February 2023

θ-angle physics of 2 color QCD

Fixed baryon charge and Near Conformal Dynamics

Based on [JHEP 11 (2022) 080] and [2208.09227]

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Winter School of Theoretical Physics and third COST Action CA18108 Training School

General Overview

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Great progress in our understanding of the structure of the space of quantum field theories (QFTs)

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Unravelling its geometry and topology $\Longrightarrow$ deep implications in mathematics, quantum gravity, string theory, cosmology and condensed matter physics.

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Unravelling its geometry and topology $\implies$ deep implications in mathematics, quantum gravity, string theory, cosmology and condensed matter physics.

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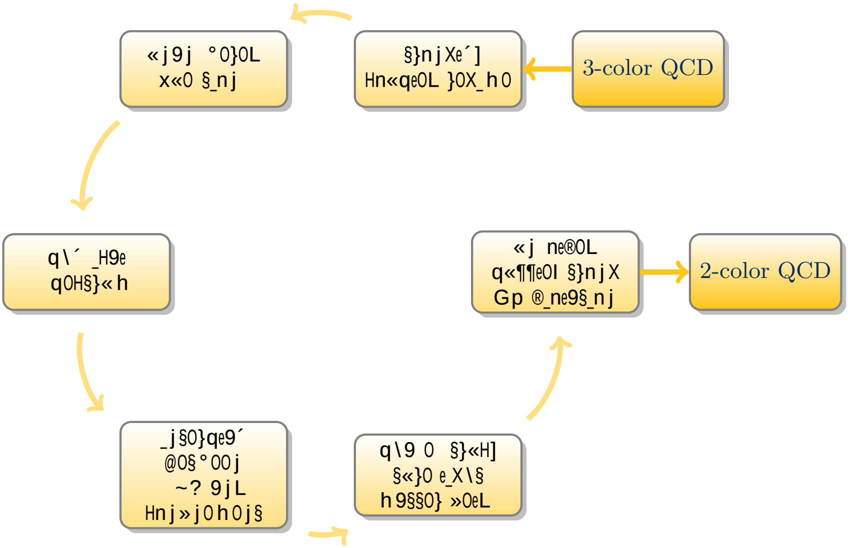
SOLVE QFT: investigate different regimes in a controlled manner and with precise results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2-color QCD

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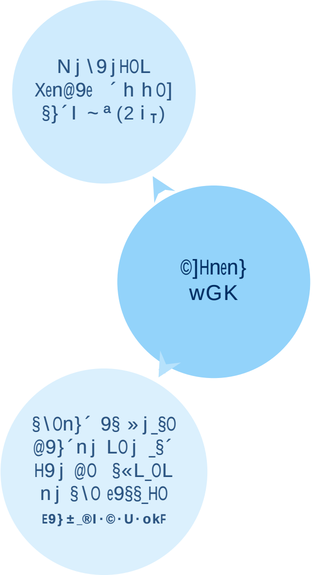
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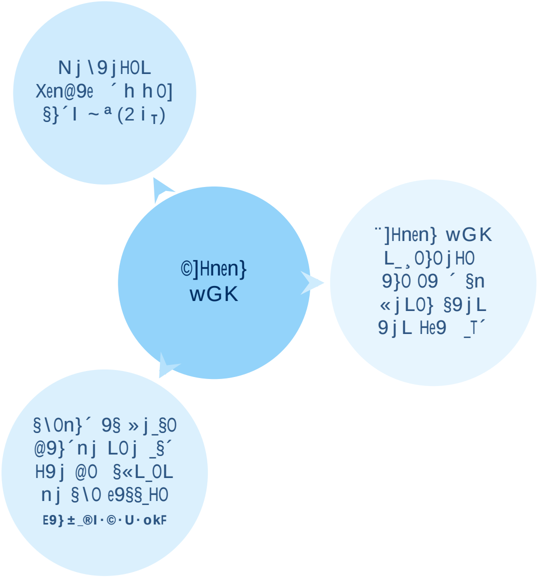

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lost of asymptotic freedom

conformal window

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confinement and SB

lost of asymptotic freedom

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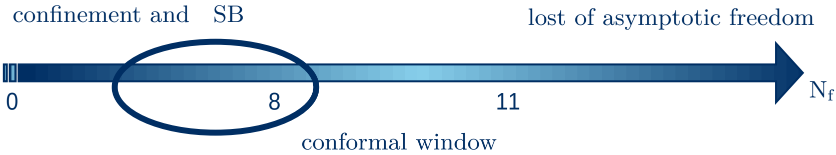

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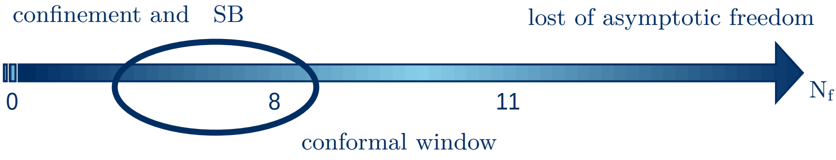

in depth analysis of the -angle physics at non-zero baryon chemical potential of 2 color QCD with SU($2N_f$) global symmetry E` [Np oo r©·©©s ·P·F

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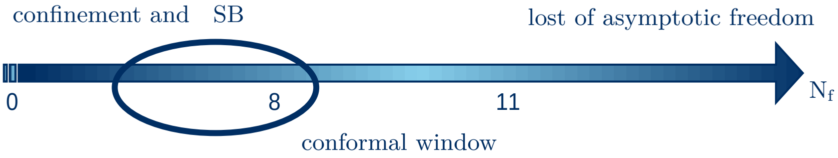

- in depth analysis of the -angle physics at non-zero baryon chemical potential of 2 color QCD with SU($2N_f$) global symmetry E`[Np oo r©·©©s ·P·F
  - 2 color effective pion Lagrangian at non-zero baryon charge including the -angle term

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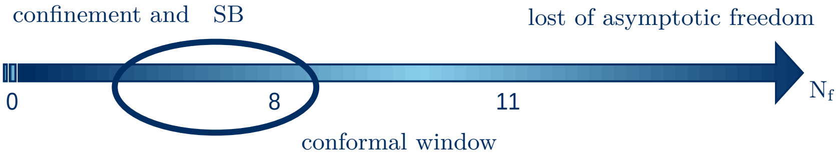

- in depth analysis of the -angle physics at non-zero baryon chemical potential of 2 color QCD with SU($2N_f$) global symmetry E`[Np oo r©·©©s ·P·F
  - 2 color effective pion Lagrangian at non-zero baryon charge including the -angle term
  - determine the vacuum structure of the theory both in the normal and superfluid phase as a function of the different number of matter fields

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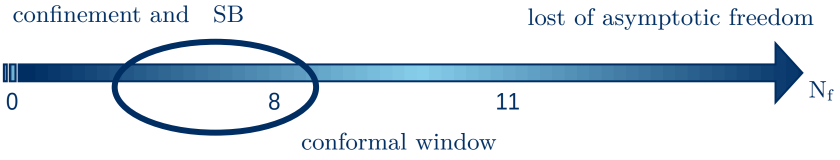

- in depth analysis of the -angle physics at non-zero baryon chemical potential of 2 color QCD with $SU(2N_f)$ global symmetry E`[Np oo r©·©©s ·P·F
  - X 2 color effective pion Lagrangian at non-zero baryon charge including the -angle term
  - X determine the vacuum structure of the theory both in the normal and superfluid phase as a function of the different number of matter fields
  - X determine the spectrum of the theory

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Theory simplifies when large/small parameter exists

~ne®_jX wS¦

Theory simplifies when large/small parameter exists → conserved charge of the internal symmetry group of the CFT →

~ne®_j X wS ¦

Theory simplifies when large/small parameter exists → conserved charge of the internal symmetry group of the CFT → d8 | WN G [8 | WN N±p8 i ~^m i

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~ne®_jX wS¦

Theory simplifies when large/small parameter exists → conserved charge of the internal symmetry group of the CFT → d8|WN G[8|WN N±p8 i~^m i

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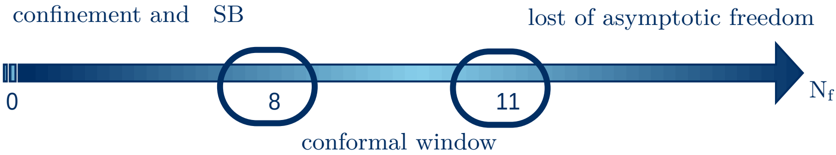
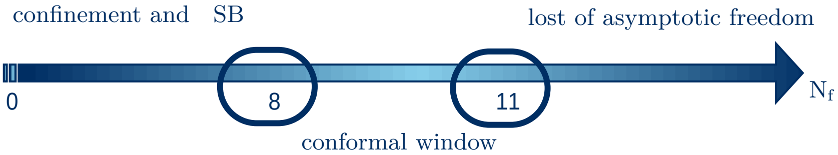
Analytic treatment of theories with global symmetries

Operators having large internal charge can be associated, via state/operator correspondence, to a superfluid phase on a cylinder

Operators having large internal charge can be associated, via state/operator correspondence, to a superfluid phase on a cylinder

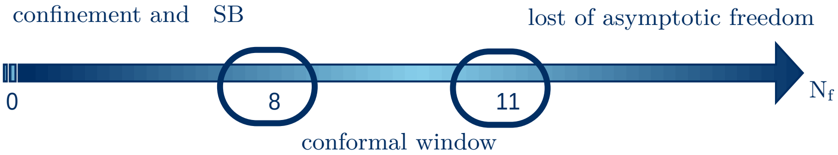


Operators having large internal charge can be associated, via state/operator correspondence, to a superfluid phase on a cylinder



near conformal invariance of the theory: dressing the lagrangian with the dilaton

Operators having large internal charge can be associated, via state/operator correspondence, to a superfluid phase on a cylinder



near conformal invariance of the theory: dressing the lagrangian with the dilaton

near conformal dynamics of the dressed theory being in a superfluid phase on a cylinder

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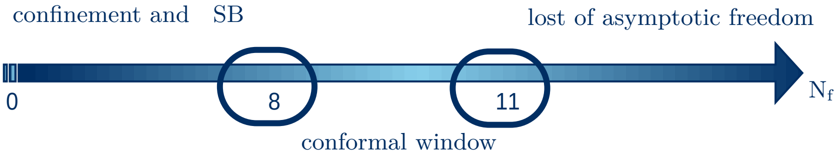
Operators having large internal charge can be associated, via state/operator correspondence, to a superfluid phase on a cylinder



near conformal invariance of the theory: dressing the lagrangian with the dilaton

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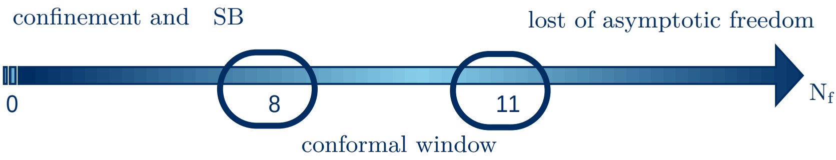
X -dependence of the ground state energy

Operators having large internal charge can be associated, via state/operator correspondence, to a superfluid phase on a cylinder



near conformal invariance of the theory: dressing the lagrangian with the dilaton

near conformal dynamics of the dressed theory being in a superfluid phase on a cylinder

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X -dependence of the ground state energy

X -dependence of the near-conformal scaling dimension of the baryon charged operators on R^4

-angle physics in the near conformal window

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-angle physics in the near conformal window

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-angle physics in the near conformal window

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-angle physics in the near conformal window

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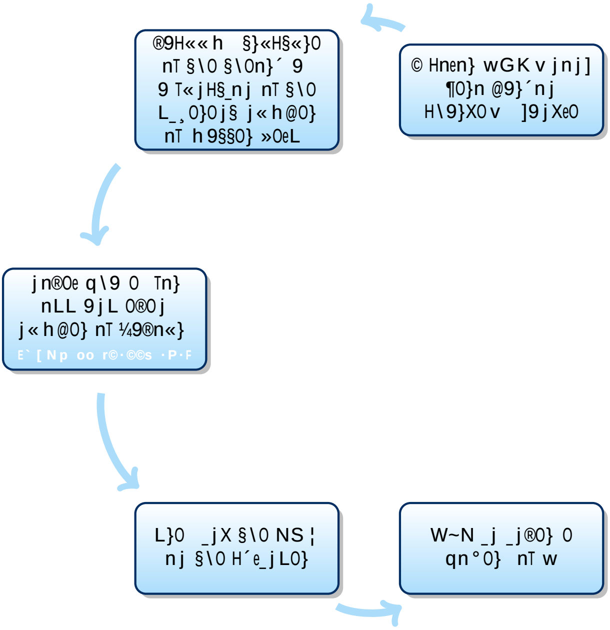
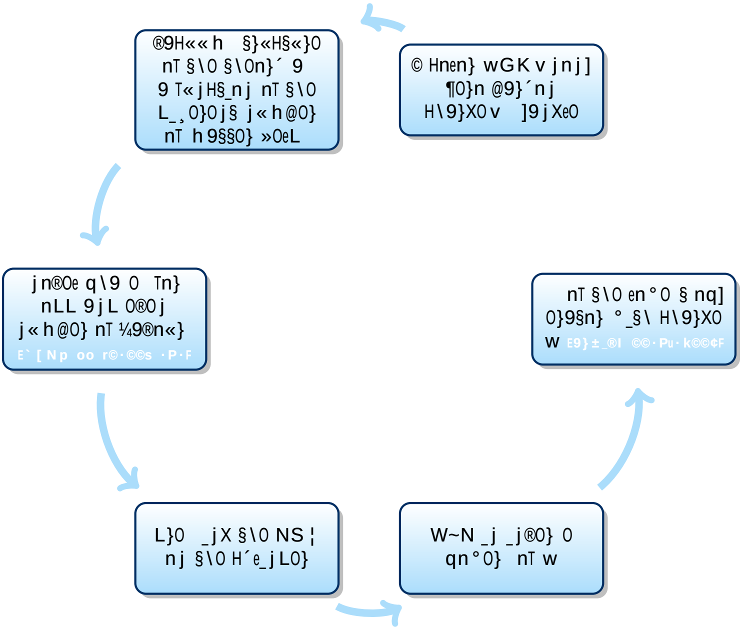
-angle physics in the near conformal window



-angle physics in the near conformal window



Results

in collaboration with J. Bersini, F. Sannino & M. Torres

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Starting from 2-color QCD EFT with SU($2N_f$) global symmetry E9}±_®I· · ·oo¢oF

$$L = {}^2\mathrm{Tr} f @ \quad @ \quad {}^y g + 4 \quad {}^2 \mathrm{Tr} f \mathrm{B} \quad {}^y @_0 \quad g + \mathrm{m}^2 \quad {}^2 \mathrm{Tr} f \mathrm{M} \quad + \mathrm{M}^y \quad {}^y g$$

$$+ 2 \quad {}^2 \quad {}^2 \ \mathrm{h} \ \mathrm{Tr} f \quad \mathrm{B}^{\mathrm{T}} \quad {}^y \mathrm{B} g + \mathrm{Tr} f \mathrm{BB} g \ \mathrm{i} \quad \mathrm{a} \quad {}^2 \quad \frac{\mathrm{i}}{4} \mathrm{Tr} f \log \quad \log \quad {}^y g \quad {}^2 \qquad (1)$$

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Starting from 2-color QCD EFT with SU$(2N_f)$ global symmetry E9}±_®I···oo¢oF

$$
\begin{aligned}
L \;=\;& {}^2\mathrm{Tr}f@ \quad @ \quad {}^y g + 4 \quad {}^2\mathrm{Tr}f\mathrm{B} \quad {}^y @_0 \quad g + m^2 \quad {}^2\mathrm{Tr}f\mathrm{M} \quad + \mathrm{M}^y \quad {}^y g \\
& + 2 \quad {}^2 \quad {}^2 \; \mathrm{h} \; \mathrm{Tr}f \quad \mathrm{B}^T \quad {}^y \mathrm{B} g + \mathrm{Tr}f\mathrm{BB}g \; \mathrm{i} \quad a \quad {}^2 \quad \frac{\mathrm{i}}{4}\mathrm{Tr}f\log \quad \log \quad {}^y g \quad {}^2
\end{aligned} \tag{1}
$$

using the ansatz $_0 = \mathrm{U}(\;_\mathrm{i})\;_\mathrm{c}\,; \qquad \mathrm{U}(\;_\mathrm{i}) \quad \mathrm{diag}fe^{\mathrm{i} \quad 1};:::;e^{\mathrm{i} \quad \mathrm{i}_T};e^{\mathrm{i} \quad 1};:::;e^{\mathrm{i} \quad \mathrm{i}_T}g$

$$
_\mathrm{c} = \begin{pmatrix} 0 & 1_{N_T} \\ 1_{N_T} & 0 \end{pmatrix} \cos ' + \mathrm{i} \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix} \sin ' \quad \text{where} \quad I = \begin{pmatrix} 0 & 1_{N_T/2} \\ 1_{N_T/2} & 0 \end{pmatrix} ; \tag{2}
$$

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Starting from 2-color QCD EFT with SU(2N$_f$) global symmetry E9}±_®I···oo¢oF

$$L = {}^2\mathrm{Tr}f@ \; @ \; {}^y g + 4 \; {}^2\mathrm{Tr}f\mathrm{B} \; {}^y @_0 \; g + \mathrm{m}^2 \; {}^2\mathrm{Tr}f\mathrm{M} \; + \mathrm{M}^y \; {}^y g$$

$$+ 2 \; {}^2 \; {}^2 \; \mathrm{h} \; \mathrm{Tr}f \; \mathrm{B}^T \; {}^y\mathrm{B}g + \mathrm{Tr}f\mathrm{BB}g \; \mathrm{i} \; \mathrm{a} \; {}^2 \; \frac{\mathrm{i}}{4}\mathrm{Tr}f\log \; \log \; {}^y g \; {}^2 \quad (1)$$

using the ansatz $_0 = \mathrm{U}(\;_i)\;_c ; \quad \mathrm{U}(\;_i) \; \mathrm{diag}fe^{\;i\;1};:::;e^{\;i\;\;i_T};e^{\;i\;1};:::;e^{\;i\;\;i_T}g$

$$_c = \begin{pmatrix} 0 & 1_{N_T} \\ 1_{N_T} & 0 \end{pmatrix} \cos ' + i \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix} \sin ' \quad \text{where} \quad I = \begin{pmatrix} 0 & 1_{N_T/2} \\ 1_{N_T/2} & 0 \end{pmatrix} ; \quad (2)$$

the energy of the system is

$$E = \; {}^2 \; 4\mathrm{m}^2 X \; \mathrm{a} \; {}^2 \; ; \qquad \text{normal phase } (' = 0) \quad (3)$$

$$E = \; {}^2 \; 2\frac{N_f^2 \; {}^4 + \mathrm{m}^4 X^2}{N_f \; {}^2} \; \mathrm{a} \; {}^2 \; ; \qquad \text{superfluid phase } \cos ' = \frac{\mathrm{m}^2}{N_f \; {}^2} X \quad (4)$$

with $X = \sum_i^{N_T} \cos \;_i$

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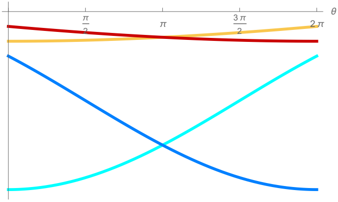

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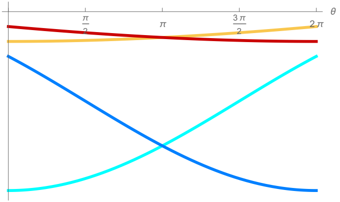

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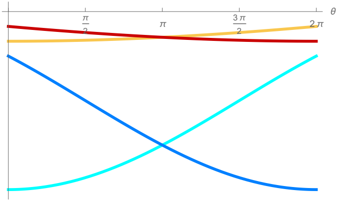

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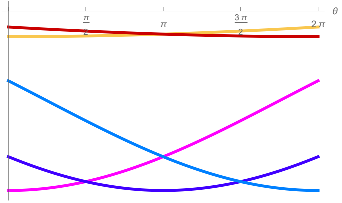

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$SU(2N_f)$ $U(1)_A$ $2N_T^2$ N_T $Sp(2N_f)$

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$SU(N_f)_V$ $U(1)_B$

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$\frac{i_T^2 \quad i_T}{2}$

$Sp(N_f)_V$

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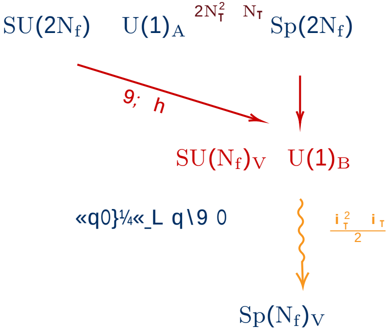

$$!_1^2 = c^2 + \ {}^2 ;$$

$$!_2^2 = c^2 + \frac{h^4 \pm^2}{2 \ i_T^2} ;$$

$$!_3^2 = c^2 + \frac{2 \quad {}^4 i_T^2 + 3h^4 \pm^2}{i_T^2 \quad {}^2} + 8 ;$$

$$!_4^2 = c^2 + \frac{2 \quad {}^4 i_T^2 + 3h^4 \pm^2}{i_T^2 \quad {}^2} \quad 8 ;$$

$$!_5^2 = c^2 + g_{\sim}^2 ;$$

$\frac{1}{2} i_T(i_T + 1)$

$\frac{1}{2} i_T(i_T \quad 1) \quad 1$

$+ \ \frac{1}{2} i_T(i_T \quad 1)$

$+ \ \frac{1}{2} i_T(i_T \quad 1)$

1

where

$$A = \frac{2}{N_f^2 \quad {}^2} \, q \overline{N_f^2 \quad {}^4 + 3m^4 X^2 \quad {}^2 + 4N_f^2 \quad {}^2 m^4 k^2 X^2} ; \qquad (5)$$

$$M_S^2 = \frac{a \quad {}^4 N_f^3 + 2 \quad {}^2 m^4 X^2}{2 \quad {}^4 N_f^2 \qquad 2m^4 X^2} \quad 1 \qquad \frac{m^4 X^2}{{}^2 N_f^2} \qquad (6)$$

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smoothly approach the conformal phase of the theory $\Longrightarrow$ dressing our Lagrangian via a dilaton field (x)

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smoothly approach the conformal phase of the theory $\implies$ dressing our Lagrangian via a dilaton field (x)

$$x \mapsto e\ x \implies \mapsto \frac{}{f} \implies O_k \mapsto e^{(k\ 4)\ f}\, O_k$$

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smoothly approach the conformal phase of the theory =) dressing our Lagrangian via a dilaton field (x)

$$x \mapsto e\ x \implies \quad \mapsto \quad \frac{}{f} \implies O_k \mapsto e^{(k\ \ 4)\ \ f} O_k$$

$$L_{;} = {}^2 ¦\}f@ \quad @ \quad {}^y g\, 0\ {}^{2\ f} + 4\ {}^2 ¦\}f?\ {}^y @_0\ g\, 0\ {}^{2\ f} + h^2\ {}^2 ¦\}f\, g + g^y\ {}^y g\, 0\ {}^{(3\ \ )\ f} +$$

$$+ 2\ {}^2\ {}^2\ \overset{h}{¦\}f}\ ?^T\ {}^y ?g\, 0\ {}^{2\ f} + \overset{i}{¦\}f??g}\ 9\ {}^2 \quad \frac{=}{4} ¦\}f enX \quad enX\ {}^y g\ \overset{2}{0}\ {}^{4\ f} + \quad r¢s$$

$$+ \frac{1}{2} @ \quad @ \quad \frac{R}{6T^2}\ 0\ {}^{2\ f}\ \frac{h^2}{16T^2}\ 0\ {}^{4\ f} + 4\ T \quad 1 \quad \frac{4}{0}\ 0\ {}^{4\ f}$$

Replacing this vacuum ansatz, the Lagrangian (7) becomes

$$\mathcal{L}_{;}[\;_0;\;_0] = e^{4f\;_0}\;^4 \quad \frac{m^2}{16f^2} \quad \frac{m^2\;4f\;_0 + e^{4f\;_0}\;1}{16f^2} \quad \frac{R\,e^{2f}}{12f^2} + \qquad (8)$$

$$+\,4m^2\;^2 X \cos{}' \; e^{f\;_0 y} + 2\;^2 N_f\;^2 e^{2f\;_0} \sin^2{}' \quad a\;^2 e^{4f\;_0}\;^2 ;$$

where

$$\sum_i^{N_f}\;_i ; \quad X \quad \sum_i^{N_f} \cos\;_i ; \quad ^4 \quad ^4_0 + \frac{m^2}{16f^2} : \qquad (9)$$

Replacing this vacuum ansatz, the Lagrangian (7) becomes

$$L_{;}\,[\;_0;\;_0] = \; e^{4f\;_0}\;^4\,\frac{m^2}{16f^2}\;\frac{m^2\;4f\;_0 + e^{4f\;_0}\;1}{16f^2}\;\frac{R\,e^{2f}}{12f^2} + \quad (8)$$

$$+\,4m^2\;^2 X\cos\,'\,e^{f\;_0 y} + 2\;^2 N_f\;^2 e^{2f\;_0}\sin^2\,'\;\;a\;^2 e^{4f\;_0}\;^2\,;$$

where

$$\sum_i^{N_f}\;_i\,;\quad X\;\sum_i^{N_f}\cos\;_i\,;\quad\;^4\;\;^4_0 + \frac{m^2}{16f^2}\,: \quad (9)$$

The respective equations of motion are

$$N_f\;^2 e^{2f}\;\cos\,'\;\;m^2 X e^{f\;y} = 0 \quad (10)$$

$$ae^{4f}\;\;2m^2\sin\;_i\cos\,'e^{f\;y} = 0\,;\quad i = 1;:::;N_f \quad (11)$$

$$\frac{Re^{2f}}{6f} + 4af\;^2 e^{4f}\;Y^2 + 4f\;^4_0 e^{4f}\;\frac{m^2\;1\;\;e^{4f}}{4f} +$$

$$4f\;^2 N_f\;^2 e^{2f}\;\sin^2\,'\;\;4fm^2\;^2 yX\cos\,'e^{f\;y} = 0 \quad (12)$$

$$4\;N_f\;^2 e^{2f}\;\sin^2\,' = \frac{Q}{V}\,: \quad (13)$$

X large-charge quasi-conformal Ground State Energy as function of the dilaton, fermion mass and background geometry to include the impact of the angle physics

- large-charge quasi-conformal Ground State Energy as function of the dilaton, fermion mass and background geometry to include the impact of the angle physics

N 1 = H4/3 w^4/3 / ≃1/3 + w^2/3 ≃1/3 (H2/3 † ±00^2 / 4 2 i τ^3 H^4_4/3 9h^2/32 2 " 1 2/3 enX w ±10/±00

enX 32 i τ 2 2H4/3 ≃2/3 / 3 ! ! #) ≃ enX w (16/9 2 i τ H2/3 H4/3 2 h 2 / 3 2 i τ^4 H^5_4/3 9h^2/32 2

" 5 / 8 2H^4_4/3 i τ^2 9h^2/32 2 ±00^4 H2/3 † i τ ±00^2 + 9±00 ±01 / 32H4/3 #) + w^0

N^1 1 = H4/3 w^4/3 / ≃1/3 + H2/3 w^2/3 † ≃1/3 9(1) ±00^2 h^4 ≃ enX w / 64H^3_4/3 i τ^2 16/9 2 h 2 i τ H2/3 H4/3 2 ≃ enX w + w^0 ;

- large-charge quasi-conformal Ground State Energy as function of the dilaton, fermion mass and background geometry to include the impact of the angle physics

$$N^{1} = \frac{H_{4/3} w^{4/3}}{\simeq^{1/3}} + w^{2/3} \simeq^{1/3} \Big(H_{2/3} \dagger \; \frac{\pm_{00}^{2}}{4\;^{2}\, i_{T}^{3} H_{4/3}^{4}} \; \frac{9h^{2}}{32}\;^{2} \; 1 \; \frac{2}{3}\, \text{enX}\, w \; \frac{\pm_{10}}{\pm_{00}} \; \text{enX} \; \frac{32\, i_{T}\;^{2}\;^{2} H_{4/3} \simeq^{2/3}}{3} \; ! \; ! \; \# \Big) \simeq \text{enX}\, w \Big(\frac{16}{9}\;^{2}\; i_{T} H_{2/3} H_{4/3}\;^{2} h^{2} \; \frac{}{3\;^{2}\, i_{T}^{4} H_{4/3}^{5}} \; \frac{9h^{2}}{32}\;^{2} \; \frac{5}{8\;^{2} H_{4/3}^{4}\, i_{T}^{2}} \; \frac{9h^{2}}{32}\;^{2} \; \pm_{00}^{4} \; H_{2/3} \dagger \, i_{T} \pm_{00}^{2} + \frac{9\pm_{00}\pm_{01}}{32 H_{4/3}} \; \# \Big) + w^{0}$$

$$N^{1}\;^{1} = \frac{H_{4/3} w^{4/3}}{\simeq^{1/3}} + H_{2/3} w^{2/3} \dagger \simeq^{1/3} \; \frac{9(1 \quad) \pm_{00}^{2} h^{4} \simeq \text{enX}\, w}{64 H_{4/3}^{3}\, i_{T}^{2}} \; \frac{16}{9}\;^{2} h^{2}\; i_{T} H_{2/3} H_{4/3}\;^{2} \simeq \text{enX}\, w + w^{0} \; ;$$

where we introduced

$$H_{4/3} = \frac{3}{8} \left(\frac{2}{i_{f}\;^{2}} \right)^{2/3} ; \quad H_{2/3} = \frac{1}{4T^{2}} \left(\frac{2}{i_{f}\;^{2}\;^{4}} \right)^{1/3} ; \quad \dagger = \frac{|}{6}\; 9jL \quad \simeq = \frac{-}{2\;^{2}} ; \qquad \text{roVs}$$

- large-charge quasi-anomalous dimension as function of the dilaton, fermion mass and background geometry to include the impact of the angle physics

- large-charge quasi-anomalous dimension as function of the dilaton, fermion mass and background geometry to include the impact of the angle physics

$$\frac{}{} = 1 \left(\frac{9h^2}{32} \right)^2 \frac{1 \quad enX \quad \frac{3 \quad ^{2/3}}{16(2 \quad ^2)^{1/3} H_{4/3} \quad ^2 i_T}}{4H^5_{4/3}\, i_T} Hn^2 \frac{+2 \quad c}{i_T} \frac{1}{2 \quad ^2}^{2/3}$$

$$+ \frac{}{H^6_{4/3}\, i_T} Hn^2 \frac{+2 \quad c}{i_T} \left(\frac{27h^4 \quad j^2 \frac{+2 \quad c}{i_T}}{256\, 2^{2/3} \quad ^{4/3} 9\, H^3_{4/3}\, i^2_T} + \frac{5 \frac{9h^2}{64} \quad ^2 Hn^2 \frac{+2 \quad c}{i_T}}{6H^4_{4/3}\, i_T} \frac{H_{2/3}}{2} \frac{}{2 \quad ^2 w}^{2/3} \right)^1$$

$$\left(\frac{9h^2}{32} \right)^2 \frac{1}{2 \quad ^2}^{4/3} enX\, w \quad \frac{16}{9} \, ^2 H_{2/3} \, ^2 i_T h^2 \frac{1}{2 \quad ^2}^{4/3} enX\, w$$

X large-charge quasi-anomalous dimension as function of the dilaton, fermion mass and background geometry to include the impact of the angle physics

$$1$$

$$\frac{}{} = 1 \quad \frac{9h^2}{32} \quad !^2 \; \frac{1 \quad \text{enX} \quad \frac{3 \quad ^{2/3}}{16(2 \quad ^2)^{1/3} H_{4/3} \quad ^2 \, i_T}}{4H^5_{4/3} \; i_T} \; Hn^2 \quad \frac{+ 2 \quad c}{i_T} \quad \frac{1}{2 \quad ^2} \; ^{2/3}$$

$$+ \frac{}{H^6_{4/3} \; i_T} Hn^2 \quad \frac{+ 2 \quad c}{i_T} \quad \overset{0}{\underset{@}{B}} \frac{27h^4 \quad \_j^2 \; \frac{+2 \quad c}{i_T}}{256 \; 2^{2/3} \quad ^{4/3} 9 \; H^3_{4/3} \; i_T^2} + \frac{5 \quad \frac{9h^2}{64} \quad ^2 \; Hn^2 \; \frac{+2 \quad c}{i_T}}{6H^4_{4/3} \; i_T} \quad \frac{H_{2/3}}{2} \quad \frac{}{2 \quad ^2 w} \; ^{2/3} \overset{1}{\underset{A}{C}}$$

$$\frac{9h^2}{32} \; !^2 \quad \frac{1}{2 \quad ^2} \; ^{4/3} \; \text{enX } w \quad \frac{16}{9} \; ^2 H_{2/3} \; ^2 \; i_T h^2 \quad \frac{1}{2 \quad ^2} \; ^{4/3} \; \text{enX } w$$

$$(1 \quad) \quad 1$$

$$\frac{}{} = 1 \quad \frac{9h^4}{64H^4_{4/3}} (1 \quad) \; Hn^2 \quad \frac{+ 2 \quad c}{i_T} + \frac{16}{9} \; ^2 H_{2/3} \; ^2 \; i_T h^2 \; ! \quad \frac{1}{2 \quad ^2} \; ^{4/3} \; \text{enX } w$$

~q0H§}« h

$$SU(2N_f) \quad U(1)_A \quad {}^{2N_T^2} \quad {}^{N_T} \; Sp(2N_f) \quad ! \quad SU(N_f)_V \quad U(1)_B \quad \frac{i_T^2 \quad i_T}{2} \quad Sp(N_f)_V \tag{15}$$

~q0H§}«h

$$SU(2N_f) \quad U(1)_A \; \xrightarrow{2N_T^2 \quad N_T} \; Sp(2N_f) \; ! \; SU(N_f)_V \quad U(1)_B \; \frac{i_T^2 \quad i_T}{2} \; Sp(N_f)_V \tag{15}$$

Having in mind the hierarchy of scales m $\quad p\,\frac{}{a} \quad 4$, we focus on the spectrum of light modes

aT2+i`mK

$$a\,l(2L_7) \quad l(1) \quad 2L_7^2 \quad L_7 \quad a\,T(2L_7)\,! \qquad a\,l(L_7)\,o \quad l(1)\,\text{"}\,\frac{L_7^2 \quad L_7}{2} \quad a\,T(L_7)\,o \qquad UR8$$

> pBM; BM KBM/ i?2 ?B2` `+?v Q7 b+4H2 b r 2K7Q+mb QM i?2 bT2 HB;?i KQ/2b

$\frac{1}{2}L_7(L_7 \quad 1)$ K bbH2bb :QH/biYQM2 7 (aLT) $p\_$

1 Tb2m/Q@:QH/bi QM2 7 (aLT) r Bi? K /bb $p\_$

aT2+i`mK

a l(2 L7) l(1) 2 L27 L 7 aT(2 L7) ! a l(L7) o l(1) " L27 L 7 / 2 aT(L7) o UR8

> pBM; BM KBM/ i?2 ?B2` `+?v Q7 b+4H2 b r?2`2 K7Q+mb QM i?2 bT2 p –

HB;?i KQ/2b

1/2 L7(L7 1) K bbH2bb :QH/biQM2 ?Qbib (aT) p –

1 Tb2m/Q@:QH/biQM2 r Bi? K /bb (aT)

i?2 bT2+i`mK +? M;2b r?2M UM2 `V+QM7Q`K H /vM KB+b Bb `2 HBx2/ i?`Qm;? i?2 /BH iQM /`2bb

aT2+i`mK

$$a\,l(2\,L_7) \quad l(1)^{2L_7^2 \quad L_7} \quad a\,T(2\,L_7)\,! \quad a\,l(L_7)_o \quad l(1)^{"\frac{L_7^2 \quad L_7}{2}} \quad a\,T(L_7)_o \qquad UR8$$

> pBM; BM KBM/ i?2 ?B2` `+?v Q7 b+4 H2 b r 2K7Q+mb QM i?2 bT2 $p\_$

HB;?i KQ/2b

$\frac{1}{2}\,L_7(L_7 \quad 1)$ K bbH2bb :QH/biQM2 b (aLT) $p\_$

1 Tb2m/Q@:QH/biQM2 (aLT) r Bi? K /b b

i?2 bT2+i`mK +? M;2b r?2M UM2 `V+QM7Q`K H /vM KB+b Bb `2 HBx2/ i?`Qm;? i?2 /BH iQM /`2bb

r2 2tT M/ `QmM/ i?2 p +mmK bQHmiBQM b 7QHH

$$= \quad 2^B_0\,2^B_i \quad r?2`2 = \quad 0_0^{\;i} \quad + \sim a\,\begin{matrix}1_{L_7} & 0\\ 0 & 1_{L_7}\end{matrix}\;; \quad \sim \quad p\frac{1}{2\,L_7}; \quad = \quad /BK\,X_{=0}\frac{l(L_7)}{a\,T(L_7)} \quad h$$

$$\frac{L}{4^{2}\, b B'M 2^{2}\, {}_{0} 7} = {}^{0}\wedge\, a\, .\, {}^{1} @ \wedge \underset{a}{A} + \overset{0\quad{}_{0}\,1\quad /BK}{\underset{=1}{X}}\; @\quad @\quad U R e$$

$$\frac{L}{4\ ^2\ bB^2M\ 2^2\ {}_0\ 7} = {}^0\ \hat{}\ a\ .\ {}^1 @\ \hat{}\ A + \ldots\ @\ @ \qquad$$

U R e

rBi? i?2 BMp2`b2 T`Q T/ 2;} IMQ2`/ . b

$$.\ {}^1 = @\ B\ ...\ \frac{!^2}{8^2\ bB^2M}\ \ldots\ J^2\ \ldots\ \frac{1}{2} A_b\ \ldots\ A\ ;\quad A_a = \frac{p\,\overline{2}\,7^2\,K^4\,p\,\overline{L_7}\,s\,v\,w}{K^4\ s^2\ \ \ 4\ L_7^2\ 2^2\ 7\ {}_0(\ v\ \ 2)}$$

U R d

$$\frac{L}{4\ {}^2\ bB^2M\ 2^2\ {}_0\ 7} = {}^0\ \wedge\ a\ .\ {}^1 @\ \wedge\ \underset{a}{A} + \overset{0\ {}_0\ 1}{\underset{=1}{\overset{/BK}{X}}}\ @\ @$$ URe

rBi? i?2 BMp2`b2 T`Q`T2;}`Q2`/ . b

$$. {}^1 = \underset{0}{\overset{0}{B @}}\ \frac{!^2\ F^2}{B\ \sqrt{7\,2\,L_7}}\ \frac{B\ \sqrt{7\,2\,L_7}}{\frac{1}{2}\ A_b}\ \frac{\frac{!^2\ F^2}{8^2\ bB^2M}\ J^2}{\frac{!^2\ F^2}{bB^2M}}\ \overset{0}{\frac{1}{2}\ A_b}\ \overset{1}{\underset{J^2_b}{C A}}\ ;\quad A_a = \frac{\sqrt{2\,7^2\,K^4}\ \sqrt{L_7} s\,v\,w}{K^4\ s^2\ \ 4\ L_7^2\ 2^2\ 7_0(v\ \ 2)}$$ URd

r?2`2 w $P^{L_7}_{B=1}$ bBM M/ i?2 G ;` M;B M K bb2b 7Q` i?2 /BH iQM@}2H/ M/

$$J^2 = \frac{7^2\ {}^2\ L_7 2^6\ 7_0\ \ {}^2\ K^4\ s^2\ \ v^2\ \ 2\ \ 2^6\ 7_0 + 2\ \ {}^4\ {}^2\ L_7^2\ 2^2\ 7_0(v+1)\ \ 4\ \ {}^4\ {}^2\ L_7 2^2\ 7_0\ v}{2\ {}^2\ \ {}^4\ L_7^2\ 2^2\ 7_0(v\ \ 2)\ \ K^4\ s^2}$$ UR3

$$J^2_a = \frac{{}^4\ L_7^3\ 2^2\ 7_0(v\ \ 1) + 2\ \ {}^2\ K^4\ s^2\ 2^4\ 7_0}{2\ \ {}^4\ L_7^2\ 2^2\ 7_0\ v\ \ \ 2\,K^4\ s^2\ 2^4\ 7_0} :$$ URN

+QM7Q`K H BMp ` /B+i i2b i?2 2tBb i2M+2 Q7 K bbH KQ/2 rBi? bT22/

$p_{\cdot} = \sqrt{\frac{1}{/ \;\; 1}} = \sqrt{\frac{1}{3}}$

(P`H M/Q,kyRNbF?)

KBtBM; #2ir22M i?2 bBM;H2i KQ/ $_0$ rBi? i?2 /BH iQ

+? M;2p2@ $_0$ HQ+Biv 7`QK $p_{\cdot} = 1$! $p_{\cdot} = \sqrt{\frac{1}{3}}$

+QM7Q`K H BMp ` /B+i i2b i?2 2tBb i2M+2 Q7 K bbH KQ/2 rBi? bT 22/

p. = p 1 / / 1 = p1 / 3

(P`H M/Q,kyRNbF?)

KBtBM; #2ir22M i?2 bBM;H2i KQ/ 0 rBi? i?2 /BH iQ

+? M;2p2@ 0 HQ+Biv 7`QK p. = 1 ! p. = p1 / 3

+QM7Q`K H BMp `
/B+i i2b i?2 2tBb
i2M+2 Q7 K bbH
KQ/2 rBi? bT22/
$p_: = p\frac{1}{/ \ 1} = p\frac{1}{3}$
(P`H M/Q,kyRNbF?)

KBtBM; #2ir22M
i?2 bBM;H2i KQ/
0 rBi? i?2 /BH iQ

+? M;2p2@
HQ+Biv 7`QK
$p_: = 1 \ ! \ p_: = p\frac{1}{3}$

+QM7Q`K H BMp ` /B+i i2b i?2 2tBb i2M+2 Q7 K bbH KQ/2 rBi? bT 22/

$p_{\cdot} = \frac{p_{1}}{/ \;\; 1} = \frac{p_{1}}{3}$

(S?vbX_2pX. RyR UkykyV e)

KBtBM; #2ir22M i?2 bBM;H2i KQ/ $_0$ rBi? i?2 /BH iQ

+? M;2p2@ $_0$ HQ+Biv 7`QK $p_{\cdot} = 1$! $p_{\cdot} = \frac{p_1}{3}$

AM i?2 H `;2@+? `;2 HBKBi- i?2 #Qp2 `2/m+2b

$$1: \quad !_2 = F \; p\frac{1}{\overline{3}} + \frac{p\,\overline{3}\,\mathsf{s}_{00}{}^2}{(2 \;\; ^2)^{2/3} \; + \frac{5}{4/3} \; L^3_7} \; \frac{9\,K^2}{128} \;^2 \; \frac{o}{Z} \;^{2/3} + ::: + O \; F^2$$

$$(1 \quad) \quad 1: \quad !_2 = F \; p\frac{1}{\overline{3}} + 1 \; \frac{2^{5/3} \; + \;^2 \; K^2}{3\,p\,\overline{3} \;^{2/3}} + \frac{9\,p\,\overline{3}\,K^4\,\mathsf{s}_{00}{}^2}{128\,p_3\,\overline{2} \;^{8/3} + \frac{4}{4/3} \; L^2_7} \; \frac{o}{Z} \;^{4/3} + ::: + O \; F^2$$

h? MFvQm

G `;2 +? `;2 b2imT

r2 rBHH +QMbB/2` Qm` bvbi2K QM i? K MB7QH/ M/ +m`p im`2 _ bmM/2`HvBM; M2r b+ H2 Q7 i?2 i?2Q`v Bb

$$z = (Z/ o)^{1/3}$$ UkyV

r?2`2 Z Bb i?2 }t2/ # `vQM +? `;2X

*QM+`2i2Hv- r2 rBHH i F2 Qm` K MB7QH/ iQ #2

$$M = R \quad a/ 1$$ UkRV

bm+? i? i r2 + M +QMbB/2` M TT`QtBK i2 bi i2@QT2` iQ` +Q``2bT

$$z = \tilde{o}^{1/3} \, 1_z ; \qquad 1_z = Z L$$ UkkV

r?2`2 $z$ Bb i?2 b+ HBM; /BK2MbBQM Q7 i?2 HQr2bi@HvBM; QT2` iQ

;`QmM/ bi i2 2M2`;v QM }t2/ +? `;2 Bb i?2 ` /BmbX Q7 a

G `;2 +? `;2 2tT MbBQM M/ 2tT2`BK2Mi H @Q M; ?Q H7 2 i ?2 vbB+b

r2 /Qm#H2@2tT M/2/ sM} ` 2M` 1/Q bH7 QHHQrb

$$s = s_0 + s_1 + {}^2 ;$$

$$s_F = s_{F0} + \frac{s_{F1}}{Z^{2/3}} + Z^{4/3} ; \quad 7Q` \quad 1$$

$$s = s_0 + s_1(1 \quad) + (1 \quad)^2 ;$$

$$s_F = s_{F0} + \frac{s_{F1}}{Z^{4/3}} + Z^2 ; \quad 7Q` \quad 1 :$$

r?2`2

$$s_{00} = L_7 + Q b \frac{+2 \ F}{L_7}$$

$$s_{01} = \frac{9 K^4 \ b B^2 M \frac{+2 \ F}{L_7} + Q b \frac{+2 \ F}{L_7}}{8 \ 2^{2/3} \ {}^{4/3} \ {}^2 + {}^{4/3}}$$

$$s_{10} = 0$$

$$s_{11} = 0$$

$${}_{00} = 0$$

$${}_{01} = \frac{K^2 \ s_{00} \ b B M \frac{+2 \ F}{L_7}}{L_7}$$

$${}_{10} = 0$$

$${}_{11} = \frac{3 K^2 \ b B M^2 \left(\frac{+2 \ F}{L_7} \right) \ H Q ; \frac{8192 \ {}^2 + {}^{3}_{4/3} L_7^3 p^6}{27 \ Z^2}}{32 \ 2^{2/3} \ {}^{4/3} \ {}^2 + {}^{4/3}}$$

1PJb 7Q` i?2 qBii2M p `B #H2b

h?2 2[m iBQMb Q7 KQiBQM `2 /

$$b B M L_7 + Q b \frac{K^2}{2} s = 0$$ UkjV

$$2 K^2 \; b B M_B + Q b = ; \quad B = 1 ; ::: ; L_7$$ Uk9V

M/ i?2 2M2`;v Q7 i?2 bvbi2K Bb

$$1 = {}^2 \; 4 K^2 s \; {}^2 ;$$ MQ`K H T(? = 0 2) Uk8V

$$1 = {}^2 \; 2 \frac{L_7^2 \; {}^4 + K^4 s^2}{L_7 \; {}^2} \; {}^2 ;$$ bmT2`~mB/ T+?Q b2 $\frac{K^2}{L_7 \; 2}$ s : UkeV

AM i?2 MQ`K H T? b2- i?2 qBii2M p `B #H2b `2 H2Hr@F2M QQM 2[m iB

$$2 K^2 \; b B M_B = \; = \sum_{B}^{L_7} ! \; {}_B :$$ UkdV

6Q` i?2 ;2M2` H bQHmiBQM r2 K mbi ?b B2H7 Q`M XMv

hQ bQHp2 7Q`_B 2i ? 2Q Mb B/2` i?2 2tT MbBQM B M $\frac{K^2}{}$ i 1 2XT ` K2i2`

1PJb 7Q` i?2 qBii2M p `B #H2b

i i?2 H2 /BM; Q`/2` QM2 M22/b iQ M b Q?Hp 2M7 H2b iBb 7v

(; B= 1;:::; M
B=
; B= M+1;:::; L7

Uk3V

r?2`2 Bb i?2 bQHmiBQM Q7 i?2 7QHHQrBM; KQ/mH ` 2[m iBQM

M( )+(L7 M) = JQ/2 :

UkNV

h?2 KQ/mHQ +QK2b 7`QK i?2 7 +ifi?g iQ B7 2[XQKdV BQM7QmM/- i?2M
#mBH/ MQi?2` bQHmiBQM b 7QHHQrb

1(+2)= 1( )+2 ; (+2)= ( ); B=2;:::; L7:

UjyV

>Qr2p2`- bBM+2 i?2 T?vbB+b /2T-2iM?2b/QM HKvB+QMB2b BMp !`B +Mi M/2
h?2 bQHmiBQM Q7 2[XUkNV + M #2 r`Bii2M b

$$= \frac{+(2\ F\ M)}{(L_7\ 2M)};\quad F=0;:::;L_7\ 2M\ 1;\quad M=0;:::;\frac{L_7\ 1}{2}:$$

UjRV

h?2 ` M;2 7Q` F #Qp2 2K2`;2b #L2+2Mbr2 72Q`T2Fi i?2 bQHmiBQM 7Q`

QM2 + M bF r?2M irQ /Bz2`2Mi bQHmiBQMb Q7 i?2 2[m iBQM Q7 KQ
+Q``2bTQM/b iQ `2[mB`BM;

$$+Qb \frac{+2 \quad F_1}{L_7} = +Qb \frac{+2 \quad F_2}{L_7} \; ; \qquad MQ`K H T? b2$$

$$+Q^2 b \frac{+2 \quad F_1}{L_7} = +Q^2 b \frac{+2 \quad F_2}{L_7} \; ; \qquad bmT2`~mB/: T? b2 \; UjjV$$

"Qi? +QM/BiBQMb `2 b iBb}2/ r?2M $F_1 = F_2 + M L_7 F$

F_1 M/ F_2 `2 BMi2;2`b

Ai Bb bm{+B2Mi iQ +QMbB/2` i?2 +Q`` 2bTQM/2M+2 H +Q``2bTQM/b iQ $F_1 = B$ i?i 2 [Q;] 2 FMi2`p H +Q``2bTQM/b iQ
bi i2 2M2`;v- 7m`i?2 `KQ`2 i?2 7Q` +2b i?2 b2+QM/ b $F_2 = HL_7$ iBQM iQ #2

amT2`~mQ7Q`B//L

r2 ? p2 i?2 bQHmiBQM $1 = iB (Q+M \frac{L_7}{2} F$ — r?B+? + M #2 `2 HBx2/ 7Q`

= $\frac{}{L_7}$ = $\frac{}{L_7}$ + = $\frac{2}{L_7}$

*S #`2 FBM;

LQi2 i? i r?2M 6M -Mi?2 p +mmK bTQMi M2Q(mbH#v2#+`2mH6b2 aQT7 i?2 /Bz
T? b2b 7Q` 2 +? [m `F ~ pQm`X

*S Bb T`2b2`p2/=r0?X2 M6Q` 2[m H K bb [m `Fb b +QMbB/2`2/ ?2`2
K =0 Q`=0 X

6Q`= i?2 G ;` M;B M TQbb2bb *S bvKK2i`v #mi BM i?2 MQ`K H
bTQMi M2QmbHv #`QF2M #v i?2 p +mmK

(. b?2M,RNdy2i-.Bo2++?B ,kyRjbr -: BQiiQ,kyRdiM2-.Bo2++?B ,kyRdtTmb) -H2 /BM; iQ b@`/Q T2M;M/2M
M2 `= X

*S #`2 FBM;

bbmKBM; i? i i?2 ;`QmM/ bi i2 /Q2b MQi #`2 F i?2 bvKK2i`v, rBi? M QT2` iQ` Q7 +?`; 2 +QMD@
(: BQiiQ,kyRdiM2)

$$l(B) = 2^{\frac{B+2F}{L7}} 1_{2L7}:$$ Uj9V

6Q`= QM2 ? b +Qb (2F+1)/L7 - r?B+? Bb K tBK Bx2/ r?2M F = L7 1- i? i Bb

$$l(B) = 2^{\frac{B}{L7}} 1_{2L7}; \quad l(B) = 2^{\frac{B}{L7}} 1_{2L7}:$$ Uj8V

h?2 irQ bQHmiBQMb `2 `2H i2/ #v *S i`MbT`Q7 bQHmiBQMb T QMi M2Q

*S #`2 FB∀M2 L

6Q` 7L> 2 i?2 KBMBK `2 b2T ` i2/ #v M 2M2`;v #7= `2B L?`2r PHH2H/2B MQ`
[m `F@K bb BM/m+2/ TQi2MiB H p MBT?`2bMiHv H2 /BM; iQ T ` /Q
++Q`/BM; iQ r?B+? QM2 ? b K bbH2bb TBQMb M/ MQ 2tTHB+Bi #`

U V@/2T2M/2M+2 Q7 i?2 2M2`;v BM i?2 MQ`K H T? b2 7=Q`XL

U #V@/2T2M/2M+2 Q7 i?2 2M2`;v BM i?2 bmT2`~mB/ T? b2 7=Q`X L

6B;m`2@/2T2M/2M+2 Q7 i?2 2M2`;v 7Q` L

h` Mb7Q`K iBQM T`QT2`iB2b Q7 i?2 }2H

	[a l(2)]	a l(L) G	a l(L) _	l (1) o	l (1)
[G			R	Y R	Y R
B2 2 [_		R		R	Y R

	[a l(2)]	a l(2 L)	l (1)
Q			Y R

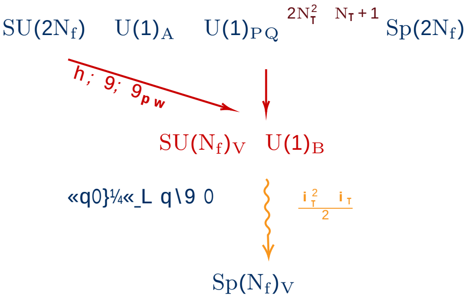
h #H,2h` Mb7Q`K iBQM T`QT-2B2iB[2b M7Q[mM/2` i?2 +iBQM Q7 i?2 bvKK2i

8 ² _nj

We denote by $_{PQ}$ the scale of $U(1)_{PQ}$ spontaneous symmetry breaking and by a_{PQ} the coefficient of the $U(1)_{PQ}$ anomalous term.

$$L_9 = {}^2 ¦ \} f @ \quad @ \quad {}^y g + {}^2_{pw} @ \quad i @ \quad i^y + 4 \quad {}^2 ¦ \} f ? \quad {}^y @_0 \quad g + h^2 \quad {}^2 ¦ \} f g \quad + g^y \quad {}^y g$$

$$+ 2 \quad {}^2 \quad {}^2 \quad ¦ \} f \quad ? ¦ \quad {}^y ? g + ¦ \} f ? ? g \quad 9 \quad {}^2 \quad \frac{-}{4} ¦ \} f enX \quad enX \quad {}^y g \quad \frac{-}{4} 9_{pw} (enX \; i \quad enX \; i^y)^2 \; : \quad r¨£s$$



$$D^{1} = @ \begin{pmatrix} \frac{!^2 \quad k^2}{\sin^2 \; '} \quad M_S^2 & \frac{p_a \; \overline{N_T a_{pw}}}{4 \; \frac{p}{2} \; _{pw} \sin^2 \; '} \\ \frac{p_a \; \overline{N_T a_{pw}}}{4 \; \frac{p}{2} \; _{pw} \sin^2 \; '} & \frac{!^2 \quad k^2}{4 \; ^2 \sin^2 \; '} \quad M_{\acute{a}}^2 \end{pmatrix} A \; ; \tag{37}$$

where

$$M_S^2 = \frac{a \quad {}^4 N_f + 2 \quad {}^2 m^4}{2 \quad {}^4 \quad 2m^4} \tag{38}$$

$$M_{\acute{a}}^2 = \frac{a \quad {}^4 a_{PQ}^2}{16 \; {}^2_{PQ} \; (\quad {}^4 \quad m^4)} \; : \tag{39}$$