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Coupling Metric-Affine Gravity to a Higgs-Like Scalar Field

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1. Different Formulations of Gravity

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1. Different Formulations of Gravity

2. Breaking the Equivalence Between the Different Formulations

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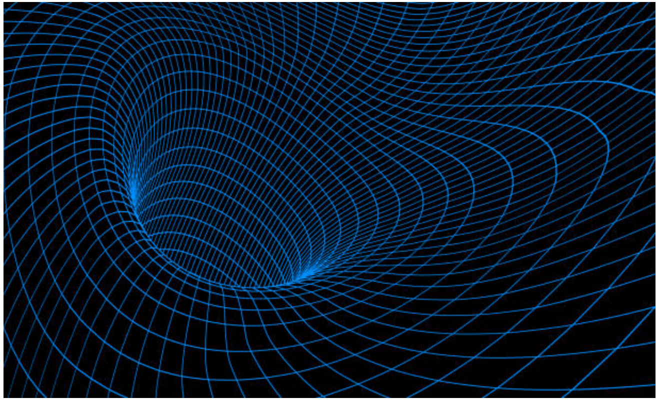
1. Different Formulations of Gravity
2. Breaking the Equivalence Between the Different Formulations
3. Phenomenology of Higgs Inflation

I. DIFFERENT FORMULATIONS OF GRAVITY

A. Metric Gravity



Degrees of freedom: $g_{\mu\nu}$

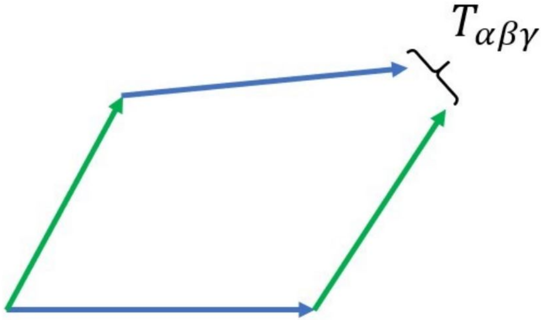
The connection is **uniquely** determined by the metric:

$$\mathring{\Gamma}^{\alpha}{}_{\beta\gamma} = \frac{1}{2} g^{\alpha\mu} \left(\partial_\beta g_{\mu\gamma} + \partial_\gamma g_{\mu\beta} - \partial_\mu g_{\beta\gamma} \right)$$

Degrees of freedom: $\{g_{\mu\nu}, \Gamma^{\alpha}{}_{(\beta\gamma)}\}$

The connection is no longer determined by the metric, they are **a priori** independent.

C. Einstein-Cartan Gravity



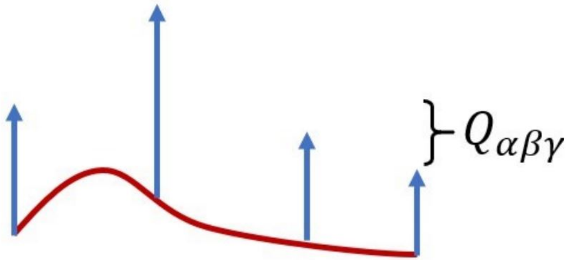
Degrees of freedom: $\{g_{\mu\nu}, \Gamma^{\alpha}{}_{\beta\gamma}\}$

$\Gamma^{\alpha}{}_{\beta\gamma}$ need not be symmetric in the last indices

$\Rightarrow$ **Torsion**:

$$T^{\alpha}{}_{\beta\gamma} = \Gamma^{\alpha}{}_{[\beta\gamma]}$$

D. Metric-Affine Gravity



Degrees of freedom: $\{g_{\mu\nu}, \Gamma^{\alpha}{}_{\beta\gamma}\}$

Most general formulation of gravity

$\Rightarrow$ **Non-metricity**:

$$Q_{\alpha\beta\gamma} = \nabla_\alpha g_{\beta\gamma}$$

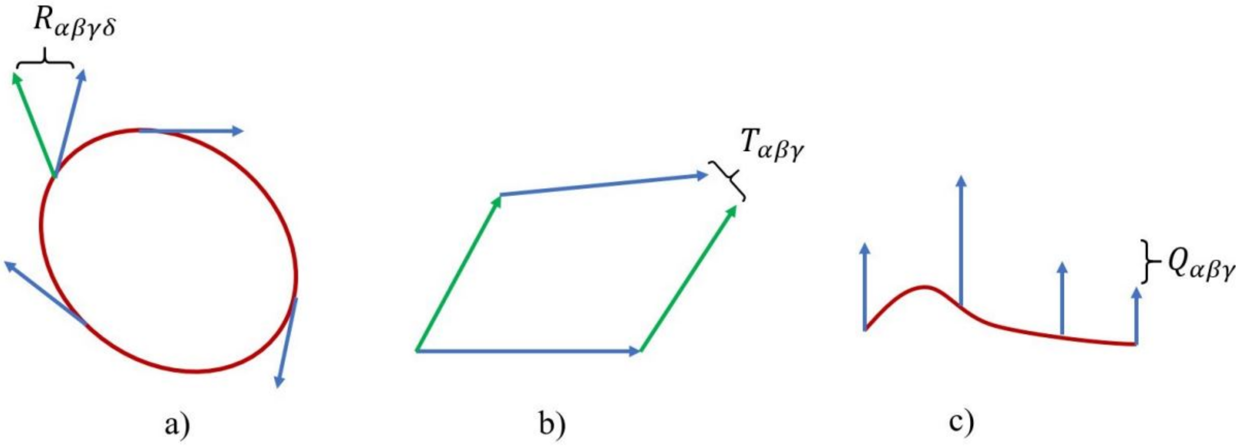


Figure 1: Schematic representation of the change of a vector under parallel transport due to the presence of: a) curvature b) torsion c) non-metricity.

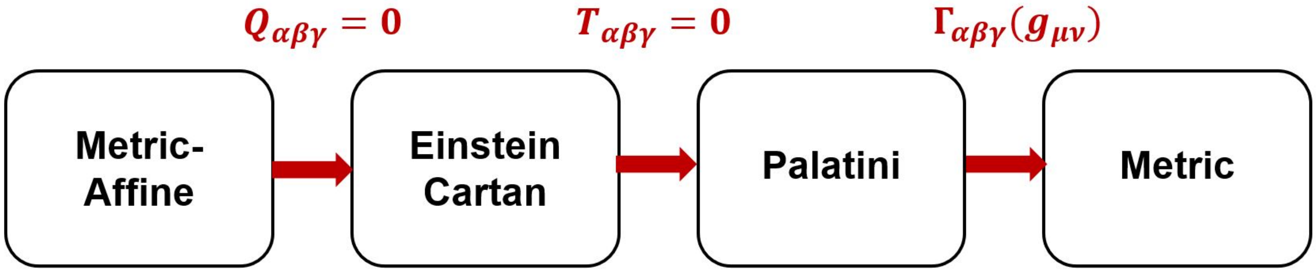


Figure 2: Relation between the different formulations of gravity.

II. BREAKING THE EQUIVALENCE BETWEEN THE DIFFERENT FORMULATIONS

Are they equivalent?

If not, what are the phenomenological consequences? Can we measure it?

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$$S_{\text{E-H}} = \int d^4x\sqrt{-g}R$$

↓

All formulations are equivalent.

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So, Why do we care?...

II. BREAKING THE EQUIVALENCE BETWEEN THE DIFFERENT FORMULATIONS

A. Higher derivatives

$$S = \int d^4x\sqrt{-g}[R + a_1 R^2 + a_2 R^{\mu\nu} R_{\mu\nu} + ...$$

Schematically, the equivalence is broken due to symmetry properties of the Riemann tensor :

II. BREAKING THE EQUIVALENCE BETWEEN THE DIFFERENT FORMULATIONS

A. Higher derivatives

	Metric	Einstein-Cartan	Metric-affine
$R_{ab[cd]}$	✓	✓	✓
$R_{[ab]cd}$	✓	✓	X
$R_{(ab)(cd)}$	✓	X	X
$R_{a[bcd]} = 0$	✓	X	X

Table I: Properties of Riemann tensor

e.g: In the metric-affine formalism, we can write a term like :

$$S = \int d^4x\sqrt{-g}R^{\alpha}{}_{\alpha\beta\gamma}R_{\mu}{}^{\mu\beta\gamma}$$

II. BREAKING THE EQUIVALENCE BETWEEN THE DIFFERENT FORMULATIONS

A. Higher derivatives

$$S = \int d^4x\sqrt{-g}[R + a_1R^2 + a_2R^{\mu\nu}R_{\mu\nu} + ...$$

Few remarks:

(a) New propagating d.o.fs $\Rightarrow$ quite a big deviation from GR.

(b) Some may be healthy, some unhealthy (ghosts or tachyons) [1]

[1] Lin, Y., Hobson, M.P., Lasenby, A.N. (2020). Power-counting renormalizable, ghost-and-tachyon-free Poincare gauge theories. *Physical Review D*

$$S = \int d^4x \sqrt{-g}(1 + \xi\phi^2)R + S_\phi$$

Few remarks:

(a) Non-minimal coupling terms come naturally when considering renormalization properties of a scalar field in a curved spacetime background [2].

(b) Gravity sector stays the same ⇒ no new *propagating* d.o.fs ⇒ minimal deformation to GR.

[2] N. D. Birrell and P. C. W. Davies, "Quantum Fields in Curved Space". *Cambridge Univ. Press*, (1984)

Conclusion: different formulations are no longer equivalent when the action is more complicated.

III. PHENOMENOLOGY OF HIGGS INFLATION

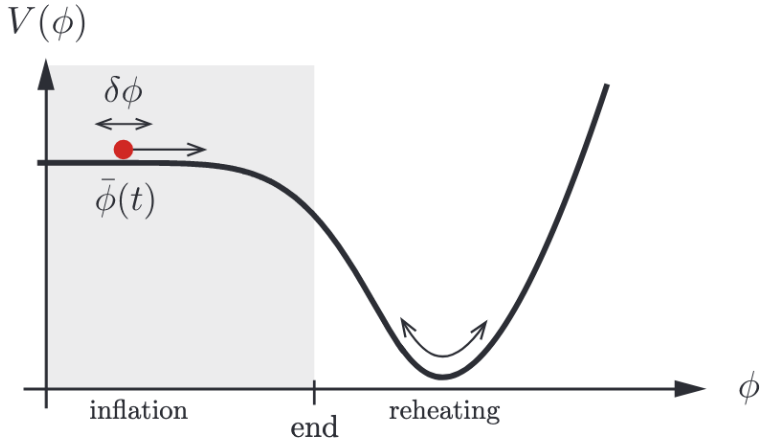
A. Motivation

- Matter field $\Rightarrow$ different formulations are no longer equivalent.
- Choose the most general formulation, i.e metric-affine.

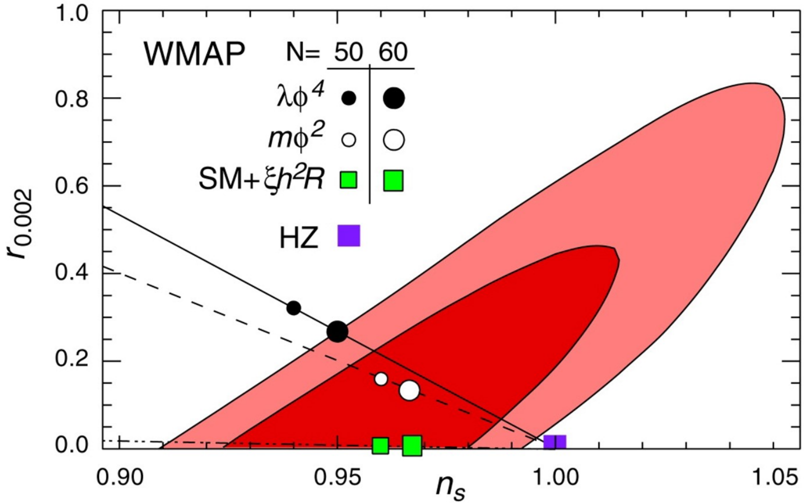
Presence of curvature, torsion and non-metricity

- Torsion and non-metricity are non-dynamical (no kinetic terms).
- They correspond to high energy effects.

Inflation



$$V(h) \simeq \frac{\lambda h^4}{4}$$



$$S \supset \int d^4x(1+\xi h^2)R$$

[3] F. Bezrukov, M. Shaposhnikov, The Standard Model Higgs boson as the inflaton, 0710.3755

Steps followed in the paper:

1. Write down the most general action including torsion and non-metricity.

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4. Perform a conformal transformation of the metric to get rid of the non-minimal coupling

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$$S = \int \mathrm{d}^4x\sqrt{-g}[\frac{1}{2}R - \frac{1}{2}K(h)\partial_\alpha h\partial^\alpha h - \frac{\lambda h^4}{(1+\xi h^2)^2}] \, .$$

- modified kinetic term for the Higgs field:

$$K(h) = \frac{1}{(1+\xi h^2)} \left[1 + \frac{h^2}{(\sum_{m=0}^{4} O_m h^{2m})^2} \sum_{n=0}^{7} P_n h^{2n} + \frac{6\xi^2 h^2}{(1+\xi h^2)} \right] .$$

- modified potential:

$$V(h) = \frac{\lambda h^4}{(1+\xi h^2)^2}$$

C. The theory, and its consequences

Steps followed in the paper:

1. Write down the most general action including torsion and non-metricity.
2. Find solution for torsion $T_{\alpha\beta\gamma}$ and non-metricity $Q_{\alpha\beta\gamma}$.
3. Plug them back in the action
4. Perform a conformal transformation of the metric to get rid of the non-minimal coupling

Conclusion: Flattened potential and new higher mass dimension self-interaction terms for the Higgs

IV. CONCLUSION

There exists different formulations of gravity.

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Lead to different predictions at high energy.

IV. CONCLUSION

New phenomenology at high energy:

- Flatter potential and new self-interactions for the Higgs field. [this paper]
- Production of Dark Matter through fermions coupled to gravity [4][to appear...].
- Different behaviour for singularities inside black holes [5]

[4] M. Shaposhnikov, A.Shrekin, I.Timiryasov and S.Zell, *Einstein-Cartan Portal to Dark Matter*, 2008.11686

[5] J.A.R.Cembranos, J. Gigante Valcarcel, and F.J. Maldonado Torralba, *Singularities and n-dimensional black holes in torsion theories*, 1609.07814

V. BIBLIOGRAPHY

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[0] Template for the slides: D. Backhouse.

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[2] N. D. Birrell and P. C. W. Davies, "Quantum Fields in Curved Space". *Cambridge Univ. Press*, (1984)

[3] F. Bezrukov, M. Shaposhnikov, The Standard Model Higgs boson as the inflaton, 0710.3755

[4] M. Shaposhnikov, A.Shrekin, I.Timiryasov and S.Zell, Einstein-Cartan Portal to Dark Matter, 2008.11686

[5] J.A.R.Cembranos, J. Gigante Valcarcel, and F.J. Maldonado Torralba, Singularities and n-dimensional black holes in torsion theories, 1609.07814

Further Reading:

[6] Beltran Jimenez, J., Heisenberg, L., Koivisto, T. S. (2019). The geometrical trinity of gravity. *Universe.*

[7] Karananas, G. K., Shaposhnikov, M., Shkerin, A., Zell, S. (2021). Matter matters in Einstein-Cartan gravity. Physical Review D, 104(6), 064036.

VI. APPENDIX

A. The action

$$
\begin{aligned}
S = \int \mathrm{d}^4x\sqrt{-g}\Big[&\frac{1}{2}(1+\xi h^2)\mathring{R} - \frac{1}{2}\tilde{K}(h)g^{\alpha\beta}\partial_\alpha h\partial_\beta h - V(h) \\
&+ A_1(h)\mathring{\nabla}_\alpha\hat{T}^\alpha + A_2(h)\mathring{\nabla}_\alpha T^\alpha + A_3(h)\mathring{\nabla}_\alpha\hat{Q}^\alpha + A_4(h)\mathring{\nabla}_\alpha Q^\alpha \\
&+ B_1(h)Q_\alpha Q^\alpha + B_2(h)\hat{Q}_\alpha\hat{Q}^\alpha + B_3(h)Q_\alpha\hat{Q}^\alpha + B_4(h)q_{\alpha\beta\gamma}q^{\alpha\beta\gamma} + B_5(h)q_{\alpha\beta\gamma}q^{\beta\alpha\gamma} \\
&+ C_1(h)T_\alpha T^\alpha + C_2(h)\hat{T}_\alpha\hat{T}^\alpha + C_3(h)T_\alpha\hat{T}^\alpha + C_4(h)t_{\alpha\beta\gamma}t^{\alpha\beta\gamma} \\
&+ D_1(h)\epsilon_{\alpha\beta\gamma\delta}t^{\alpha\beta\lambda}t^{\gamma\delta}{}_\lambda + D_2(h)\epsilon_{\alpha\beta\gamma\delta}q^{\alpha\beta\lambda}q^{\gamma\delta}{}_\lambda + D_3(h)\epsilon_{\alpha\beta\gamma\delta}q^{\alpha\beta\lambda}t^{\gamma\delta}{}_\lambda \\
&+ E_1(h)T_\alpha Q^\alpha + E_2(h)\hat{T}_\alpha Q^\alpha + E_3(h)T_\alpha\hat{Q}^\alpha + E_4(h)\hat{T}_\alpha\hat{Q}^\alpha + E_5(h)t^{\alpha\beta\gamma}q_{\beta\alpha\gamma}\Big].
\end{aligned}
$$

VI. APPENDIX

A. The action

Since torsion and non-metricity each carry three tensor indices, it is convenient to split them further into vector- and pure tensor-parts. This is done by contracting all possible indices following the symmetry properties. For torsion, this gives:

$$\text{the trace vector: } T^{\alpha} = g_{\mu\nu} T^{\mu\alpha\nu} \ , \tag{30}$$

$$\text{the pseudo trace axial vector: } \hat{T}^{\alpha} = \epsilon^{\alpha\beta\mu\nu} T_{\beta\mu\nu} \ , \tag{31}$$

$$\text{the pure tensor part: } t^{\alpha\beta\gamma} \text{ that satisfies } g_{\mu\nu} t^{\mu\alpha\nu} = 0 = \epsilon^{\alpha\beta\mu\nu} t_{\beta\mu\nu} \ . \tag{32}$$

Torsion can be be reconstructed in terms of these irreducible pieces as:

$$T_{\alpha\beta\gamma} = -\frac{2}{3} g_{\alpha[\beta} T_{\gamma]} + \frac{1}{6} \epsilon_{\alpha\beta\gamma\nu} \hat{T}^{\nu} + t_{\alpha\beta\gamma} \ . \tag{33}$$

Similarly, we can split further non-metricity into three contributions:

$$\text{a first vector: } Q^{\gamma} = g_{\alpha\beta} Q^{\gamma\alpha\beta} \ , \tag{34}$$

$$\text{a second vector: } \hat{Q}^{\gamma} = g_{\alpha\beta} Q^{\alpha\gamma\beta} \ , \tag{35}$$

$$\text{the pure tensor part: } q^{\alpha\beta\gamma} \text{ that satisfies } g_{\alpha\beta} q^{\gamma\alpha\beta} = 0 = g_{\alpha\beta} q^{\alpha\gamma\beta} \ . \tag{36}$$

In terms of the components of (34) to (36), non-metricity can expressed as:

$$Q_{\alpha\beta\gamma} = \frac{1}{18}[g_{\beta\gamma}(5Q_{\alpha} - 2\hat{Q}_{\alpha}) + 2g_{\alpha(\beta}(4\hat{Q}_{\gamma)} - Q_{\gamma)})] + q_{\alpha\beta\gamma} \ . \tag{37}$$

$$Q^\alpha = \frac{V}{Z}\partial^\alpha h \,, \qquad \hat{Q}^\alpha = \frac{W}{Z}\partial^\alpha h \,, \qquad T^\alpha = \frac{X}{Z}\partial^\alpha h \,, \qquad \hat{T}^\alpha = \frac{Y}{Z}\partial^\alpha h \,, \qquad t_{\alpha\beta\gamma} = q_{\alpha\beta\gamma} = 0 \,. \tag{38}$$

And the common denominator reads

$$\begin{aligned} Z = {} & B_3^2(4C_1C_2 - C_3^2) + 4B_2C_2E_1^2 - 4B_2C_3E_1E_2 + 4B_2C_1E_2^2 - E_2^2E_3^2 + 2E_1E_2E_3E_4 \\ & - E_1^2E_4^2 + B_3(-4C_2E_1E_3 + 2C_3E_2E_3 + 2C_3E_1E_4 - 4C_1E_2E_4) + 4B_1(B_2(-4C_1C_2 + C_3^2) \\ & + C_2E_3^2 - C_3E_3E_4 + C_1E_4^2) \,, \end{aligned} \tag{39}$$

VI. APPENDIX

C. Equivalent Metric Theory

$$S = \int \mathrm{d}^4x\sqrt{-g}\Big[\frac{1}{2}(1+\xi h^2)\mathring{R} - \frac{1}{2}\hat{K}(h)g^{\alpha\beta}\partial_\alpha h\partial_\beta h - V(h)$$

$$+A_1(h)\mathring{\nabla}_\alpha\hat{T}^\alpha + A_2(h)\mathring{\nabla}_\alpha T^\alpha + A_3(h)\mathring{\nabla}_\alpha\hat{Q}^\alpha + A_4(h)\mathring{\nabla}_\alpha Q^\alpha$$

$$+ B_1(h)Q_\alpha Q^\alpha + B_2(h)\hat{Q}_\alpha\hat{Q}^\alpha + B_3(h)Q_\alpha\hat{Q}^\alpha + B_4(h)q_{\alpha\beta\gamma}q^{\alpha\beta\gamma} + B_5(h)q_{\alpha\beta\gamma}q^{\beta\alpha\gamma}$$

$$+ C_1(h)T_\alpha T^\alpha + C_2(h)\hat{T}_\alpha\hat{T}^\alpha + C_3(h)T_\alpha\hat{T}^\alpha + C_4(h)t_{\alpha\beta\gamma}t^{\alpha\beta\gamma}$$

$$+ D_1(h)\epsilon_{\alpha\beta\gamma\delta}t^{\alpha\beta\lambda}t^{\gamma\delta}{}_\lambda + D_2(h)\epsilon_{\alpha\beta\gamma\delta}q^{\alpha\beta\lambda}q^{\gamma\delta}{}_\lambda + D_3(h)\epsilon_{\alpha\beta\gamma\delta}q^{\alpha\beta\lambda}t^{\gamma\delta}{}_\lambda$$

$$+ E_1(h)T_\alpha Q^\alpha + E_2(h)\hat{T}_\alpha Q^\alpha + E_3(h)T_\alpha\hat{Q}^\alpha + E_4(h)\hat{T}_\alpha\hat{Q}^\alpha + E_5(h)t^{\alpha\beta\gamma}q_{\beta\alpha\gamma}\Big].$$

VI. APPENDIX

D. Decomposition of the scalar curvature

$$
\begin{aligned}
R = \mathring{R} + \mathring{\nabla}_\alpha(Q^\alpha - \hat{Q}^\alpha + 2T^\alpha) - \frac{2}{3}T_\alpha(T^\alpha + Q^\alpha - \hat{Q}^\alpha) + \frac{1}{24}\hat{T}^\alpha\hat{T}_\alpha + \frac{1}{2}t^{\alpha\beta\gamma}t_{\alpha\beta\gamma} \\
- \frac{11}{72}Q_\alpha Q^\alpha + \frac{1}{18}\hat{Q}_\alpha\hat{Q}^\alpha + \frac{2}{9}Q_\alpha\hat{Q}^\alpha + \frac{1}{4}q_{\alpha\beta\gamma}(q^{\alpha\beta\gamma} - 2q^{\gamma\alpha\beta}) + t_{\alpha\beta\gamma}q^{\beta\alpha\gamma},
\end{aligned}
$$

VI. APPENDIX

E. Decomposition of the Holst term

$$\epsilon^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma} = \frac{1}{3}\hat{Q}^{\alpha}\hat{T}_{\alpha} - \frac{1}{3}Q^{\alpha}\hat{T}_{\alpha} - \frac{2}{3}\hat{T}^{\alpha}\hat{T}_{\alpha} + \mathring{\nabla}_{\alpha}T^{\alpha} - \frac{1}{2}\epsilon_{\beta\gamma\delta\mu}t_{\alpha}{}^{\delta\mu}t^{\alpha\beta\gamma} - \epsilon_{\alpha\gamma\delta\mu}q^{\alpha\beta\gamma}t_{\beta}{}^{\delta\mu}$$