Determinism at a black hole singularity

Martina Adamo

PhD advisors: Flavio Mercati, Ángel Ballesteros Castañeda

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Gravitational singularities and determinism

"A singularity can be regarded as a place where there is a breakdown of the classical concept of spacetime as a manifold with a pseudo-Reimannian metric. Because all known laws of physics are formulated on a classical spacetime background, they will all break down at a singularity. This is a great crisis for physics because it means that **one cannot predict the future**. One does not know what will come out of a singularity."

S. W. Hawking, "Breakdown of Predictability in Gravitational Collapse", Phys. Rev. D14, 246 (1976)

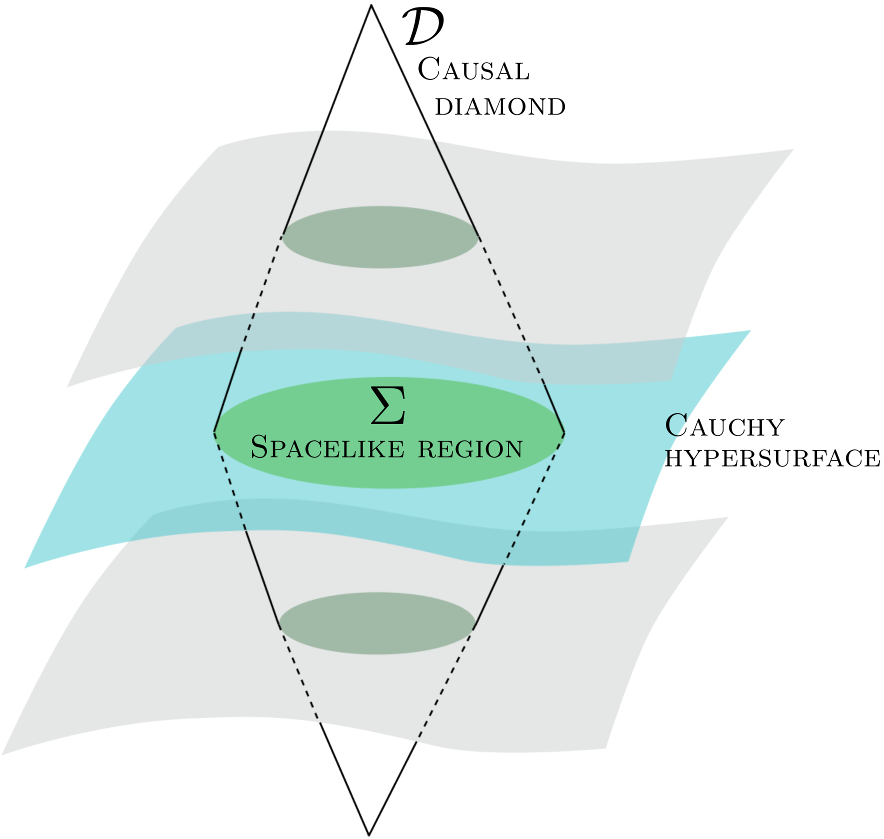
Gravitational singularities and determinism

Gravitational singularities are regions of spacetime where geometry or other fundamental physical structures become meaningless, and this happens in a **coordinate-independent way**

- the volume goes to zero
- some eigenvalues of the energy-momentum tensor diverge
- some curvature invariants diverge
- the geodesic equations are singular

But does this imply that dynamics is not well-defined?
Is this enough to give up on classical determinism?

Classical determinism in General Relativity



GR:

- Infinite number of DOFs
- Einstein's equations are a system of hyperbolic PDEs

Determinism:

Given all field values within Σ, it is possible to predict uniquely their values anywhere within $\mathcal{D}$

Classical determinism in General Relativity

Homogeneous cosmologies:

- Infinite → finite number of DOFs
- PDEs → ODEs

Determinism:

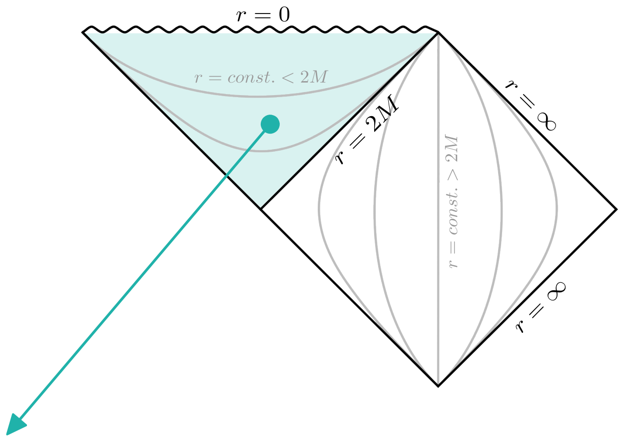
Picard–Lindelöf theorem of existence and uniqueness
under a certain set of conditions for the ODEs, an initial-value problem has a unique solution

GR is a gauge Hamiltonian system: **not all degrees of freedom are physical**, and determinism fails only if there is no way to evolve uniquely all physical DOFs

Interior Schwarzschild black hole

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\Omega^2$$

Schwarzschild metric

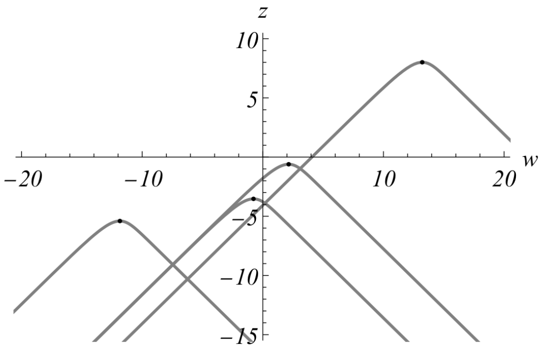


$$ds^2 = -N(\sigma)^2 d\sigma^2 + A(\sigma)^2 d\rho^2 + B(\sigma)^2 d\Omega^2$$

Kantowski–Sachs metric

Interior Schwarzschild black hole

$$\mathcal{H} = \frac{1}{2}\left(p_w^2 - p_z^2\right) - e^{\sqrt{\frac{2}{3}}(2z-w)}$$



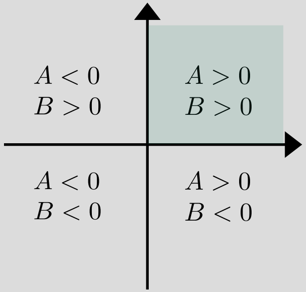
	z **scale** variable	w **shape** variable
BH singularity $r = 0^+$	$z \to -\infty$	$w \to -\infty$
horizon $r = (2M)^-$	$z \to -\infty$	$w \to +\infty$

Shape space and orientation

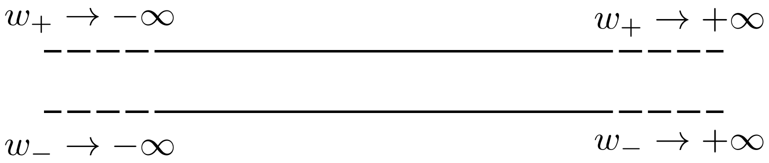
shape space

$$(w, z) \to (A, B)$$

$$\begin{cases} A = e^{\frac{z-2w}{\sqrt{6}}} \\ B = e^{\frac{w+z}{\sqrt{6}}} \end{cases}$$

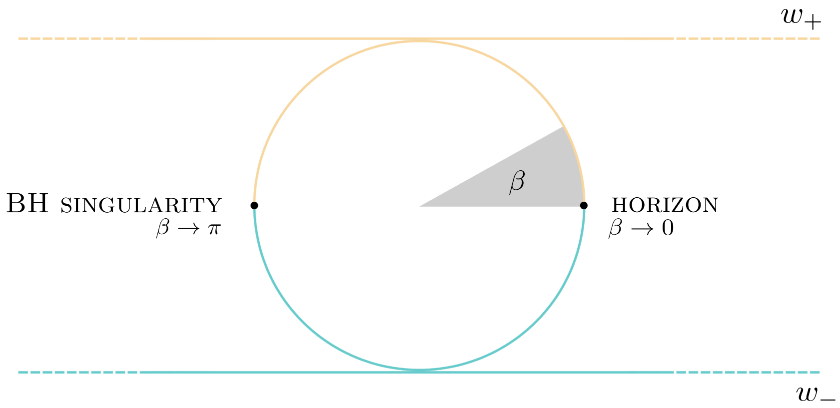


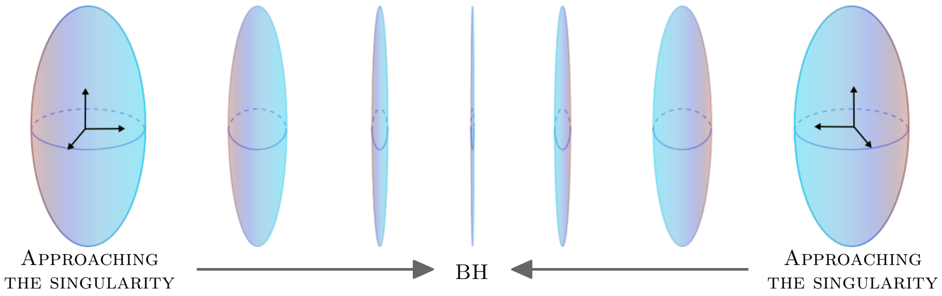
$v = AB^2 \quad \Rightarrow$ The sign of A defines two opposite orientations



extended shape space

Shape space and orientation

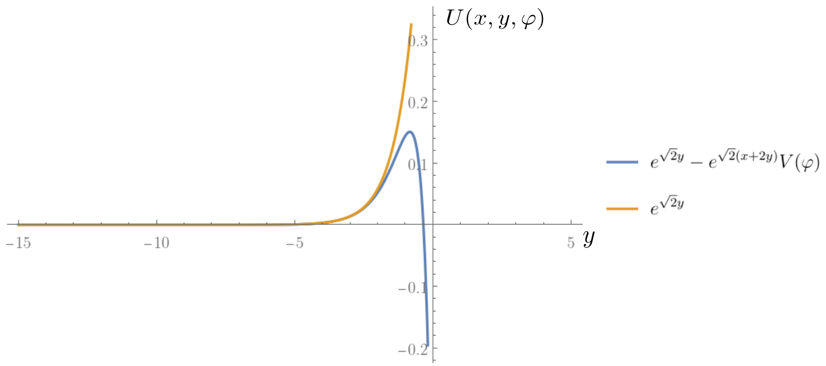




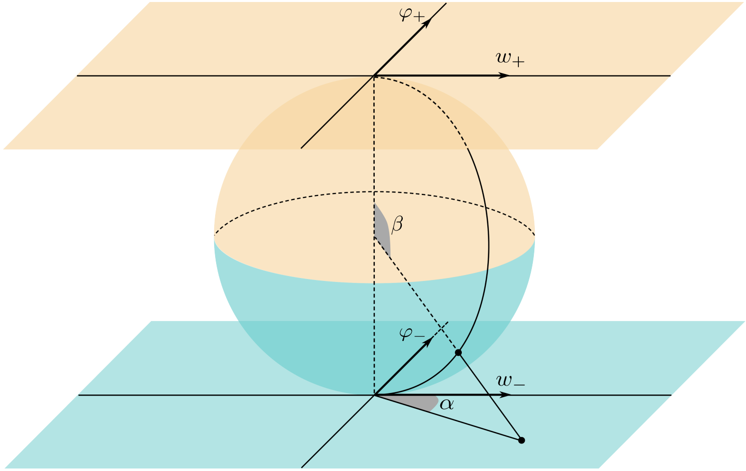
Schwarzschild-scalar model

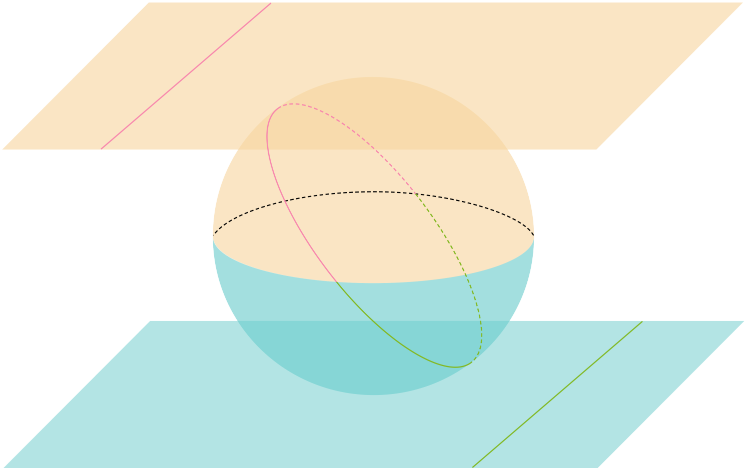
Adding an **homogeneous spherically-symmetric scalar field**
whose potential does not grow too fast

$$\mathcal{H} = \frac{1}{2}\left(p_w^2 + p_\varphi^2 - p_z^2\right) - e^{\sqrt{\frac{2}{3}}(2z-w)} + e^{\sqrt{6}z}V(\varphi)$$



Shape space and orientation





Continuation through the singularity

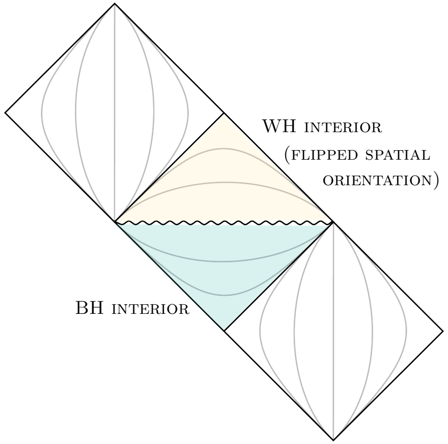
The equations of motion satisfy the Picard–Lindelöf Theorem

$$\frac{dy_i}{d\beta} = f_i(y)$$

$f_i(y)$ are differentiable functions (a stronger property than the Lipschitz-continuity required by the theorem)

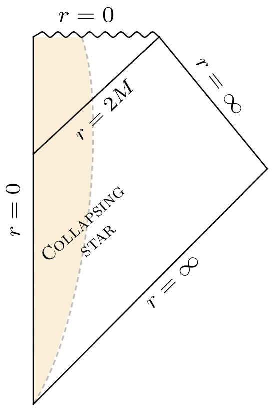
Each solution reaching the singularity from one hemisphere **is associated with one and only one solution** reaching the same point on the equator from the other hemisphere

Conclusions and future perspectives



Penrose diagrams make sense as an effective description of the causal relations between test particles propagating in a background spacetime

Ongoing work: non-spherically symmetric homogeneous scalar field (small perturbations)



Schwarzschild spacetime represents an eternal black hole, while realistic black holes are created through the collapse of matter

Future work: matter collapse model (*e.g.*, thin shell model)

Thank you for the attention
(and some references...)

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