Getting hot without accelerating:

vacuum thermal effects from conformal quantum mechanics

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In quantum field theory the notion of **vacuum state** has no universal meaning

"Before the 70s nobody thought very much about "for whom" the vacuum state appears devoid of "stuff"..."

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the **vacuum state** for **inertial observers** is perceived
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- **vacuum state** ambiguity = different possible choices of **time-like Killing vectors**
- needed to decompose the **space of solutions** of the equations of motion into **positive and negative energy subspaces**
- used to define the **one-(anti)particle Hilbert space, Fock space and vacuum**

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This talk: give a unified, group-theoretical description of **Milne and diamond temperature** using a correspondence between **radial conformal symmetries** in Minkowski space-time and **time evolution in conformal quantum mechanics**

MA, JHEP **05**, 072 (2020) [arXiv:2002.01836 [gr-qc]], JHEP **07**, 003 (2021) [arXiv:2103.07228 [hep-th]]

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Minkowski metric in spherical coordinates

$$ds^2 = -dt^2 + dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$$

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imposing that

$$\mathcal{L}_\xi \eta_{\mu\nu} \propto \eta_{\mu\nu}$$

implies that ξ is independent of θ and ϕ and it has the general form

$$\xi = \Big(a(t^2 + r^2) + bt + c\Big)\,\partial_t + r(2at + b)\,\partial_r$$

with a, b, c real constants **(Herrero and Morales, J. Math. Phys. 40, 3499 (1999))**

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1st key observation: this conformal Killing vector can be written as

$$\xi = aK_0 + bD_0 + cP_0\,,$$

where K_0, D_0 and P_0 generate, respectively, special conformal transformations, dilations and time translations

Families of radial conformal Killing vectors

The generators K_0, D_0 and P_0

$$P_0 = \partial_t \,, \qquad D_0 = r\,\partial_r + t\,\partial_t \,, \qquad K_0 = 2tr\,\partial_r + (t^2 + r^2)\,\partial_t$$

close the $\mathfrak{sl}(2,\mathbb{R})$ Lie algebra

$$[P_0, D_0] = P_0 \,, \qquad [K_0, D_0] = -K_0 \,, \qquad [P_0, K_0] = 2D_0$$

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- $\Delta < 0$: **elliptic transformations** ($\mathfrak{sl}(2,\mathbb{R}) \simeq \mathfrak{so}(2,1) \to$ rotations)

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α **is a constant with dimensions of length**, crucial in what follows...

Milne and diamond times

D_0 generates **conformal time evolution** in a Milne universe (**the future cone**)

$$ds^2 = -d\bar{t}^{\,2} + \bar{t}^{\,2}\left(d\chi^2 + \sinh\chi^2 d\Omega^2\right)$$

with $t = \bar{t}\cosh\chi$ and $r = \bar{t}\sinh\chi$ (notice similarity with Rindler coordinates...)

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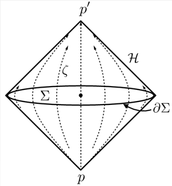

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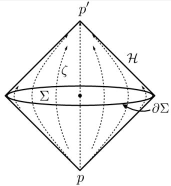

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S_0 generates evolution in **diamond time**:
the proper time of **uniformly accelerated observers with finite lifetime**

Worldline conformal time evolution

Along $r = \text{const}$ worldlines and on the light cones $u = t - r = \text{const}$, $v = t + r = \text{const}$

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covering **half time line** ($\tau > 0$ or $\tau < 0$)

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- $S_0 = \frac{1}{2\alpha} \left(\alpha^2 - \tau^2 \right) \partial_\tau$ generates translation in **"diamond time"** σ: $S_0/\alpha = \partial_\sigma$

  $$\tau = \alpha \, \tanh \sigma / 2\alpha$$

  covering the region $|\tau| < \alpha$: the "diamond"

Conformal quantum mechanics

As it turns out

$$G = i\xi = i\left(a\,\tau^2 + b\,\tau + c\right)\partial_\tau$$

is the most general **generator of time evolution in "conformal quantum mechanics"**.

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Starting from the Lagrangian

$$\mathcal{L} = \frac{1}{2}\left(\dot{q}(t)^2 + \frac{g}{q(t)^2}\right)\,, \qquad g > 0$$

the $\mathfrak{sl}(2,\mathbb{R})$ algebra can be canonically realized

$$\begin{aligned} H = iP_0 &= \frac{1}{2}\left(p^2 + \frac{g}{q^2}\right) \\ D = iD_0 &= t\,H - \frac{1}{4}\left(pq + qp\right) \\ K = iK_0 &= -t^2\,H + 2t\,D + \frac{1}{2}q^2 \end{aligned}$$

Conformal quantum mechanics as a CFT_1

Tha dAFF model can be interpreted as CFT_1

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$$L_\pm = \frac{1}{2}\,\left(\frac{K}{\alpha} - \alpha\,H\right)\,\pm i\,D\,, \qquad L_0 = \frac{1}{2}\,\left(\frac{K}{\alpha} + \alpha\,H\right)$$

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CFT_1 two-point function

The $|\tau\rangle$ states can be characterized by their overlap with $|n\rangle$ states

$$\langle \tau | n \rangle = (-1)^n \left[\frac{\Gamma(2r_0 + n)}{n!} \right]^{\frac{1}{2}} \left(\frac{\alpha - i\tau}{\alpha + i\tau} \right)^{r_n} \left(1 + \frac{\tau^2}{\alpha^2} \right)^{-r_0}$$

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For $r_0 = 1$: two-point function of a **massless scalar field in Minkowski space-time**, evaluated along the **worldline of an inertial observer** sitting at the origin

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For $r_0 = 1$: two-point function of a **massless scalar field in Minkowski space-time**, evaluated along the **worldline of an inertial observer** sitting at the origin

This is reminiscent of the $SL(2,\mathbb{R})$-invariant **wordline quantum mechanics** for **static patch observers** in de Sitter space-time

(Anninos, Hartnoll and Hofman, Class. Quant. Grav. 29, 075002 (2012))

A bi-partite vacuum state

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so that

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the ground state $|n = 0\rangle$ has a **bi-partite structure**!

CFT_1 vacua

Notice now that the Lie algebra

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can be realized via another combination of H, D and K, namely

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as in the "real world" the Hartle-Hawking vacuum is a **thermofield double state** built on the bi-partite Boulware vacuum

The thermofield double of CFT_1

With simple manipulations

$$
\begin{aligned}
|\tau = 0\rangle &= \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \left(a_L^\dagger a_R^\dagger\right)^n |0\rangle_L \otimes |0\rangle_R = \sum_{n=0}^{\infty} (-1)^n |n\rangle_L \otimes |n\rangle_R \\
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mini-detour: given a set of eigenstates $H|k\rangle = E_k|k\rangle$ for a quantum system, the **thermofield double state** is built by "doubling" the system

$$|TFD\rangle = \frac{1}{Z(\beta)} \sum_{k=0}^{\infty} e^{-\beta E_k/2} |k\rangle_L \otimes |k\rangle_R$$

tracing over the degrees of freedom of one copy $\Rightarrow$ thermal density matrix at $T = 1/\beta$

$$Tr_L(|TFD\rangle\langle TFD|) = e^{-\beta H}$$

Diamond temperature

The inertial vacuum

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has the structure of a thermofield double state with temperature

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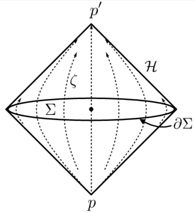
This is just the **diamond temperature** for diamond observers at the origin

(Su and Ralph, Phys. Rev. D 93, no.4, 044023 (2016))



From the diamond to Milne

S and D belong to the same class of generators of **hyperbolic time evolution**

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note: this is the map from the **causal diamond to the Rindler wedge** used to derive the **diamond modular Hamiltonian** from the Rindler one (in light-cone coordinates)

(Casini, Huerta and Myers, JHEP 05, 036 (2011))

The Milne temperature

The conformal map

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the "inertial" vacuum $|\tau=0\rangle$ is the thermofield double for the Hamiltonian D/α at the **Milne temperature** **(Olson and Ralph, PRL 106, 110404 (2011), arXiv:1003.0720)**

$$T_D = \frac{1}{2\pi\alpha}$$

Getting hot without accelerating

Observers whose worldlines are **integral curves of time-like RCKV**

$$\xi = aK_0 + bD_0 + cP_0$$

are **accelerated** **(Herrero and Morales, J. Math. Phys. 40, 3499 (1999))**

$$|\mathbf{a}| = \frac{2|a|}{\sqrt{\omega - \Delta}}$$

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You can get hot without accelerating!
(if you enjoy conformal symmetry...)

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$\Longrightarrow$ **thermodynamic** properties of the **Milne "patch"** and of **causal diamonds** are deeply connected...
new tools for studying **entanglement in Minkowski space-time?**


THANK YOU!

APPENDIX

- AdS_2: H and S as different generators of time evolution

(Järvelä, Keränen, Keski-Vakkuri, Phys. Rev. D**93**, no.4, 046002 (2016) [arXiv:1509.01092 [hep-th]])

vacua of AdS_2 black holes

(Spradlin and Strominger, JHEP **11**, 021 (1999), [arXiv:hep-th/9904143 [hep-th]])