Noncommutative correction to the entropy of charged BTZ black hole [1]

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February 14, 2023

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- Brick wall method for calculating black hole entropy
- Correction to the equations of motion due to noncommutative geometry

Brick wall method

Any field theoretic calculation of the black hole entropy leads to UV divergences [2, 3]

Brick wall method, developed by 't Hooft [4, 5], is one appropriate regularization based on the WKB approximation.

Brick wall method

D+2 dimensional, spherically symmetric black hole

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_D^2 \tag{1}$$

Calculate the Klein-Gordon equation

$$\Box_g \phi = 0 \tag{2}$$

Obtain the radial equation of motion

$$R'' + \left[\frac{V^2}{\hbar^2} - \Delta\right] R = 0 \tag{3}$$

Brick wall method

Apply the WKB approximation to the radial equation

$$R = \frac{c}{\sqrt{P(r)}} e^{\frac{i}{\hbar} \int^r P(x) dx} \tag{4}$$

And expand $P(r)$ into a power series in $\hbar^2$

$$P(r) = \sum_{n=0}^{\infty} \hbar^{2n} P_{2n}(r) \tag{5}$$

which yields

$$\begin{aligned} P_0(r) =& V(r) \\ P_2(r) =& \frac{3}{8P_0} \left(\frac{P_0'}{P_0} \right)^2 - \frac{P_0''}{4P_0} - \frac{\Delta}{2P_0} \\ P_4(r) =& \ldots \end{aligned} \tag{6}$$

Brick wall method

Imposing the Born-Sommerfeld quantization

$$\int_{r_H}^{\infty} P(r)dr = N\pi \tag{7}$$

gives the number of states

$$N(E) = \frac{1}{\pi}\int_{r_H+h}^{L} P(r)dr \tag{8}$$

where we have introduced the brick wall and IR regularization (h and L)

Brick wall method

We calculate the free energy F

$$F = -\frac{1}{\beta} \int_0^\infty \frac{N(E)}{e^{\beta E} - 1} dE \tag{9}$$

and finally obtain the entropy

$$S = \beta^2 \frac{\partial^2 F}{\partial \beta^2} \tag{10}$$

Noncommutative correction

Introduce a noncommutative $\star$ product

$$f \star g = m\left(\mathcal{F}^{-1} f \otimes g\right) = fg + \frac{ia}{2}\left(\frac{\partial f}{\partial t}\frac{\partial g}{\partial \phi} - \frac{\partial f}{\partial \phi}\frac{\partial g}{\partial t}\right) + \mathcal{O}(a^2) \qquad (11)$$

and use it to deform the geometric and tensor structure over the spacetime manifold $\mathcal{M}$ (see, e.g., [1])

Consider the noncommutative field theory

$$S[\Phi] = \int d^3x \sqrt{-g}\left(g^{\star\mu\nu}\left(D^\star_\mu \Phi\right)^+ \star D^\star_\nu \Phi - \frac{\mu^2}{\hbar^2}\Phi^+ \star \Phi\right) \qquad (12)$$

and expand the equation of motion in terms of commutative quantities

Corrected entropy

Apply the brick wall method to the new, modified equation of motion. The resulting entropy is of the form

$$S = S_{BH} + \sum_n C_n(A) \ln^n \left(\frac{A_{\mathrm{BTZ}}}{l} \right) \tag{13}$$

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