COST ACTION QGMM — TRAINING SCHOOL – BELGRADE

COURSE 5 – GAMMA-RAY DATA COLLECTION AND ANALYSIS

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This exercice was initially prepared by M. de Naurois from LLR, École Polytechnique, and adapted by J. Bolmont.

1. From the Heitler model to photo-electrons

1.1. The shower

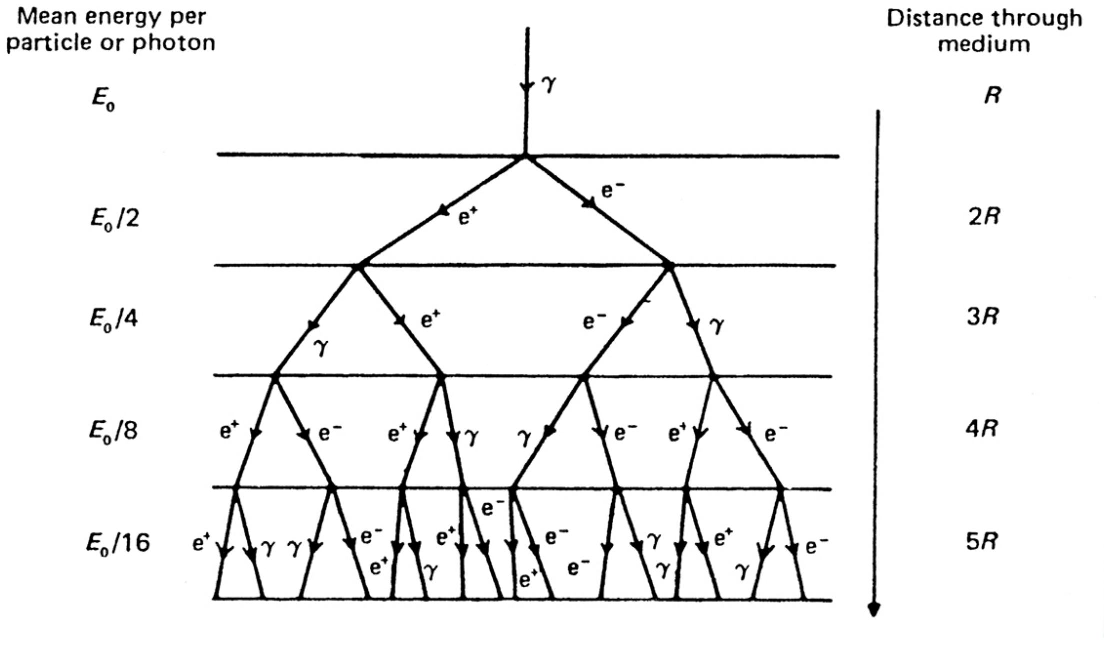

FIGURE 1 – The Heitler model for an electromagnetic shower. From Longair Vol. 1, second edition, Cambridge University Press, 1992.

The Heitler model for an electromagnetic (EM) shower relies on the hypothesis that very high energy photons interact only through pair production, and that each electron interacts only through Bremsstrahlung, transferring its energy to a single photon. So, the primary gamma-ray interacts creating an electron-positron pair, each of which generate high energy photons, which interact through pair creation, etc. Figure 1 gives an illustration of this process. At each interaction, the energy of each particle is spread equally between the two particles produced.

Both Bremsstrahlung and pair production can be considered having the same characteristic length, which is the *radiation length*, noted X_0. In dry air, $X_0 \approx 36.7\ \mathrm{g\,cm^{-2}}$. According to the PDG Booklet edition of 2014 (p. 258), radiation length is "both (a) the mean distance over which a high-energy electron loses all but $1/e$ of its energy by Bremmstrahlung, and (b) $7/9$ of the mean free path for pair production by a high energy photon". Considering $7/9 \approx 1$, electrons and photons average energy will decrease following the relation

$$E(x) = E_0\, e^{-x/X_0},$$

where x is the range of penetration ($\mathrm{g\,cm^{-2}}$) in the medium, in our case the atmosphere, and where E_0 is the energy of the primary photon.

1.1 — From the indications available on Fig. 1, give the relation between the range (or depth) R traveled through the medium and X_0.

1.2 — $n \geqslant 0$ being an integer, what is the number of particles N (electrons, positrons and photons) in the shower at a depth $X = nR$? Give the expression as a function of X and R, and as a function of X and X_0.

1.3 — What is the energy of individual particles? Give the expression as a function of X and R, and as a function of X and X_0.

The above considerations are valid only when Bremsstrahlung and pair creation processes dominate. However, since particles lose their energy at each step, there will be a point where this condition will not be true anymore. Instead of Bremsstrahlung, electrons will lose their energy through ionisation losses. Photons will interact by Compton scattering and photo-electric absorption rather than by pair production. The (smooth) transition happens at the so-called "critical energy", noted E_c. In the following, we'll take $E_c = 87$ MeV, corresponding to dry air at 1 a.t.m. Note that in the Heitler model, once the critical energy is reached, ionisation losses become catastrophically high, leading to the fact that all electrons and positions quickly disappear. Therefore the shower has no tail after it reaches its maximum...

1.4 — What is the depth of maximum shower development X_{max}?

1.5 — Give the number of particles $N(X_{\mathrm{max}})$ in the shower at the maximum development. It is possible to show that $\sim 2/3$ of the total number of particles are positrons and electrons, while $1/3$ are photons.

We now want to convert the depth of maximum development into an altitude. For this, we will consider atmosphere is a hydrostatic perfect gas, with pressure and density given by

$$P = P_0\, e^{-z/z_0}, \text{ and}$$

$$\rho = \rho_0\, e^{-z/z_0},$$

with $\rho_0 = 1.2\ \mathrm{kg\,m^{-3}}$ at $P_0 = 1$ a.t.m., and taking $z_0 = RT/gM = 8.4$ km.

1.6 — Give the expression of depth X as a function of altitude z, ρ_0 and z_0. Remember the depth is expressed in $\mathrm{g\,cm^{-2}}$.

1.7 — Give the altitude of the maximum shower development. Give the numerical value for $E_0 =$ 1 TeV and 10 TeV.

2. Cherenkov emission

Charged particles traveling faster than the speed of light in a medium of refractive index n emit Cherenkov photons on a cone of angle θ such as

$$\cos\theta = \frac{c}{n\,v} \approx \frac{1}{n},$$

with $v \approx c$ being the velocity of the charged particle. The distribution of Cherenkov photons over the wavelength λ and charged particle track length x is given by

$$\frac{d^2N}{dxd\lambda} = 2\pi\,\alpha\,\frac{\sin^2\theta}{\lambda^2}.$$

We will consider the temperature is 288.15 K. The refractive index will be taken as

$$n - 1 = 2.92 \times 10^{-4}\,\frac{P}{P_0}\,\frac{288.15\ \mathrm{K}}{T}.$$

2.1 — From the expressions given above, and considering that $T = 288.15$ K, give the expression for the energy spectrum of Cherenkov photons as a function of z and z_0. Note that θ is usually very small, as we will see later.

2.2 — Give the expression of the Cherenkov angle (noted θ_c) as a function of z and z_0.

2.3 — Deduce the approximate expression of the radius of the Cherenkov light pool R on the ground at sea level as a function of z and z_0. Give also the maximum $R_{\max}$ of this expression. Give the numerical value for a 1 TeV primary photon.

2.4 — From the above, deduce the number of Cherenkov photons emitted by charged particles above wavelength λ_0. The integration over the track length will be simplified using a substitution with $t = X/X_0$.

2.5 — Considering the photo detectors usually used in Cherenkov telescopes are sensitive only above 300 nm, give the number of Cherenkov photons as a function of E_0 and E_c. Give the numerical value for $E_0 = 1$ TeV.

3. From Cherenkov light to photo electrons

The Heitler model is obviously over-simplified, because it only takes into account only pair production and Bremsstrahlung, thus ignoring shower tails. In a more accurate formalism such as the semi analytical model by Greisen, the predicted number of Cherenkov photons is 2.9×10^8 photons per TeV. As seen above, all these photons will fall into a circular area with a radius $R_{\max}$ (question 2.3). However, some of them will be absorbed in the lower layers of the atmosphere due to Rayleigh scattering. The corresponding average transmission above 300 nm is about $\epsilon_{ray} = 75\%$.

3.1 — Estimate the number of photo electrons produced by a primary photon of 1 TeV in the camera of a 100 m^2 telescope with an optical efficiency of $\epsilon_o = 10\%$.