Lecture 4: The black hole that evaporates (2/2)

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Major consequences:

- Black holes must have entropy
- Black holes evaporate

Entropy: ask Clausius: an object with energy E radiating at Temperature T has an entropy S that satisfies

$$dS = \frac{dE}{T}$$

The black hole has $E = M$ $\quad T = \frac{\hbar\kappa}{2\pi} = \frac{\hbar}{8\pi GM}$ $\quad (c=1)$

$$dS = \frac{8\pi G}{\hbar} M\, dM \Rightarrow S = \frac{4\pi G}{\hbar} M^2$$

$$= \frac{A_H}{4G\hbar} \qquad A_H = 4\pi (2GM)^2$$

$$\sim \frac{\text{Horizon area}}{\text{Planck area}}$$

Bekenstein-Hawking: $\boxed{S_{BH} = \frac{A_H}{4G\hbar}}$

Ask Boltzmann: BHs must have many microstates!

Black holes: simplest classical objects in the Universe

most complex quantum " " " "

Evaporation rate and black hole lifetime. The black hole will evaporate by emitting radiation like a blackbody. The radiating power of a blackbody of area A and temperature T is

$$\frac{dE}{dt} = \sigma A T^4 \tag{2.3.1}$$

where σ is the Stefan-Boltzmann factor, which depends on the specific (massless) fields that are being radiated. For a real scalar field, $\sigma = \pi^2/60$.

As a first approximation we can take $A \simeq A_H$. For a Schwarzschild black hole,

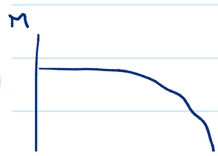
$$A_H = 16\pi M^2, \qquad T = \frac{1}{8\pi M} \tag{2.3.2}$$

and since $dE = -dM$ we can obtain the evaporation rate of the black hole as

$$\frac{dM}{dt} \simeq -\frac{\sigma}{256\pi^3}\frac{1}{M^2} \propto M^{-2} \tag{2.3.3}$$

so the total evaporation time

$$\int_0^{t_{evap}} dt \propto -\int_M^0 dM' M'^2 \tag{2.3.4}$$

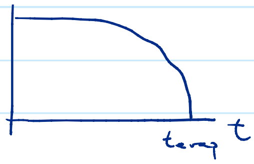


is

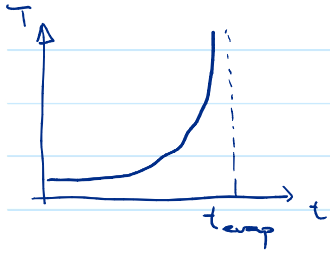
$$\int_0^{t_{evap}} dt \propto -\int_M^0 dM' \, M'^2 \tag{2.3.4}$$

is

$$t_{evap} \propto M^3 \,. \tag{2.3.5}$$



The black hole evaporates in a finite time. This calculation is of course very rough since it neglects the backreaction effect that the emission of radiation and loss of mass have on the black hole geometry and on the radiation process itself. These effects should be small when the energy of emitted quanta is much smaller than the mass of the black hole,



$$T_H \ll M \tag{2.3.6}$$

i.e., as long as $M \gg M_{Planck}$. Therefore, for most of the black hole lifetime the approximation is good, and its mass will reach Planck size in a time $\propto M^3$.

Astrophysical (ir)relevance of Hawking evaporation. Restoring full units,

$$T_H = \frac{\hbar c^3}{8\pi k_B G M} = 6 \times 10^{-8} \frac{M_\odot}{M} \,\mathrm{K} \tag{2.3.7}$$

so for a solar-mass black hole, $T_H \sim 10^{-7}$ K. This is much colder than the temperature (~ 3 K) of the CMB! Thus the accretion of CMB photons alone is a stronger effect for these black holes than Hawking evaporation. Of course it gets even worse for supermassive black holes.

The smallness of the effect should not be surprising: it is a quantum effect and therefore one expects it to be small for macroscopic objects (and for a black hole, macroscopic means larger than the Planck scale).

The initial mass of a black hole that started evaporating in the early universe and ends its evaporation today, so $t_{evap} \sim 10^{10}$ yr, is $M \sim 10^{15}$ g. This is roughly the mass of a kilometer-high mountain. Black holes with these masses are necessarily primordial, *i.e.*, formed by density fluctuations in the early universe, since astrophysical collapse cannot yield black holes with masses much below a solar mass (Chandrasekhar limit).

On the other hand, the entropy of astrophysical black holes is enormous:

$$S_{BH} = \frac{c^3}{\hbar G}\frac{\mathcal{A}_H}{4} \simeq 10^{76}\left(\frac{M}{M_\odot}\right)^2 . \tag{2.3.8}$$

A single galactic black hole, with $M \sim 10^{6-9} M_\odot$, has more entropy than all the matter and radiation in the universe ($S_{CMB} \sim N_{\text{photons}} \sim$ volume of universe in mm$^3 \sim 10^{87}$).

Black holes are small radiators. The wavelength of Hawking quanta is

$$\lambda_H \sim \frac{\hbar}{T_H} \sim GM \tag{2.3.9}$$

which is comparable to the Schwarzschild radius of the black hole (it had to be, since this is the only scale in the system). Including numerical factors, one in fact finds $\lambda_H \gtrsim R_{Schw}$.

The black hole is therefore a small radiator, with a size comparable to or smaller than the wavelength of the radiation. Thus Hawking radiation cannot be traced to any point on the horizon. The image one forms of a black hole from its Hawking quanta is a blurred one. This is unlike, *e.g.*, the Sun, whose size ($\sim 10^9$ m) is much larger than the wavelength of the radiation it emits (10^{2-3} nm), and therefore we can use it to get a detailed image of the star. Another consequence is that black holes radiate mostly in low-multipole waves.

Observe that the typical frequency and wavelength of Hawking quanta is the same as that of the classical quasinormal vibrations of the black hole.

The difference is that in quasinormal ringdown the occupation numbers are very large, while in Hawking emission they are of order one.

Scanning all the particle spectrum. During the black hole evaporation, T_H increases as M decreases, all the way until Hawking's approximations break down. Thus, in its evaporation the black hole will produce any particle that is permitted by local conservation laws, with mass possibly all the way up to the Planck energy. Thus, initially the black hole will radiate mostly photons, gravitons, and then neutrinos, and as it reaches different mass thresholds, all other particles will be produced: electrons and positrons around $T_H \sim 1$ MeV, mesons at $T_H \sim O(100)$ MeV, nucleons at $T_H \sim 1$ GeV, Higgs bosons at ~ 126 GeV, then X? at ???ev etc.

However, given that $t_{evap} - t \sim M^3$, we have $T_H \sim (t_{evap} - t)^{-1/3}$, and therefore the black hole spends most of its lifetime, and releases most of its energy, emitting low-energy quanta in copious quantities. By the time it reaches the threshold to produce more interesting massive stuff, little energy is left and few of these particles are produced.

Negative specific heat. The specific heat of the black hole is negative:

$$C = \frac{dE}{dT} = \frac{dM}{dT_H} = -8\pi M^2 < 0 . \tag{2.3.10}$$

This means that the black hole is thermodynamically unstable. The black hole heats up by radiating energy, and cools down by absorbing it. So if we try to keep it in equilibrium with a radiation bath at temperature T_H, then if the black hole absorbs a little more energy than required, it will become cooler than the bath, and then it will tend to absorb more and get further away from equilibrium. Conversely, if it absorbs less energy than demanded by equilibrium, it will heat up and radiate even more.

This is unlike conventional thermodynamic systems in equilibrium, but it is in fact typical of gravitating systems. For instance, a star that emits radiation reduces its pressure and contracts, which raises its temperature[16]. In fact, this property of gravitating systems to increase their entropy by becoming more concentrated is *absolutely crucial* for the universe to evolve structure from an initial thermal, almost homogeneous state.

In these arguments we have been considering the Schwarzschild black hole. Indeed the thermodynamic instability is typical of vacuum black holes. But there do exist other black hole solutions with positive specific heat, which are thermodynamically stable. One way to achieve this change is by adding charge to the black hole: the Reissner-Nordstrom solution near its extremal limit has positive specific heat. Another possibility is to put the black hole in a box, say by limiting the radial coordinate to be smaller than some $r_{\mathrm{box}} > r_{Schw}$. For a given temperature of the box, there are two possible black holes with that temperature, a small and a large one. The small one hardly feels the presence of the box and is qualitatively similar to the one in asymptotically flat space. The large one feels, so to speak, that radiation cannot easily fit in the box and is thermodynamically stable. A covariant version of this box is provided by Anti-deSitter spacetime, which has a confining effect on radiation. Black holes in AdS with a size larger than the cosmological radius have positive specific heat and play an important role in the AdS/CFT correspondence.