THE TIME PROBLEM IN PRIMORDIAL PERTURBATIONS*

Winter School of Theoretical Physics

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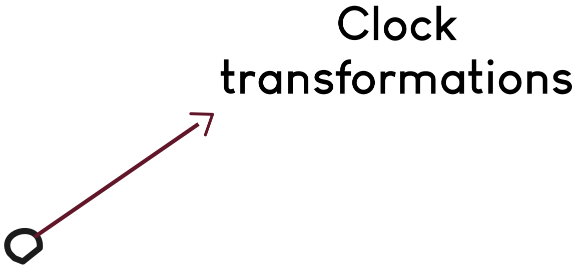
14/02/2023

* "The time problem in primordial perturbations"- **A.B.**, P. Peter, P. Małkiewicz, soon on ArXiv

Time problem in physics

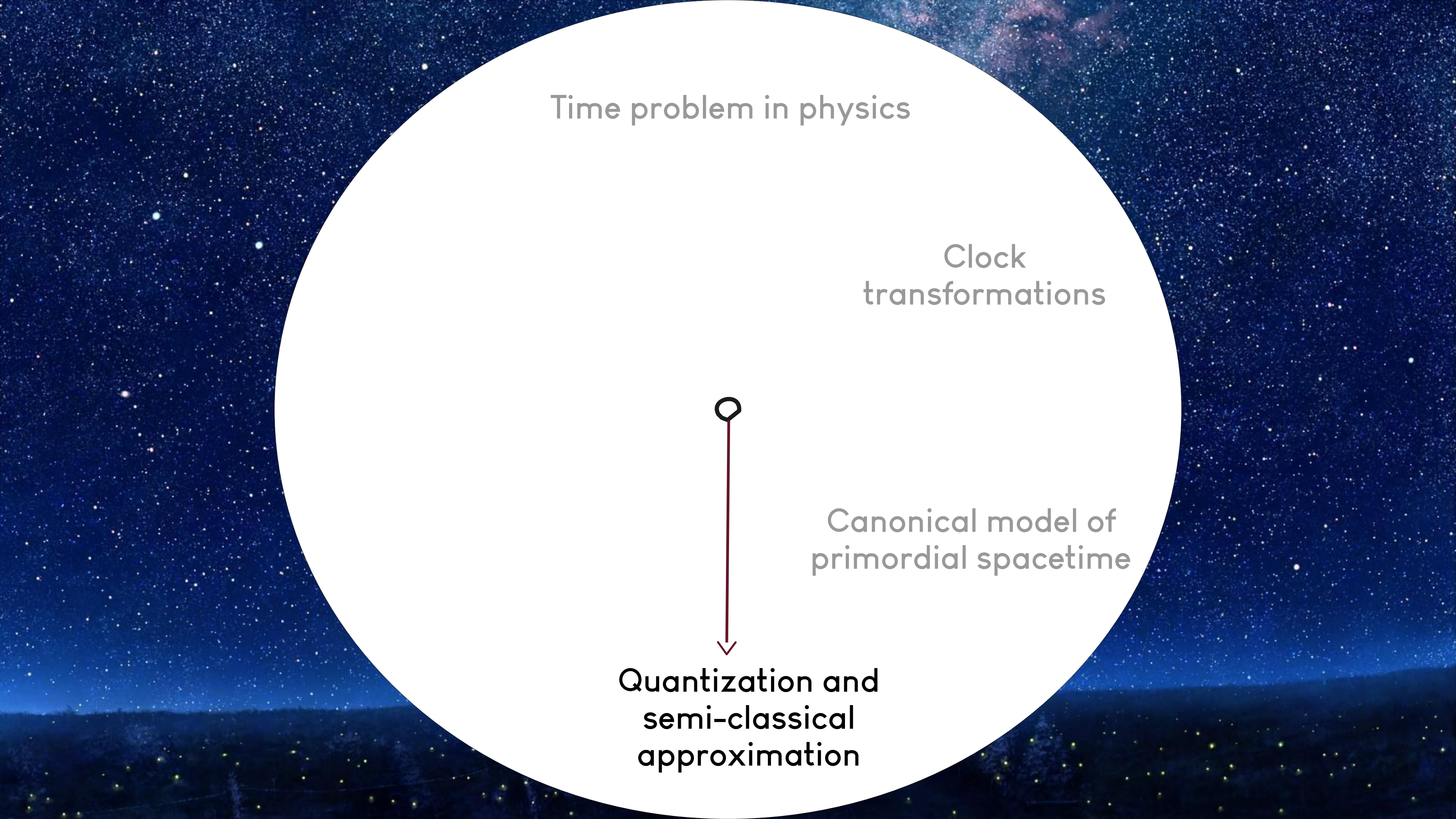
Time problem in physics



Time problem in physics

Clock transformations

Canonical model of primordial spacetime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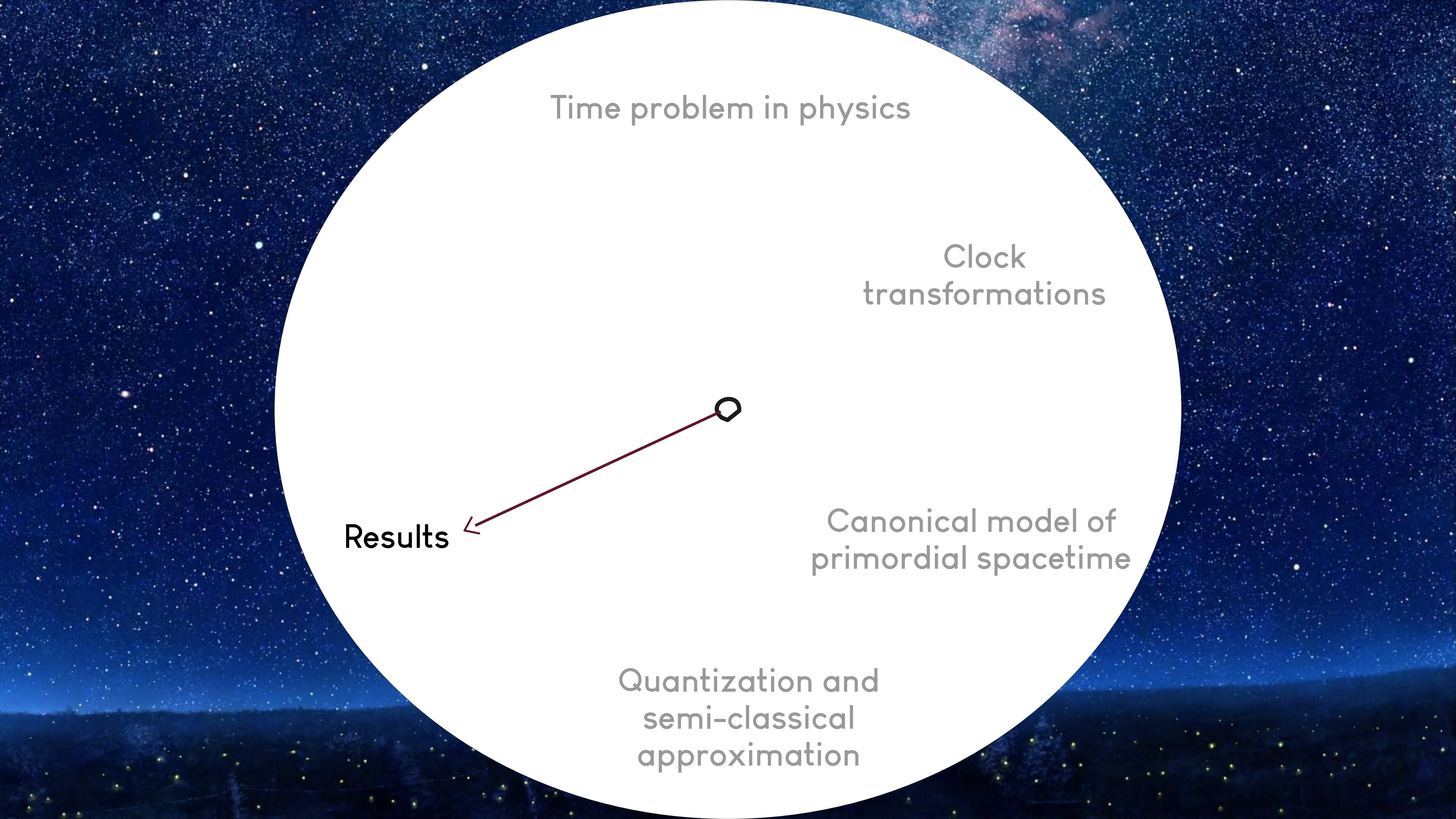
Quantization and semi-classical approximation


Time problem in physics
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Results

Time problem in physics

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Results

Conclusions

TIME PROBLEM IN PHYSICS

GR is **diffeomorphism invariant,** thus the free choice of internal time variable has no physical consequence.

Upon passing to quantum theory, however, different choices of internal time variables are known to produce unitarily in equivalent quantum models.

The problem of finding the correct interpretation of these non-equivalent models is commonly known as **the time problem**.

CLOCK TRANSFORMATIONS

Hamiltonian constraint

$$C(q_I, p^J) = 0$$

Assume q_0 varies monotonically with the evolution generated by the constraint

$$\{q_0, C\}_{\text{PB}} \neq 0$$

we can assign to it the role of the internal clock in which the evolution of the remaining variables occurs

The dynamics reads

$$\frac{\mathrm{d}q_I}{\mathrm{d}q_0} = \frac{\partial H}{\partial p_I} \qquad \frac{\mathrm{d}p^I}{\mathrm{d}q_0} = -\frac{\partial H}{\partial q^I}$$

Let's call $q_0 = t$

CLOCK TRANSFORMATIONS

We can define a new clock as function of the old clock and the old canonical variables

$$\tilde{t} = \tilde{t}(q_I, p^I, t)$$

There must exist an invertible map between the **old** and the **new** variables.
We find a complete set of **canonical constants of motion**, denoted by

$$D_I = D_I(q_J, p^J, t)$$

KEY FORMULA

$$\tilde{t} = \tilde{t}(q_I, p^I, t), \quad D_I(q_J, p^J, t) = D_I(\tilde{q}_J, \tilde{p}^J, \tilde{t})$$

CANONICAL MODEL OF PRIMORDIAL SPACETIME

$$\mathrm{d}s^2 = -N^2(t)\mathrm{d}t^2 + a^2(t)(\delta_{ij} + h_{ij}(x,t))\mathrm{d}x^i\mathrm{d}x^j$$

Flat Friedmann universe filled with radiation and perturbed by gravitational waves

$$H = H^{(0)} + \sum_k H_k^{(2)}$$

$$H^{(0)} = \frac{1}{2}p^2$$

Hamiltonian in reduced phase space

$$H_k^{(2)} = -\frac{1}{2}|\pi_\pm(\vec{k})|^2 - \frac{1}{2}\left(k^2 - \frac{\ddot{a}}{a}\right)|\mu_\pm(\vec{k})|^2$$

Expansion of the gravitational constraint up to second order

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Derivatives with respect to conformal time

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$$\mu_\pm = a h_\pm$$

Expansion of the gravitational constraint up to second order

CANONICAL MODEL OF PRIMORDIAL SPACETIME

We find the constants of motion that form canonical pairs

$$\frac{\mathrm{d}D}{\mathrm{d}t} = \frac{\partial D}{\partial t} + \{D, H^{(0)} + H^{(2)}\} = 0$$

Zeroth order

$$D_1 = a - pt, \quad D_2 = p$$

First order

$$\delta D_{1,\pm} = \mu_\pm \sqrt{k} \sin(\omega) - \pi_\pm \frac{1}{\sqrt{k}} \cos(\omega)$$

$$\delta D_{2,\pm} = \mu_\pm \sqrt{k} \cos(\omega) + \pi_\pm \frac{1}{\sqrt{k}} \sin(\omega)$$

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$$\omega = k(t + q/p)$$

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$$\omega = k(t + q/p)$$

They form canonical pairs

$$\{D_1, D_2\} = 1, \quad \{\delta D_{1,\pm}, \delta D_{2,\pm}\} = \delta_{\pm,\pm}$$

CANONICAL MODEL OF PRIMORDIAL SPACETIME

Clock transformation ⟶ $t \to \tilde{t} = t + \Delta(q, p, t)$

Delay function

1) The clock must move forward

$$\frac{\partial(\tilde{q}, \tilde{p})}{\partial(q, p)} = \begin{vmatrix} \frac{\partial \tilde{q}}{\partial q} & \frac{\partial \tilde{q}}{\partial p} \\ \frac{\partial \tilde{p}}{\partial q} & \frac{\partial \tilde{p}}{\partial p} \end{vmatrix} > 0$$

2) Preserves the ranges of the background variables

$$\lim_{p \to \pm\infty} \tilde{p}(q, p, t) = \pm\infty$$

$$\tilde{q}(q, p, t)\big|_{q=0} = 0$$

CANONICAL MODEL OF PRIMORDIAL SPACETIME

The Dirac observables are invariant under the clock transformation

Thus dynamical varaibles are not invariant, i.e.

$(\tilde{a}, \tilde{p}) \in \mathbb{R}_+ \times \mathbb{R}$

$$\tilde{a} = a + p\Delta, \quad \tilde{p} = p,$$

$$\begin{bmatrix} \tilde{\mu}_k \\ k^{-1}\tilde{\pi}_k \end{bmatrix} = \begin{bmatrix} \cos k\Delta & -\sin k\Delta \\ \sin k\Delta & \cos k\Delta \end{bmatrix} \begin{bmatrix} \mu_k \\ k^{-1}\pi_k \end{bmatrix}$$

The two frameworks generate the same physical dynamics of the system

QUANTIZATION AND SEMI-CLASSICAL APPROXIMATION

Semi-classical background Hamiltonian

$$H_{\text{sem}} = \frac{1}{2}\left(p^2 + \frac{\hbar^2 \mathfrak{K}}{a^2}\right) \quad \rightarrow \mathfrak{K} > 0$$

$$\dot{a} = \frac{\partial H_{\text{sem}}}{\partial p}$$

$$\dot{p} = -\frac{\partial H_{\text{sem}}}{\partial a}$$

Hamilton equations

Physical perturbation Hamiltonian

$$H_k^{(2)} = -\frac{1}{2}|\pi(\vec{k})|^2 - \frac{1}{2}\left(k^2 - \frac{\ddot{a}}{a}\right)|\mu(\vec{k})|^2$$

QUANTIZATION AND SEMI-CLASSICAL APPROXIMATION

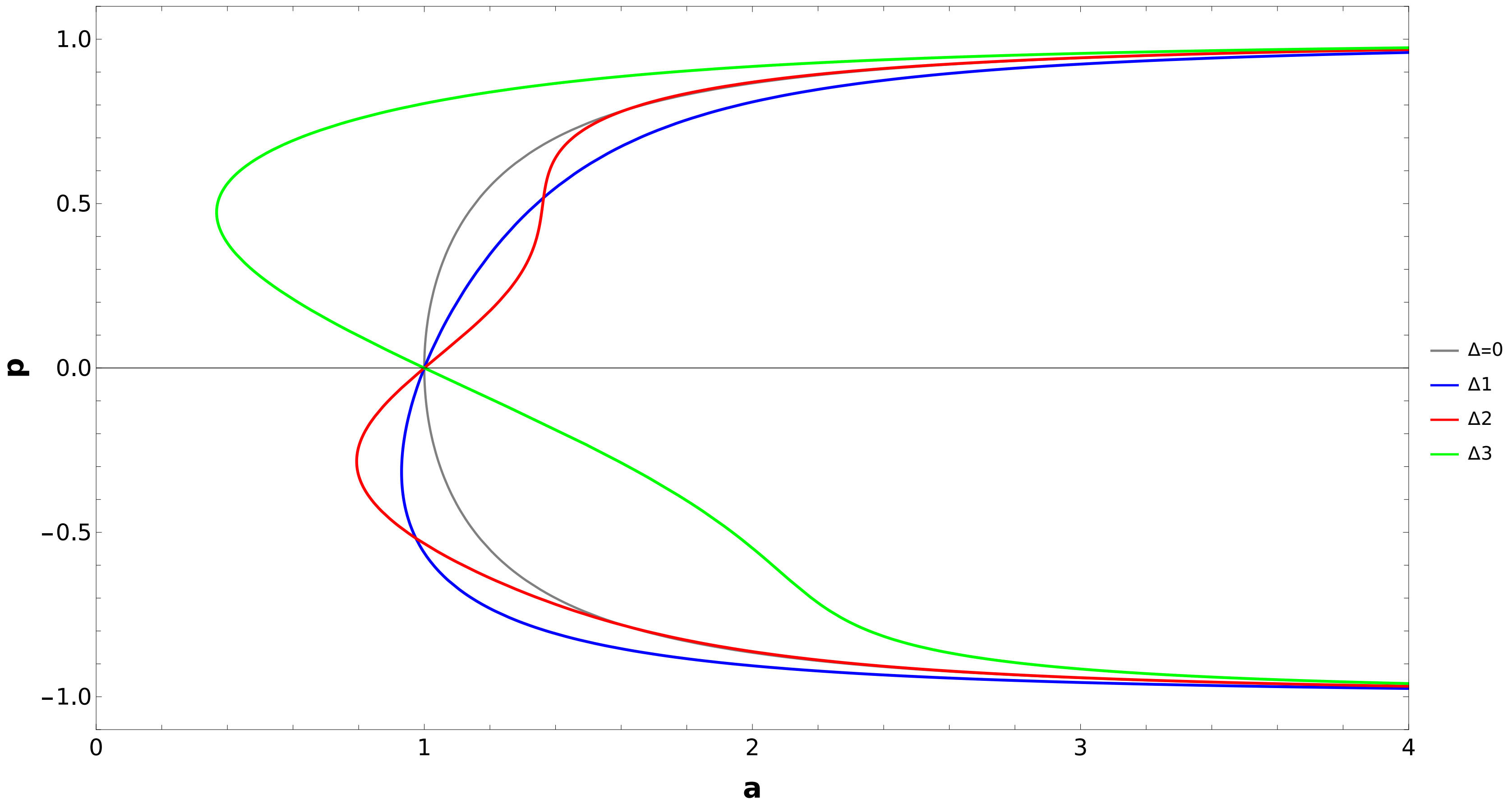
Quantization of the perturbation

$$\mu_{\boldsymbol{k}} \mapsto \widehat{\mu}_{\boldsymbol{k}} = \sqrt{\frac{\hbar}{2}}\left[a_{\boldsymbol{k}}\bar{\mu}_k(\tau) + a^\dagger_{-\boldsymbol{k}}\mu_k(\tau)\right] \qquad \pi_{\boldsymbol{k}} \mapsto \widehat{\pi}_{\boldsymbol{k}} = \sqrt{\frac{\hbar}{2}}\left[a_{\boldsymbol{k}}\dot{\bar{\mu}}_k(\tau) + a^\dagger_{-\boldsymbol{k}}\dot{\mu}_k(\tau)\right]$$

Equation of motion in Heisenberg picture

$$\boxed{\frac{\mathrm{d}^2\mu_k}{\mathrm{d}\eta^2} + \left(k^2 + \frac{\hbar^2\mathfrak{K}}{a^4}\right)\mu_k = 0}$$

RESULTS



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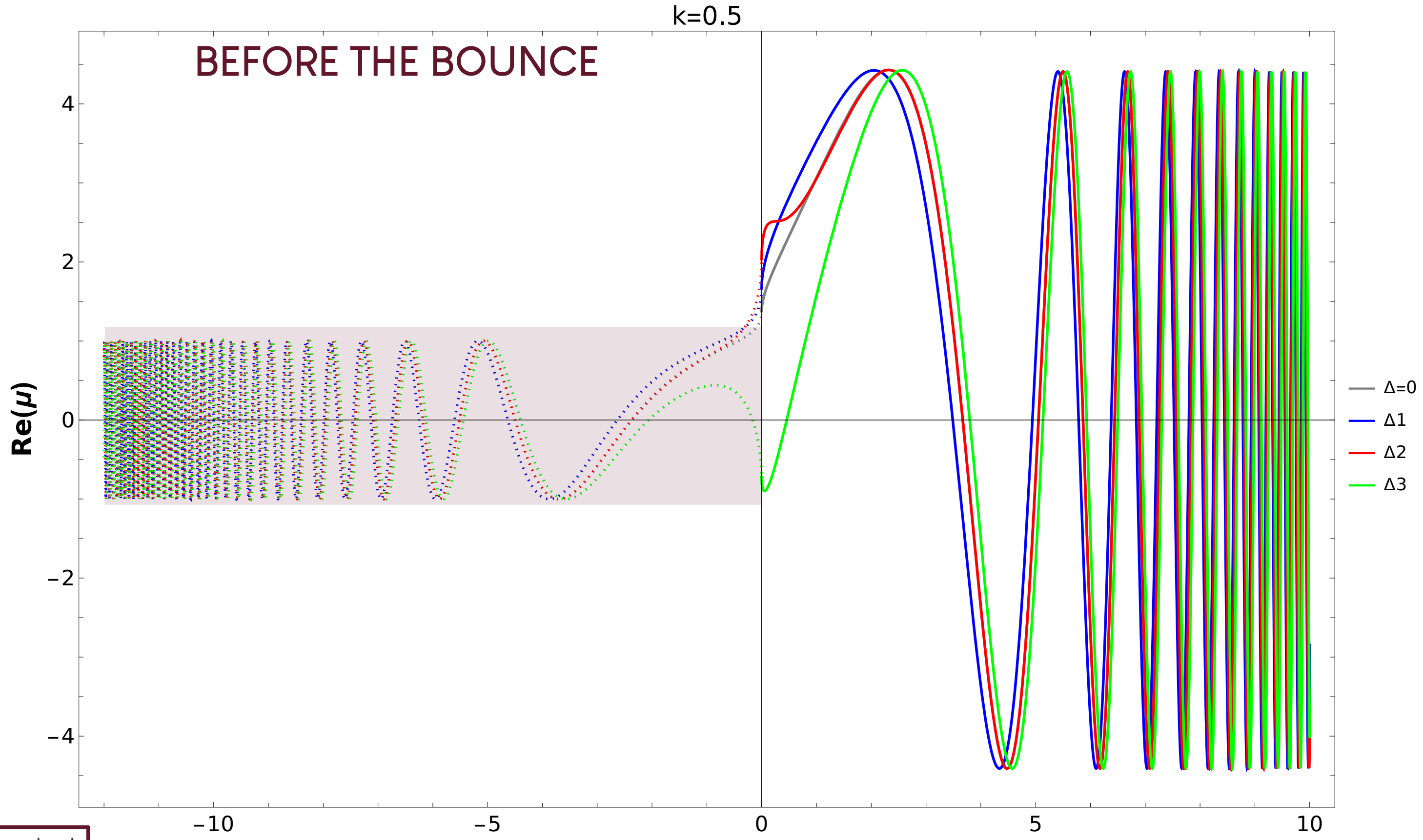
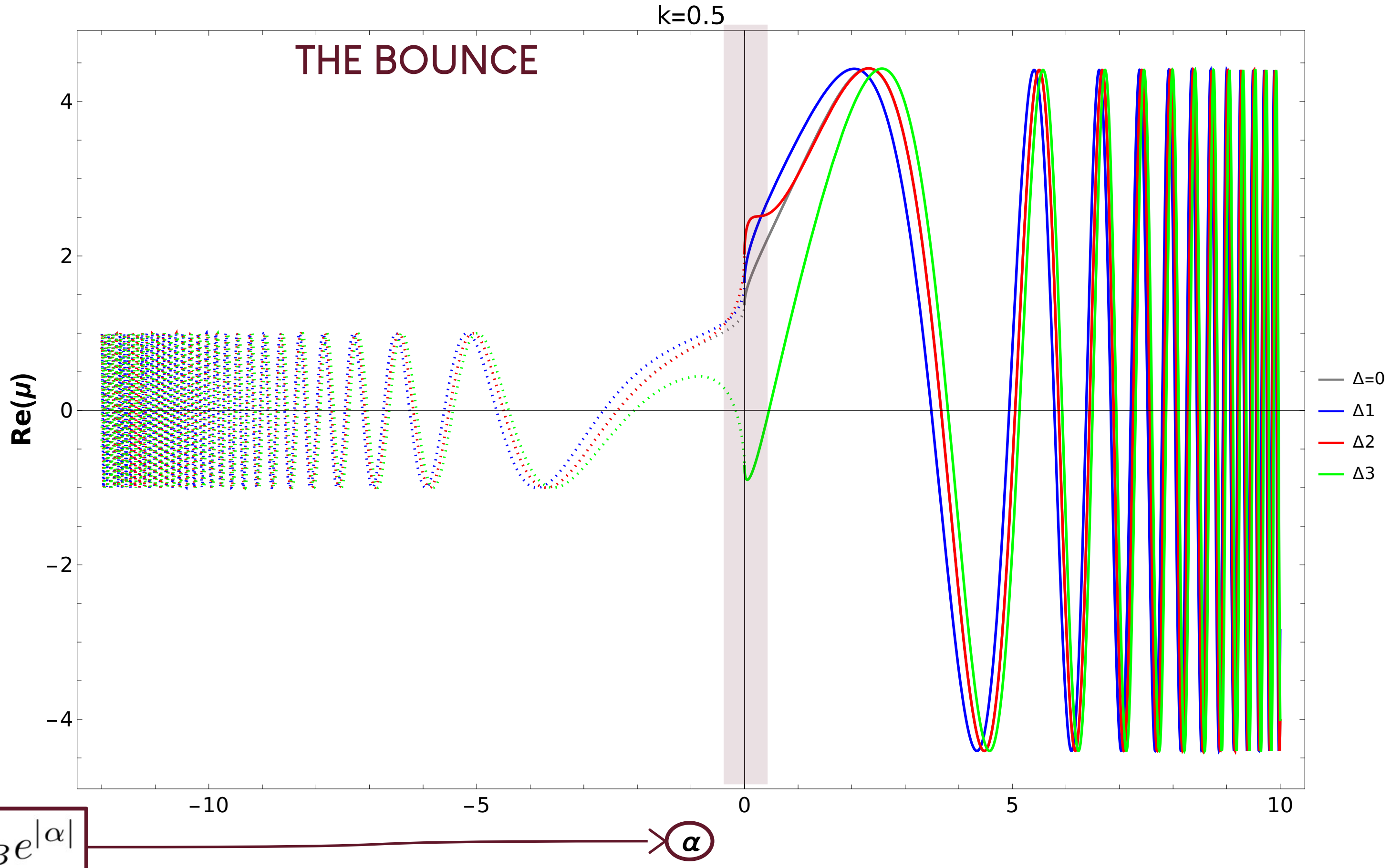


$$a = a_B e^{|\alpha|}$$

α

RESULTS



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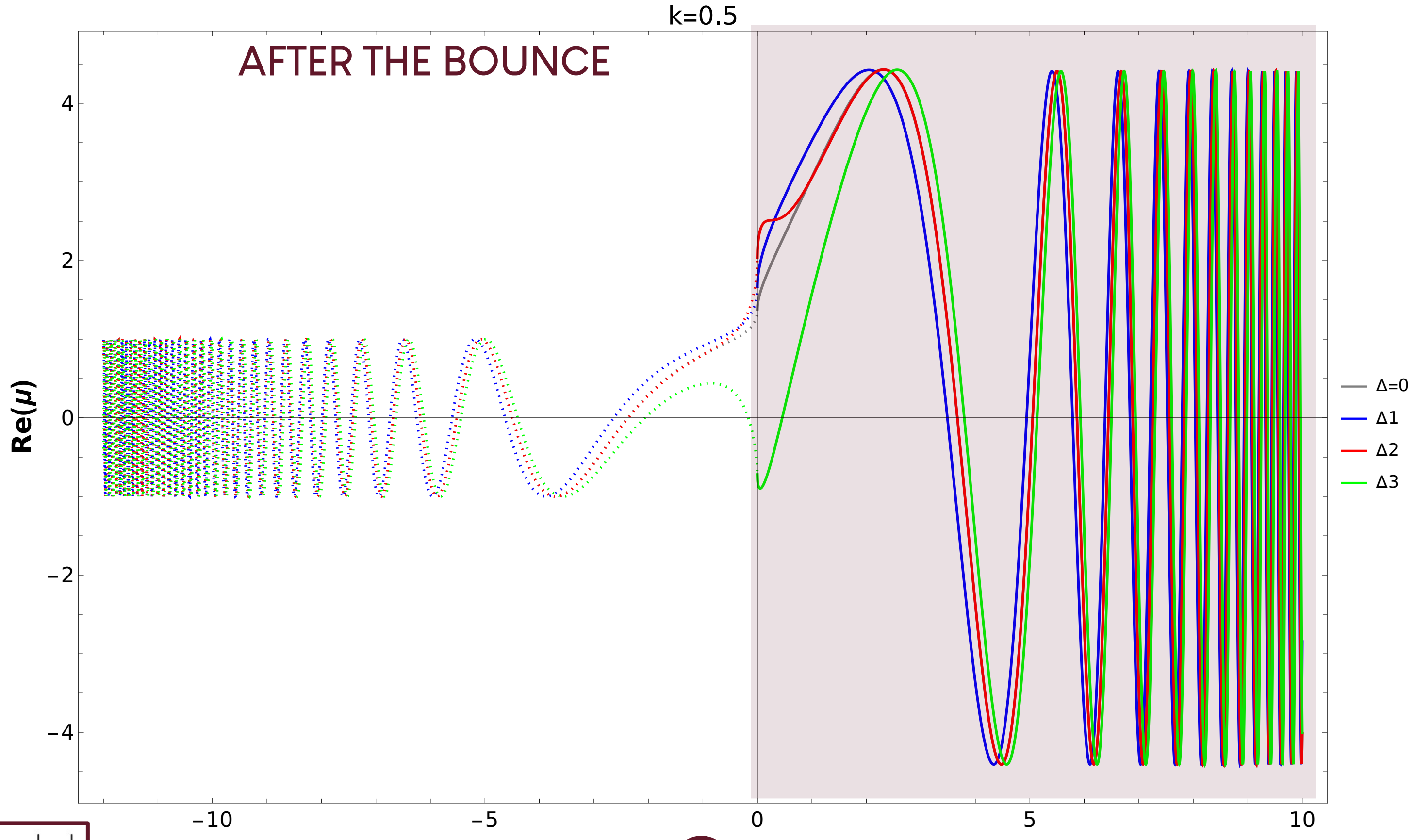

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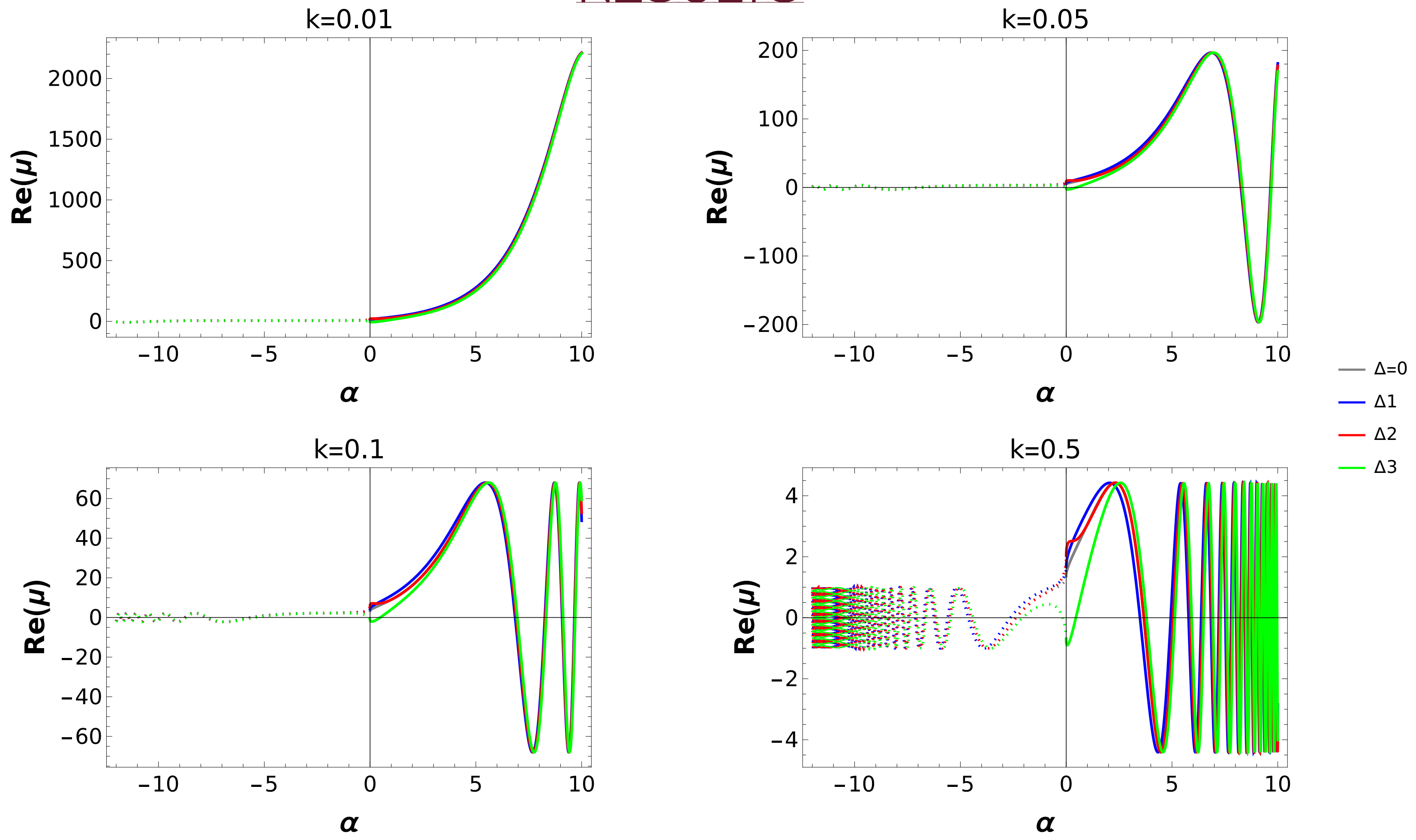
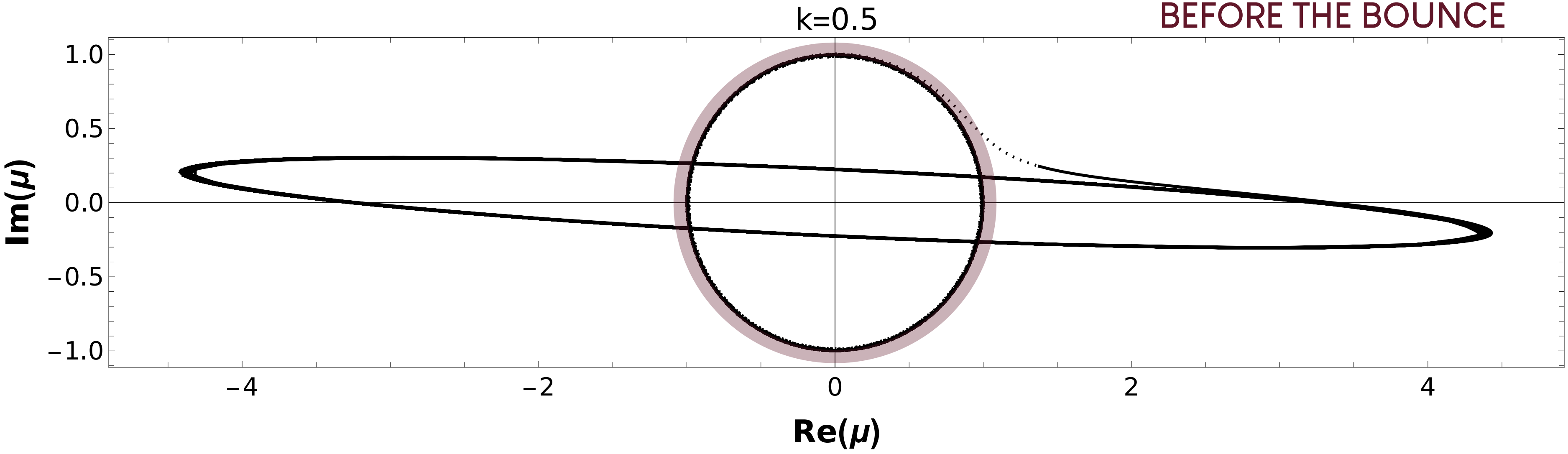


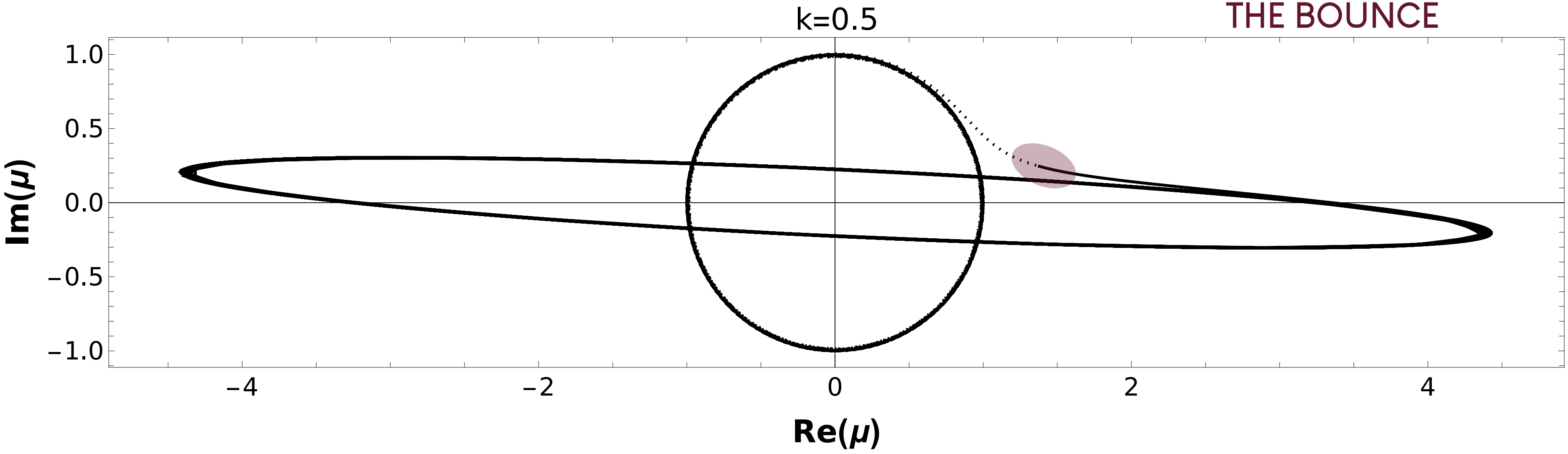
RESULTS

BEFORE THE BOUNCE



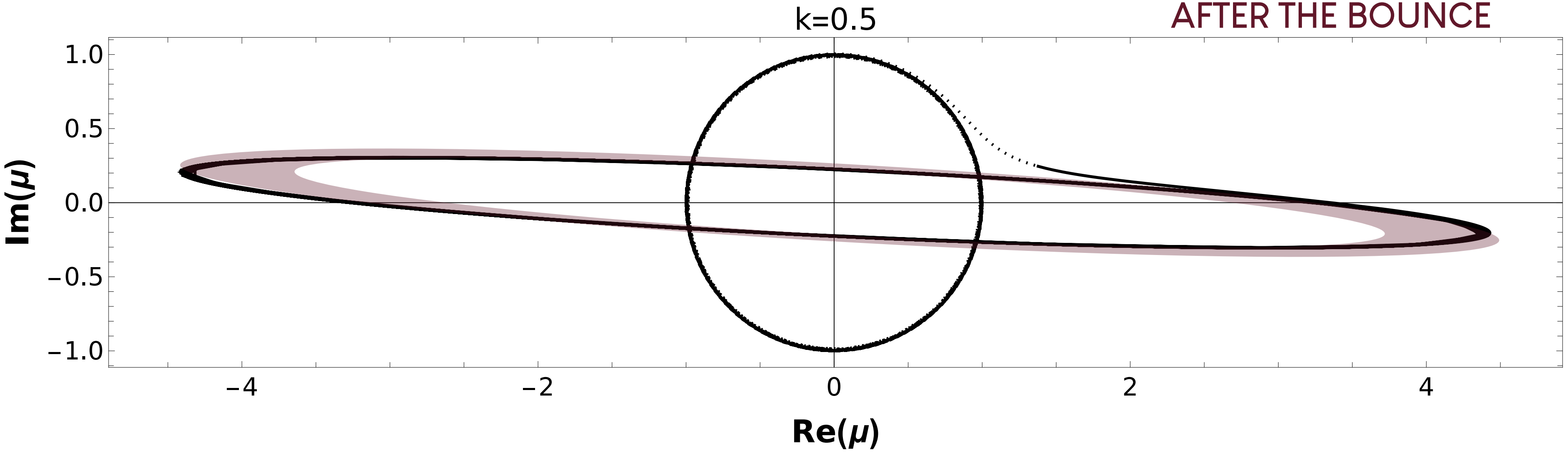
RESULTS

THE BOUNCE



RESULTS

AFTER THE BOUNCE



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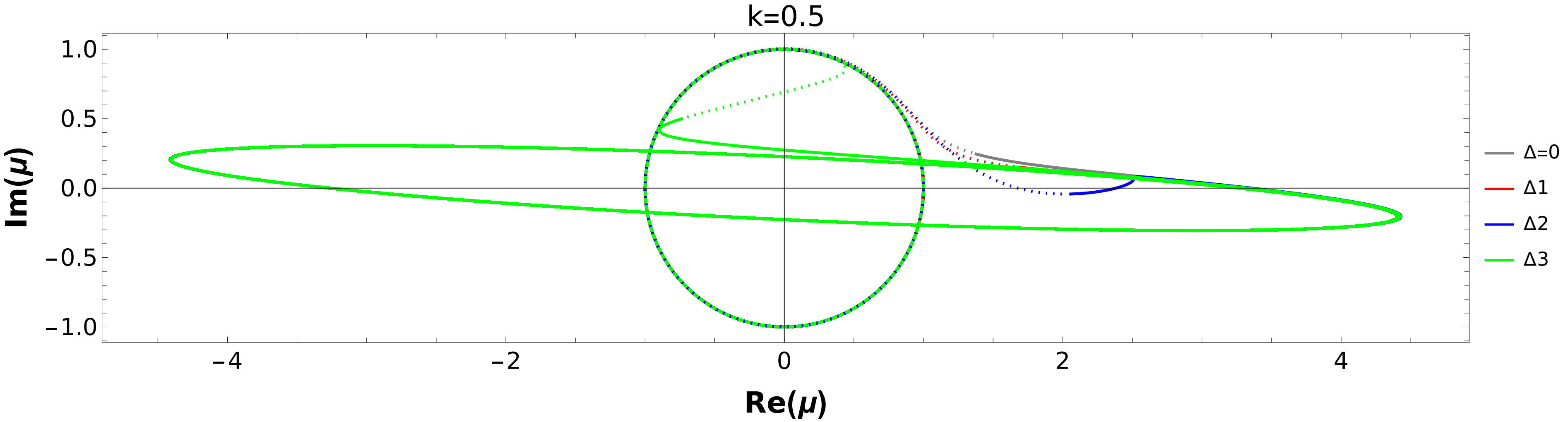

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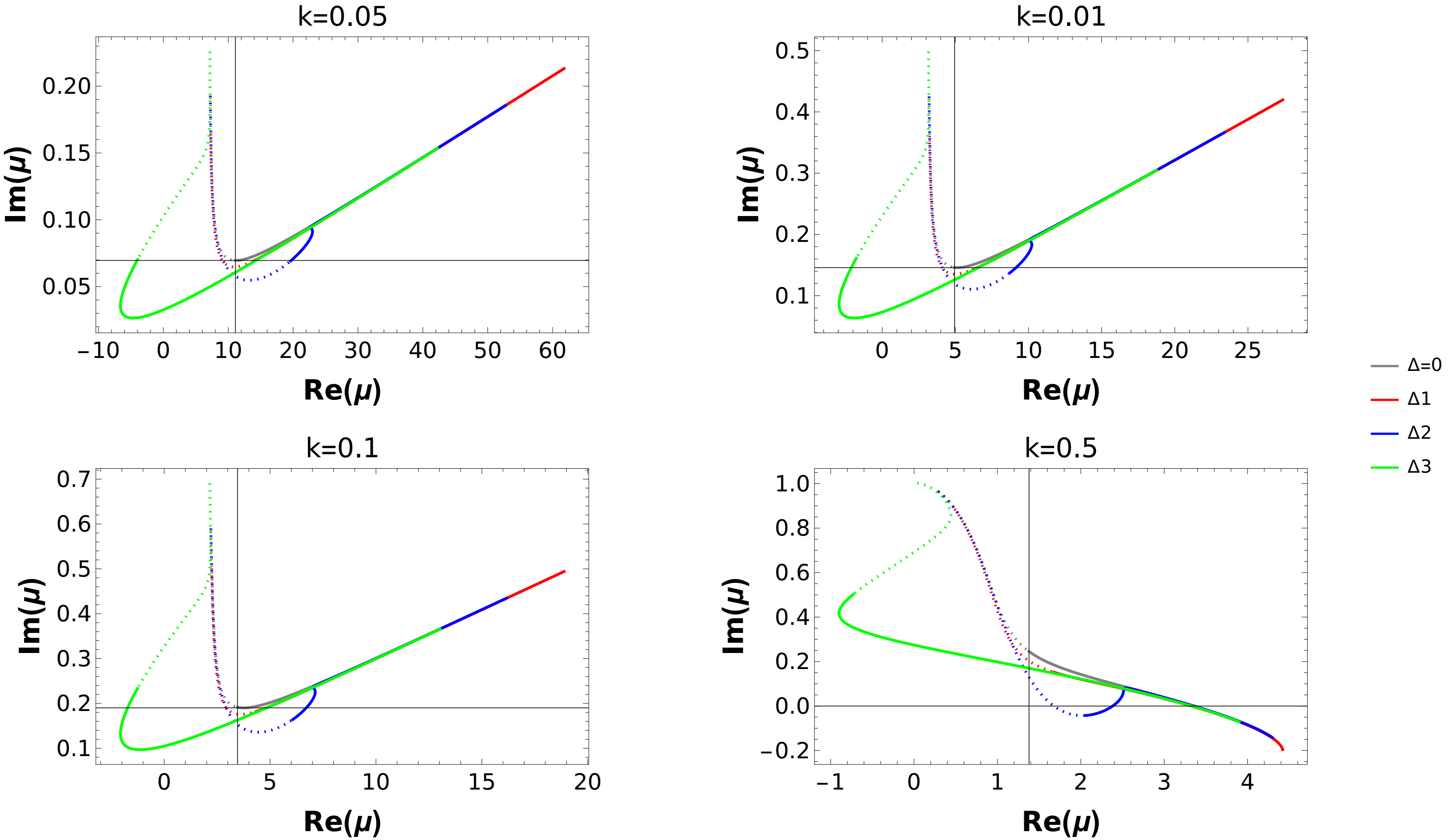
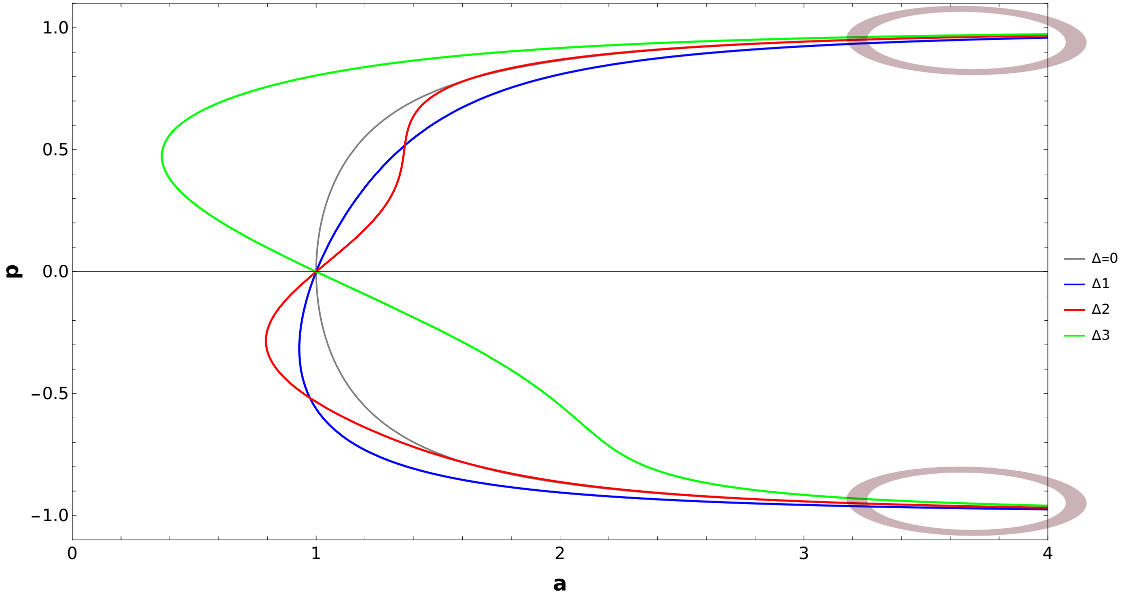


CONCLUSIONS

The expectation value of the dynamical variables is invariant under clock transformations away from the bounce where the behavior of the expectation values becomes classical.



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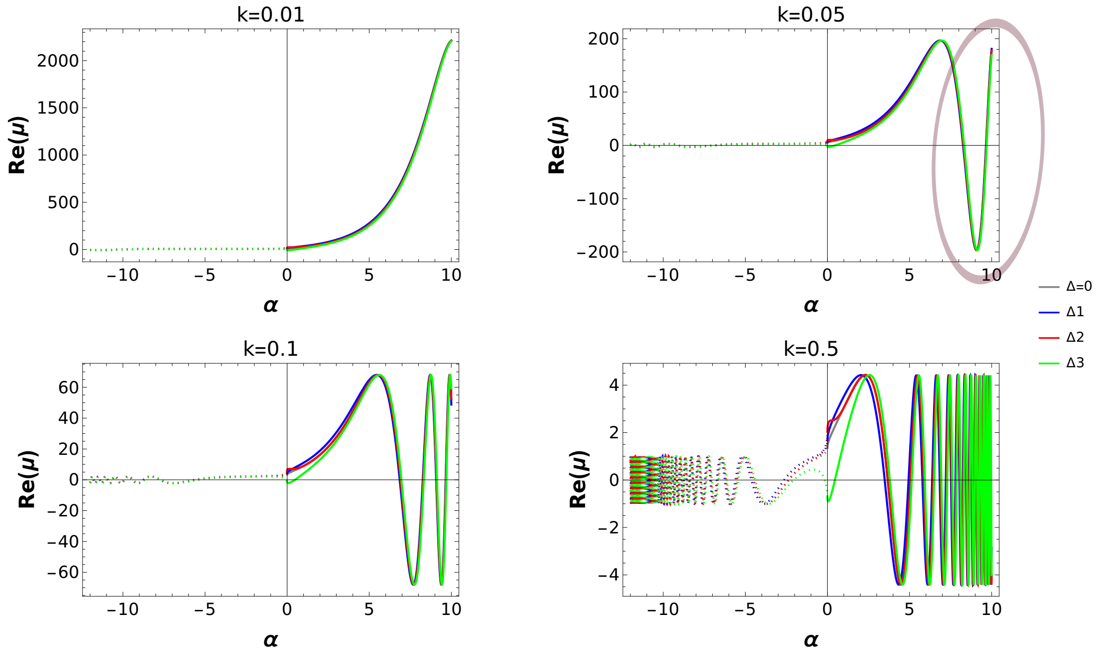
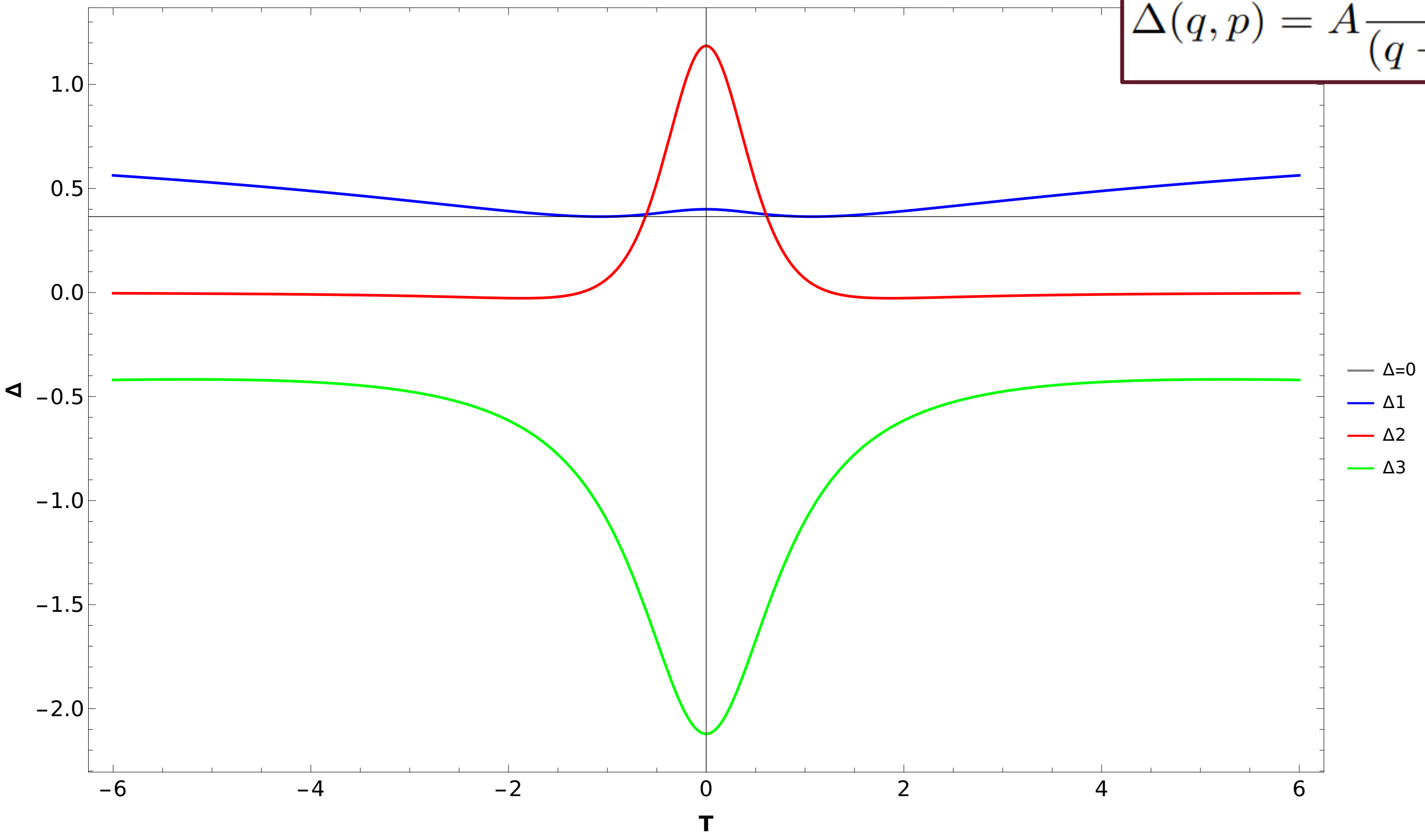


For asymptotically large universes the dynamical predictions of quantum gravity do not depend on the clock

THANK YOU

DELAY FUNCTIONS

$$\Delta(q,p) = A\frac{q^B}{(q+C)^D}\frac{\sin(Ep)}{p}$$



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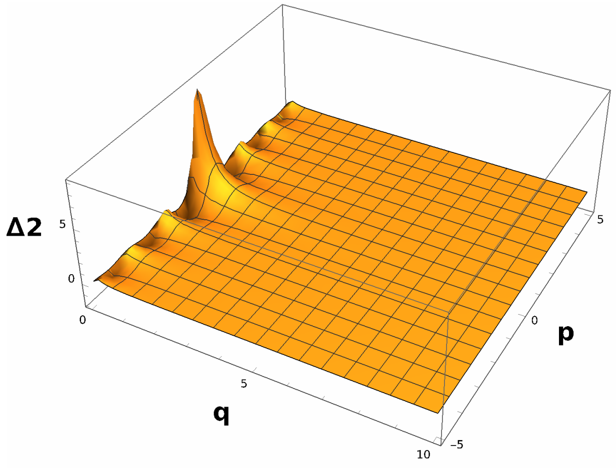
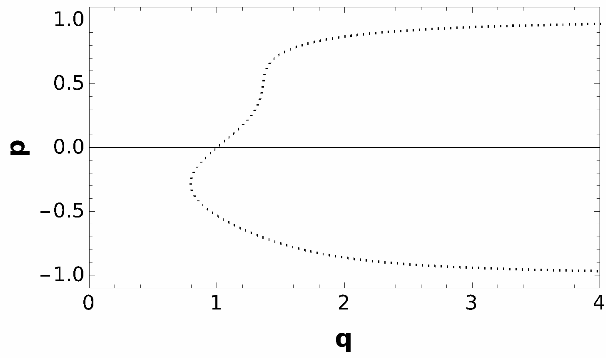
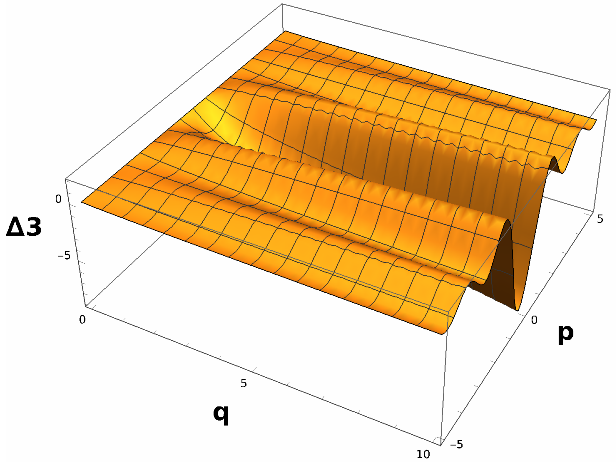
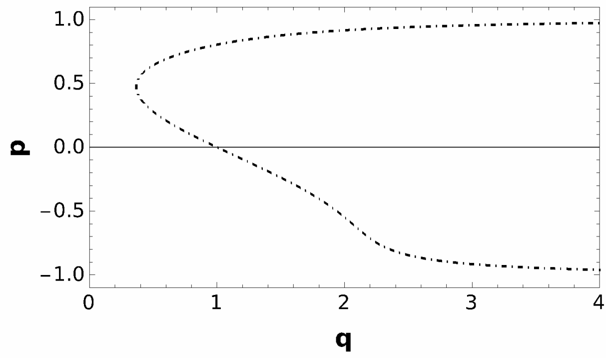

Δ2
5
0
0
5
q
10
5
0
p
-5


1.0
0.5
0.0
−0.5
−1.0
p
0
1
2
3
4
q


Δ3
0
-5
0
5
q
10
5
0
p
-5


1.0
0.5
0.0
−0.5
−1.0
p
0
1
2
3
4
q

CLOCK TRANSFORMATIONS

Hamiltonian constraint

$$C(q_I, p^J) = 0$$

Assume q_0 varies monotonically with the evolution generated by the constraint

$$\{q_0, C\}_{\mathrm{PB}} \neq 0$$

we can assign to it the role of the internal clock in which the evolution of the remaining variables occurs

Hamiltonian constraint $$C \propto p_0 + H$$

Reduced Hamiltonian formalism

$$\Omega\big|_{C=0} = \left(\mathrm{d}q_I \wedge \mathrm{d}p^I + \mathrm{d}t \wedge \mathrm{d}p^0\right)\big|_{C=0}$$
$$= \mathrm{d}q_I \wedge \mathrm{d}p^I - \mathrm{d}t \wedge \mathrm{d}H$$

The dynamics reads

$$\frac{\mathrm{d}q_I}{\mathrm{d}q_0} = \frac{\partial H}{\partial p_I} \qquad \frac{\mathrm{d}p^I}{\mathrm{d}q_0} = -\frac{\partial H}{\partial q^I}$$