5D Chern-Simons Gravity, RS Braneworld and Holography

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Five-Dimensional Chern-Simons gravity

The action is given by

$$S_{CS} = k \int \varepsilon_{ABCDE} \left(\frac{1}{l} \hat{R}^{AB} \hat{R}^{CD} \hat{e}^{E} + \frac{2}{3l^3} \hat{R}^{AB} \hat{e}^{C} \hat{e}^{D} \hat{e}^{E} + \frac{1}{5l^5} \hat{e}^{A} \hat{e}^{B} \hat{e}^{C} \hat{e}^{D} \hat{e}^{E} \right).$$

Two (different) perspectives. Chern-Simons gravity as a gauge theory for an AdS Lie group, vs Lovelock Chern-Simons theory, as a special point in the space of Lovelock theories. Chern-Simons gravity is defined only in an odd number of dimensions. For phenomenology, we stick to $D = 5$ dimensions.
Motivation: relate this theory to some more phenomenologically acceptable [Zanelli, Izaurieta,Rodriguez,...]

$SO(4,2)$ Chern-Simons gravity

In an odd number of dimensions, one can define Chern-Simons action as $\mathrm{d}L_{CD} \sim \langle F^{\frac{D+1}{2}} \rangle$, where F is a curvature two form for a given gauge connection. Choosing a gauge group $SO(4,2)$, and a gauge connection decomposition [Zanelli, Chamseddine]

$$A = \frac{1}{2}\hat{\omega}^{AB} J_{AB} + \frac{1}{l}\hat{e}^{A} P_{A},$$

with

$$\begin{aligned}
[J_{AB}, J_{CD}] &= G_{AD} J_{BC} + G_{BC} J_{AD} - (C \leftrightarrow D), \\
[J_{AB}, J_{C5}] &= G_{BC} J_{A5} - G_{AC} J_{B5}, \\
[J_{A5}, J_{C5}] &= J_{AC}.
\end{aligned}$$

Chern Simons action $\int\limits_{\mathcal{N}} \langle F^2 A - \frac{1}{2} F A^3 + \frac{1}{5} A^5 \rangle$ gives the promised formula.

Chern-Simons gravity

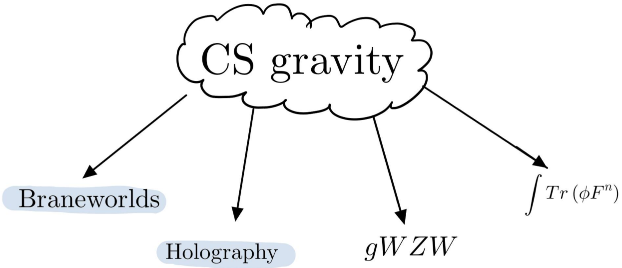
The last equality holds up to a boundary term. From now on, we consider a given Lagrangian that is obviously a particular choice of Lovelock Lagrangians. In a more familiar form

$$S_{CS} = \frac{1}{16\pi G}\int \mathrm{d}^5x\sqrt{-g}\Big[R - 2\Lambda + \frac{l^2}{4}\left(R^2 - 4R^{\mu\nu}R_{\nu\mu} + R^{\mu\nu\rho\sigma}R_{\rho\sigma\mu\nu}\right)\Big]$$

We set $l = 1$ for simplicity. Varying with respect to independent fields $\hat{e}^A$ and $\hat{\omega}^{AB}$, we obtain equations of motion. However, note that the relative values of coefficients multiplying different terms are fixed by l, and that we cannon choose this parameter to obtain EH term as dominant (cf. effective gravity action in string theory). Equations of motion allows for a nonzero torsion. Also, AdS spacetime is a solution to those equations ($\hat{R}^{AB} + \hat{e}^2\hat{e}^B = 0$).

CS Gravity Story

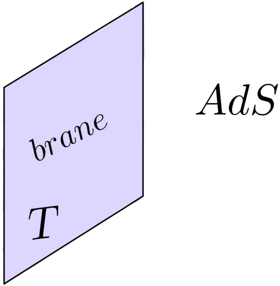
Torsion: General relativity was formulated on Riemannian manifolds, where torsion is set to zero. An alternative approach to classical GR would be to consider action $\int \varepsilon_{abcd} R^{ab} e^c e^d$ in the first order formalism, with e^a and ω^a independent, and then to set $T^a = 0$ through the equations of motion for a spin-connection. Examples where torsion is important: PGT, teleparalel gravity, $SO(2,3)$ NC gravity, NC Chern-Simons gravity, coupling with fermions, SUGRA (higher order curvature corrections),...
[Blagojević, Cvetković, Radovanović, Dimitrijević Ćirić, Gočanin, Nikolić, Djordjević,...]



RS models

Braneworld scenarios [ADD, RS, KR,...].
The initial motivation to cure the hierarchy problem: RS I model (two branes).
RS II model involves only one brane. Two sided version: analogue of a delta function potential in QM. [Randall, Sundrum, Karch,...]

$$\frac{1}{16\pi G}\int_N \mathrm{d}x^5\,\sqrt{-g}(R-2\Lambda)+T\int_Q \mathrm{d}x^4\,\sqrt{-h} + GHY$$



One can also put matter fields on the brane.
(BTW, this is a string theory inspired model and is not a product of string theory.)
Use of AdS space connect this model with the AdS/CFT.

RS setup

We will not assume that the manifold is not Riemannian. This is the most important difference from most previous considerations. Action is given by

$$S_{CS} + S_{brane}.$$

In the one-sided version, the brane is used as an IR cutoff, and integration is performed only over one part of the manifold. We then use holographic renormalization [Banados, Miskovic, Theisen, Cvetkovic, Simic,...]. Usual motivation: on-shell action suffers from the infrared divergences (and dual CFT has UV divergences) AdS/CFT prescription [Witten, Maldacena,...]

$$\delta S_{ren} = \int \tau^a \delta e_a + \frac{1}{2}\sigma^{ab}\delta\omega_{ab}.$$

Here, we have a cutoff CFT - a finite part that is not determined solely by the boundary fields. Its variations contains further ϵ finite terms.

Holographic renormalization

Using appropriate gauge choice [Banados, Miskovic, Theisen, Miskovic, Cvetkovic, Simic], Fefferman-Graham expansion is finite

$$\mathrm{d}s^2 = \frac{\mathrm{d}\rho^2}{4\rho^2} + \frac{1}{\rho}(g_{\alpha\beta} + 2\rho k_{(ab)} + \rho^2 k^a_\alpha k_{a\beta})\mathrm{d}x^\alpha \mathrm{d}x^\beta.$$

CS Lagrangian is gauge invariant up to boundary terms, so different asymptotic gauge conditions lead to different theories. Fields e^a, ω^{ab} and k^a are boundary fields. e^a and ω^{ab} are interpreted as sources in a dual CFT, while k^a is connected with one-point functions and is not determined from the CFT geometry. They have to satisfy identities coming from the bulk equations. They give rise to a holographic Ward identities in a dual language.

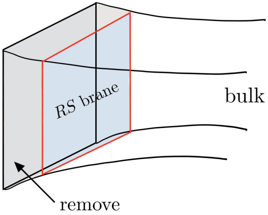
Tension

We place a brane at $\rho = \epsilon$. The tension term added to the brane Lagrangian is

$$S_{brane} = -T \int_{\mathcal{Q}} \varepsilon_{abcd} \hat{e}^a \hat{e}^b \hat{e}^c \hat{e}^d$$

Here, T is poportional to the brane tension. One could add matter fields confined to the brane, that would make the setup more realistic, but we stick to the simplest choice. Constraints that have to be satisfied are

$$\varepsilon_{abcd}(R^{ab} + 4e^a k^b)(R^{ab} + 4e^a k^b) = 0,$$
$$\varepsilon_{abcd}(R^{bc} + 4e^b k^c)T^d = 0,$$
$$\varepsilon_{abcd}(R^{bc} + 4e^c k^c)Dk^d = 0,$$
$$\varepsilon_{abcd}\big((R^{cd} + 4e^c k^d)e^e k_e + 2T^c Dk^d\big) = 0,$$



They have to be introduced via Lagrange multipliers to make variations of boundary fields independent on the brane.

AdS/CFT and RS braneworld

Equations of motion are obtained from

$$\delta\Bigg[W_{CFT} + 4k\int \varepsilon_{abcd}R^{ab}k^c e^d + \int \varepsilon_{abcd}\Big(\frac{2}{3}k - T\Big)e^a e^b e^c e^d$$
$$+ \int \varepsilon_{abcd}\Big(4k - 6T\Big)e^a k^b k^c k^d - 2k\int \varepsilon_{abcd}R^{ab}e^c e^d$$
$$- \int \varepsilon_{abcd}\Big(\frac{8}{3}k - 4T\Big)e^a e^b e^c k^d - T\int \varepsilon_{abcd}\Big(4e^a k^b k^c k^d + k^a k^b k^c k^d\Big)\Bigg].$$

Here, we consider "Minkowski" branes, tuning the brane tension to be $T = \frac{2}{3}k$. We recognise the EH term in the last expression (but note the minus sign). Fields are rescaled to be physical, and we don't use induce metrics here.

Equations

$$
\begin{aligned}
&k\varepsilon_{abcd}\Big(-4R^{ab}k^c - 16e^a k^b k^c - 4R^{ab}e^c\Big)\\
&+\varepsilon_{abcd}\Big(8\phi_1(R^{ab}+4e^a k^b)k^c - 4\phi_2^a k^b T^c - D\phi_2^a(R^{bc}+4e^b k^c) - 4\phi_2^a T^b k^c + 4\phi_2^a e^b Dk^c - 4\phi_3^a k^b Dk^c\\
&\qquad +4\phi_4^{ab}k^c e^m k_m + 2D\phi_4^{ab}Dk^c + 2\phi_4^{ab}R^c{}_m k^m\Big) - \varepsilon_{abcm}\phi_4^{ab}(R^{cm}+4e^c k^m)k_d = 0,
\end{aligned}
$$

$$
\begin{aligned}
&k\varepsilon_{abcd}\Big(4R^{ab}e^c + 4R^{ab}k^c - \frac{16}{3}k^a k^b k^c\Big)\\
&+\varepsilon_{abcd}\Big(8\phi_1(R^{ab}+4e^a k^b)e^c - 4\phi_2^a e^b T^c - 4\phi_3^a e^b Dk^c - D\phi_3^a(R^{bc}+4e^b k^c)\\
&\qquad -4\phi_3^a D(e^b k^c) - 2D(\phi_4^{ab}T^c) + 4\phi_4^{ab}e^c e^m k_m\Big) + \varepsilon_{abcm}\phi_4^{ab}(R^{cm}+4e^c k^m)e^d = 0,
\end{aligned}
$$

$$
\begin{aligned}
&-4\varepsilon_{abcd}\Big(Dk^c e^d + k^c T^d + e^c T^d + Dk^c k^d\Big)\\
&+\varepsilon_{abcd}\Big(-2D\phi_1(R^{cd}+4e^c k^d) - 8\phi_1 T^c k^d + 8\phi_1 e^c Dk^d D\phi_2^c T^d + \phi_2^c R^d{}_m e^m - D\phi_3^c Dk^d\\
&\qquad +\phi_3^c R^d{}_m k^m - D\phi_4^{cd}e^m k_m + \phi_4^{cd}T^m k_m - \phi_4^{cd}e^m Dk_m\Big)\\
&+\varepsilon_{amcd}\Big(\phi_2^c(R^{dm}+4e^d k^m)e_b + \phi_2^c(R^{dm}+4e^d k^m)k_b - 2\phi_4^{cd}Dk^m e_b + 2\phi_4^{cd}T^m k_b\Big) = 0.
\end{aligned}
$$

pp-waves

Solutions: Minkowski spacetime on the brane (trivial, but good for consistency), pp waves without torsion.

pp-waves correspond to a specific spacetime model designed to describe the propagation of different planar waves. We have

$$\mathrm{d}s^2 = 2H\mathrm{d}u^2 + 2\mathrm{d}u\mathrm{d}v + \mathrm{d}x^2 + \mathrm{d}y^2$$

Einstein equations: $\partial_x \partial_x H + \partial_y \partial_y H = 0$. Given spacetime also solves the constraint $\varepsilon_{abcd} R^{ab} R^{cd} = 0$.
In five dimensions, similar equations and conclusions have been obtained from a different perspective [Edelstein, Hassaıne, Troncoso, Zanelli]. This opens a possible connection between our model and those models seeking to obtain GR from CS gravity.

Conclusion and future directions

1. Braneworld scenario can be realised in the first order formalism, starting from the CS action. It would be interesting to compare with the Riemannian theory.
2. Obtained equations are complicated but have some simple solutions.
3. It would be interesting to find more complicated solutions (Minkowski and pp waves).
4. One can consider other brane tensions, and add more complicated amtter fields.
5. Recently, there has been an enormous amount of work on information paradox, using the Island formula [Maldacena, Engelhardt, Almheiri,...]. This formula is most easily seen in the AdS/BCFT [Takayanagi,...]. This type of duality emerged from the braneworld considerations [Karch, Randall]. AdS/BCFT is our motive for exploring this theory (CFT dual of our bulk may not be unitary [Banados, Schwimmer, Theisen, Olea]).

Thank you for your attention!