Collinear approximation

Decay of superluminal neutrinos in a LIV scenario

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Contents

Introduction

The search for a Quantum Gravity theory leads to change our conception of spacetime.

This way, one finds the need to go beyond Lorentz Invariance, either considering a breaking of the symmetry,

- Lorentz Invariance Violation (LIV),

or a deformation of it,

- Doubly Special Relativity (DSR).

Introduction

In those scenarios, one expect deviations from Special Relativity (SR) that stem from the particles' kinematics,

> like time delays, or the appearance, disappearance, or shifting of threshold energies of some interactions,

and dynamics,

> such as changes in the interaction rates and energy distribution of some interactions.

In order to study the dynamics, one needs a well defined Quantum Field Theory (QFT) to derive the Feynman rules in order to compute the modified cross sections and decays widths.

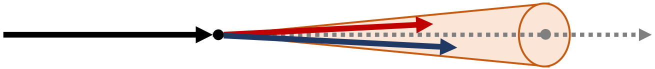
- In the case of DSR, a dynamical framework does not exist yet.
- In the case of LIV, one can use the Standard Model Extension (SME), which is constructed adding Lorentz violating terms to the Standard Model (SM) Lagrangian.

In SR one can simplify the calculation of cross sections and decays widths using the Center of Mass (CM) reference frame.

In the case of LIV, there exists one preferred system of reference in which the laws of physics must be studied. Then, the calculations must be done in that specific reference system.

Considering the study of very-high-energy astrophysical messengers, one can make an approximation that can simplify the calculations, both in SR and in the LIV scenario,

the collinear approximation.



Under this approximation, one uses the property that the particles produced after some interaction follow “almost” the same direction as the parent particle.

Using this approximation, we will present a general analysis of a neutrino three body decay.

Then we will apply the results to the two neutrino vacuum decays allowed in a LIV scenario:

- the production of electron-positron pairs,

$$\boxed{\nu \ \rightarrow \ \nu \, e^- \ e^+},$$

- and of neutrino-antineutrino pairs,

$$\boxed{\nu \ \rightarrow \ \nu \, \nu \, \bar{\nu}}.$$

Collinear approximation: neutrino three body decay

Collinear approximation: neutrino three body decay

We consider a scenario in which the only effect of LIV is a Lorentz violating term in the free neutrino Lagrangian,

$$\mathcal{L}^{(\nu)}_{\text{LIV}} = -\frac{1}{\Lambda^n}\,\overline{\nu}_L\gamma^0\,(i\partial_0)^{n+1}\,\nu_L\,, \tag{1}$$

where we assume no flavor dependence. This way, the LIV term is completely defined by the scale of new physics Λ and the order of correction $n = 1, 2$.

One can introduce a plane wave expansion for the neutrino field

$$\nu_L(t,\vec{x}) \,=\, \int d^3\vec{p}\, \left[b_{\vec{p}}\, u(\vec{p})\, e^{-iE_- t+i\vec{p}.\vec{x}} + d^\dagger_{\vec{p}}\, v(\vec{p})\, e^{iE_+ t-i\vec{p}.\vec{x}}\right]\,, \tag{2}$$

where $u(\vec{p})$ and $v(\vec{p})$ are Dirac spinors that, in order to solve the equations of motion, must obey

$$\left[\gamma^0\,\tilde{E}_- \,-\, \vec{\gamma}.\vec{p} \,-\, \gamma^0\,\frac{E_-^{n+1}}{\Lambda^n}\right]\, u(\vec{p}) \,=\, 0\,, \tag{3}$$

$$\left[\gamma^0\,\tilde{E}_+ \,-\, \vec{\gamma}.\vec{p} \,+\, (-1)^{n+1}\gamma^0\,\frac{E_+^{n+1}}{\Lambda^n}\right]\, v(\vec{p}) \,=\, 0\,. \tag{4}$$

Identifying E_- and E_+ with the energy of a neutrino and antineutrino, respectively, one gets that their modified energy-momentum relations are:

$$|\vec{p}| \approx E_- \left[1 - \left(\frac{E_-}{\Lambda}\right)^n\right], \tag{5}$$

$$|\vec{p}| \approx E_+ \left[1 + (-1)^{n+1} \left(\frac{E_+}{\Lambda}\right)^n\right]. \tag{6}$$

Collinear approximation: neutrino three body decay

In the case of a very-high-energy three-body decay,

$$A(\vec{p}) \to B_1(\vec{p}_1) \,+\, B_2(\vec{p}_2) \,+\, B_3(\vec{p}_3)\,, \tag{7}$$

in which the parent particle is a neutrino, one would have (neglecting masses)

$$|\vec{p}_i| \;\approx\; E_i \,\left[1 + \alpha_i \left(\frac{E_i}{\Lambda}\right)^n\right]\,, \tag{8}$$

for the particles of the final state ($\alpha_i = (-1)$ if it is a neutrino, $(-1)^{n+1}$ if it is an antineutrino, and zero if it is another particle).

Collinear approximation: neutrino three body decay

The decay width will be given by

$$\Gamma = \frac{1}{2E_0} \int_{\mathcal{PS}} |\mathcal{A}|^2 d\mathcal{PS} \tag{9}$$

where E_0 is the energy of the parent particle, $|\mathcal{A}|^2$ is the spin averaged squared decay amplitude

$$|\mathcal{A}|^2 \approx \mathcal{N}\, |\vec{p}|\, |\vec{p}_1|\, |\vec{p}_2|\, |\vec{p}_3|\, (1 - \widehat{p}.\widehat{p}_1)\, (1 - \widehat{p}_2.\widehat{p}_3)\,, \tag{10}$$

and $d\mathcal{PS}$ is a differential region of the phase space.

Collinear approximation: neutrino three body decay

Under the collinear approximation, the angles between the particles of the final state are "very small". More explicitly,

$$(1 - \widehat{p}.\widehat{p}_1)\,(1 - \widehat{p}_2.\widehat{p}_3) \sim O(1/\Lambda^{2n})\,. \tag{11}$$

Then, $|\mathcal{A}|^2 \sim O(1/\Lambda^{2n})$ and the phase space can be written at dominant order,

$$d\mathcal{PS} \;\approx\; \left[\prod_{i=1}^{3} \frac{d^3\vec{p}_i}{(2\pi)^3\, 2|\vec{p}_i|}\right]\,(2\pi)^4\,\delta(E - \sum_i E_i)\,\delta^3(\vec{p} - \sum_i \vec{p}_i)\,. \tag{12}$$

Substituting in the decay width, one obtains,

$$\Gamma \approx \frac{\mathcal{N}}{16} \frac{1}{(2\pi)^5} \int \left[\prod_{i=1}^{3} d^3\vec{p}_i\right] \delta(E - \sum_i E_i)\, \delta^3(\vec{p} - \sum_i \vec{p}_i) \\ (1 - \widehat{p}.\widehat{p}_1)\,(1 - \widehat{p}_2.\widehat{p}_3)\,, \tag{13}$$

where some additional simplifications can be applied, related with the fact that all the momenta are almost parallel,

$$\vec{p} - \vec{p}_1 = \vec{p}_2 + \vec{p}_3 \quad \rightarrow \quad |\vec{p}| - |\vec{p}_1| \approx |\vec{p}_2| + |\vec{p}_3|\,. \tag{14}$$

Collinear approximation: neutrino three body decay

Then, one finally finds that the decay width can be written as

$$\Gamma \approx \frac{\mathcal{N}}{96} \frac{E^5}{(2\pi)^3} \left(\frac{E}{\Lambda}\right)^{3n} \int \left[\prod_i dx_i\right] \delta(1-\sum_i x_i)(1-x_1)^2 \left[1+\sum_i \alpha_i x_i^{n+1}\right]^3 , \quad (15)$$

where $x_i = E_i/E_0$ is the fraction of the initial energy inherited by the i-th particle of the final state.

We will use this result in the following sections.

Neutrino Vacuum Pair Production

Neutrino Vacuum Pair Production

The electron-positron pair can be emitted through a neutral channel or a charged channel:

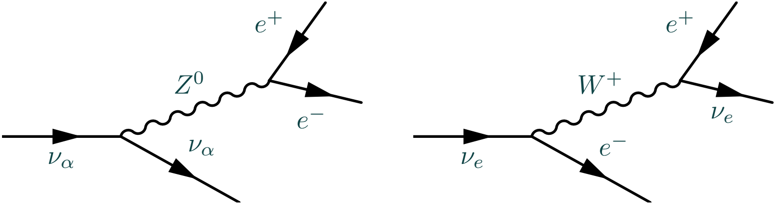


Figure 1: Disintegration of a neutrino into a pair electron-positron and another neutrino, mediated by a Z^0 boson (left) or by a W^+ boson (right).

The spin averaged squared decay amplitude of muon and tau neutrinos to left-handed and right-handed electrons are given by

$$\left|\mathcal{A}_{\nu_{\mu,\tau}(\vec{p})\to\nu_{\mu,\tau}(\vec{p}\,')e^-_L(\vec{p}_-)e^+_L(\vec{p}_+)}\right|^2 = \left(\frac{g^2}{M_W^2}\right)^2 (s_W^2 - 1/2)^2\, 4\,(p.p_+)(p'.p_-)\,, \tag{16}$$

$$\left|\mathcal{A}_{\nu_{\mu,\tau}(\vec{p})\to\nu_{\mu,\tau}(\vec{p}\,')e^-_R(\vec{p}_-)e^+_R(\vec{p}_+)}\right|^2 = \left(\frac{g^2}{M_W^2}\right)^2 (s_W^2)^2\, 4\,(p.p_-)(p'.p_+)\,, \tag{17}$$

which only includes the neutral channel.

Neutrino Vacuum Pair Production

For the electron neutrinos, the right-handed contribution is the same as in the case of muon and tau neutrinos, but the left-handed one now must include both channels, charged and neutral, i.e.

$$\left|\mathcal{A}_{\nu_e(\vec{p})\to\nu_e(\vec{p}\,')e^-_L(\vec{p}_-)e^+_L(\vec{p}_+)}\right|^2 = \left(\frac{g^2}{M_W^2}\right)^2 (s_W^2 - 3/2)^2 4(p.p_+)(p'.p_-)\,, \tag{18}$$

$$\left|\mathcal{A}_{\nu_e(\vec{p})\to\nu_e(\vec{p}\,')e^-_R(\vec{p}_-)e^+_R(\vec{p}_+)}\right|^2 = \left(\frac{g^2}{M_W^2}\right)^2 (s_W^2)^2\, 4\,(p.p_-)(p'.p_+)\,. \tag{19}$$

Using that

$$(p.p_+)(p'.p_-) \approx |\vec{p}|\,|\vec{p}'|\,|\vec{p}_-|\,|\vec{p}_+|\,(1-\hat{p}.\hat{p}_+)\,(1-\hat{p}'.\hat{p}_-)\,,$$
$$(p.p_-)(p'.p_+) \approx |\vec{p}|\,|\vec{p}'|\,|\vec{p}_-|\,|\vec{p}_+|\,(1-\hat{p}.\hat{p}_-)\,(1-\hat{p}'.\hat{p}_+)\,, \tag{20}$$

we see that the decay amplitudes have the general form assumed in the collinear approximation

$$|\mathcal{A}|^2 \approx \mathcal{N}\,|\vec{p}|\,\vec{p}_1|\,\vec{p}_2|\,\vec{p}_3|\,(1-\widehat{p}.\widehat{p_1})\,(1-\widehat{p_2}.\widehat{p_3})\,, \tag{10}$$

Using the result of the collinear approximation and adding the left-handed and right-handed contributions, one gets that for muon and tau neutrinos

$$\Gamma_{\nu_{\mu,\tau}(E)\to\nu_{\mu,\tau}\, e^- \, e^+} \approx \left(\frac{g^2}{M_W^2}\right)^2 \frac{E^5}{192\,\pi^3} \left(\frac{E}{\Lambda}\right)^{3n} \left[\left(s_W^2 - \frac{1}{2}\right)^2 + (s_W^2)^2\right] c_n^{(e)}\,, \tag{21}$$

$$c_n^{(e)} \doteq \left[\frac{1}{4} - \frac{3}{(n+2)(n+5)} + \frac{3}{(2n+3)(2n+6)} - \frac{1}{(3n+4)(3n+7)}\right], \tag{22}$$

reproducing the results of previous works [2].

Neutrino Vacuum Pair Production

And for the electron neutrinos

$$\Gamma_{\nu_e(E)\to\nu_e\, e^-\, e^+} \approx \left(\frac{g^2}{M_W^2}\right)^2 \frac{E^5}{192\,\pi^3} \left(\frac{E}{\Lambda}\right)^{3n} \left[\left(s_W^2 - \frac{3}{2}\right)^2 + (s_W^2)^2\right] c_n^{(e)}\,, \tag{23}$$

which differs from the muon and tau neutrinos only by a constant factor. As a consequence, the neutrino final energy distribution is flavour independent,

$$\mathcal{P}^{e-e+}_{\nu_e/\nu_e}(x) = \mathcal{P}^{e-e+}_{\nu_\mu/\nu_\mu}(x) = \mathcal{P}^{e-e+}_{\nu_\tau/\nu_\tau}(x) \approx \frac{1}{3\,c_n^{(e)}}\,(1-x^{n+1})^3\,(1-x^3)\,. \tag{24}$$

Neutrino Splitting

Neutrino Splitting

The neutrino-antineutrino pair can be emitted through a neutral channel only:

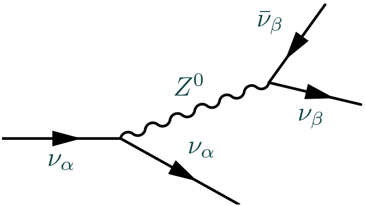


Figure 2: Disintegration of a neutrino into two neutrinos and one antineutrino, mediated by a Z^0 boson.

Neutrino Splitting

The spin averaged squared decay amplitude is given by

$$\left|\mathcal{A}_{\nu_\alpha(\vec{p})\to\nu_\alpha(\vec{p}')\,\nu_\beta(\vec{p}_-)\,\bar{\nu}_\beta(\vec{p}_+)}\right|^2 = \left(\frac{g^2}{M_W^2}\right)^2 (p.p_+)(p'.p_-)\,, \tag{25}$$

which, once more, has the general form assumed by the collinear approximation,

$$|\mathcal{A}|^2 \approx \mathcal{N}\,|\vec{p}|\,\vec{p}_1|\,\vec{p}_2|\,\vec{p}_3|\,(1-\widehat{p}.\widehat{p}_1)\,(1-\widehat{p}_2.\widehat{p}_3)\,. \tag{10}$$

Neutrino Splitting

Using the result of the collinear approximation, one gets that

$$\Gamma_{\nu_\alpha(E)\to\nu_\alpha\nu_\beta\bar{\nu}_\beta} \approx \left(\frac{g^2}{M_W^2}\right)^2 \frac{E^5}{192\,\pi^3} \left(\frac{E}{\Lambda}\right)^{3n} c_n^{(\nu)}\,, \tag{26}$$

where

$$c_n^{(\nu)} = \frac{(n+1)^3}{4}\left[\frac{1}{3n+1} - \frac{3}{3n+2} + \frac{7}{2(3n+3)} - \frac{2}{3n+4} \right. \tag{27}$$

$$\left. + \frac{3}{5(3n+5)} - \frac{1}{10\,(3n+6)} + \frac{1}{140\,(3n+7)}\right]. \tag{28}$$

And the final energy distribution for the two neutrinos and the antineutrino, for $n = 1, 2$, is given by

$$\mathcal{P}_{\nu_\alpha \nu_\beta \bar{\nu}_\beta / \nu_\alpha}(x', x_-, x_+) \approx \frac{(n+1)^3}{4\, c_n^{(\nu)}} \delta(1 - x' - x_- - x_+)\,(1 - x')^3\,(1 - x_-)^3\,(1 - x_+)^{3n-1}\,. \tag{29}$$

Discussion

Discussion

In case of very-high-energy particles (negligible masses) and almost one dimensional propagation of fluxes, the collinear approximation can simplify the computation of cross sections and decay widths.

One could extend the use of this approximation to DSR scenarios, in order to simplify the kinematic analysis.

The explicit forms of cross sections and decay widths, as well as the differential energy distributions, are a crucial ingredient in order to put bounds in the parameters of new physics from observations of very-high-energy astrophysical messengers.

References

[1] J. M. Carmona et al. “Decay of superluminal neutrinos in the collinear approximation”. Work in progress.

[2] J. M. Carmona, J. L. Cortes, and D. Mazon. “Uncertainties in Constraints from Pair Production on Superluminal Neutrinos”. In: *Phys. Rev. D* 85 (2012), p. 113001. DOI: *10.1103/PhysRevD.85.113001*. arXiv: *1203.2585 [hep-ph]*.