PAGE CURVE IN DREH MODEL OF 2D DILATON GRAVITY

Stefan Đorđević
Faculty of Physics
University of Belgrade

COST Workshop
2. september 2022
Belgrade, Serbia

TABLE OF CONTENTS

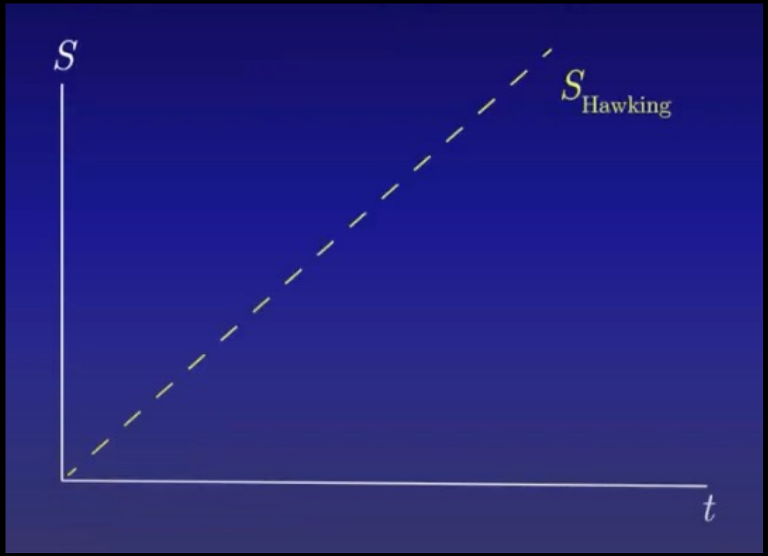
arXiv: 2207.07409

A LITTLE BIT OF HISTORY

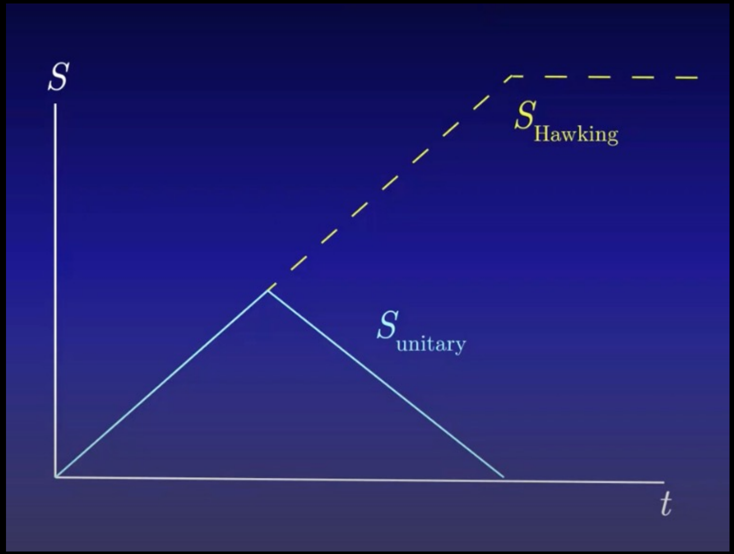
- 1974. Hawking discovered that black holes emit radiation. The main problem was that his calculation was in dispute with quantum mechanics' unitary evolution. Hence we have information loss paradox
- 1992. Don Page proposes a curve that entropy should follow if the evolution was indeed unitary. Many attempts were made in the following years to reproduce that curve. They were unsuccessful until late 2010s
- During the nineties many toy models were created to study black hole evaporation. Most prominent being JT and RST/BPP models. In JT model Page's curve was, finally, reproduced in 2019, while in the RST/BPP it was reproduced in 2020-2021 with help of a new formula for fine-grained entropy in gravitational systems developed by Maldecena in 2013

INFORMATION PARADOX

- Hawking's curve:



- Page's curve:



REPLICA WORMHOLES

- A method for calculating fine-grained entropy in gravitational systems based on euclidean gravitational path integral:

$$\mathcal{Z} = \int \mathcal{D}g_{\mu\nu}\mathcal{D}\phi e^{-S_E[g_{\mu\nu},\phi]} = e^{-S_E\left[g_{\mu\nu}^{(0)},\phi^{(0)}\right]}$$

- It is a method developed by Hawking. He used the exact same method to derive generalized entropy formula as course-grained entropy during the seventies:

$$S_{gen} = \frac{A}{4G} + S_{outside}$$

REPLICA WORMHOLES

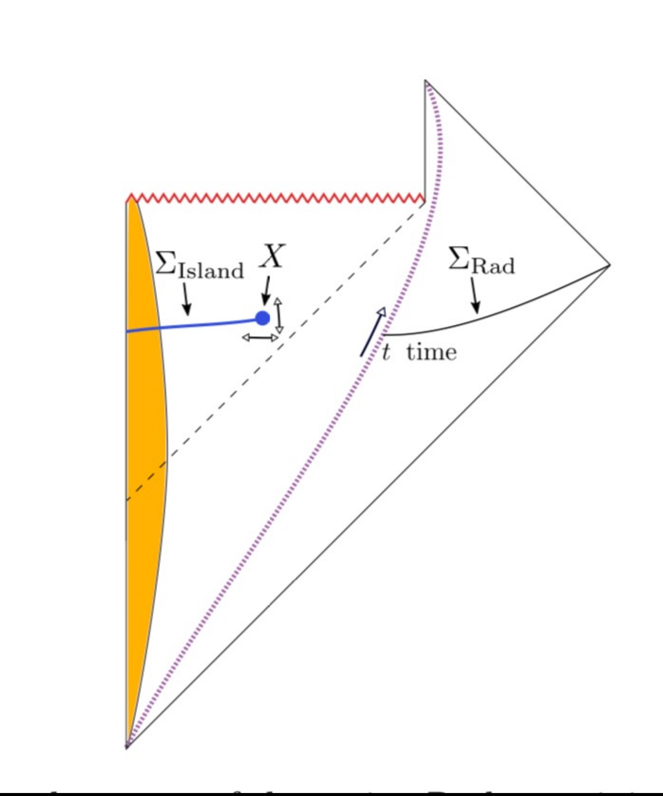
- Hawking's calculation for the fine-grained entropy of quantum fileds, outside the black hole, results into fine-grained entropy as always increasing function of time. That is why we have information paradox!
- What is wrong with Hawking's calculation?

  Answer: He used wrong saddle point in gravitational path integral!
- When you consider summing over topologies, you get two saddle points in gravitational path integral. One corespondes to Hawking's result, and the other coresponds to replica wormholes result. The one that is a global minimum is used when calculating path integral. Over time the global minimum changes between these two saddle points. That point in time coresponds to a phase transition which occures at Page's time.

REPLICA TRICK

- Formula for fine-grained entropy in gravitational systems:

$$S_{FG} = \min_{I} \left\{ \underset{I}{\mathbf{ext}} \left[\frac{A[I]}{4G} + S_{semi-cl} \left(\Sigma_I \cup \Sigma_{rad} \right) \right] \right\}$$



CLASSICAL DREH ACTION

- We start from 4D Einstein-Hilbert action:

$$S_{\mathrm{EH}} = \frac{1}{16\pi G_{\mathrm{N}}^{(4)}} \int \mathrm{d}^4x \sqrt{-g^{(4)}} R^{(4)}$$

- Consider the following spherically symmetric ansatz:

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu + \lambda^{-2} e^{-2\phi} \left[d\theta^2 + \sin^2\theta d\varphi^2 \right]$$

- Using this ansatz the 4D scalar curvature becomes:

$$R^{(4)} = R + 2(\nabla\phi)^2 + 2\lambda^2 e^{2\phi} - 2e^{2\phi} \Box e^{-2\phi}$$

CLASSICAL DREH ACTION

- The action is given by:

$$
\begin{aligned}
& S_{\mathrm{DREH}} = S_\phi + S_m \\
& = \frac{1}{4G_{\mathrm{N}}} \int \mathrm{d}^2 x \sqrt{-g} \left[e^{-2\phi} \left(R + 2(\nabla\phi)^2 \right) + 2\lambda^2 \right] \\
& - \frac{1}{2} \int \mathrm{d}^2 x \sqrt{-g} \, (\nabla f)^2 .
\end{aligned}
$$

- Varying this action we get the following equations of motion:

$$
\begin{aligned}
\left[2\nabla_\mu \nabla_\nu \phi - 2\nabla_\mu \phi \nabla_\nu \phi + g_{\mu\nu} \left(3(\nabla\phi)^2 - 2\Box\phi - \lambda^2 e^{2\phi} \right) \right] e^{-2\phi} &= 2G_{\mathrm{N}} T^{(f)}_{\mu\nu,\mathrm{class}}, \\
(\nabla\phi)^2 - \Box\phi &= \frac{R}{2}, \\
\Box f &= 0.
\end{aligned}
$$

CLASSICAL SOLUTION

- We expact to find the Schwarzschild black solution! $ds^2 = -h_0(r)dt^2 + \frac{dr^2}{h_0(r)}$ $\frac{1}{\lambda}e^{-\phi} = r$
- In this ansatz, EoM reduce to: $\partial_r^2 h_0 + \frac{2}{r}\partial_r h_0 = 0$ and $h_0(r) = 1 - r\partial_r h_0(r)$.
- There is a unique solution to this system of equations:

$$ds^2 = -\left(1 - \frac{2MG_{\mathrm{N}}}{\lambda^2 r}\right)dt^2 + \frac{dr^2}{1 - \frac{2MG_{\mathrm{N}}}{\lambda^2 r}}$$

- This is of course the Schwarzschild black hole solution!

QUANTUM CORRECTIONS

- Now we add quantum corrections in the form of Polyakov-Liouville action:

$$S_{PL} = -\frac{\hbar}{96\pi}\int d^2x \int d^2y \sqrt{-g(x)}\sqrt{-g(y)}\mathcal{R}(x)G(x-y)\mathcal{R}(y)$$

- This is a non-local action. We can localise it by introducion an auxiliary field:

$$S_{\mathrm{PL}} = -\frac{\hbar}{96\pi}\int \mathrm{d}^2x\sqrt{-g}\left[2R\psi + (\nabla\psi)^2\right] \qquad \Box\psi = R.$$

- Dilaton equation does not change. But the equation for the metric does:

$$\left[2\nabla_\mu\nabla_\nu\phi - 2\nabla_\mu\phi\nabla_\nu\phi + g_{\mu\nu}\left(3(\nabla\phi)^2 - 2\Box\phi - \lambda^2 e^{2\phi}\right)\right]e^{-2\phi} = 2G_{\mathrm{N}}\left\{\nabla_\mu f\nabla_\nu f - \frac{1}{2}g_{\mu\nu}(\nabla f)^2 + \frac{\hbar}{48\pi}\left[-2\nabla_\mu\nabla_\nu\psi + \nabla_\mu\psi\nabla_\nu\psi + g_{\mu\nu}\left(2\Box\psi - \frac{1}{2}(\nabla\psi)^2\right)\right]\right\}$$

QUANTUM CORRECTED SOLUTION

- We want to keep: $\frac{1}{\lambda}e^{-\phi} = r$ So the ansatz is: $ds^2 = -h(r)e^{2\epsilon\varphi(r)}dt^2 + \frac{dr^2}{h(r)}$ $h(r) = h_0(r) + \frac{\epsilon m(r)}{r}$

- The equations for the corrections are then:
- Here T is proportional to the quantum corrected energy-momentum tensor.

$$\frac{dm}{dr} = -\frac{\tilde{T}^{(f)}_{tt}}{h_0},$$

$$2rh_0\frac{d\varphi}{dr} = h_0\tilde{T}^{(f)}_{rr} + \frac{\tilde{T}^{(f)}_{tt}}{h_0}$$

- Solving the equation for the auxiliary field we get: $\partial_r\psi = \frac{C - \partial_r h}{h}$
- Corrected energy-momentum tensor (EMT) is then:

$$\langle \Delta T^{(f)}_{tt} \rangle = \epsilon\left[\frac{C^2}{2} - \frac{1}{2}(\partial_r h_0)^2 + 2h_0\partial_r^2 h_0\right]$$

$$\langle \Delta T^{(f)}_{rr} \rangle = \epsilon\frac{C^2 - (\partial_r h_0)^2}{2h_0^2}$$

ETERNAL BLACK HOLE SCENARIO

- The correct state of the system is Hartle-Hawking (HH) state. That means we need to normally order EMT in coordinates which are well defined at the horizon (Kruskal x - coordinates). First we introduce tortoise coordinate:

$$
\begin{aligned}
ds^2 &= -h(r)e^{2\epsilon\varphi(r)}dt^2 + \frac{dr^2}{h(r)} \\
&= h(r)e^{2\epsilon\varphi(r)}\left(-dt^2 + \frac{dr^2}{h^2(r)e^{2\epsilon\varphi(r)}}\right) \\
&= h(r)e^{2\epsilon\varphi(r)}\left(-dt^2 + dr_*^2\right),
\end{aligned}
$$

$$r_* = \int \frac{e^{-\epsilon\varphi(r)}}{h(r)}dr$$

- Then we introduce lightcone coordinates: $\sigma^{\pm} = t \pm r_*$ And, with the help of the surface gravity, we get to the Kruskal coordinates: $\kappa x^{\pm} = \pm e^{\pm\kappa\sigma^{\pm}}$
- Metric is then: $ds^2 = -e^{2\rho}dx^+dx^-$ $\rho(x) = \frac{1}{2}\ln h + \epsilon\varphi - \kappa r_*$.

TRANSFORMATIONS OF THE EMT

- In conformal gauge, energy-momentum tensor is given by:

$$\langle \Delta T^{(f)}_{\pm\pm} \rangle = 4\epsilon \left[\partial^2_\pm \rho - (\partial_\pm \rho)^2 - t_\pm(x^\pm) \right]$$
$$\langle \Delta T^{(f)}_{+-} \rangle = -4\epsilon \partial_+ \partial_- \rho,$$

- Under conformal transformations function t tarnsforms as follows:

$$t_\pm(y^\pm) = \left(\frac{dx^\pm}{dy^\pm} \right)^2 \left[t_\pm(x^\pm) - \frac{1}{2} D_{x^\pm}[y^\pm] \right]$$

$$D_{x^\pm}[y^\pm] = \frac{(y^\pm)'''}{(y^\pm)'} - \frac{3}{2} \left(\frac{(y^\pm)''}{(y^\pm)'} \right)^2$$

- By the methods of QFT in curved space-time, the normally ordered part of energy-momentum tensor transforms as:

$$: \hat{T}^{(f)}_{\pm\pm}(y^\pm) := \left(\frac{dx^\pm}{dy^\pm} \right)^2 \left[: \hat{T}^{(f)}_{\pm\pm}(x^\pm) : + \frac{\hbar}{24\pi} D_{x^\pm}[y^\pm] \right]$$

=>

$$\langle 0, x | : \hat{T}^{(f)}_{\pm\pm}(x^\pm) : | 0, x \rangle = -\epsilon t_\pm(x^\pm),$$

HH STATE

- We know that in Kruskal coordinates EMT has to be normally ordered. By transforming EMT back to original coordinates we get:

$$
\begin{aligned}
\langle \Delta T^{(f)}_{rr} \rangle &= \frac{\partial x^{\mu}}{\partial r}\frac{\partial x^{\nu}}{\partial r}\langle \Delta T^{(f)}_{\mu\nu} \rangle \\
&= \left(\frac{\partial x^{+}}{\partial r}\right)^2 \langle \Delta T^{(f)}_{++} \rangle + \left(\frac{\partial x^{-}}{\partial r}\right)^2 \langle \Delta T^{(f)}_{--} \rangle \\
&+ 2\frac{\partial x^{+}}{\partial r}\frac{\partial x^{-}}{\partial r}\langle \Delta T^{(f)}_{+-} \rangle = \frac{\epsilon}{2l^2}\left[\frac{1}{r_0^2} - \left(\frac{r_0}{r^2}\right)^2\right]
\end{aligned}
$$

- Comparing with previous result we see that constant C is fixed as: $C = \frac{1}{r_0}$. Now we can solve equations. We get:

$$
m(r) = \frac{G_{\mathrm{N}}}{\lambda^2 r_0}\left[-\frac{7}{2}\left(\frac{r_0}{r}\right)^2 + \frac{r_0}{r} + \ln\frac{r_0}{r} - \frac{r}{r_0}\right] + C_1
$$

$$
\varphi(r) = \frac{G_{\mathrm{N}}}{\lambda^2 r_0^2}\left[-\frac{3}{2}\left(\frac{r_0}{r}\right)^2 - 2\frac{r_0}{r} - \ln\frac{r_0}{r}\right] + C_2.
$$

TORTOISE COORDINATE

- We need to explicitly calaculate tortoise coordinate. The horizon position is defined by: $g^{rr}(r_H) = 0$ => $r_H = r_0 - \epsilon m(r_0)$
- Surface gravity is defined using the redshift factor: $V = \sqrt{-g_{\mu\nu}\xi^\mu\xi^\nu} = \sqrt{-g_{tt}} = \sqrt{he^{2\varepsilon\varphi}}$

$$\kappa = \sqrt{(\nabla V)^2} = \sqrt{g^{rr}}\partial_r V = \frac{1}{2}\partial_r h e^{\epsilon\varphi} \tag{38}$$
$$= \frac{1}{2r_0}\left[1 + \epsilon\left(\varphi(r_0) + \partial_r m(r_0) + \frac{m(r_0)}{r_0}\right) + \mathcal{O}(\epsilon^2)\right].$$

- The unknown function is then:

$$\alpha(r) = \frac{G_{\mathrm{N}}}{\lambda^2 r_0^2}\left[\frac{5}{2}\frac{r}{r_0} + \left(1 - \frac{r}{r_0} - \frac{r_0}{r - r_0}\right)\ln\frac{r}{r_0}\right]$$

The tortoise coordinate has a pole at the horizon position, so we write:

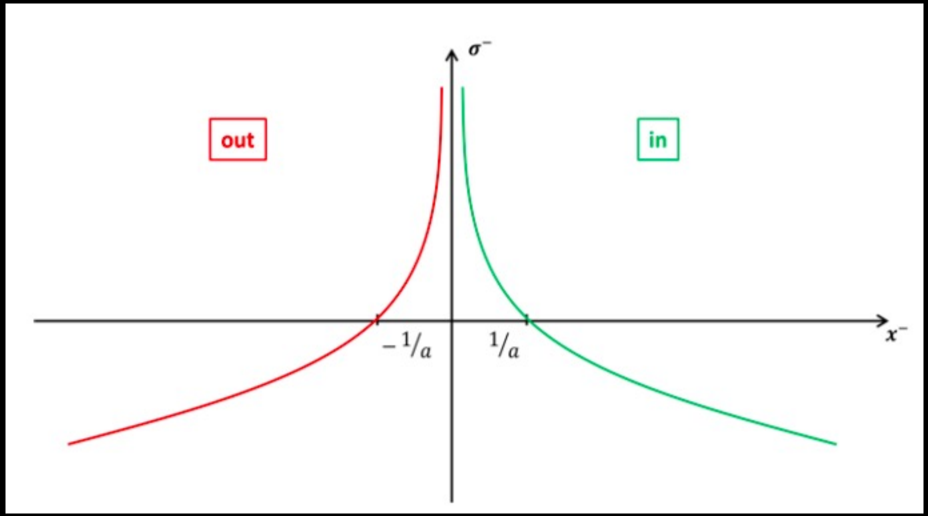
$$r_* = \frac{1}{2\kappa}\int\left[\frac{1}{r - r_H} - \left(\frac{1}{2}\frac{h''}{h'} + \epsilon\frac{d\varphi}{dr}\right)\Big|_H + o(r - r_H)\right]dr$$
$$= \frac{1}{2\kappa}\left[\frac{r}{r_H} + \ln\left(\frac{r}{r_H} - 1\right) + \epsilon\alpha(r)\right], \tag{41}$$

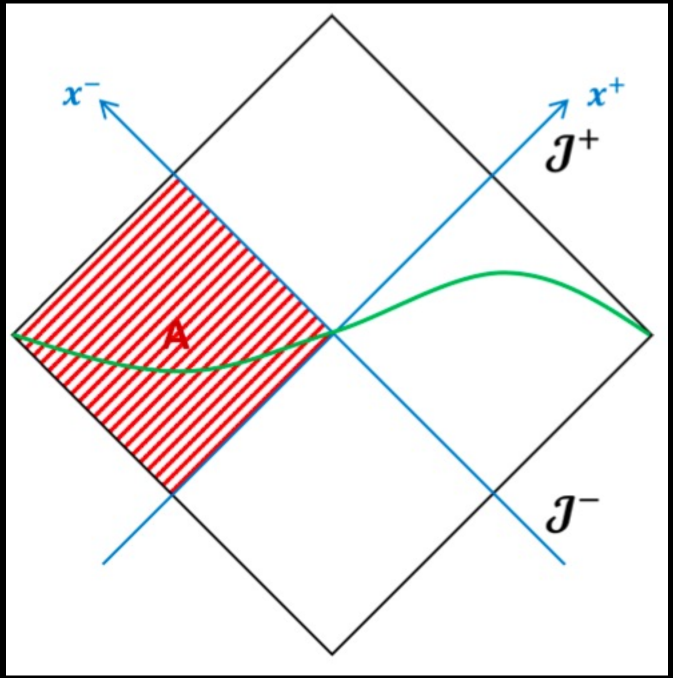
UNRUH EFFECT (REVIEW)

- We need to find coordinates that corespond to the region of the space-time that our observer has acces to:

$$\sigma^-_{out} = -\frac{1}{a}\ln\left(-ax^-\right) \;\wedge\; \sigma^-_{in} = -\frac{1}{a}\ln\left(ax^-\right)$$

Rindler coordinates!





We can now analiticly continue these Rindler modes to the whole space-time by expressing them in terms of Minkowski coordinates:

$$\phi^\omega_{in} = \frac{1}{\sqrt{4\pi\omega}}\theta(x^-)e^{i\omega\sigma^-_{in}}$$

$$\phi^\omega_{out} = \frac{1}{\sqrt{4\pi\omega}}\theta(-x^-)e^{-i\omega\sigma^-_{out}}$$

UNRUH EFFECT (REVIEW)

- This yields the follwing result. This is a famous Unruh trick for calculationg Bogolubov coefficients!

$$\phi_1^\omega = \frac{1}{\sqrt{2\sinh\frac{\pi\omega}{a}}}\left[\phi_{out}^\omega e^{\frac{\pi\omega}{2a}} + \phi_{in}^{\omega *} e^{-\frac{\pi\omega}{2a}}\right]$$

$$\phi_2^\omega = \frac{1}{\sqrt{2\sinh\frac{\pi\omega}{a}}}\left[\phi_{in}^\omega e^{\frac{\pi\omega}{2a}} + \phi_{out}^{\omega *} e^{-\frac{\pi\omega}{2a}}\right]$$

- For a quantum state we choose Minkowski vacuum state and express it in bases of in/out functions. We get thermo-field double state:

$$|0_M\rangle = \sqrt{\prod_{k=1}^{s}\left(1 - e^{-\frac{2\pi\omega_k}{a}}\right)} \exp\left\{\sum_{k=1}^{s} e^{-\frac{\pi\omega_k}{a}} a_{out}^\dagger(\omega_k) \otimes a_{in}^\dagger(\omega_k)\right\} |0_{out}\rangle \otimes |0_{in}\rangle$$

- Then we trace out in states to get density matrix of the out region:

$$\hat{\rho}_{out} = \left[\prod_{k=1}^{s}\left(1 - e^{-\frac{2\pi\omega_k}{a}}\right)\right] \sum_{n_1,\ldots,n_s=0}^{\infty} e^{-\frac{2\pi}{a}\sum_{k=1}^{s} n_k\omega_k} |n_1,\ldots,n_s,out\rangle \langle n_1,\ldots,n_s,out|$$

UNRUH EFFECT (REVIEW)

- We have: $$\hat{\rho}_{out} = \frac{1}{\mathcal{Z}} e^{-\beta H} \quad \Longrightarrow \quad \mathcal{Z} = \prod_{k=1}^{s} \frac{1}{1 - e^{-\frac{2\pi\omega_k}{a}}} \quad \wedge \quad T = \frac{a}{2\pi}$$
- It is easy to calculate entanglement entropy now:

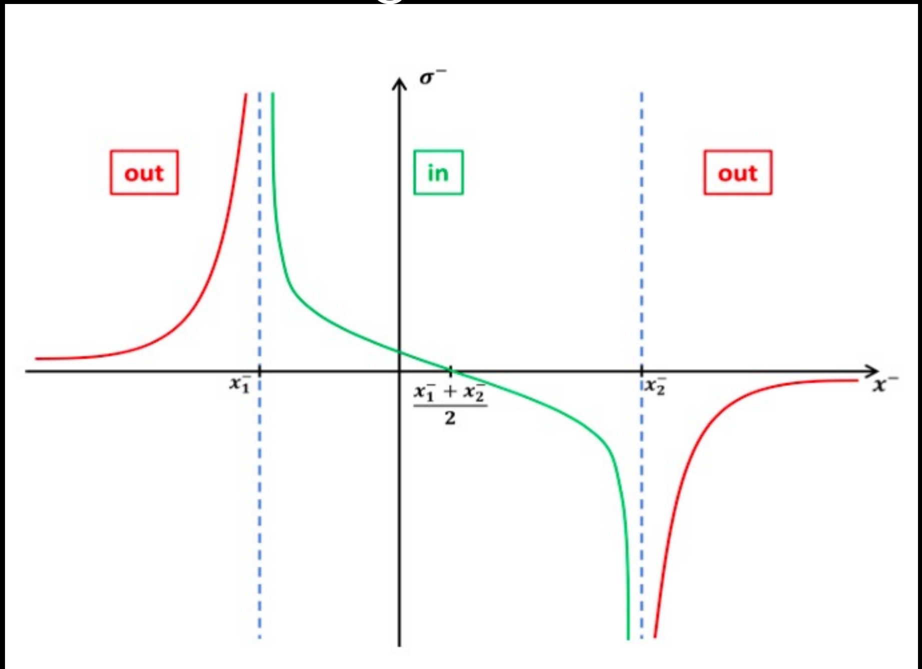
$$S_{FN} = \frac{2L}{a} \int_0^\infty \frac{\omega d\omega}{e^{\frac{2\pi\omega}{a}} - 1} = \frac{2L}{a} \left(\frac{a}{2\pi}\right)^2 \int_0^\infty \frac{\xi d\xi}{e^\xi - 1} = \frac{aL}{2\pi^2} \frac{\pi^2}{6} = \frac{aL}{12} \Longrightarrow \boxed{S_{FN} = \frac{\pi LT}{6}}.$$

$$S_{FN} = \frac{aL}{12} = \frac{a}{12} \left[\max\{\sigma_{in}^-\} - \min\{\sigma_{in}^-\}\right] = \frac{\cancel{a}}{12} \frac{1}{\cancel{a}} \ln \frac{X_{\max}^-}{X_{\min}^-} = \frac{1}{12} \ln \frac{X_{\max}^-}{X_{\min}^-}.$$

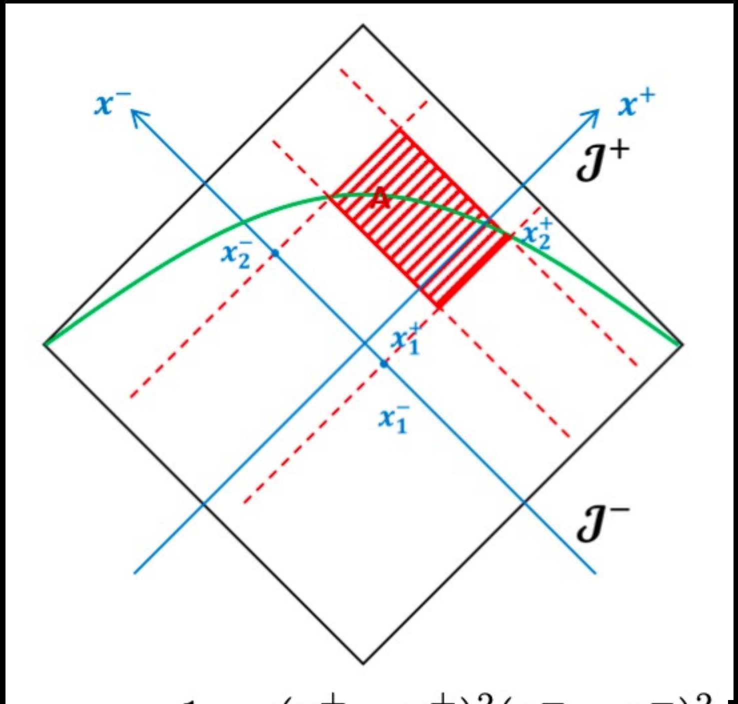
- Now we would like to generalize this result to a causal diamond!

GENERALIZATION TO CAUSAL DIAMOND

- It is straight forward to generalize this formula to a causal dimanod in wich live degrees of freedom that an observer has acces to:



$$\sigma^- = -\frac{1}{a}\ln\left|\frac{x^- - x_1^-}{x_2^- - x^-}\right|$$



$$S_{FN} = \frac{a}{12}\left[\sigma_{in}^-(x_1^- + \delta_1^-)) - \sigma_{in}^-(x_2^- - \delta_2^-)\right] = \frac{1}{12}\left[\ln\frac{x_2^- - x_1^-}{\delta_2^-} - \ln\frac{\delta_1^-}{x_2^- - x_1^-}\right]$$

$$S_{FN} = \frac{1}{12}\ln\frac{(x_2^+ - x_1^+)^2(x_2^- - x_1^-)^2}{\delta_1^+\delta_1^-\delta_2^+\delta_2^-}$$

GENERALIZATION TO CURVED SPACE-TIME

- First we generalize to different coordinates y which are x coordinates transformed by a confomal transformation of Minkowski metric:

$$ds^2 = -dx^+ dx^- = -\frac{dx^+}{dy^+}\frac{dx^-}{dy^-}dy^+ dy^- = -e^{2\rho} dy^+ dy^-$$

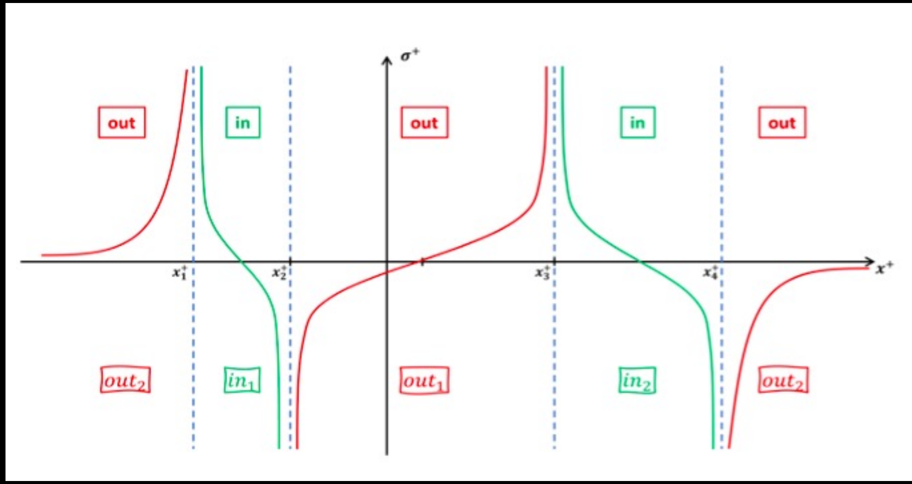
- Since cut-offs are lengths we know how they transform and get:

$$S_{FN} = \frac{1}{12} \ln \frac{(y_2^+ - y_1^+)^2 (y_2^- - y_1^-)^2}{\delta^4 e^{-2\rho_1} e^{-2\rho_2}}$$

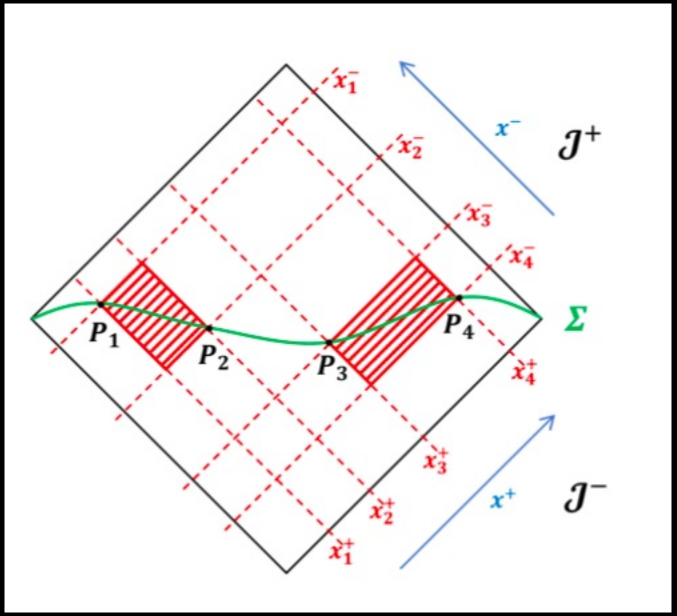
- This formula holds in curved space-time as well!

GENERALIZATION TO TWO DISJOINT REGIONS

- For Page curve calculation we need this formula for two disjoint regions. The pocedure is straigth forward:



$$\boxed{\sigma^- = -\frac{1}{a}\ln\left|\frac{x^- - x_1^-}{x_2^- - x^-}\frac{x^- - x_3^-}{x_4^- - x^-}\right|}.$$



$$\boxed{S_{FN} = \frac{1}{6}\ln\frac{d_{12}^2 d_{23}^2 d_{14}^2 d_{34}^2}{\delta^4 d_{24}^2 d_{13}^2 e^{-\rho_1}e^{-\rho_2}e^{-\rho_3}e^{-\rho_4}} \quad \wedge \quad d_{ij}^2 = (x_i^+ - x_j^+)(x_i^- - x_j^-)}.$$

PAGE CURVE – NO ISLAND CASE

- Formula that we need to use is:

$$S_{matter} = \frac{1}{12} \ln \frac{(x_R^+ - x_L^+)^2 (x_R^- - x_L^-)^2}{\delta^4 e^{-2\rho_R} e^{-2\rho_L}}.$$

- Coordinates are given by:

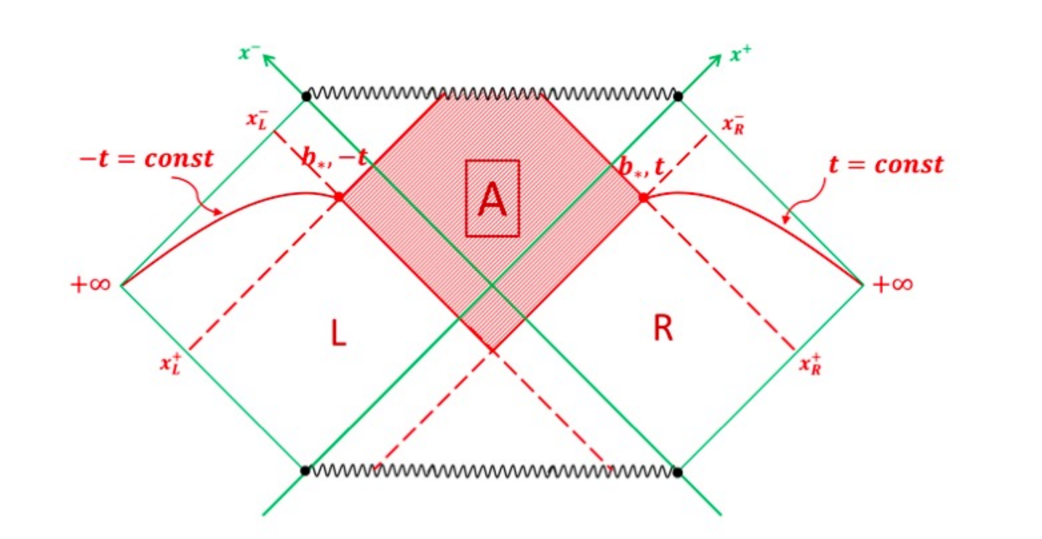
$$\begin{aligned} \kappa x_R^+ &= e^{\kappa(t+b_*)} \\ \kappa x_R^- &= -e^{-\kappa(t-b_*)} \\ \kappa x_L^+ &= -e^{\kappa(-t+b_*)} \\ \kappa x_L^- &= e^{-\kappa(-t-b_*)}. \end{aligned}$$

=>

$$S_{FG} = S_{matter} = \frac{1}{6} \ln \frac{4h(b)\cosh^2(\kappa t)}{(\kappa\delta)^2 e^{-2\epsilon\varphi(b)}}$$

=>

$$S_{FG} \approx \frac{1}{3}\kappa t - \frac{1}{3}\ln 2.$$



- Hawking's result!

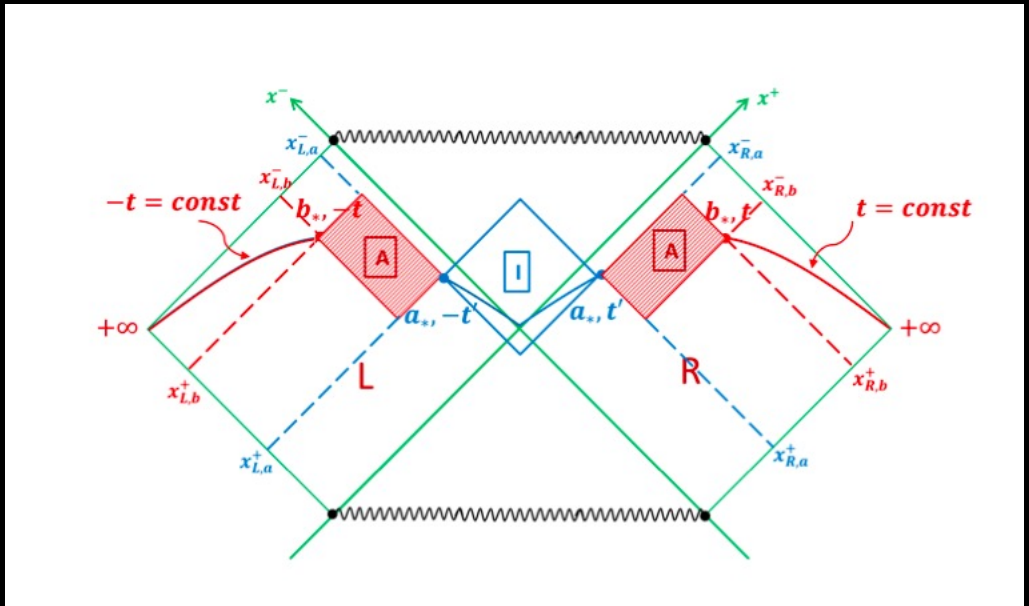
PAGE CURVE – ISLAND CASE

- In this case we have to disjoint regions:

$$S_{matter} = \frac{1}{6} \ln \frac{d_{12}^2 d_{23}^2 d_{14}^2 d_{34}^2}{\delta^4 d_{24}^2 d_{13}^2 e^{-\rho_1} e^{-\rho_2} e^{-\rho_3} e^{-\rho_4}},$$

- Coordinates are given by:

$$\begin{aligned}
\kappa x_{Rb}^+ &= e^{\kappa(t+b_*)}, \qquad \kappa x_{Rb}^- = -e^{-\kappa(t-b_*)}, \\
\kappa x_{Lb}^+ &= -e^{\kappa(-t+b_*)}, \qquad \kappa x_{Lb}^- = e^{-\kappa(-t-b_*)}, \\
\kappa x_{Ra}^+ &= e^{\kappa(t'+a_*)}, \qquad \kappa x_{Ra}^- = -e^{-\kappa(t'-a_*)}, \\
\kappa x_{La}^+ &= -e^{\kappa(-t'+a_*)}, \qquad \kappa x_{La}^- = e^{-\kappa(-t'-a_*)}.
\end{aligned}$$



- In this coordinates we have: $\frac{d_{23}^2 d_{14}^2}{d_{24}^2 d_{13}^2} \to 1$ => $S_{matter} = \frac{1}{6} \ln \left(\frac{d_{12}^4}{\delta^4} e^{2\rho_a} e^{2\rho_b} \right)$

PAGE CURVE – ISLAND CASE

- After substituting we get:

$$S_{matter} = \frac{1}{6} \ln \frac{h(a)h(b)\left(e^{\kappa(t+b_*)} - e^{\kappa(t'+a_*)}\right)^2 \left(-e^{-\kappa(t-b_*)} + e^{-\kappa(t'-a_*)}\right)^2}{(\kappa\delta)^4 e^{2\kappa(b_*+a_*)} e^{-2\varepsilon(\varphi(a)+\varphi(b))}}.$$

- First we extremize with respect to primed time. It is equal to t. nWe need to add an area term as in formula for fine-grained entropy. The funcional that needs to be extremized is then given by:

$$\begin{aligned} S_{gen} = \frac{2\pi\lambda^2 a^2}{G_{\mathrm{N}}\hbar} + \frac{1}{3}(\rho_a + \rho_b) + \frac{2}{3}\ln\left(e^{\kappa b_*} - e^{\kappa a_*}\right) \\ - \frac{2}{3}\ln(\kappa\delta). \end{aligned}$$

PAGE CURVE – ISLAND CASE

- The extremisation procedure gives the following equation:

$$\left[a + \frac{4\epsilon G_N}{\lambda^2}\frac{d\rho(a)}{dr}\right] h(a)e^{\epsilon\varphi(a)} = \frac{8\epsilon G_N}{\lambda^2}\frac{\kappa}{e^{\kappa(b_* - a_*)} - 1}$$

- This equation can't be easily solved perturbativly. But, there is a theorem that states that, in case of eternal black hole, an island should appear near the horizon, but outside the horizon. So we expand the position of the island around horizon position $a = r_H + x$ and try to solve for x. This needs to be done carefully becouse of divergence of tortoise coordinate at the horzion. After carefully applying this procedure we get:

$$x = \frac{1}{r_H}\frac{\left(\frac{4\epsilon G_N}{\lambda^2 r_H}\right)^2 e^{1-2\kappa b_* + \epsilon\alpha(r_H)}}{\left[1 + \frac{4\epsilon G_N}{\lambda^2 r_H}\left(\left.\frac{d\rho}{dr}\right|_H - \frac{1}{r_H}e^{1-2\kappa b_* + \epsilon\alpha(r)}\right)\right]^2}$$

FINE-GRAINED ENTROPY

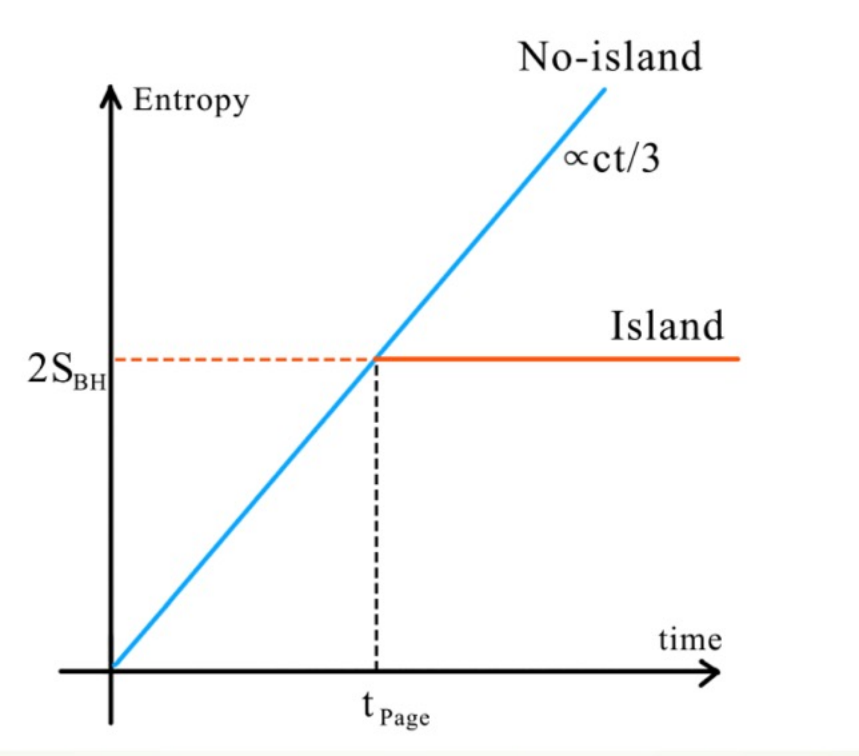
- After substituting all functions in this formula, for the position of the island we get:

$$a = r_H + \frac{1}{r_H}\left(\frac{\hbar G_N}{12\pi\lambda^2 r_H}\right)^2 e^{1-2\kappa b_*}\left[1 + \frac{\hbar G_N}{6\pi\lambda^2 r_H^2}\left(\frac{19}{16} + e^{1-2\kappa b_*}\right)\right].$$

- The second term is really much smaller then the first!
- The fine-grained entropy is then given by the formula:

$$S_{FG} = \min\left\{\frac{1}{3}\kappa t, 2S_{BH}\right\}$$

- We have reproduced Page curve!!



CONCLUSION

- We have reproduced the Page curve in physicaly relevant DREH model in the case of an eternal black hole
- Next step could be to reproduce the Page curve in case of an evaporating black hole. This work is in progress!
- Another idea is to reproduce Page curve in case of a charged black hole
- The talk was based on our paper: **arXiv: 2207.07409**

Thank you for your attention!