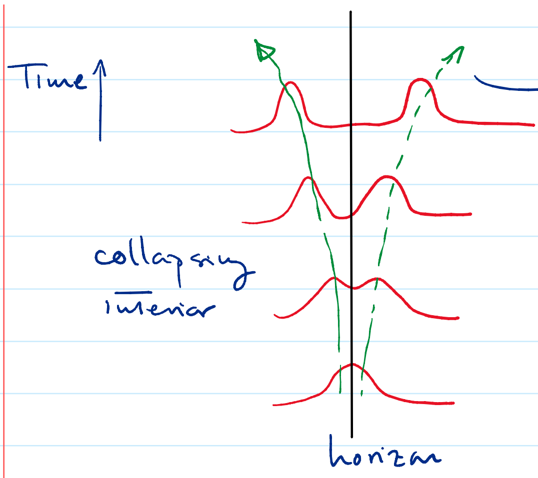
$$\delta M = E_2 = -E_1\,, \tag{2.2.13}$$

consistent with conservation of the energy as measured by outside observers.

- Wavefunctions probe the interior of the bh, which is dynamically collapsing in a time $\sim \frac{1}{\kappa}$

Then, even if the exterior of the black hole is static, and so normally wouldn't be able to excite a quantum field, a quantum wavefunction can have support simultaneously in the exterior and the interior, and therefore be sensitive to the interior time dependence.
The characteristic time of the interior collapse is $\sim 1/\kappa$, so the characteristic frequency of the field excitations will be $\sim \kappa$.

Indeed, the part of the wavefunction in the interior will be dragged on by the collapse, and be stretched:



Hawking radiation. Clearly, the Hawking radiation quanta will be entangled with quanta in the interior, since they're part of the same wavefunction.

This also shows that in order that Hawking radiation is emitted, the interior must be time-dependent.

The extremal RN black hole has a time-independent interior, but it also has $\kappa = 0$ and therefore $T=0$ so no radiation.

Pair production consists of an external field polarizing the wavefunction, eg electron-positron pair production. In the case of the gravitational field, around the horizon the wavefunction can be polarized into positive and negative energy components