Numerical computation of Schwarzschild Quasi Normal Modes

1. Integrate the Regge-Wheeler equation, which describe axial perturbations of a Schwarzschild black hole:

$$\frac{d^2R}{dr_\star^2} + [\omega^2 - V(r)]R = 0 \ , \tag{1}$$

where $r_\star = r + 2M \log[\frac{r}{2M} - 1]$ is the tortoise coordinate, and the potential $V(r)$ is given by:

$$V(r) = \left(1 - \frac{2M}{r}\right)\left(\frac{\ell(\ell+1)}{r^2} - \frac{6M}{r^3}\right) \ . \tag{2}$$

using a direct integration method. Compute the real and imaginary part of the QNMs for different values of the multipole number $\ell \geq 2$, as roots of the Wronskian built with homogenous solutions of Eq. (2).

2. Compare the values found with the direct integration method with those obtained analytically through the WKB approach.