February 2023

θ -angle physics of 2 color QCD

Fixed baryon charge and Near Conformal Dynamics Based on [JHEP 11 (2022) 080] and [2208.09227]

🐣 Alessandra D'Alise

General Overview

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General Overview

Great progress in our understanding of the structure of the space of quantum field theories $$(\rm QFTs)$$







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Unravelling its geometry and topology \implies deep implications in mathematics, quantum gravity, string theory, cosmology and condensed matter physics.







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Q How do we tackle the task?







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SOLVE QFT: investigate different regimes in a controlled manner and with precise results







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Enhanced global symmetry: SU(2N_f) 2-color QCD







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Enhanced global symmetry: $SU(2N_f)$ 2-color QCD theory at finite baryon density can be studied on the lattice [arXiv:0205019]







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(Near) Conformal Window [arXiv: 0107099, 0611341]

0 8 11 N_f







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(Near) Conformal Window $_{\scriptscriptstyle [arXiv: 0107099, 0611341]}$









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(Near) Conformal Window [arXiv: 0107099, 0611341]









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 $\fbox{0} \end{tabular} \text{ of 1 color} \\ \end{tabular} QCD \end{tabular} with $SU(2N_f)$ global symmetry $$ [JHEP 11 (2022) 080] $$ \end{tabular} $$ \end{tabular} $$ \end{tabular} \end$







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 $\fbox{0.2} \hline \texttt{(o)} in depth analysis of the θ-angle physics at non-zero baryon chemical potential of 2 color $$QCD with $SU(2N_f)$ global symmetry $$_{[JHEP 11 (2022) 080]}$ }$

 $\checkmark~2$ color effective pion Lagrangian at non-zero baryon charge including the $\theta\text{-angle term}$







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(Near) Conformal Window $_{\scriptscriptstyle [arXiv: 0107099, 0611341]}$



() in depth analysis of the θ -angle physics at non-zero baryon chemical potential of 2 color QCD with SU(2N_f) global symmetry [JHEP 11 (2022) 080]

- $\checkmark~2$ color effective pion Lagrangian at non-zero baryon charge including the $\theta\text{-angle term}$
- $\checkmark~$ determine the vacuum structure of the theory both in the normal and superfluid phase as a function of the different number of matter fields







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(Near) Conformal Window [arXiv: 0107099, 0611341]



() in depth analysis of the θ -angle physics at non-zero baryon chemical potential of 2 color QCD with SU(2N_f) global symmetry [JHEP 11 (2022) 080]

- $\checkmark~2$ color effective pion Lagrangian at non-zero baryon charge including the $\theta\text{-angle term}$
- $\checkmark~$ determine the vacuum structure of the theory both in the normal and superfluid phase as a function of the different number of matter fields
- $\checkmark~$ determine the spectrum of the theory







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Theory simplifies when large/small parameter exists







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ü Analytic treatment of theories with global symmetries







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Operators having large internal charge can be associated, via state/operator correspondence, to a superfluid phase on a cylinder







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 \checkmark near conformal invariance of the theory: dressing the lagrangian with the dilaton







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 ${\ensuremath{\mathcal{V}}}$ near conformal invariance of the theory: dressing the lagrangian with the dilaton

② near conformal dynamics of the dressed theory being in a superfluid phase on a cylinder [arXiv: 2208.09227]







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 $\checkmark~$ $\theta\text{-dependence}$ of the ground state energy







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- $\checkmark~\theta\text{-dependence}$ of the ground state energy
- $\checkmark~\theta\text{-dependence}$ of the near-conformal scaling dimension of the baryon charged operators on \mathbb{R}^4







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 $\theta\text{-angle physics}$ in the near conformal window

 $\begin{array}{c} 2 \ {\rm color} \ {\rm QCD+non-} \\ {\rm zero} \ {\rm baryon} \\ {\rm charge+} \ \theta{\rm -angle} \end{array}$







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Results

in collaboration with J. Bersini, F. Sannino & M. Torres

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The $\theta\text{-angle}$ physics of two-color QCD at fixed baryon charge







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The θ -angle physics of two-color QCD at fixed baryon charge Starting from 2-color QCD EFT with SU(2N_f) global symmetry [arXiv:0001171]

$$\mathcal{L}_{\theta} = \nu^{2} \mathrm{Tr} \{ \partial_{\mu} \Sigma \partial^{\mu} \Sigma^{\dagger} \} + 4\mu \nu^{2} \mathrm{Tr} \{ \mathrm{B} \Sigma^{\dagger} \partial_{0} \Sigma \} + \mathrm{m}_{\pi}^{2} \nu^{2} \mathrm{Tr} \{ \mathrm{M} \Sigma + \mathrm{M}^{\dagger} \Sigma^{\dagger} \} + 2\mu^{2} \nu^{2} \left[\mathrm{Tr} \{ \Sigma \mathrm{B}^{\mathrm{T}} \Sigma^{\dagger} \mathrm{B} \} + \mathrm{Tr} \{ \mathrm{B} \mathrm{B} \} \right] - \mathrm{a} \nu^{2} \left(\theta - \frac{\mathrm{i}}{4} \mathrm{Tr} \{ \log \Sigma - \log \Sigma^{\dagger} \} \right)^{2}$$
(1)







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(1)

using the ansatz $\Sigma_0 = U(\alpha_i)\Sigma_c$, $U(\alpha_i) \equiv diag\{e^{-i\alpha_1}, \dots, e^{-i\alpha_N_f}, e^{-i\alpha_1}, \dots, e^{-i\alpha_{N_f}}\}$

$$\Sigma_{\rm c} = \begin{pmatrix} 0 & 1_{\rm N_f} \\ -1_{\rm N_f} & 0 \end{pmatrix} \cos \varphi + i \begin{pmatrix} \mathcal{I} & 0 \\ 0 & \mathcal{I} \end{pmatrix} \sin \varphi \quad \text{where} \quad \mathcal{I} = \begin{pmatrix} 0 & -1_{\rm N_f/2} \\ 1_{\rm N_f/2} & 0 \end{pmatrix} , \qquad (2)$$







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the energy of the system is

$$\begin{split} \mathbf{E} &= -\nu^2 \left[4\mathbf{m}_\pi^2 \mathbf{X} - \mathbf{a}\bar{\theta}^2 \right] \,, \qquad \text{normal phase } (\varphi = 0) \qquad (3) \\ \mathbf{E} &= -\nu^2 \left[2 \frac{\mathbf{N}_{\mathrm{f}}^2 \mu^4 + \mathbf{m}_\pi^4 \mathbf{X}^2}{\mathbf{N}_{\mathrm{f}} \mu^2} - \mathbf{a}\bar{\theta}^2 \right] \,, \qquad \text{superfluid phase } \left(\cos \varphi = \frac{\mathbf{m}_\pi^2}{\mathbf{N}_{\mathrm{f}} \mu^2} \mathbf{X} \right) \qquad (4) \end{split}$$

with $X = \sum_i^{N_{\rm f}} \cos \alpha_i$







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$\theta\text{-dependence of the energy}$ [jhep 11 (2022) 080]







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$\theta\text{-dependence of the energy}_{[JHEP 11 (2022) 080]}$ even $N_{\rm f}$







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$\theta\text{-dependence of the energy}_{[JHEP 11 (2022) 080]}$ even $N_{\rm f}$



normal phase:

$$0 \leq \frac{\theta}{N_{\rm f}} \leq \pi \leq \frac{\theta + 2\pi(N_{\rm f} - 1)}{N_{\rm f}} \leq 2\pi$$

superfluid phase: $0 \leq \cos^2 \frac{\theta}{N_{\rm f}} \leq \pi$ $\pi \leq \cos^2 \frac{\theta + 2\pi (N_{\rm f} - 1)}{N_{\rm f}} \leq 2\pi$







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heta-dependence of the energy [JHEP 11 (2022) 080] even N_f

odd N_f



normal phase:

$$0 \le \cos \frac{\theta}{N_{\rm f}} \le \pi \le \cos \frac{\theta + 2\pi (N_{\rm f} - 1)}{N_{\rm f}} \le 2\pi$$

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heta-dependence of the energy [JHEP 11 (2022) 080] even N_f

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Symmetry breaking pattern & Spectrum







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Symmetry breaking pattern & Spectrum ${}_{SU(2N_f) \, \times \, U(1)_A} \stackrel{2N_{f}^2 - N_f}{\leadsto} {}_{Sp(2N_f)}$









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Symmetry breaking pattern & Spectrum $SU(2N_f) \times U(1)_A \xrightarrow{2N_f^2 - N_f} Sp(2N_f)$ $\omega_{*}^{2} = k^{2} + \mu^{2}$ $\Box = \frac{1}{2} N_{f}(N_{f} + 1)$ a, m $\frac{1}{2}N_f(N_f-1)-1$ $\omega_2^2 = \mathbf{k}^2 + \frac{\mathbf{m}_\pi^4 \mathbf{X}^2}{\mu^2 \mathbf{N}_c^2} \,,$ $SU(N_f)_V \times U(1)_B$ $\omega_3^2 = k^2 + \frac{2\left(\mu^4 N_f^2 + 3m_\pi^4 X^2\right)}{N^2 \mu^2} + A, \qquad \bullet + \bigsqcup \ \frac{1}{2} N_f (N_f - 1)$ superfluid phase $\begin{cases} \frac{N_{f}^{2} - N_{f}}{2} \\ \omega_{4}^{2} = k^{2} + \frac{2\left(\mu^{4}N_{f}^{2} + 3m_{\pi}^{4}X^{2}\right)}{N_{r}^{2}\mu^{2}} - A, \quad \bullet + \boxed{\frac{1}{2}N_{f}(N_{f} - 1)} \end{cases}$ $\omega_r^2 = k^2 + M_o^2$ $Sp(N_f)_V$ where $A = \frac{2}{N^2 \mu^2} \sqrt{\left(N_f^2 \mu^4 + 3m_\pi^4 X^2\right)^2 + 4N_f^2 \mu^2 m_\pi^4 k^2 X^2},$ (5) $M_{\rm S}^2 = \frac{a\mu^4 N_{\rm f}^3 + 2\mu^2 m_{\pi}^4 X^2}{2\mu^4 N_{\pi}^2 - 2m^4 X^2} \left(1 - \frac{m_{\pi}^4 X^2}{\mu^2 N_{\pi}^2}\right)$ (6)







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Charging the conformal window at nonzero θ -angle [2208.09227]







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Charging the conformal window at nonzero θ -angle [2208.09227]

 \checkmark smoothly approach the conformal phase of the theory \implies dressing our Lagrangian via a dilaton field $\sigma(\mathbf{x})$







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Charging the conformal window at nonzero θ -angle [2208.09227]

 \checkmark smoothly approach the conformal phase of the theory \implies dressing our Lagrangian via a dilaton field $\sigma(\mathbf{x})$

$$x\mapsto e^{\alpha}x\implies \sigma\mapsto \sigma-\tfrac{\alpha}{f}\implies \mathcal{O}_k\mapsto e^{(k-4)\sigma f}\;\mathcal{O}_k$$







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$$x\mapsto e^{\alpha}x\implies \sigma\mapsto \sigma-\tfrac{\alpha}{f}\implies \mathcal{O}_k\mapsto e^{(k-4)\sigma f}\;\mathcal{O}_k$$

$$\mathcal{L}_{\theta,\sigma} = \nu^{2} \mathrm{Tr} \{ \partial_{\mu} \Sigma \partial^{\mu} \Sigma^{\dagger} \} e^{-2\sigma f} + 4\mu \nu^{2} \mathrm{Tr} \{ \mathrm{B} \Sigma^{\dagger} \partial_{0} \Sigma \} e^{-2\sigma f} + \mathrm{m}_{\pi}^{2} \nu^{2} \mathrm{Tr} \{ \mathrm{M} \Sigma + \mathrm{M}^{\dagger} \Sigma^{\dagger} \} e^{-(3-\gamma)\sigma f} + 2\mu^{2} \nu^{2} \left[\mathrm{Tr} \{ \Sigma \mathrm{B}^{\mathrm{T}} \Sigma^{\dagger} \mathrm{B} \} e^{-2\sigma f} + \mathrm{Tr} \{ \mathrm{B} \mathrm{B} \} \right] - \mathrm{a} \nu^{2} \left(\theta - \frac{\mathrm{i}}{4} \mathrm{Tr} \{ \log \Sigma - \log \Sigma^{\dagger} \} \right)^{2} e^{-4\sigma f} + \frac{1}{2} \left(\partial_{\mu} \sigma \partial^{\mu} \sigma - \frac{\mathcal{R}}{6\mathrm{f}^{2}} \right) e^{-2\sigma f} - \frac{\mathrm{m}_{\sigma}^{2}}{1\mathrm{6}\mathrm{f}^{2}} \left(e^{-4\sigma f} + 4\sigma f - 1 \right) - \Lambda_{0}^{4} e^{-4\sigma f}$$

$$(7)$$







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Replacing this vacuum ansatz, the Lagrangian (7) becomes

$$\mathcal{L}_{\theta,\sigma} \left[\Sigma_0, \sigma_0 \right] = -e^{-4f\sigma_0} \left(\Lambda^4 - \frac{m_\sigma^2}{16f^2} \right) - \frac{m_\sigma^2 \left(4f\sigma_0 + e^{-4f\sigma_0} - 1 \right)}{16f^2} - \frac{R}{12f^2} + \frac{1}{4m_\pi^2 \nu^2 X \cos\varphi} e^{-f\sigma_0 y} + 2\mu^2 N_f \nu^2 e^{-2f\sigma_0} \sin^2\varphi - a\nu^2 e^{-4f\sigma_0} \bar{\theta}^2 ,$$
(8)

where

$$\bar{\theta} \equiv \theta - \sum_{i}^{N_{f}} \alpha_{i} , \qquad X \equiv \sum_{i}^{N_{f}} \cos \alpha_{i} , \qquad \Lambda^{4} \equiv \Lambda_{0}^{4} + \frac{m_{\sigma}^{2}}{16f^{2}} .$$
(9)







 $\square \theta$ -angle physics of 2 color QCD

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Replacing this vacuum ansatz, the Lagrangian (7) becomes

$$\mathcal{L}_{\theta,\sigma} \left[\Sigma_0, \sigma_0 \right] = -e^{-4f\sigma_0} \left(\Lambda^4 - \frac{m_\sigma^2}{16f^2} \right) - \frac{m_\sigma^2 \left(4f\sigma_0 + e^{-4f\sigma_0} - 1 \right)}{16f^2} - \frac{R}{12f^2} + \frac{1}{4m_\pi^2 \nu^2 X \cos\varphi} e^{-f\sigma_0 y} + 2\mu^2 N_f \nu^2 e^{-2f\sigma_0} \sin^2\varphi - a\nu^2 e^{-4f\sigma_0} \bar{\theta}^2 ,$$
(8)

where

$$\bar{\theta} \equiv \theta - \sum_{i}^{N_{f}} \alpha_{i}, \qquad X \equiv \sum_{i}^{N_{f}} \cos \alpha_{i}, \qquad \Lambda^{4} \equiv \Lambda_{0}^{4} + \frac{m_{\sigma}^{2}}{16f^{2}}.$$
(9)

The respective equations of motion are

$$N_{\rm f}\mu^2 e^{-2f\sigma} \cos\varphi - m_\pi^2 X e^{-f\sigma y} = 0$$
(10)

$$ae^{-4f\sigma}\bar{\theta} - 2m_{\pi}^{2}\sin\alpha_{i}\cos\varphi e^{-f\sigma y} = 0, \qquad i = 1, .., N_{f} \qquad (11)$$

$$\frac{\operatorname{Re}^{-2f\sigma}}{6f} + 4af\nu^{2}e^{-4f\sigma}Y^{2} + 4f\Lambda_{0}^{4}e^{-4f\sigma} - \frac{m_{\sigma}^{2}\left(1 - e^{-4f\sigma}\right)}{4f} + -4f\mu^{2}N_{f}\nu^{2}e^{-2f\sigma}\sin^{2}\varphi - 4fm_{\pi}^{2}\nu^{2}yX\cos\varphi e^{-f\sigma y} = 0$$
(12)

$$4\mu N_{\rm f} \nu^2 e^{-2f\sigma} \sin^2 \varphi = \frac{Q}{V} \ . \tag{13}$$







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 $\checkmark~$ large-charge quasi-conformal Ground State Energy as function of the dilaton, fermion mass and background geometry to include the impact of the θ angle physics







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 $\checkmark\,$ large-charge quasi-conformal Ground State Energy as function of the dilaton, fermion mass and background geometry to include the impact of the θ angle physics

$$\begin{split} \mathrm{E}^{\gamma \ll 1} &= \ \frac{\mathrm{c}_{4/3} \mathrm{Q}^{4/3}}{\tilde{\mathrm{V}}^{1/3}} + \mathrm{Q}^{2/3} \tilde{\mathrm{V}}^{1/3} \left\{ \mathrm{c}_{2/3} \tilde{\mathrm{R}} - \frac{\mathrm{X}_{00}^2}{4\pi^2 \mathrm{N}_{\mathrm{f}}^3 \mathrm{c}_{4/3}^4} \left(\frac{9\mathrm{m}_\pi^2}{32\nu} \right)^2 \left[1 - \gamma \left(\frac{2}{3} \log \mathrm{Q} - \frac{\mathrm{X}_{10}}{\mathrm{X}_{00}} - \right) \right] \right] \right\} \\ &- \left[\log \left(\frac{32 \mathrm{N}_{\mathrm{f}} \nu^2 \pi^2 \mathrm{c}_{4/3} \tilde{\mathrm{V}}^{2/3}}{3} \right) \right] \right] \right\} - \tilde{\mathrm{V}} \log \mathrm{Q} \left\{ \frac{16 \pi^2}{9} \mathrm{N}_{\mathrm{f}} \mathrm{c}_{2/3} \mathrm{c}_{4/3} \nu^2 \mathrm{m}_{\sigma}^2 - \frac{\gamma}{3\pi^2 \mathrm{N}_{\mathrm{f}}^4 \mathrm{c}_{4/3}^5} \left(\frac{9\mathrm{m}_\pi^2}{32\nu} \right)^2 \mathrm{C} \right] \\ &- \left[\frac{5}{8\pi^2 \mathrm{c}_{4/3}^4 \mathrm{N}_{\mathrm{f}}^2} \left(\frac{9\mathrm{m}_\pi^2}{32\nu} \right)^2 \mathrm{X}_{00}^4 - \mathrm{c}_{2/3} \tilde{\mathrm{R}} \mathrm{N}_{\mathrm{f}} \mathrm{X}_{00}^2 + \frac{9\mathrm{X}_{00}\mathrm{X}_{01}}{32\mathrm{c}_{4/3}} \right] \right\} + \left(\mathrm{Q}^0 \right) \\ &\mathrm{E}^{1-\gamma \ll 1} = \frac{\mathrm{c}_{4/3} \mathrm{Q}^{4/3}}{\tilde{\mathrm{V}}^{1/3}} + \mathrm{c}_{2/3} \mathrm{Q}^{2/3} \tilde{\mathrm{R}} \tilde{\mathrm{V}}^{1/3} - \frac{9(1-\gamma) \mathrm{X}_{00}^2 \mathrm{m}_\pi^4 \tilde{\mathrm{V}} \log \mathrm{Q}}{64\mathrm{c}_{4/3}^3 \mathrm{N}_{\mathrm{f}}^2} - \frac{16}{9} \pi^2 \mathrm{m}_{\sigma}^2 \mathrm{N}_{\mathrm{f}} \mathrm{c}_{2/3} \mathrm{c}_{4/3} \nu^2 \mathrm{V} \log \mathrm{Q} + \left(\mathrm{Q}^0 \right) \\ \end{split}$$







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where we introduced

$$c_{4/3} = \frac{3}{8} \left(\frac{\Lambda^2}{\pi N_f \nu^2}\right)^{2/3}, \quad c_{2/3} = \frac{1}{4f^2} \left(\frac{\pi^2}{N_f \nu^2 \Lambda^4}\right)^{1/3}, \quad \tilde{R} = \frac{R}{6} \quad \text{and} \quad \tilde{V} = \frac{V}{2\pi^2}, \tag{14}$$







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 $\checkmark~$ large-charge quasi-anomalous dimension Δ as function of the dilaton, fermion mass and background geometry to include the impact of the θ angle physics

• $\gamma \ll 1$

$$\begin{split} \frac{\Delta}{\Delta^*} &= 1 - \left(\frac{9m_\pi^2}{32\pi\nu}\right)^2 \frac{1 - \gamma \log\left(\frac{3\rho^{2/3}}{16(2\pi^2)^{1/3}c_{4/3}\nu^2N_f}\right)}{4c_{4/3}^5N_f} \cos^2\left(\frac{\theta + 2\pi k}{N_f}\right) \left(\frac{1}{2\pi^2\rho}\right)^{2/3} \\ &+ \frac{\gamma}{c_{4/3}^6N_f} \cos^2\left(\frac{\theta + 2\pi k}{N_f}\right) \left(\frac{27m_\pi^4 \sin^2\left(\frac{\theta + 2\pi k}{N_f}\right)}{256\ 2^{2/3}\pi^{4/3}a\ c_{4/3}^3N_f^2} + \frac{5\left(\frac{9m_\pi^2}{64\pi\nu}\right)^2 \cos^2\left(\frac{\theta + 2\pi k}{N_f}\right)}{6c_{4/3}^4N_f} - \frac{c_{2/3}}{2}\left(\frac{\rho}{2\pi^2Q}\right)^{2/3}\right) \\ &\times \left(\frac{9m_\pi^2}{32\pi\nu}\right)^2 \left(\frac{1}{2\pi^2\rho}\right)^{4/3} \log Q - \frac{16}{9}\pi^2 c_{2/3}\nu^2 N_f m_\sigma^2\left(\frac{1}{2\pi^2\rho}\right)^{4/3} \log Q \end{split}$$







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•
$$(1 - \gamma) \ll 1$$

$$\frac{\Delta}{\Delta^*} = 1 - \left(\frac{9m_{\pi}^4}{64c_{4/3}^4}(1 - \gamma)\cos^2\left(\frac{\theta + 2\pi k}{N_f}\right) + \frac{16}{9}\pi^2 c_{2/3}\nu^2 N_f m_{\sigma}^2\right) \left(\frac{1}{2\pi^2\rho}\right)^{4/3} \log Q$$







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Spectrum

$$SU(2N_f) \times U(1)_A \xrightarrow{2N_f^2 - N_f} Sp(2N_f) \longrightarrow SU(N_f)_V \times U(1)_B \xrightarrow{\frac{N_f^2 - N_f}{2}} Sp(N_f)_V$$
(15)







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$$\mathrm{SU}(2\mathrm{N}_{\mathrm{f}}) \times \mathrm{U}(1)_{\mathrm{A}} \xrightarrow{2\mathrm{N}_{\mathrm{f}}^2 - \mathrm{N}_{\mathrm{f}}} \mathrm{Sp}(2\mathrm{N}_{\mathrm{f}}) \longrightarrow \mathrm{SU}(\mathrm{N}_{\mathrm{f}})_{\mathrm{V}} \times \mathrm{U}(1)_{\mathrm{B}} \xrightarrow{\frac{\mathrm{N}_{\mathrm{f}}^2 - \mathrm{N}_{\mathrm{f}}}{\sim}} \mathrm{Sp}(\mathrm{N}_{\mathrm{f}})_{\mathrm{V}}$$
(15)

Having in mind the hierarchy of scales m $\ll \sqrt{a} \leq \mu \ll 4\pi\nu,$ we focus on the spectrum of light modes







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 $\operatorname{SU}(2N_{\mathrm{f}}) \times \operatorname{U}(1)_{\mathrm{A}} \xrightarrow{2N_{\mathrm{f}}^{2}-N_{\mathrm{f}}} \operatorname{Sp}(2N_{\mathrm{f}}) \longrightarrow \operatorname{SU}(N_{\mathrm{f}})_{\mathrm{V}} \times \operatorname{U}(1)_{\mathrm{B}} \xrightarrow{\frac{N_{\mathrm{f}}^{2}-N_{\mathrm{f}}}{\sim}} \operatorname{Sp}(N_{\mathrm{f}})_{\mathrm{V}}$ (15)

Having in mind the hierarchy of scales m $\ll \sqrt{a} \le \mu \ll 4\pi\nu$, we focus on the spectrum of light modes

- $\frac{1}{2}N_f(N_f 1)$ massless Goldstones: -+ of $Sp(N_f)$
- 1 pseudo-Goldstone of $Sp(N_f)$ with mass $\propto \sqrt{a}$







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 $SU(2N_f) \times U(1)_A \xrightarrow{2N_f^2 - N_f} Sp(2N_f) \longrightarrow SU(N_f)_V \times U(1)_B \xrightarrow{\frac{N_f^2 - N_f}{2}} Sp(N_f)_V$ (15)

Having in mind the hierarchy of scales $m \ll \sqrt{a} \le \mu \ll 4\pi\nu$, we focus on the spectrum of

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- $\frac{1}{2}N_f(N_f 1)$ massless Goldstones: $\Box + \bullet$ of $Sp(N_f)$
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the spectrum changes when (near)conformal dynamics is realized through the dilaton dressing







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 $\operatorname{SU}(2N_{\mathrm{f}}) \times \operatorname{U}(1)_{\mathrm{A}} \xrightarrow{2N_{\mathrm{f}}^{2} - N_{\mathrm{f}}} \operatorname{Sp}(2N_{\mathrm{f}}) \longrightarrow \operatorname{SU}(N_{\mathrm{f}})_{\mathrm{V}} \times \operatorname{U}(1)_{\mathrm{B}} \xrightarrow{\frac{N_{\mathrm{f}}^{2} - N_{\mathrm{f}}}{2}} \operatorname{Sp}(N_{\mathrm{f}})_{\mathrm{V}}$ (15)

Having in mind the hierarchy of scales $m \ll \sqrt{a} \le \mu \ll 4\pi\nu$, we focus on the spectrum of

light modes

- $\frac{1}{2}N_f(N_f 1)$ massless Goldstones: $\Box + \bullet$ of $Sp(N_f)$
- 1 pseudo-Goldstone of $Sp(N_f)$ with mass $\propto \sqrt{a}$

the spectrum changes when (near)conformal dynamics is realized through the dilaton dressing

we expand around the vacuum solution as follows

$$\Sigma = e^{i\Omega} \Sigma_0 e^{i\Omega^t} \quad \text{where} \quad \Omega = \begin{pmatrix} \pi & 0 \\ 0 & -\pi^t \end{pmatrix} + \tilde{\beta} S \begin{pmatrix} 1_{N_f} & 0 \\ 0 & 1_{N_f} \end{pmatrix}, \quad \tilde{\beta} \equiv \frac{1}{\sqrt{2N_f}}, \ \pi = \sum_{a=0}^{\dim \frac{U(N_f)}{S_P(N_f)}} \pi^a T_a$$

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$$\frac{\mathcal{L}}{4\nu^2 \sin^2 \varphi \, \mathrm{e}^{-2\sigma_0 \mathrm{f}}} = \begin{pmatrix} \pi^0 & \hat{\sigma} & \mathrm{S} \end{pmatrix} \mathrm{D}^{-1} \begin{pmatrix} \pi^0 \\ \hat{\sigma} \\ \mathrm{S} \end{pmatrix} + \sum_{\mathrm{a}=1}^{\mathrm{dim}(\underbrace{\vdash})} \partial^{\mu} \pi^{\mathrm{a}} \partial_{\mu} \pi^{\mathrm{a}}$$
(16)







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$$\frac{\mathcal{L}}{4\nu^2 \sin^2 \varphi \, \mathrm{e}^{-2\sigma_0 \mathrm{f}}} = \begin{pmatrix} \pi^0 & \hat{\sigma} & \mathrm{S} \end{pmatrix} \mathrm{D}^{-1} \begin{pmatrix} \pi^0 \\ \hat{\sigma} \\ \mathrm{S} \end{pmatrix} + \sum_{\mathrm{a}=1}^{\mathrm{dim}(\square)} \partial^{\mu} \pi^{\mathrm{a}} \partial_{\mu} \pi^{\mathrm{a}}$$
(16)

with the inverse propagator D^{-1} defined as

$$D^{-1} = \begin{pmatrix} \omega^{2} - k^{2} & i\omega\mu f\sqrt{2N_{f}} & 0\\ -i\omega\mu f\sqrt{2N_{f}} & \frac{\omega^{2} - k^{2}}{8\nu^{2}\sin^{2}\varphi} - M_{\sigma}^{2} & \frac{1}{2}I_{\hat{\sigma}s} \\ 0 & \frac{1}{2}I_{\hat{\sigma}s} & \frac{(\omega^{2} - k^{2})}{\sin^{2}\varphi} - M_{s}^{2} \end{pmatrix}, \qquad I_{\hat{\sigma}S} = \frac{\sqrt{2}f\mu^{2}m_{\pi}^{4}\sqrt{N_{f}}XyZ}{m_{\pi}^{4}X^{2} - \mu^{4}N_{f}^{2}e^{2f\sigma_{0}(y-2)}}$$
(17)







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$$\frac{\mathcal{L}}{4\nu^2 \sin^2 \varphi \, \mathrm{e}^{-2\sigma_0 \mathrm{f}}} = \begin{pmatrix} \pi^0 & \hat{\sigma} & \mathrm{S} \end{pmatrix} \mathrm{D}^{-1} \begin{pmatrix} \pi^0 \\ \hat{\sigma} \\ \mathrm{S} \end{pmatrix} + \sum_{\mathrm{a}=1}^{\mathrm{dim}(\underbrace{\vdash})} \partial^{\mu} \pi^{\mathrm{a}} \partial_{\mu} \pi^{\mathrm{a}}$$
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with the inverse propagator D^{-1} defined as

$$D^{-1} = \begin{pmatrix} \omega^{2} - k^{2} & i\omega\mu f\sqrt{2N_{f}} & 0\\ -i\omega\mu f\sqrt{2N_{f}} & \frac{\omega^{2} - k^{2}}{8\nu^{2}\sin^{2}\varphi} - M_{\sigma}^{2} & \frac{1}{2}I_{\hat{\sigma}s} \\ 0 & \frac{1}{2}I_{\hat{\sigma}s} & \frac{(\omega^{2} - k^{2})}{\sin^{2}\varphi} - M_{s}^{2} \end{pmatrix}, \qquad I_{\hat{\sigma}S} = \frac{\sqrt{2}f\mu^{2}m_{\pi}^{4}\sqrt{N_{f}}XyZ}{m_{\pi}^{4}X^{2} - \mu^{4}N_{f}^{2}e^{2f\sigma_{0}(y-2)}}$$
(17)

where $Z \equiv \sum_{i=1}^{N_f} \sin \alpha_i$ and the Lagrangian masses for the dilaton-field and the S mode are given by

COSE

$$M_{\sigma}^{2} = -\frac{f^{2}\mu^{2}N_{f}e^{-6f\sigma_{0}}\left(\nu^{2}m_{\pi}^{4}X^{2}\left(y^{2}-2\right)e^{6f\sigma_{0}}+2\mu^{4}\nu^{2}N_{f}^{2}e^{2f\sigma_{0}(y+1)}-4\Lambda^{4}\mu^{2}N_{f}e^{2f\sigma_{0}y}\right)}{2\nu^{2}\left(\mu^{4}N_{f}^{2}e^{2f\sigma_{0}(y-2)}-m_{\pi}^{4}X^{2}\right)}$$
(18)

$$M_{\rm S}^2 = \frac{a\mu^4 N_{\rm f}^3 e^{2f\sigma_0(y-1)} + 2\mu^2 m_\pi^4 X^2 e^{4f\sigma_0}}{2\mu^4 N_{\rm f}^2 e^{2f\sigma_0 y} - 2m_\pi^4 X^2 e^{4f\sigma_0}} .$$
(19)





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 $\begin{array}{l} \text{conformal invariance} \\ \text{dictates the existence of a massless} \\ \text{mode with speed} \\ \text{v}_{\text{G}} = \frac{1}{\sqrt{d-1}} = \frac{1}{\sqrt{3}} \\ \\ \text{[Orlando:2019skh]} \end{array}$

mixing between the singlet mode π_0 with the dilaton







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In the large-charge limit, the above reduces to

$$\gamma \ll 1: \quad \omega_2 = k \left[\frac{1}{\sqrt{3}} + \frac{\sqrt{3} X_{00}^2}{(2\pi^2)^{2/3} c_{4/3}^5 N_f^3} \left(\frac{9m_\pi^2}{128\pi\nu} \right)^2 \left(\frac{V}{Q} \right)^{2/3} + \dots \right] + \mathcal{O}(k^2)$$

$$(1 - \gamma) \ll 1: \quad \omega_2 = k \left[\frac{1}{\sqrt{3}} + 1 \left(\frac{2^{5/3} c_{2/3} \nu^2 m_\sigma^2}{3\sqrt{3}\pi^{2/3}} + \frac{9\sqrt{3}m_\pi^4 X_{00}^2}{128\sqrt[3]{2}\pi^{8/3} c_{4/3}^4 N_f^2} \right) \left(\frac{V}{Q} \right)^{4/3} + \dots \right] + \mathcal{O}(k^2)$$







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Thank you 🖸







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Large charge setup

we will consider our system on a manifold \mathcal{M} with volume V and curvature R such that the underlying new scale of the theory is

$$\Lambda_{\mathbf{Q}} = (\mathbf{Q}/\mathbf{V})^{1/3} \tag{20}$$

where Q is the fixed baryon charge. Concretely, we will take our manifold to be

$$\mathcal{M} = \mathbb{R} \times \mathrm{S}^{\mathrm{d}-1} \tag{21}$$

such that we can consider an approximate state-operator correspondence that implies

$$\Delta_{\mathbf{Q}} = \tilde{\mathbf{V}}^{1/3} \mathbf{E}_{\mathbf{Q}} \,, \qquad \mathbf{E}_{\mathbf{Q}} = \mu \mathbf{Q} - \mathcal{L} \tag{22}$$

where $\Delta_{\rm Q}$ is the scaling dimension of the lowest-lying operator with baryon charge Q, $E_{\rm Q}$ is the ground state energy on $\mathbb{R} \times {\rm S}^{d-1}$ at fixed charge, $\tilde{\rm V}^{1/3}$ is the radius of ${\rm S}^{d-1}$.







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Large charge expansion of the θ -angle physics

we double-expanded X first in γ and then also in $1/{\rm Q}$ as follows

$$\begin{split} X &= X_0 + X_1 \gamma + \left(\gamma^2\right) \,, & X_k = X_{k0} + \frac{X_{k1}}{Q^{2/3}} + \left(Q^{-4/3}\right) \,, & \text{for } \gamma \ll 1 \\ X &= X_0 + X_1 (1-\gamma) + \left((1-\gamma)^2\right) \,, & X_k = X_{k0} + \frac{X_{k1}}{Q^{4/3}} + \left(Q^{-2}\right) \,, & \text{for } 1-\gamma \ll 1 \,. \end{split}$$

where

$$\begin{aligned} X_{00} &= N_{\rm f} \cos\left(\frac{\theta + 2k\pi}{N_{\rm f}}\right) & \theta_{00} &= 0 \\ X_{01} &= \frac{9m_{\pi}^4 \sin^2\left(\frac{\theta + 2k\pi}{N_{\rm f}}\right) \cos\left(\frac{\theta + 2k\pi}{N_{\rm f}}\right)}{8\ 2^{2/3}\pi^{4/3} {\rm a}\ c_{4/3}^2} & \bar{\theta}_{01} &= \frac{m_{\pi}^2 X_{00} \sin\left(\frac{\theta + 2\pi k}{N_{\rm f}}\right)}{{\rm aN}_{\rm f}} \\ \bar{\theta}_{10} &= 0 \\ X_{10} &= 0 \\ X_{11} &= 0 & \bar{\theta}_{11} &= \frac{3m_{\pi}^2 \sin\left(\frac{2(\theta + 2\pi k)}{N_{\rm f}}\right) \log\left(\frac{8192\pi^2 c_{4/3}^3 N_{\rm f}^3 v^6}{27Q^2}\right)}{32\ 2^{2/3}\pi^{4/3} {\rm a}\ c_{4/3}^2} \end{aligned}$$







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EOMs for the Witten variables

The equations of motion read

$$\sin\varphi \left(N_{\rm f} \cos\varphi - \frac{m_\pi^2}{\mu^2} X \right) = 0 \tag{23}$$

$$2m_{\pi}^2 \sin \alpha_i \cos \varphi = a\bar{\theta}, \quad i = 1, .., N_f$$
 (24)

and the energy of the system is

$$\begin{split} \mathbf{E} &= -\nu^2 \left[4\mathbf{m}_\pi^2 \mathbf{X} - \mathbf{a}\bar{\theta}^2 \right] \,, \qquad \text{normal phase } (\varphi = 0) \end{split} \tag{25} \\ \mathbf{E} &= -\nu^2 \left[2 \frac{\mathbf{N}_{\mathrm{f}}^2 \mu^4 + \mathbf{m}_\pi^4 \mathbf{X}^2}{\mathbf{N}_{\mathrm{f}} \mu^2} - \mathbf{a}\bar{\theta}^2 \right] \,, \qquad \text{superfluid phase } \left(\cos \varphi = \frac{\mathbf{m}_\pi^2}{\mathbf{N}_{\mathrm{f}} \mu^2} \mathbf{X} \right) \,. \end{aligned} \tag{26}$$

In the normal phase, the Witten variables are related to θ by the well-known equation

$$2m_{\pi}^{2}\sin\alpha_{i} = a\bar{\theta} = a\left(\theta - \sum_{i}^{N_{f}}\alpha_{i}\right) .$$
⁽²⁷⁾

For the general solution we must have for any $\bar{\theta}$ fixed $\sin \alpha_i = \sin \alpha_j$. To solve for the α_i we consider the expansion in the parameter $\frac{m_{\pi}^2}{a} \ll 1$.



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EOMs for the Witten variables

at the leading order one needs to solve for $\bar{\theta} = 0$ and the angles α_i satisfy

$$\alpha_{i} = \begin{cases} \pi - \alpha, & i = 1, \dots, n \\ \alpha, & i = n + 1, \dots, N_{f} \end{cases}$$
(28)

where α is the solution of the following modular equation

$$n(\pi - \alpha) + (N_f - n)\alpha = \theta \text{ Mod } 2\pi .$$
(29)

The modulo comes from the fact that if a solution $\{\alpha_i\}$ of eq.(27) is found, then it is possible to build another solution as follows

> $\alpha_1(\theta + 2\pi) = \alpha_1(\theta) + 2\pi$, $\alpha_i(\theta + 2\pi) = \alpha_i(\theta)$, $i = 2, \dots, N_f$. (30)

However, since the physics depends only on $e^{-i\alpha_i}$, the dynamics is invariant under $\theta \to \theta + 2\pi$. The solution of eq.(29) can be written as

$$\alpha = \frac{\theta + (2k - n)\pi}{(N_f - 2n)}, \quad k = 0, \dots, N_f - 2n - 1, \quad n = 0, \dots, \left[\frac{N_f - 1}{2}\right].$$
(31)

The range for k above emerges because for $k \ge N_f - 2n$ we repeat the solution for a given n.

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one can ask when two different solutions of the equation of motion can have the same energy. This corresponds to requiring

$$\cos\left(\frac{\theta + 2\pi k_1}{N_f}\right) = \cos\left(\frac{\theta + 2\pi k_2}{N_f}\right) , \qquad \text{normal phase} \qquad (32)$$
$$\cos^2\left(\frac{\theta + 2\pi k_1}{N_f}\right) = \cos^2\left(\frac{\theta + 2\pi k_2}{N_f}\right) , \qquad \text{superfluid phase} . \qquad (33)$$

• Both conditions are satisfied when
$$k_1 = -\frac{\theta}{\pi} - k_2 + N_f$$
.

- k₁ and k₂ are integers
- It is sufficient to consider the case $k_1 = 0$ that for $[0, \pi]$ interval corresponds to the ground state energy, furthermore at $\theta = \pi$ it forces the second solution to be $k_2 = N_f 1$







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Superfluid $N_{\rm f}$ odd

we have the solution
$$k_1 = -k_2 + \frac{N_f}{2} - \frac{\theta}{\pi}$$
 which can be realized for







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CP breaking

- Note that when $n \neq 0$, the vacuum spontaneously breaks $Sp(2N_f)$ because of the different phases for each quark flavour.
- CP is preserved when $\bar{\theta} = 0$. For equal mass quarks as considered here, this happens when $m_{\pi} = 0$ or $\theta = 0$.
- For $\theta = \pi$ the Lagrangian possess CP symmetry but in the normal phase the latter is spontaneously broken by the vacuum [Dashen:1970et,DiVecchia:2013swa,Gaiotto:2017tne,DiVecchia:2017xpu], leading to a strong θ -dependence near $\theta = \pi$.







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CP breaking

assuming that the ground state does not break ${\rm Sp}(2N_{\rm f})$ spontaneously (i.e. n = 0), the vacua lie at [Gaiotto:2017tne]

$$U(\alpha_i) = e^{i\frac{\theta + 2\pi k}{N_f}} \mathbb{1}_{2N_f} .$$
(34)

For $\theta = \pi$ one has $X = \cos\left(\frac{(2k+1)\pi}{N_f}\right)$, which is maximized when k = 0 and $k = N_f - 1$, that is

$$U(\alpha_i) = e^{\frac{i\pi}{N_f}} \mathbb{1}_{2N_f}, \qquad U(\alpha_i) = e^{-\frac{i\pi}{N_f}} \mathbb{1}_{2N_f}.$$
(35)

The two solutions are related by a CP transformation $U \rightarrow U^{\dagger}$ and thus CP is spontaneously broken.







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CP breaking $N_f = 2$

For $\rm N_f>2$ the minima are separated by an energy barrier while for $\rm N_f=2$ the leading order quark-mass induced potential vanishes $_{\rm [Smilga:1998dh]}$, apparently leading to a paradoxical situation according to which one has massless pions and no explicit breaking of chiral symmetry.



Figure: θ -dependence of the energy for $N_f = 2$.







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Transformation properties of the fields

	[SU(2)]	$\rm SU(N_f)_L$	×	${\rm SU}({\rm N}_{\rm f})_{\rm R}$	×	$U(1)_V$	×	$U(1)_A$
$q_{\rm L}$				1		+1		+1
$\mathrm{i}\sigma_{2} au_{2}\mathrm{q}_{\mathrm{R}}^{*}$		1		$\overline{\Box}$		-1		+1
	[SU(2)]	$SU(2N_f)$	×	$U(1)_A$				
Q				+1				

Table: Transformation properties of q_L , $i\sigma_2\tau_2q_R^*$ and Q under the action of the symmetry groups.







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Axion

We denote by ν_{PQ} the scale of U(1)_{PQ} spontaneous symmetry breaking and by a_{PQ} the coefficient of the $U(1)_{PQ}$ anomalous term.

$$\mathcal{L}_{\hat{a}} = \nu^{2} \operatorname{Tr}\{\partial_{\mu} \Sigma \partial^{\mu} \Sigma^{\dagger}\} + \nu_{PQ}^{2} \partial_{\mu} N \partial^{\mu} N^{\dagger} + 4\mu \nu^{2} \operatorname{Tr}\{B\Sigma^{\dagger} \partial_{0} \Sigma\} + m_{\pi}^{2} \nu^{2} \operatorname{Tr}\{M\Sigma + M^{\dagger} \Sigma^{\dagger}\}$$

$$+ 2\mu^{2} \nu^{2} \left[\operatorname{Tr}\{\Sigma B^{T} \Sigma^{\dagger} B\} + \operatorname{Tr}\{BB\}\right] - a\nu^{2} \left(\theta - \frac{i}{4} \operatorname{Tr}\{\log \Sigma - \log \Sigma^{\dagger}\} - \frac{i}{4} a_{PQ} (\log N - \log N^{\dagger})\right)^{2}.$$

$$(36)$$





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