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Probing gravitational theories via PBHs

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Based on

[T. Papanikolaou, C. Tzerefos, S. Basilakos, E.N. Saridakis, JCAP 10 (2022) 013]
[T. Papanikolaou, V. Vennin, D. Langlois, JCAP 03 (2021) 053]

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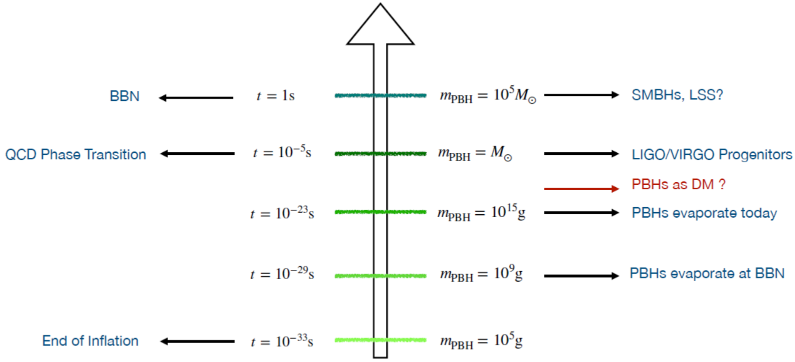
What are PBHs?

- Primordial Black Holes (PBHs) form in the early universe, before star formation, out of the collapse of enhanced energy density perturbations upon horizon reentry of the typical size of the collapsing overdensity region. This happens when $\delta > \delta_c \approx 0.1$
- PBHs can be used to probe different cosmic epochs in the cosmic history as well as to probe different physical phenomena depending on their mass.

PBHs and cosmic eras



[1]

[1] Credit: T. Papanikolaou

Poisson gas of PBHs

In our work, we considered ultra light PBHs such that:

- they are *monochromatic* aka all have the same mass m_{PBH}
- $10g < m_{PBH} < 10^9 g$
- they formed *after* inflation
- they have evaporated *before* BBN
- they the led to a transit *matter* dominated era
- they can be described in our scales of interest ($k < k_{UV} = a/\bar{r}$[2]) as a gas of randomly distributed black holes $\rightarrow$ they follow *Poissonian statistics*

[2] $\bar{r}$ is the mean seperation distance of each BH

Main idea

- The gravitational potential of this gas of randomly distributed PBHs can induce a stochastic GW background through second - order gravitational effects.

- By requiring that these scalar-induced GWs (SIGWs) are *not* overproduced ($\Omega_{\mathrm{GW,tot}}(\eta_{\mathrm{evap}}) < 1$) we can:

$\rightarrow$ Find an upper bound on the abundance of PBHs at formation time ($\Omega_{\mathrm{PBH,f}}$) as a function of their mass

$\rightarrow$ Conversely, by inputting indicative values for the parameters of PBHs we can constrain the parameters of the assumed gravitational theory.

SIGWs in GR

- PBHs are a Poisson gas:

$$\mathcal{P}_\delta(k) = \frac{k^3}{2\pi^2} P_\delta(k) = \frac{2}{3\pi}\left(\frac{k}{k_{\rm UV}}\right)^3 \Theta(k_{\rm UV} - k) \text{ [Riotto et al 2019]}$$

- Since ρ_{PBH} is *inhomogeneous* but ρ_{tot} is *homogeneous* $\rightarrow$ δ_{PBH} is an *isocurvature* perturbation
- δ_{PBH} will convert during the PBHD era to a curvature perturbation ζ_{PBH} associated to Φ for which we find:

$$\mathcal{P}_\Phi(k) = \frac{2}{3\pi}\left(\frac{k}{k_{\rm UV}}\right)^3 \left(5 + \frac{4}{9}\frac{k^2}{k_{\rm d}^2}\right)^{-2}$$

SIGWs in GR

For the 2^{nd} order tensor perturbations h_{ij} induced by the gravitational potential Φ in the Newtonian gauge we have:

$$\mathrm{d}s^2 = a^2(\eta)\left\{-(1+2\Phi)\mathrm{d}\eta^2 + \left[(1-2\Phi)\delta_{ij} + \frac{h_{ij}}{2}\right]\mathrm{d}x^i\mathrm{d}x^j\right\},$$

$$h_k^{s,\prime\prime} + 2\mathcal{H}h_k^{s,\prime} + k^2 h_k^s = 4S_k^s\,,$$

$$S_k^s = \int \frac{\mathrm{d}^3q}{(2\pi)^{3/2}} e_{ij}^s(k) q_i q_j \Big[2\Phi_q\Phi_{k-q} + \frac{4}{3(1+w)}(\mathcal{H}^{-1}\Phi_q' + \Phi_q)$$

$$(\mathrm{H}^{-1}\Phi_{k-q}' + \Phi_{k-q})\Big],$$

$$\Phi_k'' + \frac{6(1+w)}{1+3w}\frac{1}{\eta}\Phi_k' + wk^2\Phi_k = 0\,.$$

Results

Finally, the energy density of GWs is given by:

$$\rho_{GW}(\eta, x) = \frac{M_p^2}{32a^2} \overline{(\partial_\eta h_{\alpha\beta}\partial_\eta h^{\alpha\beta} + \partial_i h_{\alpha\beta}\partial^i h^{\alpha\beta})} \text{ [Maggiore - 2000]},$$

The GW spectral abundance is just the GW energy density per logarithmic comoving scale, i.e.

$$\Omega_{\rm GW} \equiv \frac{1}{\bar{\rho}_{\rm tot}} \frac{\mathrm{d}\rho_{\rm GW}(\eta, k)}{\mathrm{d}\ln k} \simeq \frac{1}{\bar{\rho}_{\rm tot}} \frac{\mathrm{d}\rho_{\rm GW,grad}(\eta, k)}{\mathrm{d}\ln k} = \frac{1}{24}\left(\frac{k}{\mathcal{H}(\eta)}\right)^2 \mathcal{P}_h(\eta, k)$$

Thus $$\boxed{\Omega_{\rm GW,tot} \leq 1 \Rightarrow \Omega_{\rm PBH,f} \leq 10^{-4}\left(\frac{10^9\mathrm{g}}{m_{\rm PBH}}\right)^{1/4}}$$

So why modified gravity?

- GR is notoriously inconsistent with QM at very high energies ($E \approx 10^{19} GeV$) $\rightarrow$ can't be the *final* theory of gravity

- Gravity theories with higher order curvature terms have improved renormalizability [Stelle - 1976]
- Proposed physics of inflation doesn't fit easily with our current fundamental physical laws

- Dark energy and dark matter persistently evade any experimental probing $\rightarrow$ it is possible they are manifestations of gravity itself

Poisson gas of PBHs in $f(R)$ gravity

The characteristic action of $f(R)$ is the following:

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,f(R) + \int d^4x\sqrt{-g}\mathcal{L}_m,$$

which yields the field equations:

$$FR_{\mu\nu} - \frac{1}{2}g_{\mu\nu}f + (g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu)F = 8\pi G T^{(m)}_{\mu\nu} \text{ with } F \equiv df(R)/dR$$

We can reformulate $f(R)$ as $GR + eff.fluid$. Then, at the scales we are interested in we will only include its effects via :

$$G_{\rm eff} \equiv \frac{G}{F}\left(\frac{1+4\frac{k^2}{a^2}\frac{F_{,\rm R}}{F}}{1+3\frac{k^2}{a^2}\frac{F_{,\rm R}}{F}}\right).$$

Power spectrum of Φ in $f(R)$

Treating again the PBH energy density fluctuations as isocurvature perturbations converting to curvature perturbations during the PBH dominated era we calculate

$$\mathcal{P}_\Phi(k) \equiv \frac{k^3}{2\pi^2} P_\Phi(k) = \frac{2}{3\pi}\left(\frac{k}{k_{\rm UV}}\right)^3 \left[5 + \frac{2}{3}\left(\frac{k}{\mathcal{H}}\right)^2 \frac{F}{\xi(a)} \left(\frac{1+3\frac{k^2}{a^2}\frac{F_{,R}}{F}}{1+2\frac{k^2}{a^2}\frac{F_{,R}}{F}}\right)\right]^{-2},$$

where

$$\xi(a) \equiv \frac{\delta_{\rm PBH}(a)}{\delta_{\rm PBH}(a_{\rm f})},$$

2^{nd} order tensor perturbations

The main difference with respect to GR is the existence of an extra massive degree of freedom, the so-called *scalaron* :

$\Box F(R) = \frac{1}{3}\left[2f(R) - F(R)R + 8\pi G\, T^{\mathrm{m}}\right] \equiv \frac{dV}{dF}$, with
$\mathrm{m}^2_{\mathrm{sc}} \equiv \frac{d^2V}{dF^2} = \frac{1}{3}\left(\frac{F}{F_{,R}} - R\right)$

This modifies the propagation equation of the tensor perturbations:

$$h_k^{s,\prime\prime} + 2\mathcal{H}h_k^{s,\prime} + (k^2 - \lambda m^2_{\mathrm{sc}})h_k^s = 4S_k^s\,,$$

where $\lambda = 0$ when $s = (+), (\times)$ and $\lambda = 1$ when $s = (\mathrm{sc})$.

$$e_{ij}^{(+)}(k) = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & 0\end{pmatrix}, e_{ij}^{(\times)}(k) = \frac{1}{\sqrt{2}}\begin{pmatrix}0 & 1 & 0\\ 1 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}$$

Case study: R^2 gravity

$$e_{ij}^{(\mathrm{sc})}(k) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

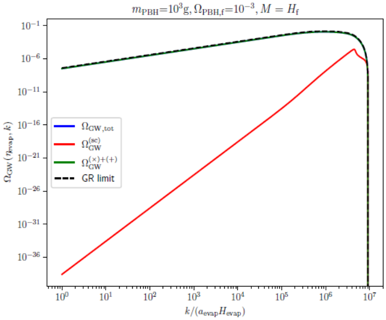
As a particular case study we shall take the gravity R^2 model:

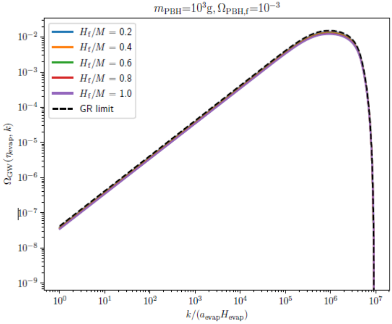
$$f(R) = R + \frac{R^2}{6M^2},$$

with a non-fixed mass scale parameter M with $H_{\mathrm{f}} \leq M \leq 10^{-5} M_{\mathrm{pl}}$ since these PBHs formed after the end of inflation.

Results

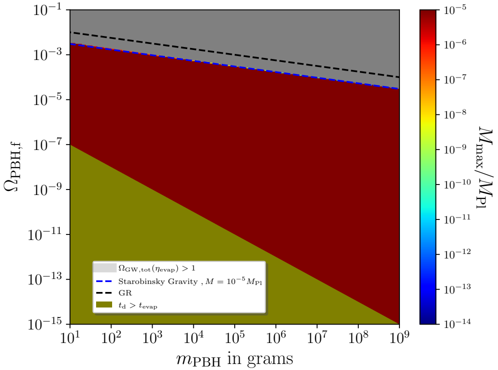
$$\Omega_{\rm GW}(\eta,k) \simeq \frac{1}{\bar{\rho}_{\rm tot}}\frac{{\rm d}\rho_{\rm GW,grad}(\eta,k)}{{\rm d}\ln k} = \frac{1}{96}\left(\frac{k}{\mathcal{H}(\eta)}\right)^2\left[2\overline{\mathcal{P}_h^{(\times)}}(\eta,k) + \overline{\mathcal{P}_h^{(\rm sc)}}(\eta,k)\right]$$





Constraints of the PBHs abundances and M

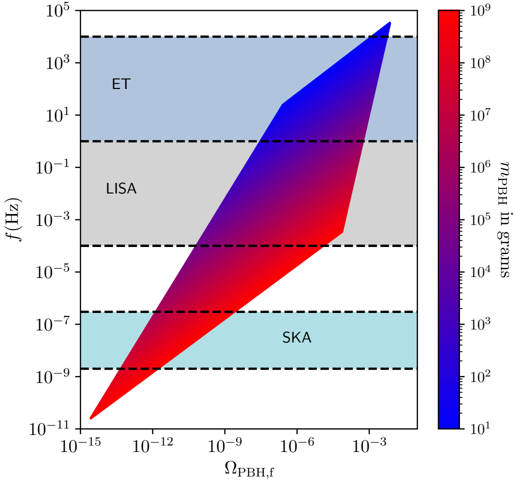
$$\Omega_{\mathrm{GW,tot}} \leq 1 \Rightarrow \Omega_{\mathrm{PBH,f}} \leq 5.5 \times 10^{-5} \left(\frac{10^9 \mathrm{g}}{m_{\mathrm{PBH}}}\right)^{1/4} \text{ (45\% tighter than GR!)}$$



Conclusions

- By investigating SIGWs produced by this Poisson gas of PBH that transiently dominated and merely demanding that they are not overproduced on can constrain the PBHs abundances or, reversely, the gravitational theory
- We demonstrated this methodology in the context of $f(R)$ via the R^2 model
- It is applicable in other gravitational theories like $f(T)$ as is shown in [T. Papanikolaou, C. Tzerefos, S. Basilakos, E.N. Saridakis, EPJC 80 (2023)]
- Currently we are working on applying it to $f(Q)$

Appendix: Observability of SIGWs in R^2 gravity



Appendix: Reformulation as an effective fluid

$$G^{\mu}_{\nu} = 8\pi G\left(T^{(m)\mu}_{\nu} + T^{(eff)\,\mu}_{\nu}\right) \equiv T^{(tot)\,\mu}_{\nu} \text{ with} \tag{1}$$

$$T^{(eff)\,\mu}_{\nu} \equiv (1-F)R^{\mu}_{\nu} + \frac{1}{2}\delta^{\mu}_{\nu}(f-R) - (\delta^{\mu}_{\nu}\Box - \nabla^{\mu}\nabla_{\nu})F \tag{2}$$

As we explained earlier, via the bianchi identities this tensor is conserved $\nabla_{\mu}T^{(eff)\,\mu}_{\nu} = 0$. Thanks to this construction, for the FLRW metric we can get the Friedmann equations with:

$$\bar{\rho}_{eff} \equiv -T^{(eff)\,0}_{0} = \frac{1}{8\pi G a^2}\left(3\mathcal{H}^2 - \frac{1}{2}\alpha^2 f + 3F\mathcal{H}' - 3\mathcal{H}F'\right) \tag{3}$$

$$\bar{p}_{eff} \equiv \frac{T^{(eff)\,i}_{i}}{3} = \frac{1}{8\pi G a^2}\left(-2\mathcal{H}' - \mathcal{H}^2 + \frac{1}{2}a^2 f - F\mathcal{H}' - 2F\mathcal{H}^2 + F'' + \mathcal{H}F'\right) \tag{4}$$

Perturbations in $f(R)$

After performing the same (scalar) perturbations as in the previous section, the set of perturbed field equations is the same as in GR with:

$$\delta\rho_{eff} \equiv -\delta T_0^{(eff)\,0} = -\frac{1}{8\pi G a^2}\Big\{(1-F)\big[-6\mathcal{H}'\Psi + k^2\Psi - 3\mathcal{H}(\Phi'+\Psi') - 3\Phi''\big]$$
$$-3\mathcal{H}'\delta F + a^2\delta f/2 - k^2\Psi + 2k^2\Phi + 6(\mathcal{H}'+\mathcal{H}^2)\Psi + 3\Phi'' + 3\mathcal{H}(\Psi'+3\Phi')$$
$$+k^2\delta F + 3\mathcal{H}\delta F' - 3F'(\Phi'+2\mathcal{H}\Psi)\Big\}, \tag{5}$$

$$\delta p_{eff} \equiv \frac{\delta T_i^{(eff)\,i}}{3} = \frac{1}{8\pi G a^2}\Big\{(1-F)\big[-k^2\Phi - \Phi'' - 3\mathcal{H}(5\Phi'+\Psi')$$
$$-(2\mathcal{H}'+4\mathcal{H}^2)\Psi - k^2(\Phi-\Psi)/3\big] - (\mathcal{H}'+2\mathcal{H}^2)\delta F + a^2\delta f/2+$$
$$+3\Phi'' + k^2(2\Phi-\Psi) + 3\mathcal{H}(\Psi'+3\Phi') + 6(\mathcal{H}'+\mathcal{H})\Psi+$$
$$+\delta F'' + 2k^2\delta F/3 + \mathcal{H}\delta F' - F'(2\Phi'+2\mathcal{H}\Psi+\Psi') - 3\Psi F''\Big\} \tag{6}$$

Perturbations in $f(R)$

$$(\bar{\rho}_{eff}+\bar{p}_{eff})\upsilon^{eff}_{,i} \equiv -\delta T^{(eff)\,0}_{i} = \frac{1}{8\pi G}\Big[2(1-F)(\Phi'+\mathcal{H}\Psi)_{,i}+$$

$$+\delta F'_{,i}+F'\Psi_{,i}-\mathcal{H}\delta F_{,i}\Big] \text{ and}$$

$$\Pi^{eff}_{ij}\bar{p}_{eff} \equiv \delta T^{(eff)\,i}_{j} = \frac{1}{8\pi G a^2}\Big[(1-F)(\Phi-\Psi)_{,ij}+\delta F_{,ij}\Big],\ i\neq j \qquad (7)$$