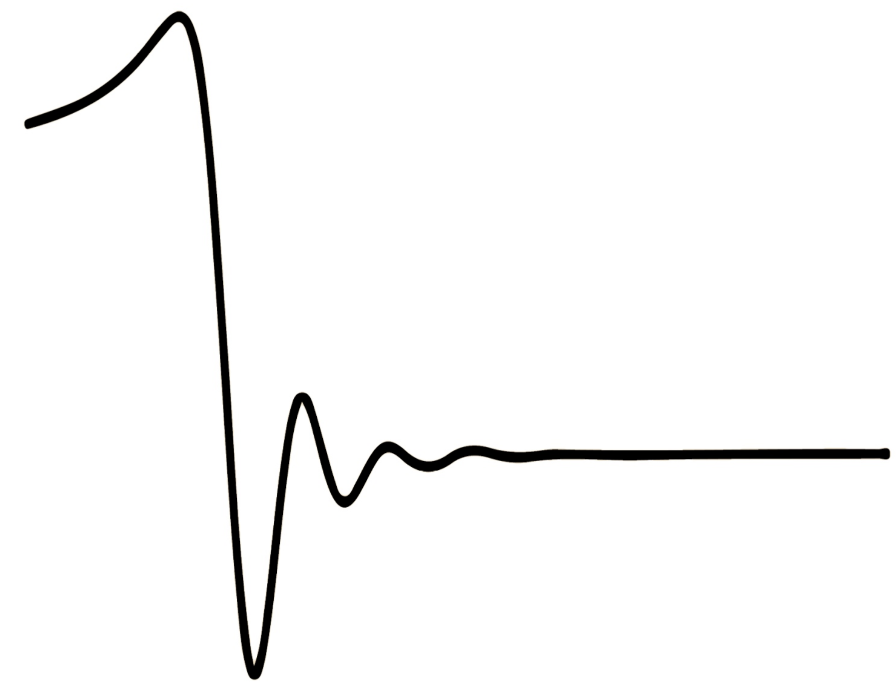
# *Astrophysical black holes: theory and observations*

**@ 59th Winter School "Gravity: Classical, Quantum and Phenomenology"**

Wroclaw, 12-21 February 2023



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Scuola Universitaria Superiore

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*Phys. Rev. D 96, 084002 (2017)*


www.blackholes.ist.utl.pt

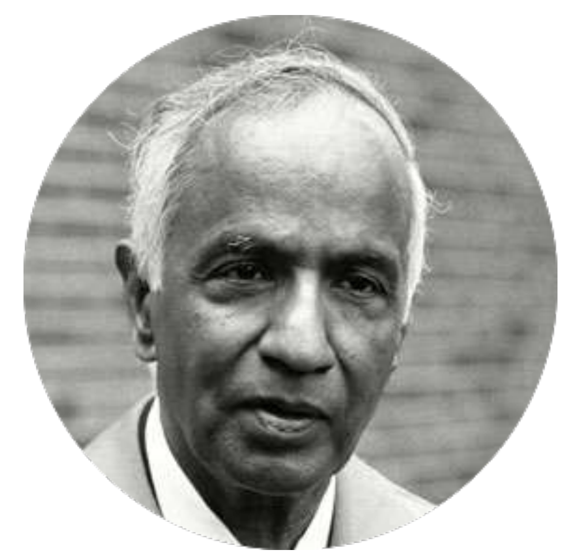
# *BH Sociology*

*"As you see, the war treated me kindly enough, in spite of the heavy gunfire, to allow me to get away from it all and take this walk in the land of your ideas: ..."*

Karl Schwarzschild to Albert Einstein
Letter dated 22 December 1915

*"In my entire scientific life, extending over forty-five years, the most shattering experience has been the realisation that an exact solution of Einstein's equations of general relativity provides the absolutely exact representation of untold numbers of black holes that populate the universe."*

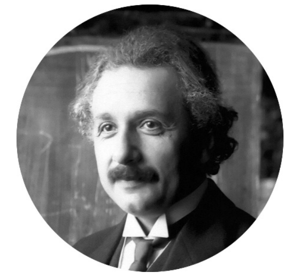
*S. Chandrasekhar*
The Nora and Edward Ryerson lecture, Chicago April 22 1975



# *History of BHs*

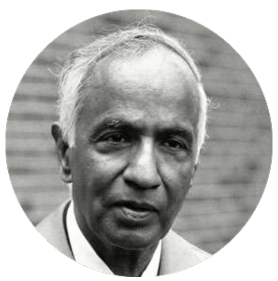
**Mitchell**

**Schwarzschild**



**Einstein**

**Eddington**



**Chandrasekhar**

**Kerr**

**Wheeler**

**Oppenheimer**

**Penrose**

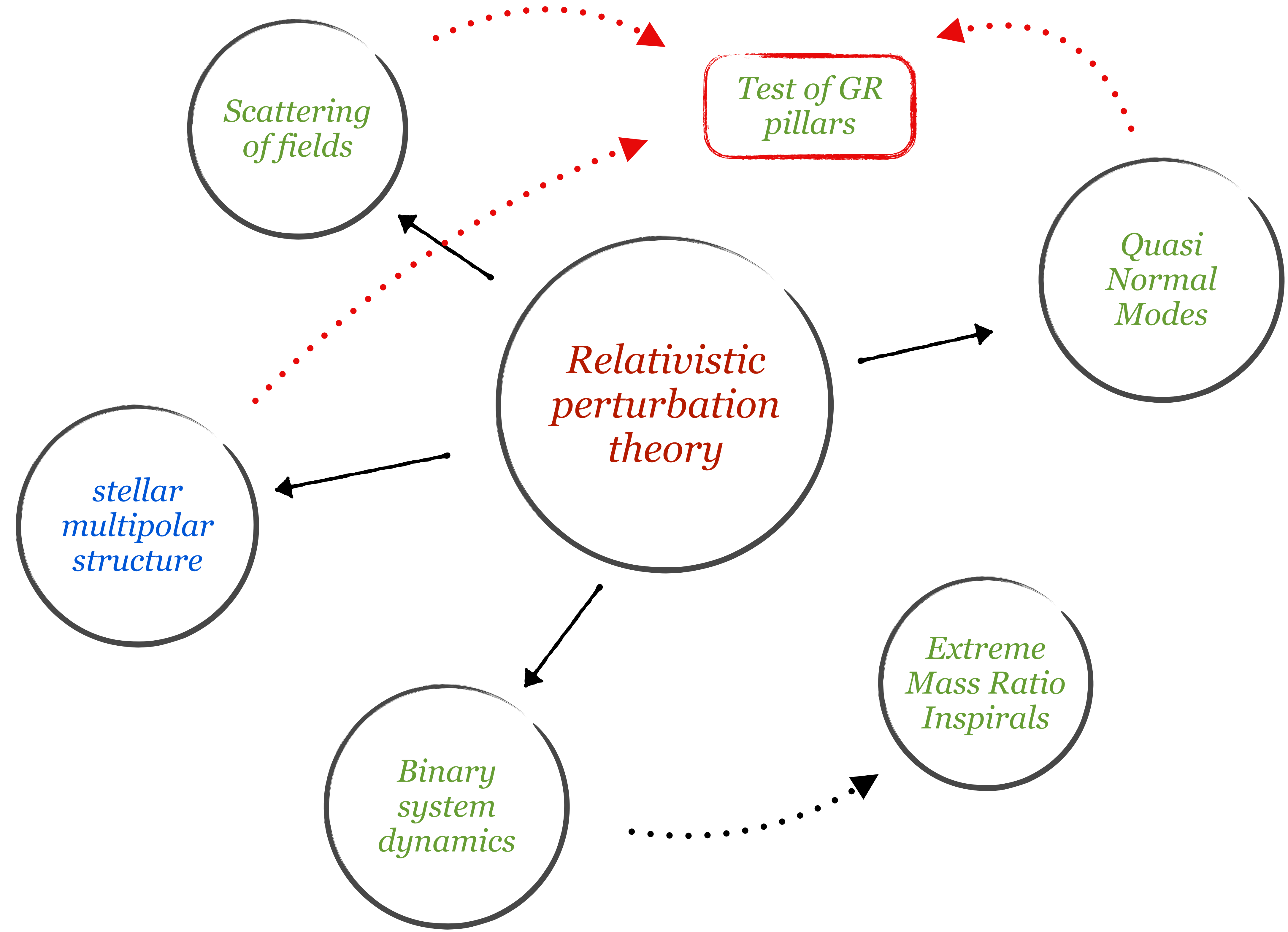
**Carter**

**Hawking**

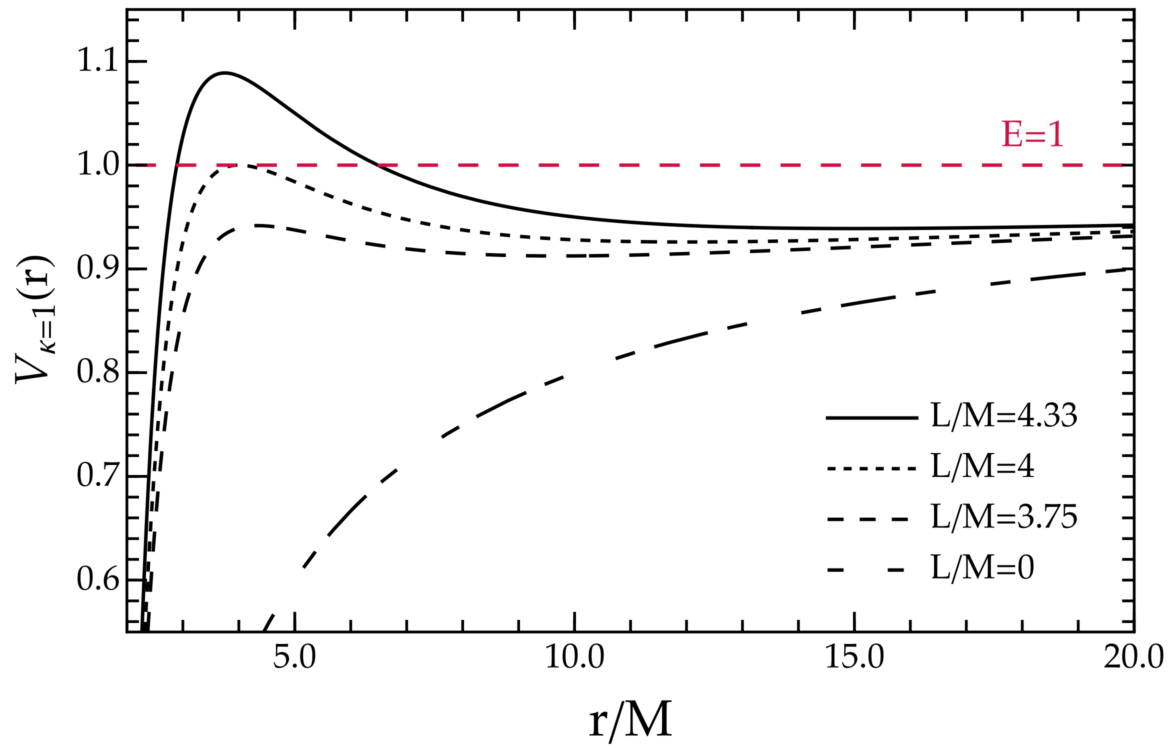
**Thorne**

*Many more…*

# BH perturbations

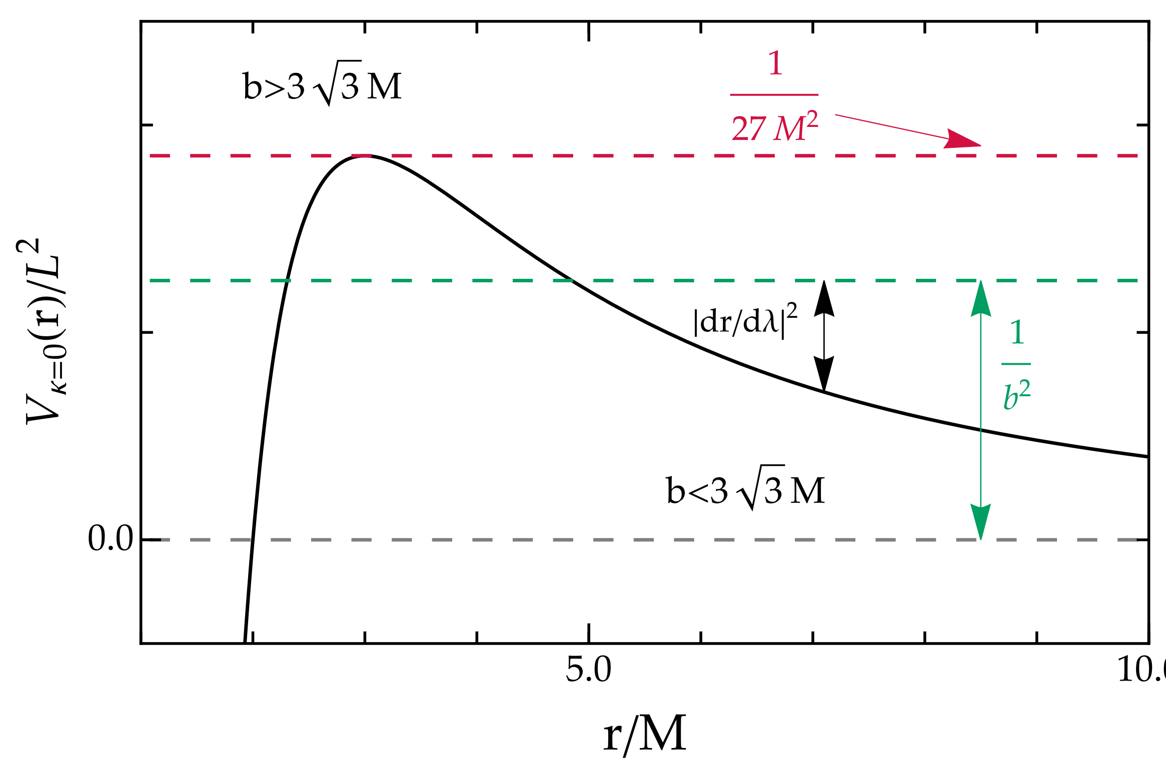


# Effective potentials




*massive particles*

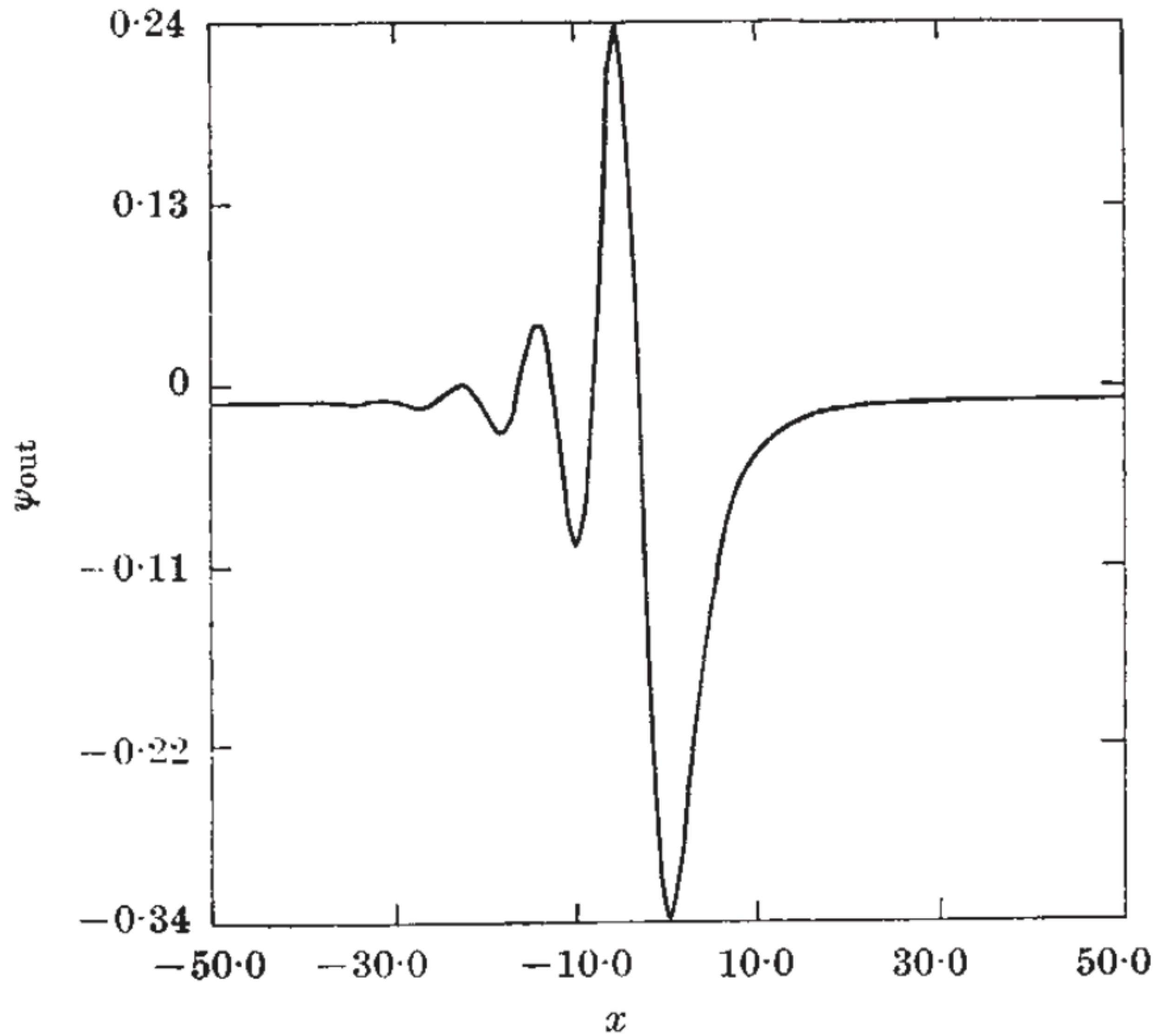
*massless particles*



# QNM universal behaviour

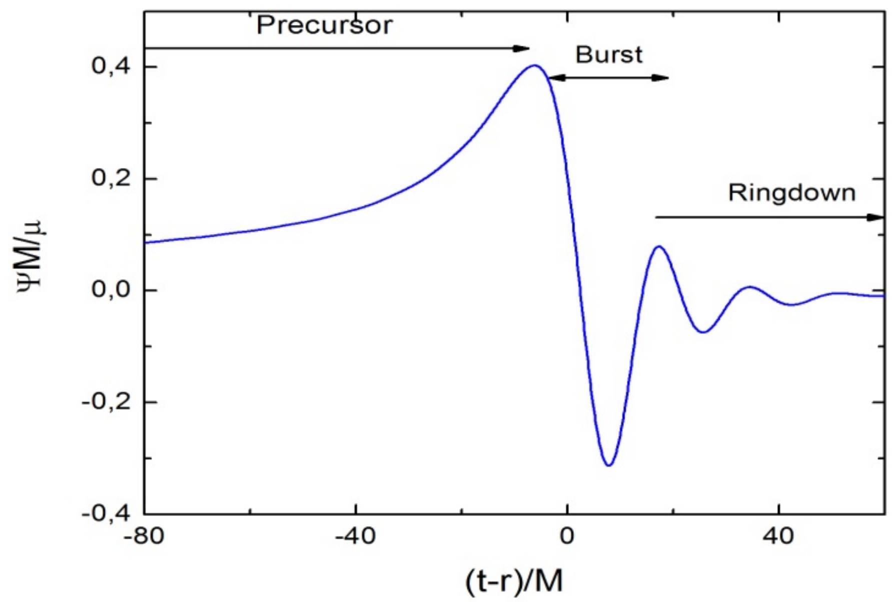
*Scattering experiments which show always the same behaviour*

*C. V. Vishveshwara, Nature 227 (1970)*

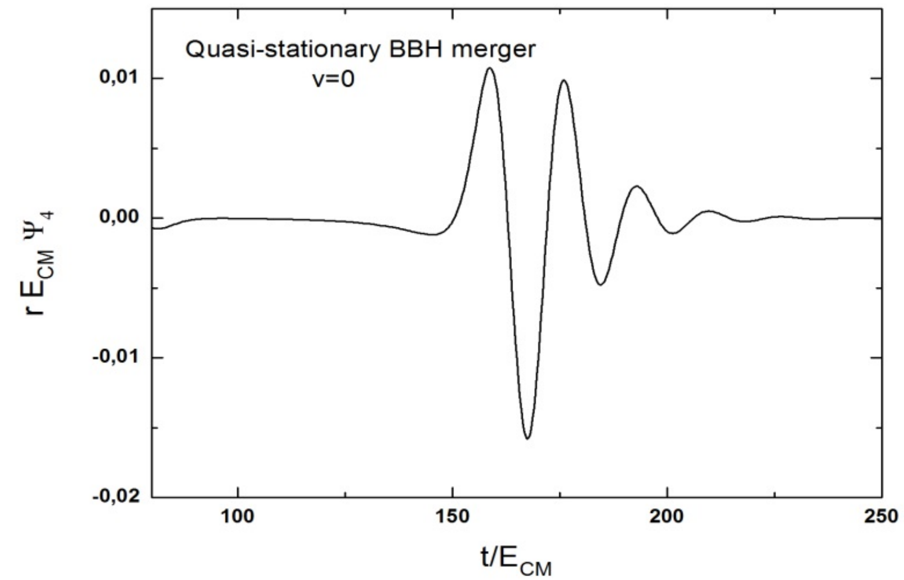



- *Scattering of initial gaussian packages on Schwarzschild BH*

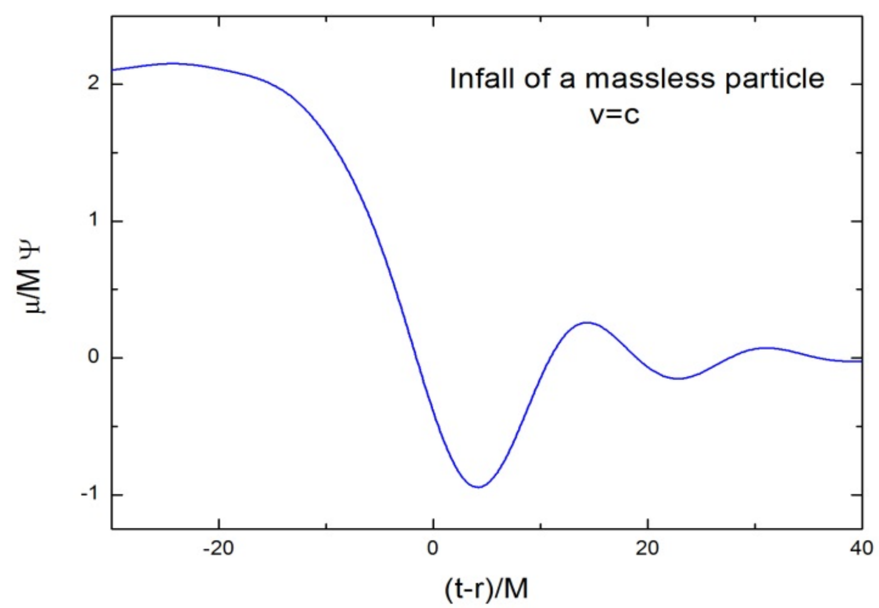
# *QNM universal behaviour*



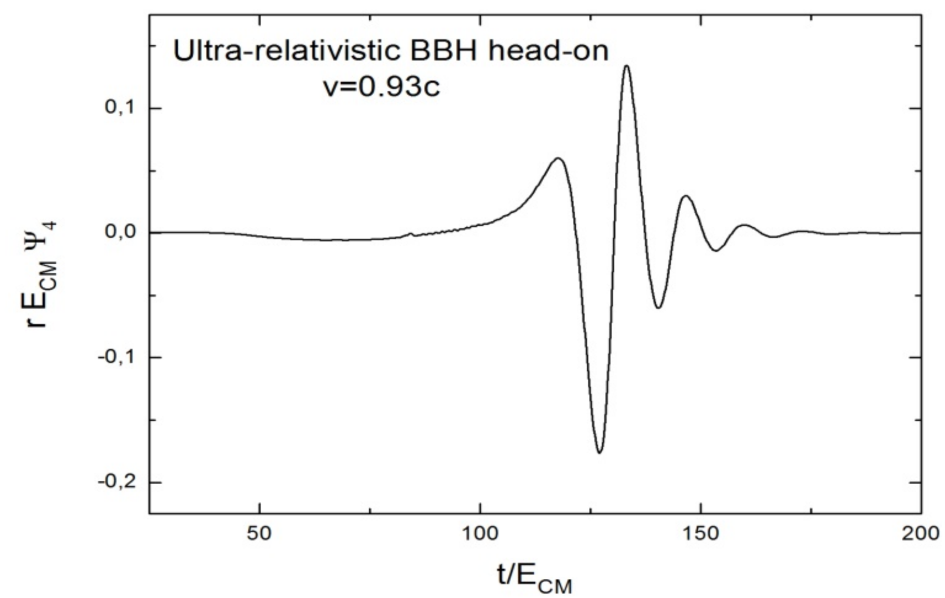

*M. Davis +, Phys. Rev. Lett. 27 (1971)*




*P. Anninos +, Phys. Rev. Lett. 71 (1993)*



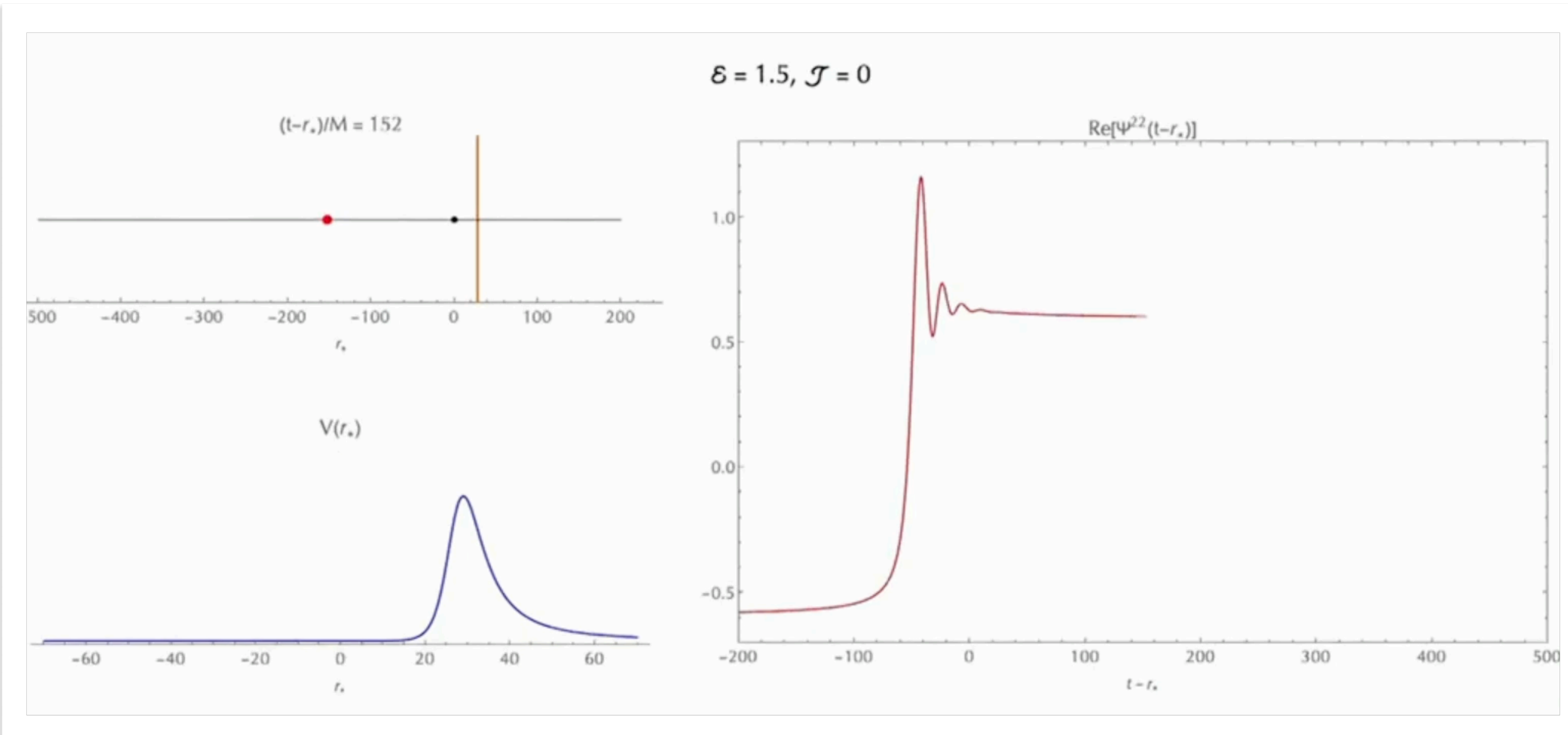

*V. Cardoso & J. Lemos, Phys. Lett. B 538 (2002)*




*U. Sperhake +, Phys. Rev. Lett. 101, 161101 (2008)*

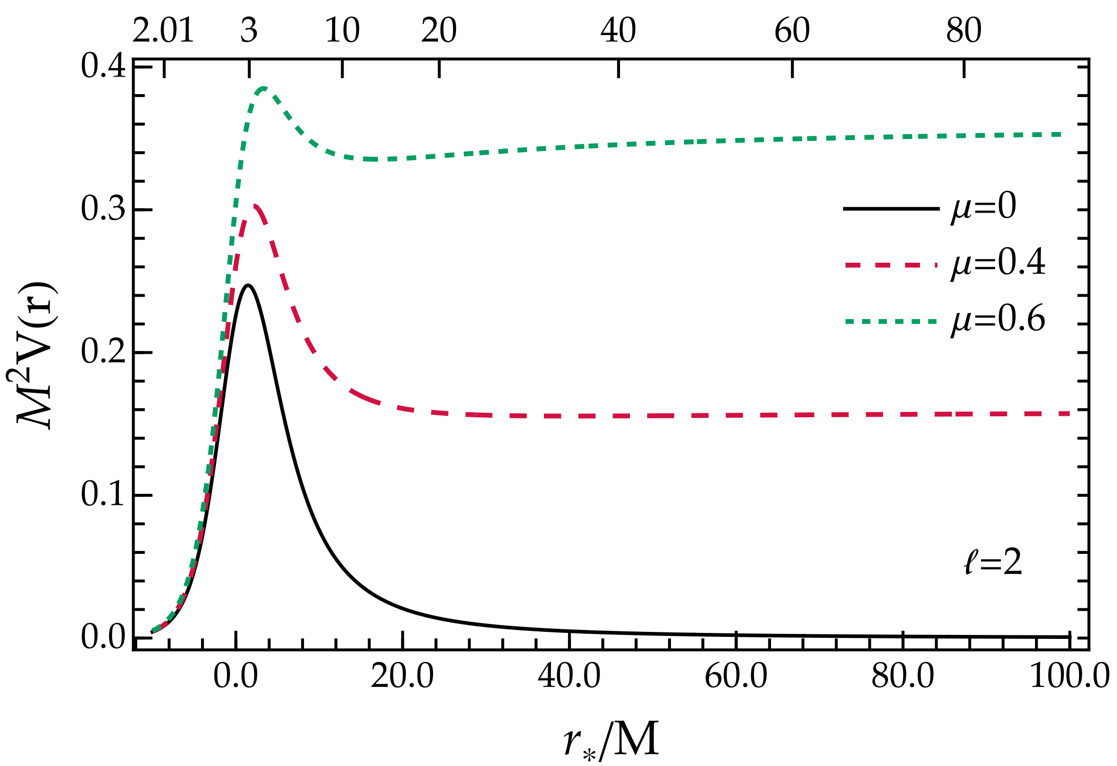
$$e^{-0.0898\, t/M_{BH}} \sin\left(0.374\, t/M_{BH}\right)$$

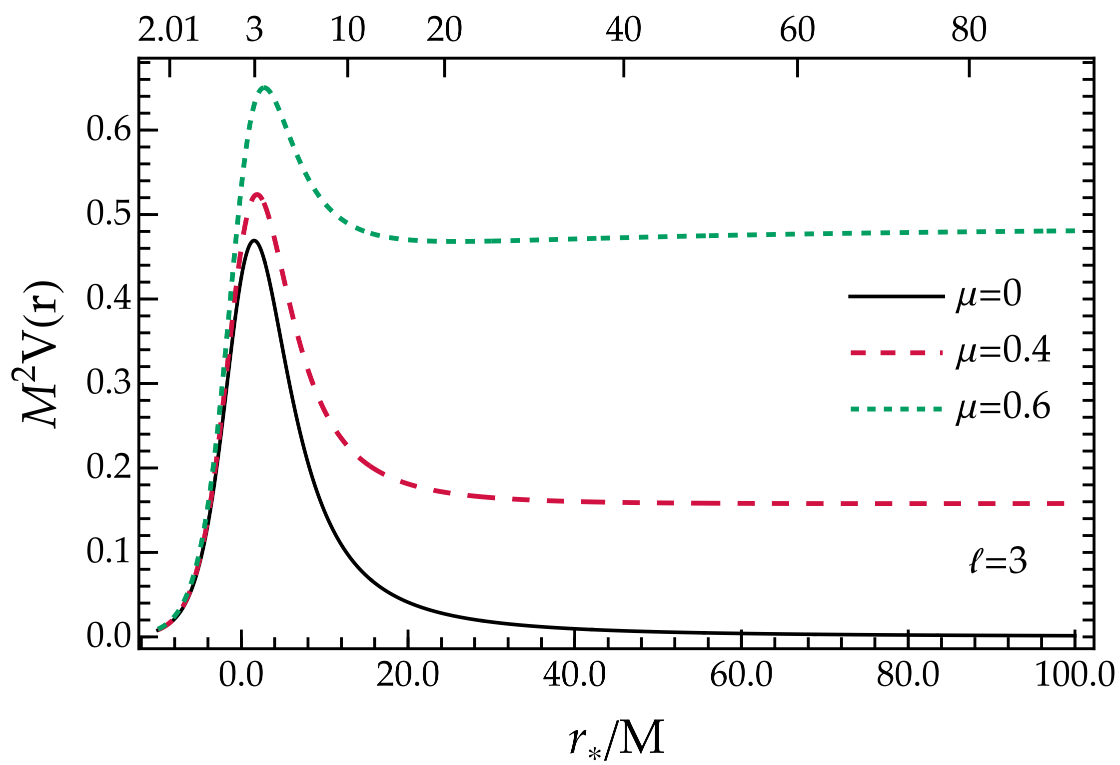
# *QNMs and the light ring*



# Scattering potentials

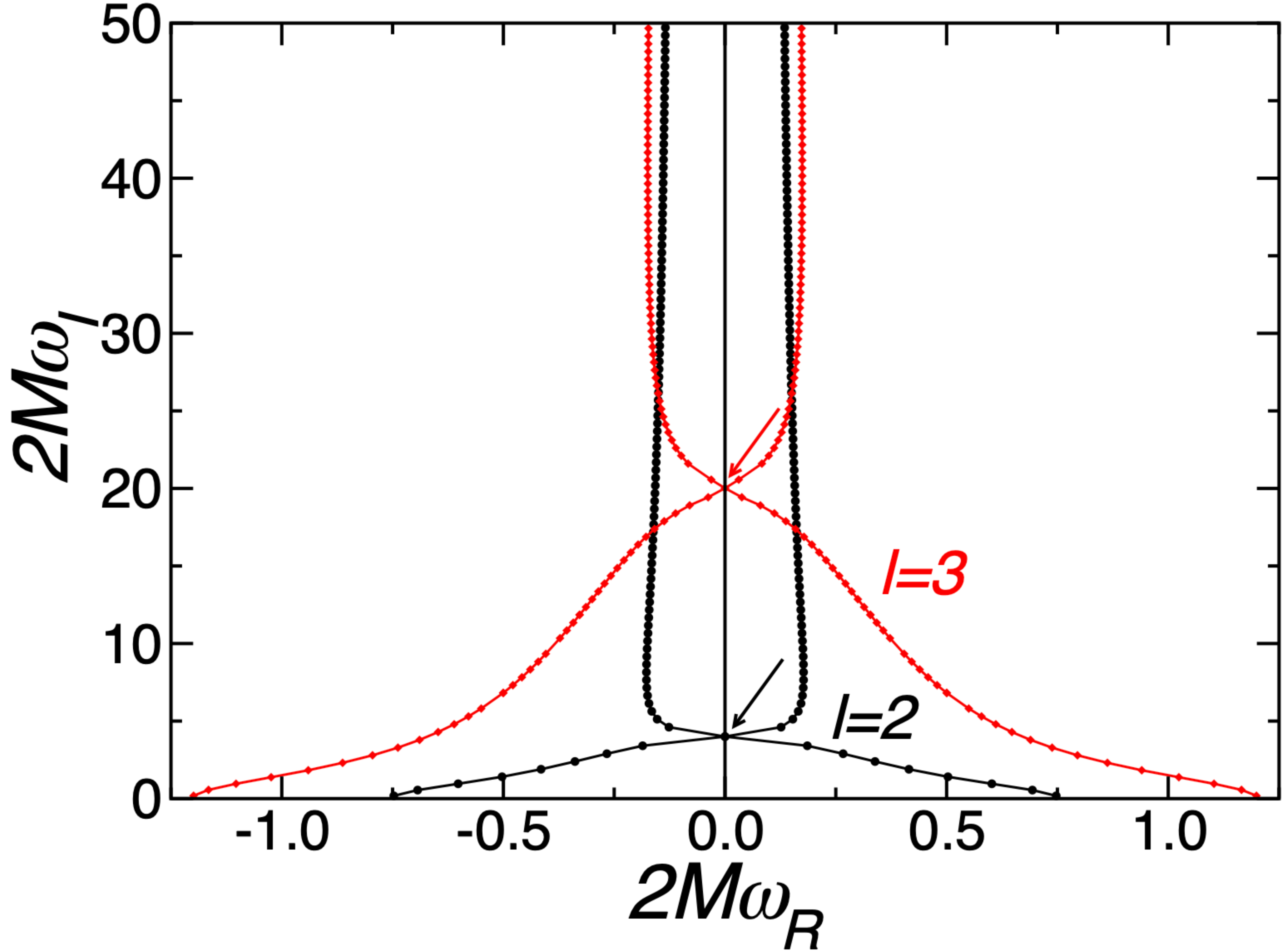
*Scattering potentials for scalar field perturbations*



# *Schwarzschild frequencies*

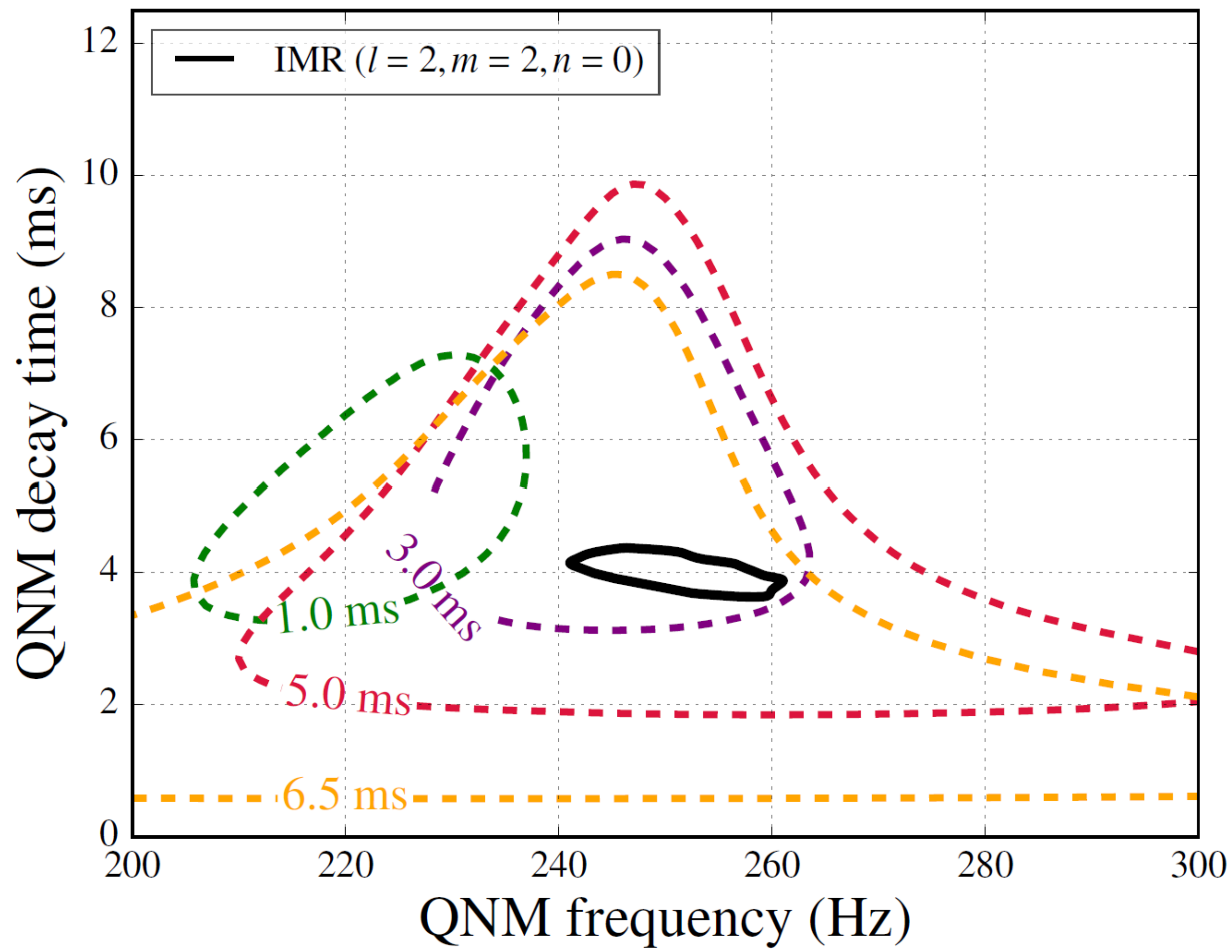
*Credit Emanuele Berti*



# The start of BH spectroscopy

*90% posterior distributions on the QNM frequencies from GW150914*

*LIGO/Virgo, Phys. Rev. Lett. 116, 221101 (2016)*




- *Black solid is 90% posterior of QNM as derived from the posterior mass and spin of remnant*

# The start of BH spectroscopy

## *Frequencies and damping times for more events*

*LIGO/Virgo, Phys. Rev. D 103, 122002 (2021)*

| Event | Redshifted frequency [Hz] | | | Redshifted damping time [ms] | | |
|---|---|---|---|---|---|---|
| | IMR | DS | pSEOB | IMR | DS | pSEOB |
| GW150914 | $248^{+8}_{-7}$ | $247^{+14}_{-16}$ | – | $4.2^{+0.3}_{-0.2}$ | $4.8^{+3.7}_{-1.9}$ | – |
| GW170104 | $287^{+15}_{-25}$ | $228^{+71}_{-102}$ | – | $3.5^{+0.4}_{-0.3}$ | $3.6^{+36.2}_{-2.1}$ | – |
| GW170814 | $293^{+11}_{-14}$ | $527^{+340}_{-332}$ | – | $3.7^{+0.3}_{-0.2}$ | $25.1^{+22.2}_{-19.0}$ | – |
| GW170823 | $197^{+17}_{-17}$ | $222^{+664}_{-62}$ | – | $5.5^{+1.0}_{-0.8}$ | $13.4^{+31.8}_{-9.8}$ | – |
| GW190408_181802 | $319^{+11}_{-20}$ | $504^{+479}_{-459}$ | – | $3.2^{+0.3}_{-0.3}$ | $10.0^{+32.5}_{-8.9}$ | – |
| GW190421_213856 | $162^{+14}_{-14}$ | – | $171^{+50}_{-16}$ | $6.3^{+1.2}_{-0.8}$ | – | $8.5^{+5.3}_{-4.2}$ |
| GW190503_185404 | $191^{+17}_{-15}$ | – | $265^{+501}_{-79}$ | $5.3^{+0.8}_{-0.8}$ | – | $3.5^{+3.4}_{-1.8}$ |
| GW190512_180714 | $381^{+33}_{-42}$ | $220^{+686}_{-42}$ | – | $2.6^{+0.2}_{-0.2}$ | $26.1^{+21.3}_{-22.9}$ | – |
| GW190513_205428 | $241^{+26}_{-28}$ | $250^{+493}_{-88}$ | – | $4.3^{+1.1}_{-0.4}$ | $5.3^{+19.2}_{-3.8}$ | – |
| GW190519_153544 | $127^{+9}_{-9}$ | $123^{+11}_{-19}$ | $124^{+12}_{-13}$ | $9.5^{+1.7}_{-1.5}$ | $9.7^{+9.0}_{-3.8}$ | $10.3^{+3.6}_{-3.1}$ |
| GW190521 | $68^{+4}_{-4}$ | $65^{+3}_{-3}$ | $67^{+2}_{-2}$ | $15.8^{+3.9}_{-2.5}$ | $22.1^{+12.4}_{-7.4}$ | $30.7^{+7.7}_{-7.4}$ |
| GW190521_074359 | $198^{+7}_{-7}$ | $197^{+15}_{-15}$ | $205^{+15}_{-12}$ | $5.4^{+0.4}_{-0.4}$ | $7.7^{+6.4}_{-3.3}$ | $5.3^{+1.5}_{-1.2}$ |
| GW190602_175927 | $105^{+10}_{-9}$ | $93^{+13}_{-22}$ | $99^{+15}_{-15}$ | $10.0^{+2.0}_{-1.4}$ | $10.0^{+17.2}_{-4.5}$ | $8.8^{+5.4}_{-3.6}$ |
| GW190706_222641 | $108^{+11}_{-10}$ | $109^{+7}_{-12}$ | $112^{+7}_{-8}$ | $10.9^{+2.4}_{-2.2}$ | $20.4^{+25.2}_{-12.9}$ | $19.4^{+7.2}_{-8.9}$ |
| GW190708_232457 | $497^{+10}_{-46}$ | $642^{+279}_{-596}$ | – | $2.1^{+0.2}_{-0.1}$ | $24.6^{+23.0}_{-22.6}$ | – |
| GW190727_060333 | $178^{+18}_{-16}$ | $345^{+587}_{-267}$ | $201^{+11}_{-21}$ | $6.1^{+1.1}_{-0.8}$ | $21.1^{+25.6}_{-17.9}$ | $15.4^{+5.3}_{-6.1}$ |
| GW190828_063405 | $239^{+10}_{-11}$ | $247^{+350}_{-15}$ | – | $4.8^{+0.6}_{-0.5}$ | $17.3^{+25.3}_{-10.4}$ | – |
| GW190910_112807 | $177^{+8}_{-8}$ | $166^{+9}_{-8}$ | $174^{+12}_{-8}$ | $5.9^{+0.8}_{-0.5}$ | $13.2^{+17.1}_{-6.2}$ | $9.5^{+3.1}_{-2.7}$ |
| GW190915_235702 | $232^{+14}_{-18}$ | $534^{+371}_{-493}$ | – | $4.6^{+0.8}_{-0.6}$ | $15.0^{+30.1}_{-13.1}$ | – |

# *The start of BH spectroscopy*

## *New tests are available once QNM spectroscopy is in full power*

*LIGO/Virgo, Phys. Rev. D 103, 122002 (2021)*

- *Masses and spins of the final object produced by the merger*

| Event | Redshifted final mass $(1+z)M_f$ [$M_\odot$] | | | | Final spin $\chi_f$ | | | |
|---|---|---|---|---|---|---|---|---|
| | IMR | $\text{Kerr}_{220}$ | $\text{Kerr}_{221}$ | $\text{Kerr}_{\text{HM}}$ | IMR | $\text{Kerr}_{220}$ | $\text{Kerr}_{221}$ | $\text{Kerr}_{\text{HM}}$ |
| GW150914 | $68.8^{+3.6}_{-3.1}$ | $62.7^{+19.0}_{-12.1}$ | $71.7^{+13.2}_{-12.5}$ | $80.3^{+20.1}_{-21.7}$ | $0.69^{+0.05}_{-0.04}$ | $0.52^{+0.33}_{-0.44}$ | $0.69^{+0.18}_{-0.36}$ | $0.83^{+0.13}_{-0.45}$ |
| GW170104 | $58.5^{+4.6}_{-4.1}$ | $56.2^{+19.1}_{-11.6}$ | $61.3^{+16.7}_{-13.2}$ | $104.3^{+207.7}_{-43.1}$ | $0.66^{+0.08}_{-0.11}$ | $0.26^{+0.42}_{-0.24}$ | $0.51^{+0.34}_{-0.44}$ | $0.59^{+0.34}_{-0.51}$ |
| GW170814 | $59.7^{+3.0}_{-2.3}$ | $46.1^{+133.0}_{-33.6}$ | $56.6^{+20.9}_{-11.1}$ | $171.2^{+268.7}_{-143.5}$ | $0.72^{+0.07}_{-0.05}$ | $0.52^{+0.42}_{-0.47}$ | $0.47^{+0.40}_{-0.42}$ | $0.54^{+0.41}_{-0.48}$ |
| GW170823 | $88.8^{+11.2}_{-10.2}$ | $73.8^{+26.8}_{-23.7}$ | $79.0^{+21.3}_{-13.2}$ | $103.0^{+133.1}_{-46.7}$ | $0.72^{+0.09}_{-0.12}$ | $0.46^{+0.40}_{-0.41}$ | $0.36^{+0.38}_{-0.32}$ | $0.74^{+0.22}_{-0.61}$ |
| GW190408_181802 | $53.0^{+3.2}_{-3.4}$ | $22.4^{+253.0}_{-11.1}$ | $46.6^{+18.8}_{-10.9}$ | $127.4^{+327.7}_{-107.6}$ | $0.67^{+0.06}_{-0.07}$ | $0.45^{+0.45}_{-0.40}$ | $0.36^{+0.46}_{-0.33}$ | $0.46^{+0.47}_{-0.41}$ |
| GW190512_180714 | $43.5^{+4.0}_{-2.8}$ | $37.6^{+48.9}_{-22.4}$ | $36.7^{+19.3}_{-24.8}$ | $99.4^{+247.6}_{-66.5}$ | $0.65^{+0.07}_{-0.07}$ | $0.41^{+0.47}_{-0.37}$ | $0.45^{+0.40}_{-0.39}$ | $0.77^{+0.20}_{-0.66}$ |
| GW190513_205428 | $70.6^{+11.5}_{-6.7}$ | $55.5^{+31.5}_{-42.1}$ | $68.5^{+28.2}_{-11.8}$ | $88.7^{+250.0}_{-41.9}$ | $0.68^{+0.14}_{-0.12}$ | $0.38^{+0.48}_{-0.34}$ | $0.31^{+0.53}_{-0.28}$ | $0.59^{+0.34}_{-0.52}$ |
| GW190519_153544 | $146.8^{+14.7}_{-15.4}$ | $120.7^{+39.7}_{-21.5}$ | $125.9^{+24.3}_{-21.7}$ | $155.4^{+84.4}_{-42.5}$ | $0.79^{+0.07}_{-0.13}$ | $0.42^{+0.41}_{-0.36}$ | $0.52^{+0.25}_{-0.40}$ | $0.70^{+0.21}_{-0.50}$ |
| GW190521 | $256.6^{+36.6}_{-30.4}$ | $282.2^{+50.0}_{-61.9}$ | $284.0^{+40.4}_{-43.9}$ | $299.3^{+57.7}_{-62.4}$ | $0.71^{+0.12}_{-0.16}$ | $0.76^{+0.14}_{-0.38}$ | $0.78^{+0.10}_{-0.22}$ | $0.80^{+0.13}_{-0.30}$ |
| GW190521_074359 | $88.0^{+4.3}_{-4.8}$ | $83.0^{+24.0}_{-17.2}$ | $86.4^{+14.1}_{-14.8}$ | $105.9^{+20.8}_{-26.4}$ | $0.72^{+0.05}_{-0.07}$ | $0.57^{+0.31}_{-0.49}$ | $0.67^{+0.17}_{-0.34}$ | $0.87^{+0.09}_{-0.39}$ |
| GW190602_175927 | $163.8^{+20.7}_{-18.3}$ | $156.4^{+71.4}_{-30.6}$ | $160.0^{+37.4}_{-31.2}$ | $261.7^{+84.4}_{-91.5}$ | $0.70^{+0.10}_{-0.14}$ | $0.34^{+0.41}_{-0.31}$ | $0.46^{+0.31}_{-0.39}$ | $0.79^{+0.14}_{-0.49}$ |
| GW190706_222641 | $171.1^{+20.0}_{-23.7}$ | $136.0^{+52.0}_{-29.3}$ | $152.5^{+37.8}_{-28.4}$ | $184.0^{+139.2}_{-55.8}$ | $0.78^{+0.09}_{-0.18}$ | $0.41^{+0.42}_{-0.37}$ | $0.55^{+0.31}_{-0.45}$ | $0.68^{+0.26}_{-0.54}$ |
| GW190708_232457 | $34.4^{+2.7}_{-0.7}$ | $28.9^{+285.4}_{-17.9}$ | $32.3^{+15.0}_{-12.2}$ | $171.9^{+307.6}_{-147.8}$ | $0.69^{+0.04}_{-0.04}$ | $0.47^{+0.45}_{-0.42}$ | $0.34^{+0.44}_{-0.31}$ | $0.43^{+0.51}_{-0.39}$ |
| GW190727_060333 | $99.2^{+10.7}_{-9.8}$ | $78.7^{+45.7}_{-66.4}$ | $88.8^{+25.7}_{-16.0}$ | $107.4^{+112.1}_{-42.7}$ | $0.73^{+0.10}_{-0.10}$ | $0.53^{+0.42}_{-0.47}$ | $0.45^{+0.39}_{-0.41}$ | $0.71^{+0.24}_{-0.59}$ |
| GW190828_063405 | $75.7^{+6.0}_{-5.2}$ | $71.2^{+35.8}_{-55.5}$ | $69.6^{+22.0}_{-17.3}$ | $99.0^{+166.0}_{-49.1}$ | $0.75^{+0.06}_{-0.07}$ | $0.72^{+0.25}_{-0.62}$ | $0.65^{+0.27}_{-0.55}$ | $0.92^{+0.06}_{-0.74}$ |
| GW190910_112807 | $97.0^{+9.3}_{-7.1}$ | $112.2^{+32.0}_{-31.7}$ | $107.7^{+28.6}_{-27.4}$ | $137.1^{+59.5}_{-31.4}$ | $0.70^{+0.08}_{-0.07}$ | $0.76^{+0.18}_{-0.55}$ | $0.75^{+0.17}_{-0.46}$ | $0.91^{+0.07}_{-0.27}$ |
| GW190915_235702 | $74.8^{+7.9}_{-7.4}$ | $38.3^{+335.1}_{-27.4}$ | $63.0^{+19.1}_{-9.9}$ | $137.3^{+324.1}_{-96.2}$ | $0.70^{+0.09}_{-0.11}$ | $0.52^{+0.43}_{-0.46}$ | $0.27^{+0.40}_{-0.24}$ | $0.55^{+0.39}_{-0.49}$ |

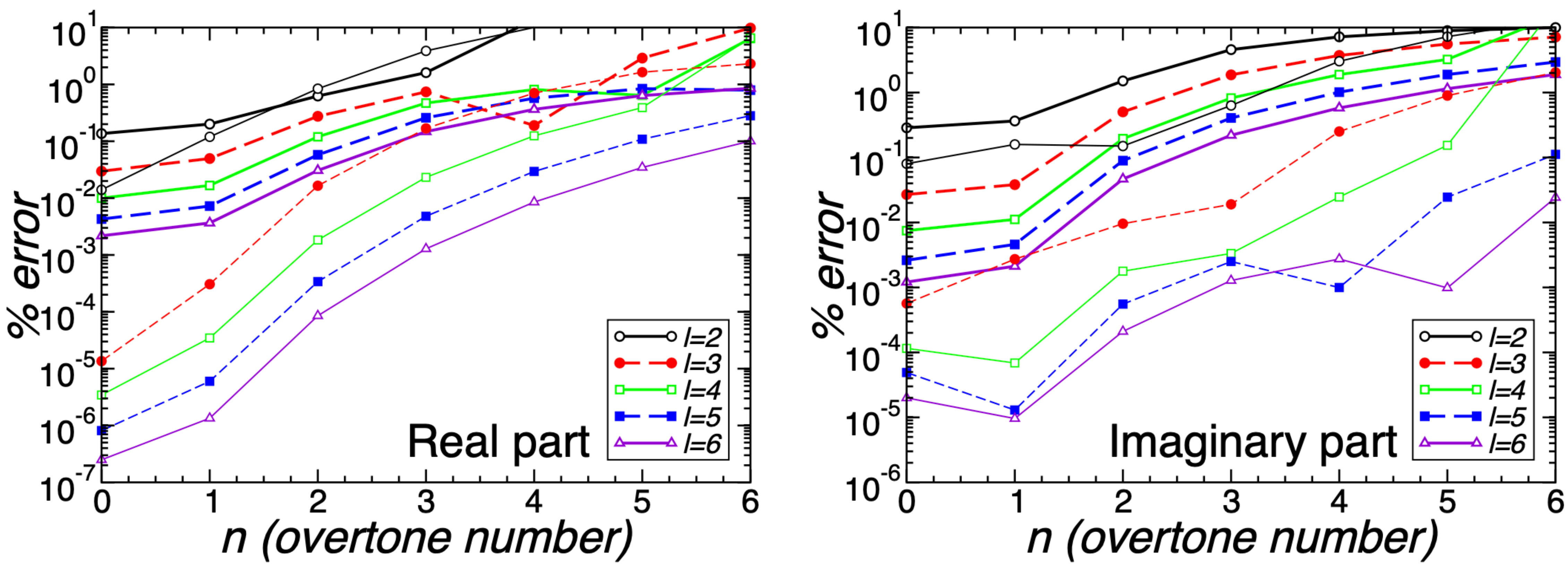
*as predicted from the inspiral*

*from the post-merger only*

# WKB vs "exact" values

*Relative error on the QNM frequencies of a Schwarzschild BH, computed through WKB and continued fraction*

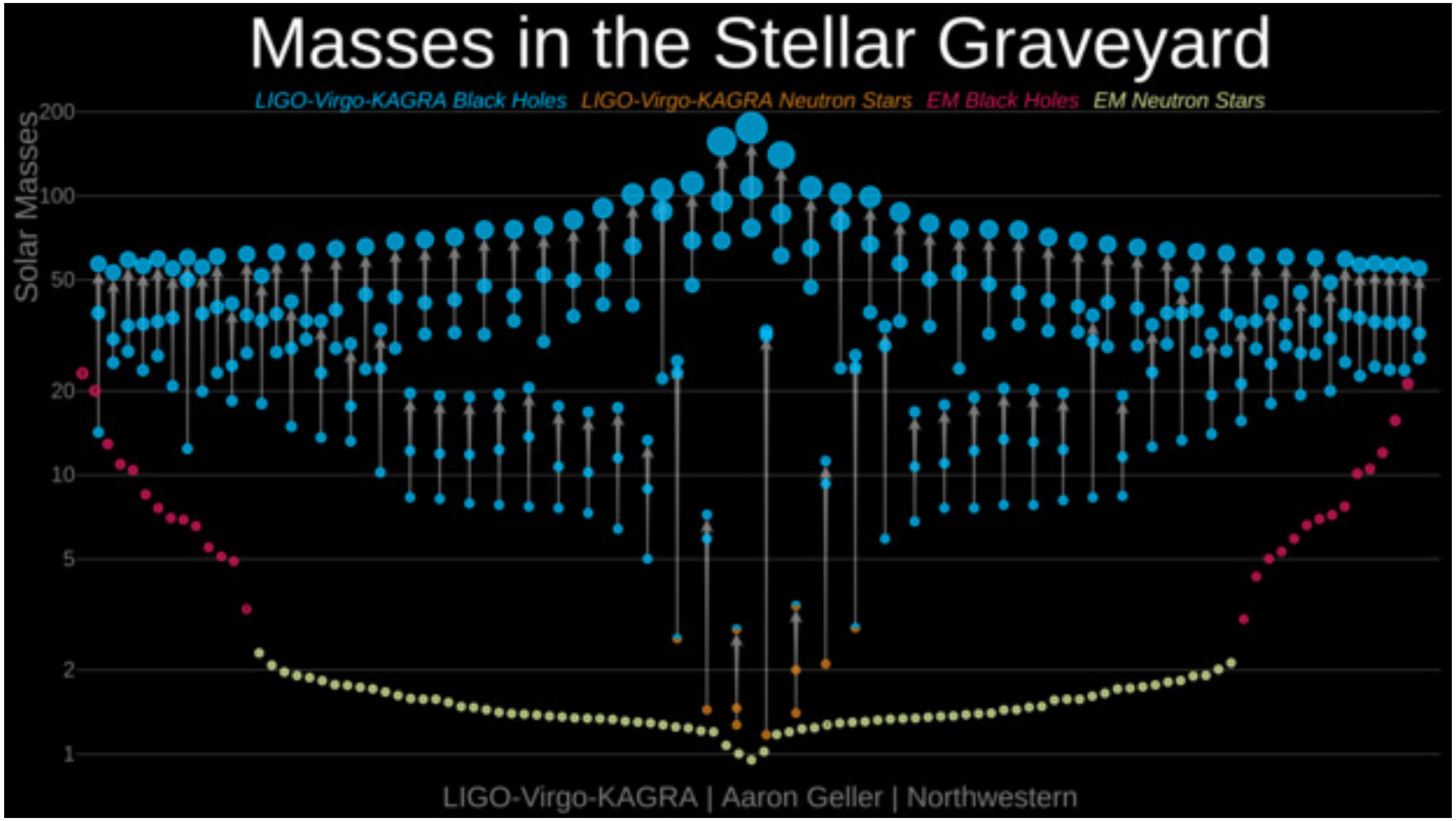
*E. Berti +, Class. Quant. Grav. 26, 163001 (2010)*



*Exotic Love*

# *Alone in the dark?*

*Are Kerr black hole alone in the Universe?*



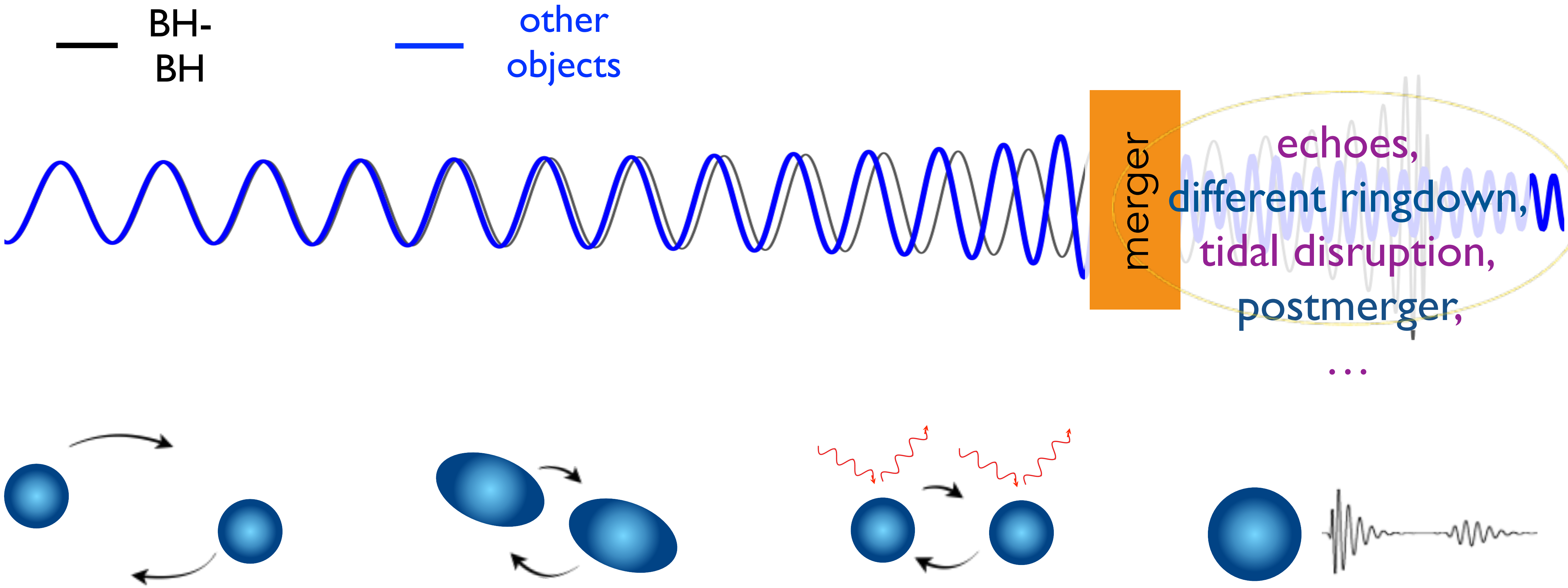
# *Alone in the dark?*

*Kerr BHs as endpoint of stellar evolution?*

- *Unexpected processes may avoid their formation*
- *Extended theories of gravity in which extra fields couple to the gravity sector can predict BHs different from Kerr solutions, with specific* ***hairs***
- *Other* ***E****xotic* ***C****ompact* ***O****bjects may be the output of stellar collapse, which form without an event horizon*
- *Can we distinguish ECOs with no horizon, compact enough to mimic a BH,* ***and/or*** *, BHs with different hairs?*

*GW from coalescing binaries may provide new answers*

# What do we look for?




*~point masses: same signal for all objects*

***tidal effects* + *spins deformations***

*absence of horizon* ***absorption effects***

***different ringdown or echoes***

# The Love number

*Tidal interactions leave the footprint of the NS structure on the GW signal*

*T. Hinderer, Astrophys. J. 677 (2008)*
*Damour & Nagar, Phys. Rev. D 80, 084035 (2009)*
*Binnington & Poisson, Phys. Rev. D 80, 084018 (2009)*

- *Deformation properties encoded within the Love numbers*

$$\mathcal{Q}_{ij} = \frac{2}{3} k_2 R^5 \mathcal{E}_{ij} = \lambda \mathcal{E}_{ij}$$

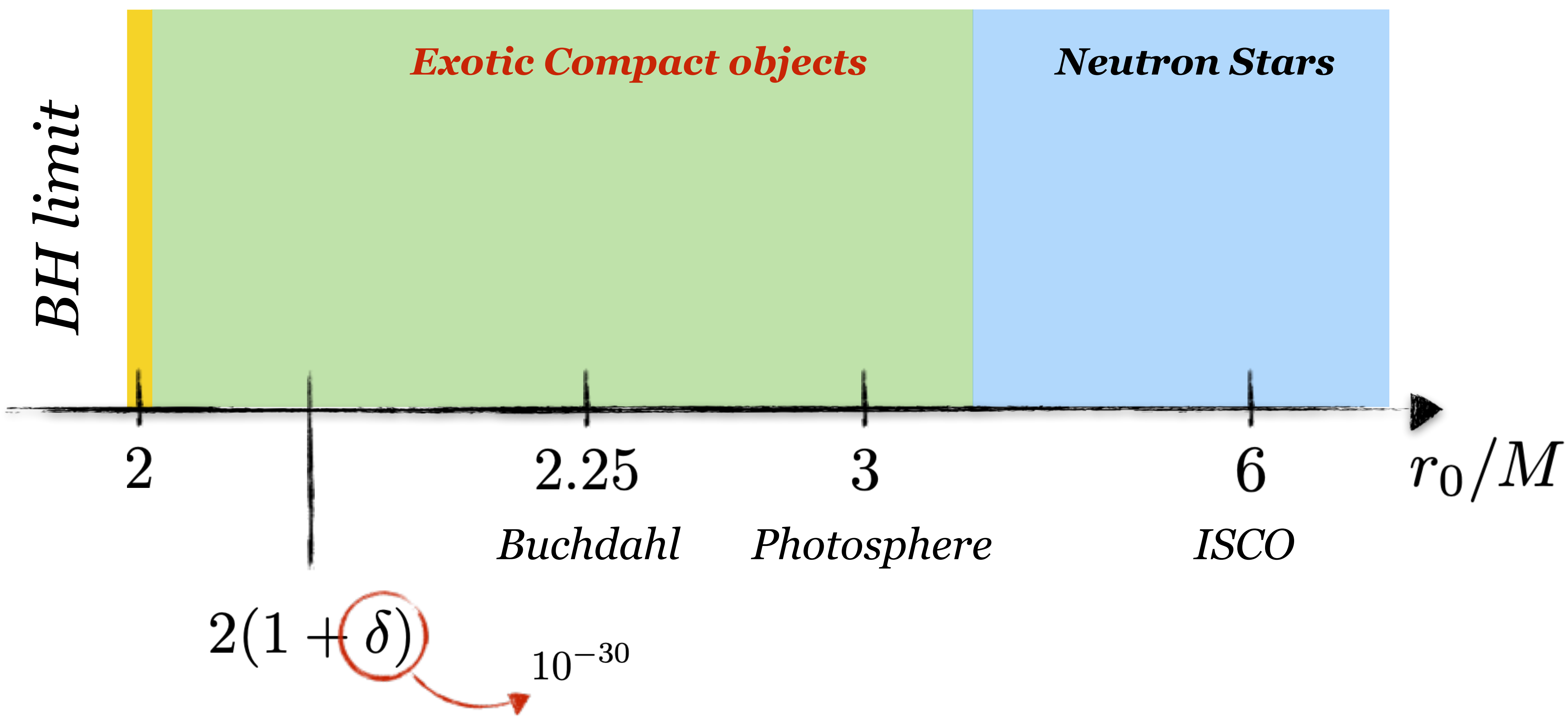
*star's quadrupole* ($\mathcal{Q}_{ij}$) — *external tidal field* ($\mathcal{E}_{ij}$)

- $\lambda$ *depends on the internal structure only, for a given compactness*
- $\lambda$ *enters within the gravitational waveform*

# ECOs vs BHs

*A first order classification*

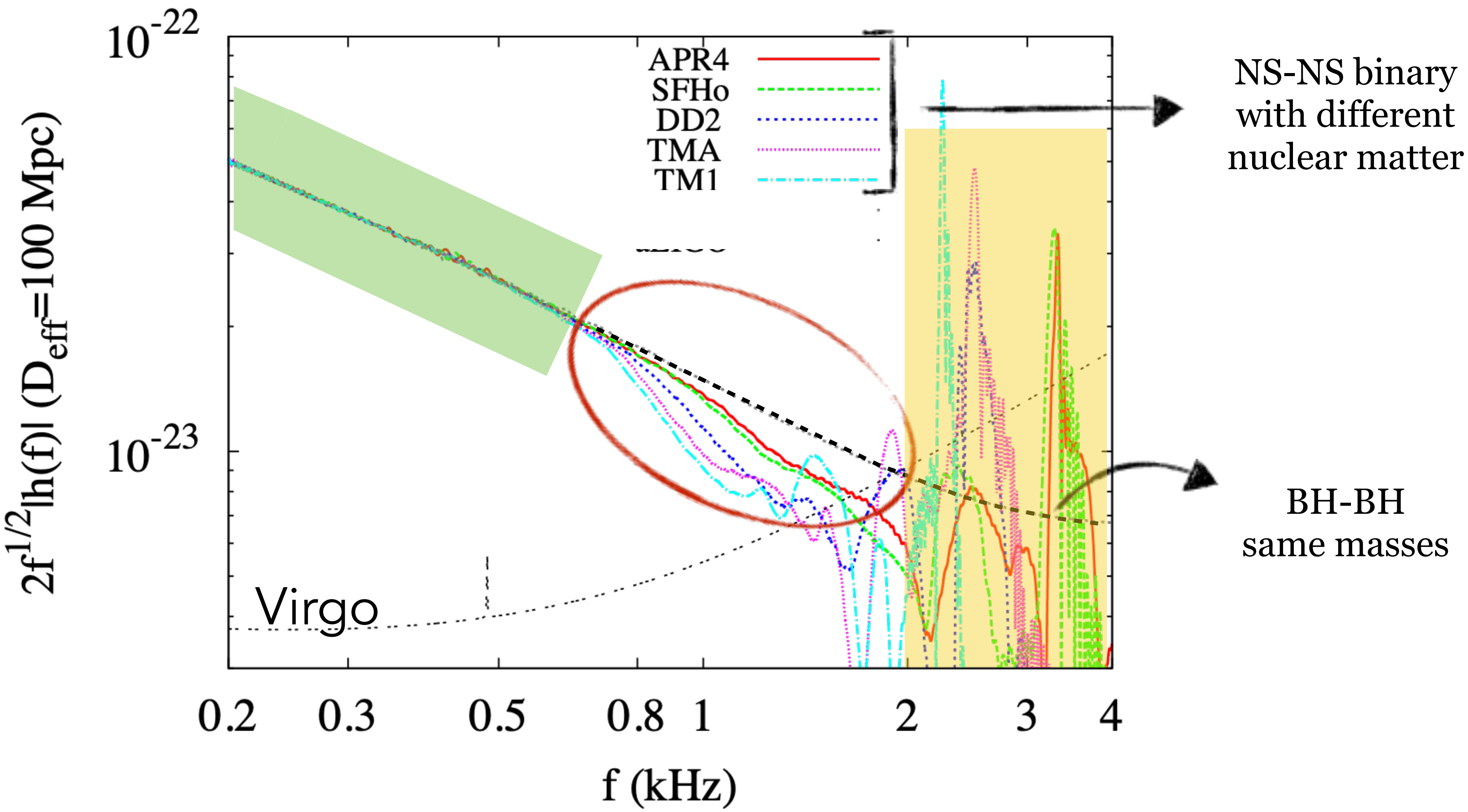
*Cardoso & Pani, Living Rev.Rel. 22, 4 (2019)*

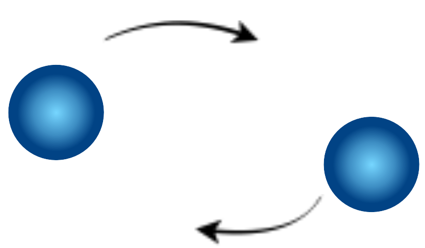



$$M_{\rm BH}/R_{\rm BH} = 0.5 \quad v.s. \quad M_{\rm ECO}/R_{\rm ECO} = 0.49(99\ldots)$$

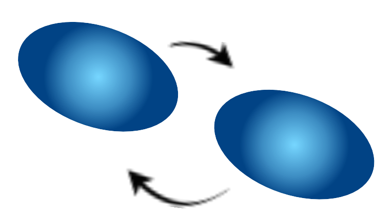
# *What do we look for?*

*K. Hotokezaka +, Phys. Rev. D 93, 064082 (2016)*

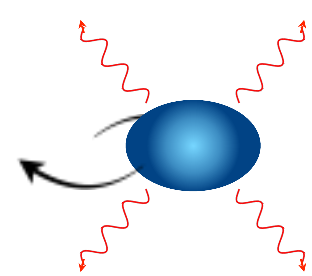


*point masses*



*tidal interactions*



*merger/post-merger*

# The recipe of Love

*Polar-electric-type perturbation of background metric*

*Regge & Wheeler, PRD 108, 1063 (1957)*
*Zerilli, PRD 2, 2141 (1970)*

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + h_{\mu\nu}$$

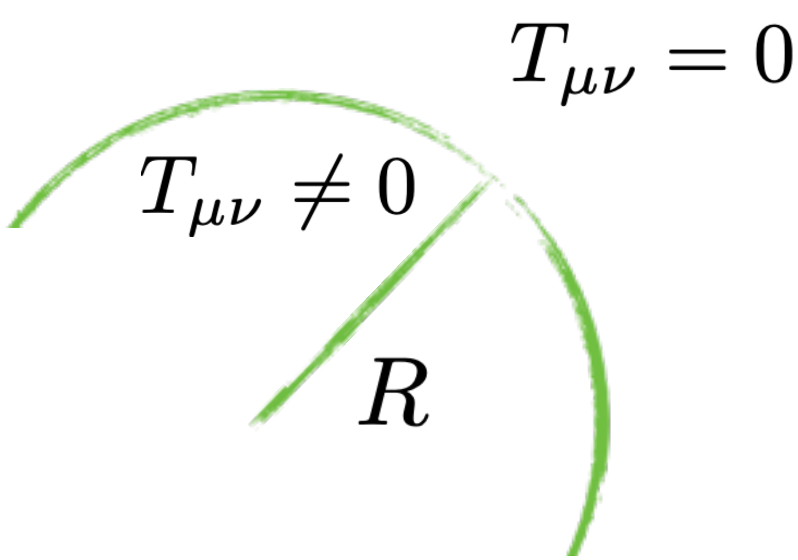
$$\begin{pmatrix} -e^{\nu(r)} & 0 & 0 & 0 \\ 0 & e^{\lambda(r)} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2\sin^2\theta \end{pmatrix} + \begin{pmatrix} -e^{\nu(r)}H_0(r) & 0 & 0 & 0 \\ 0 & e^{\lambda(r)}H_2(r) & 0 & 0 \\ 0 & 0 & r^2K(r) & 0 \\ 0 & 0 & 0 & r^2K(r)\sin^2\theta \end{pmatrix} Y_{lm}(\theta,\phi)$$

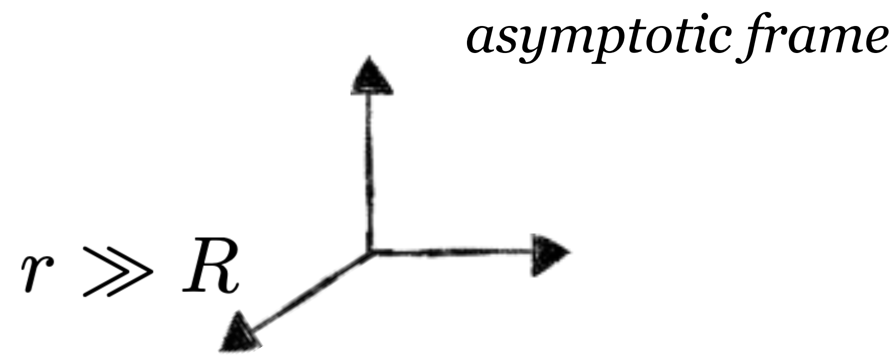
- *Solve at linear oder in the perturbations* $H_0, H_2, K$
- *Cook everything within Einstein's equations* $G_{\mu\nu} = kT_{\mu\nu}$

→ *set of sourced ODEs*

# *The recipe of Love*

- *Numerically integrate from center to outside with appropriate boundary conditions*




*read off*
*multipole moments*

$$Q_{ij} = \lambda \mathcal{E}_{ij}$$

- *Matching interior & exterior solution*

$$g_{\rm tt}^{\rm (int)} = g_{\rm tt}^{\rm (ext)} = -1 + \frac{2M}{r} + \frac{3Q_{ij}\hat{n}^i\hat{n}^j}{r^3} - \mathcal{E}_{ij}x^i x^j$$

# *ECO Love numbers*

## *Neutron stars*

$\mathcal{C} \in [0.1 \div 0.2]$

$\lambda \neq 0$

*GW170817*

*constrain the NS equation of state*

## *Kerr Black holes*

$\mathcal{C} = 1/2$

$\lambda = 0$

## *ECOs*

$\mathcal{C} < 0.5$ $\lambda \neq 0$

$\mathcal{C} \to 0.5$ $\lambda \to 0$

*signature to distinguish ECO and BH's inspiral*

*BH Beyond GR have non vanishing Love number too!*

*Landry & Poisson, Phys. Rev. D 91, 104018 (2015)*
*P. Pani +, Phys. Rev. D 92, 024010 (2017)*
*Le Tiec & Casals, Phys. Rev. Lett. 126, 131102 (2021)*

*V. Cardoso +, Phys. Rev. D 95, 084014 (2017)*
*A.M. +, Phys. Rev. Lett. 120, 081101 (2018)*
*A.M. +, Class. Quant. Grav. 36, 167001 (2010)*

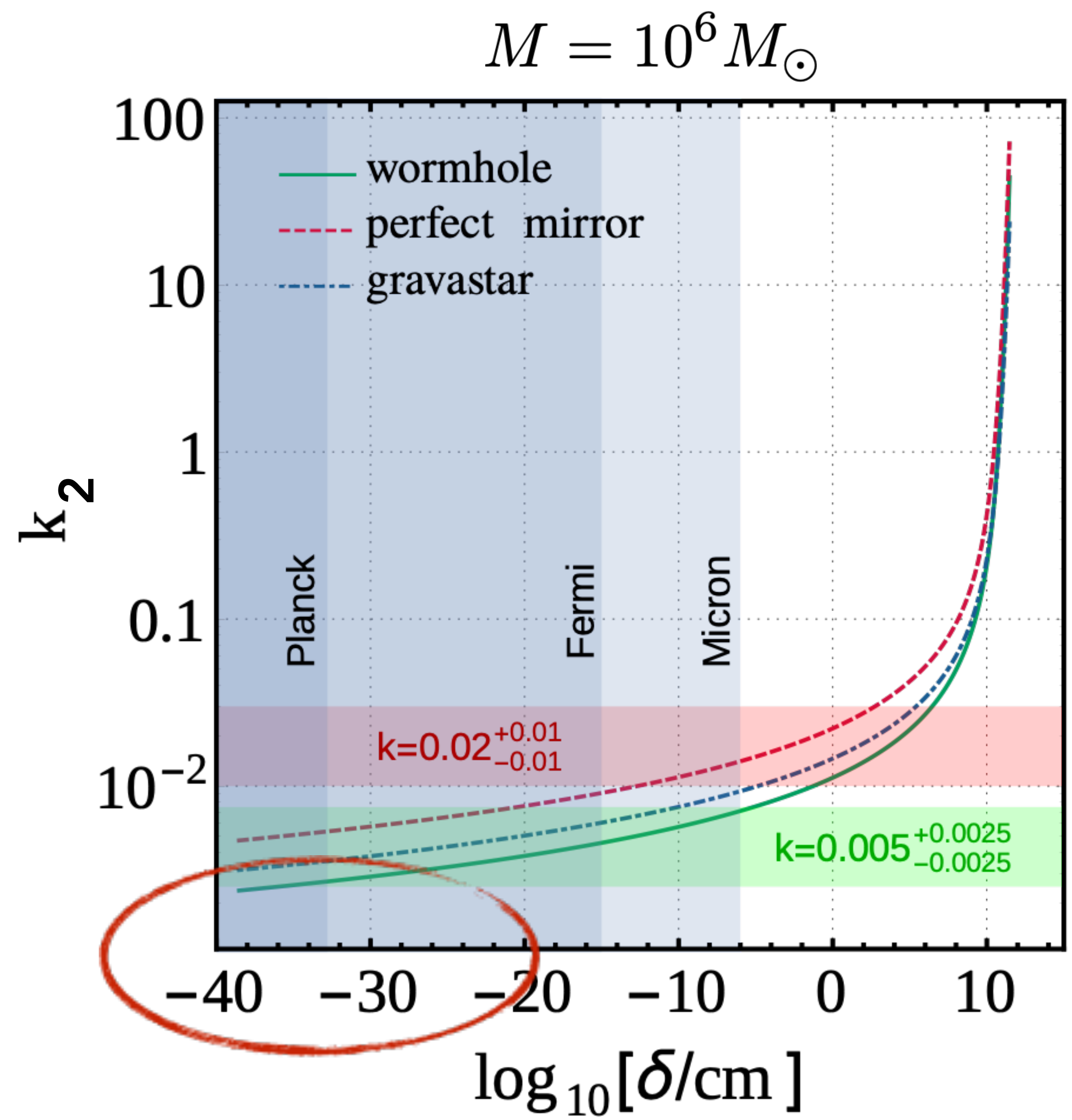
# ECO Love numbers

*The love number reflects the distance of the ECO surface from its Schwarzschild radius*

$$\delta \equiv r_0 - 2M \sim 2Me^{-1/k_2}$$

$$k_2 \simeq 0.005$$

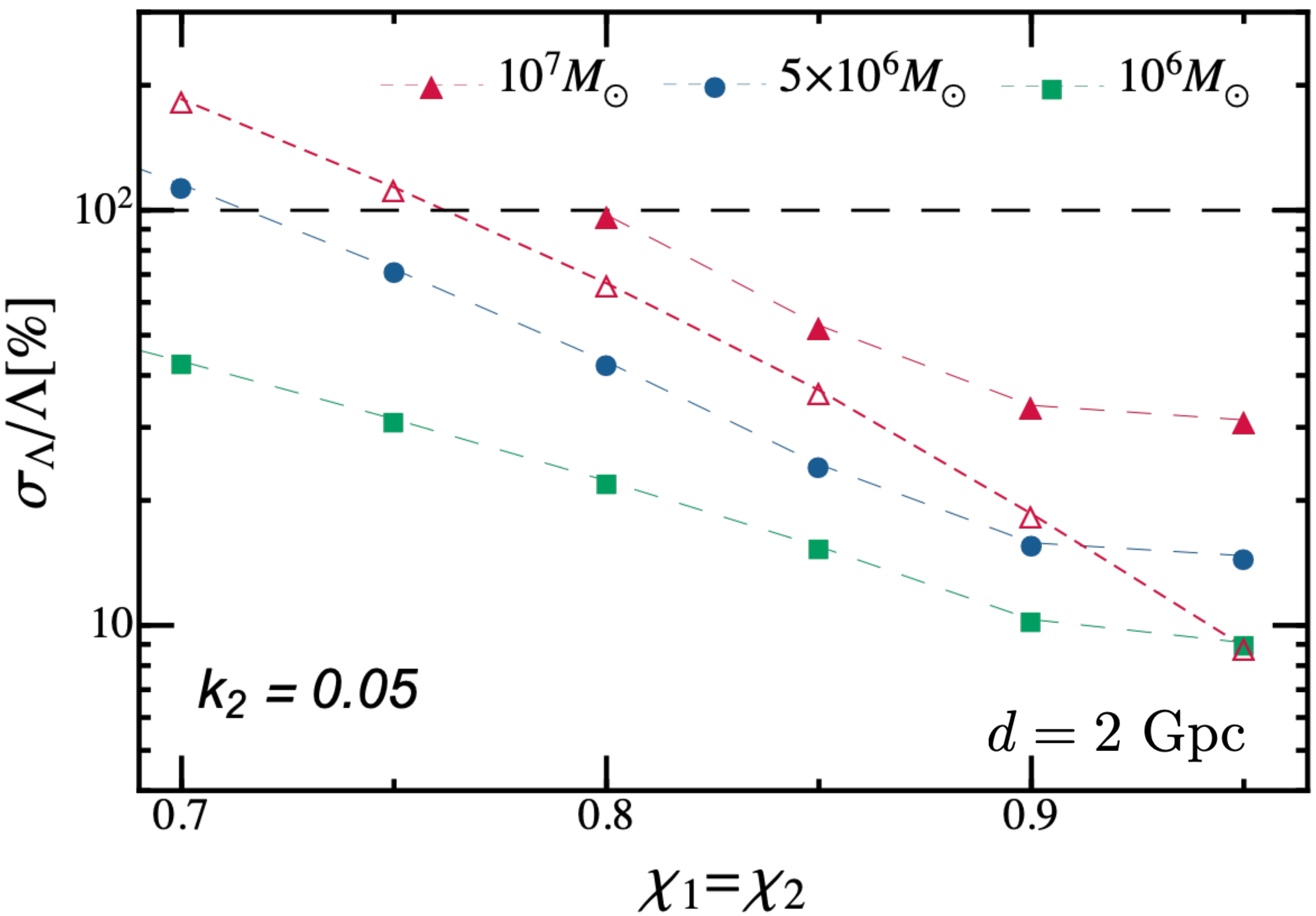
$$\delta \simeq 10^{-33}\text{cm} \sim \ell_P$$



# LISA & The uncertainty of Love

*Spinning binaries can test microphysical modifications at the* ***horizon*** *scale*

*A.M. +, Phys. Rev. Lett. 120, 081101 (2018)*




$\sigma_\Lambda/\Lambda \sim 1/k_2$

$k_2 = 0.005$ @ $d = 2$ Gpc

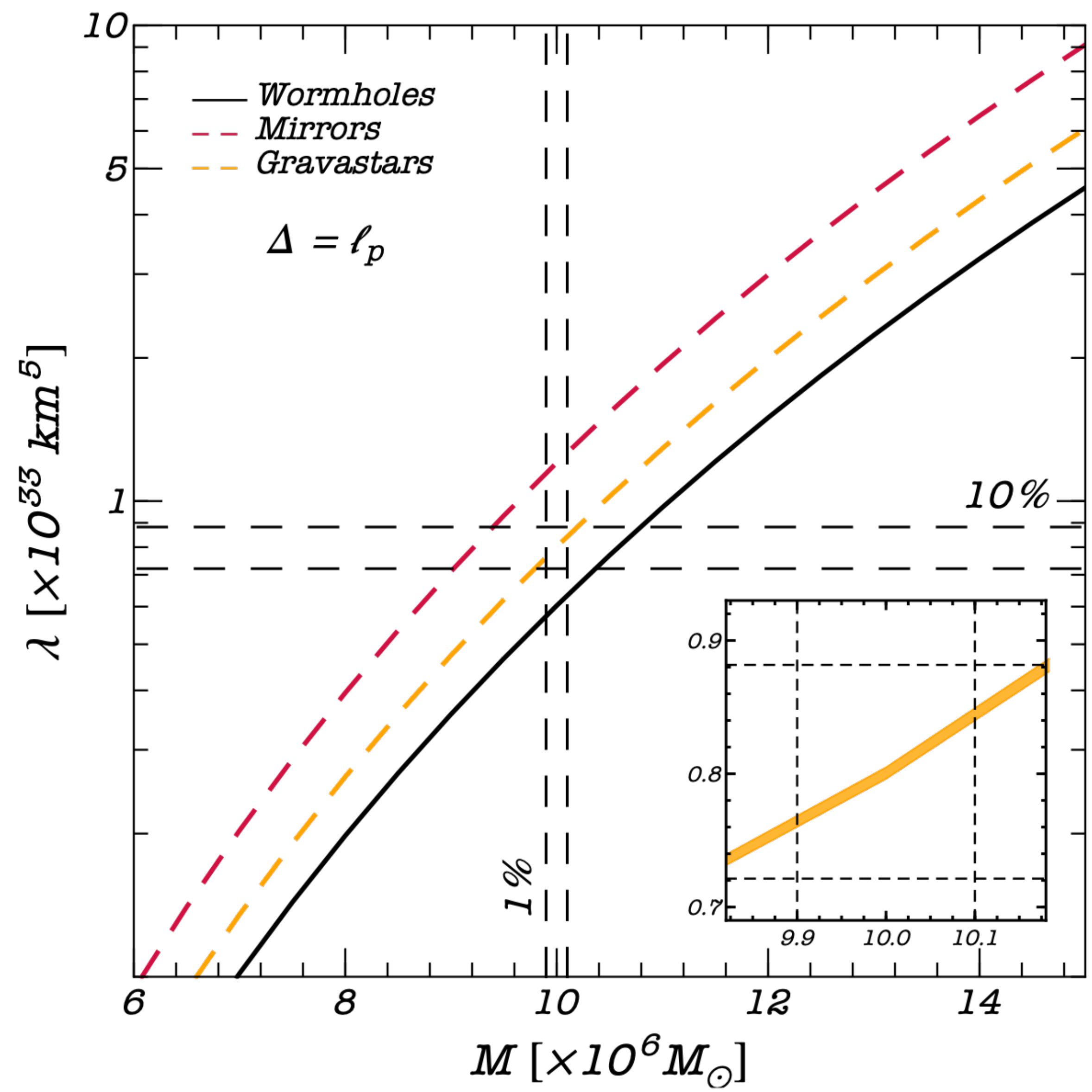
$\sigma_\Lambda/\Lambda \sim d$

$k_2 = 0.025$ @ $d = 10$ Gpc

# LISA & The uncertainty of Love

GWs can distinguish **between** models with quantum modifications

A.M. +, Class. Quant. Grav. 36, 167001 (2010)



# *Scalar fields*

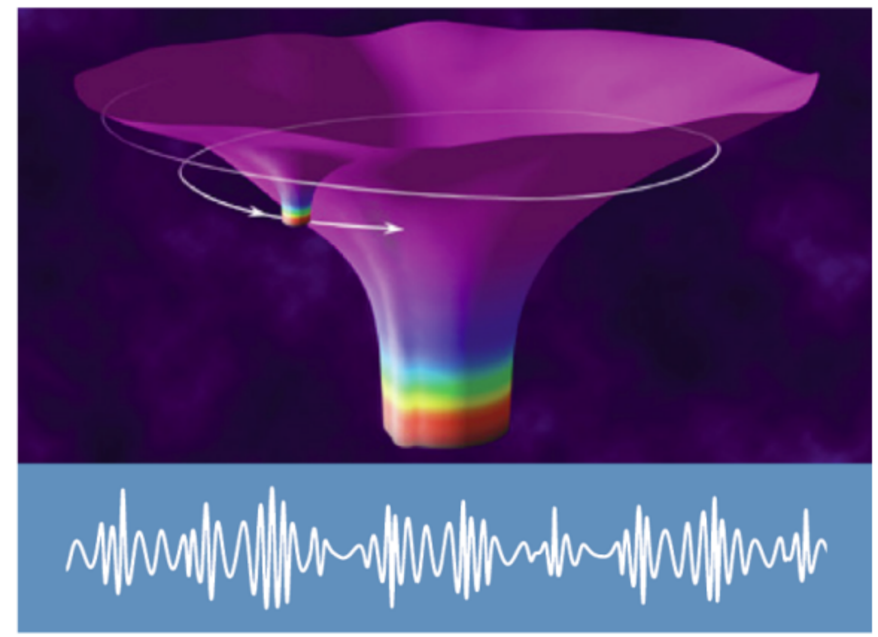
# *EMRI in nuce*

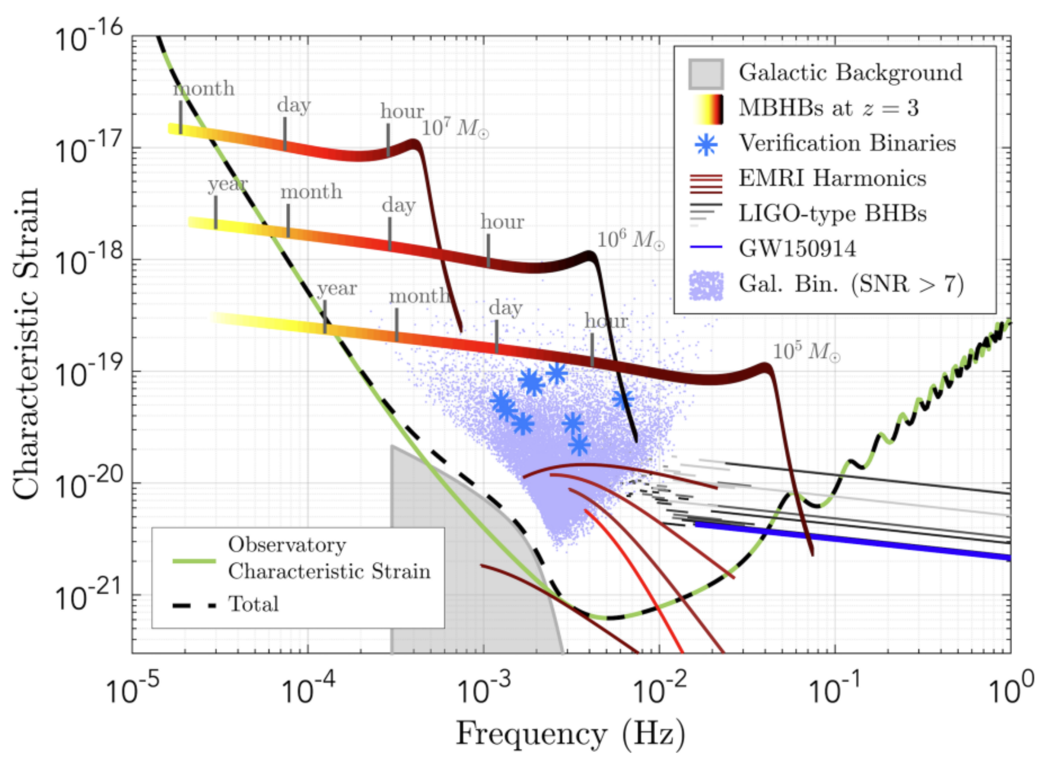
*Binary systems with a stellar-mass body inspiralling into a massive black hole*

- *Primary with* $M \sim (10^4 - 10^7) M_\odot$
- *Secondary such that the mass ratio*

$$q = m_p / M \sim (10^{-6} - 10^{-3})$$

  - *Key point of theoretical description*



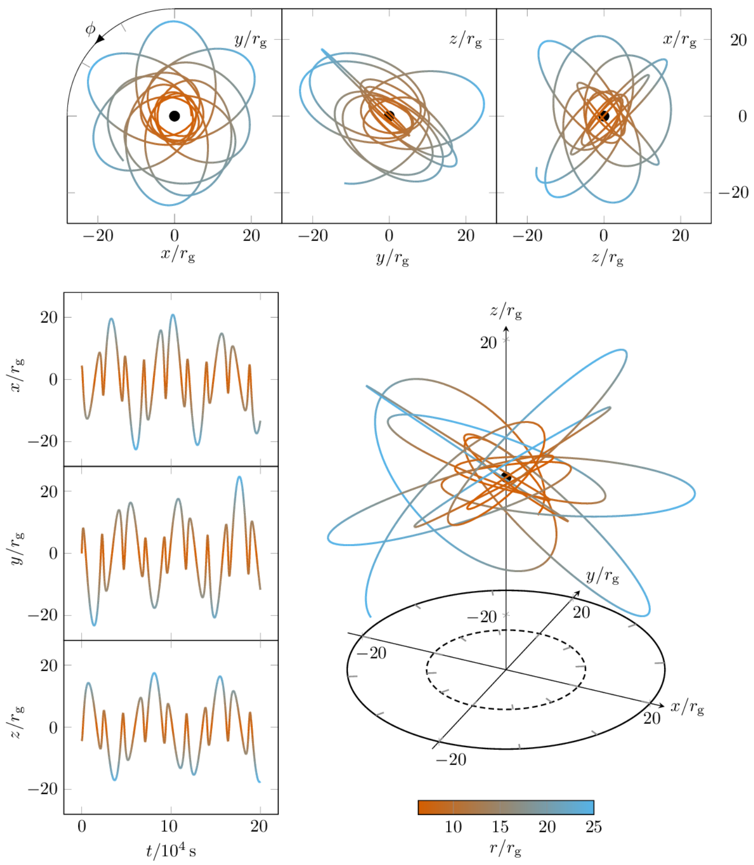



- *Emit GWs in the mHz band, golden targets for LISA, dim to ground based detectors*

*Baker +, Astro2020 1907.06482 (2019)*

# EMRI in nuce

*EMRIs provide a rich phenomenology, due to their orbital features*




Berry +, Astro2020 1903.03686 (2019)

- *Non equatorial orbits*
- *Eccentric motion*
- *Resonances*
- *Complete* $\sim (10^4 - 10^5)$ *cycles before the plunge*

***bless*** *and* ***disguise***

*Tracking EMRIs for O(year) requires accurate templates*

*Very appealing to test fundamental & astro-physics*

*Precise space-time map and accurate binary parameters*

# EMRI in GR

## How do we study EMRI in GR?

- *The asymmetric character introduces a natural parameter to study the problem in perturbation theory* $q = m_p/M \ll 1$

$$g_{\alpha\beta} = g^0_{\alpha\beta} + h_{\alpha\beta}$$

$$G_{\mu\nu} = T^{\mathrm{p}}_{\mu\nu} = 8\pi m_{\mathrm{p}} \int \frac{\delta^{(4)}(x - y_p(\lambda))}{\sqrt{-g}} \frac{dy_p^\alpha}{d\lambda} \frac{dy_p^\beta}{d\lambda} d\lambda$$

*leading* → *adiabatic*

*Regge-Wheeler-Zerilli (Schwarzschild)*

*Teukolsky (Kerr)*

- *The solution determines the phase evolution*

*adiabatic* — *first post-adiabatic*

$$\phi(t) = \phi_{\mathrm{diss}-1} + \ldots$$

$\phi_{\mathrm{diss}-1}$ → $\mathcal{O}(1/q)$

$\ldots$ → $\mathcal{O}(1)$

# The Setup

*Scalar field* $\varphi$ *non-minimally coupled to the gravity sectors*

$$S[\mathbf{g},\varphi,\Psi] = S_0[\mathbf{g},\varphi] + \alpha S_c[\mathbf{g},\varphi] + S_{\rm m}[\mathbf{g},\varphi,\Psi]$$

$$S_0 = \int d^4x \frac{\sqrt{-g}}{16\pi}\left(R - \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi\right)$$

*Non-minimal coupling*

*Matter fields*

- *Dimensionful coupling* $[\alpha] = (mass)^n$

*We assume that*

- *BH solutions are connected to GR solutions* $\alpha \to 0$
- $S_c$ *is analytic in* $\varphi$


A.M. +, Phys. Rev. Lett. 125, 141101 (2020)
A.M. +, Nature Astronomy 6, 4 464-470 (2022)
S.Barsanti. +, Phys. Rev. D 106, 044029 (2022)

# The Setup

*Key simplifications for the* ***exterior*** *space-time occur for 2 families*

1) *Theories with no-hair theorems*

2) *Theories which evade no-hair but have dimensionful coupling* $\alpha$ *with* $n \geq 1$

- *Any correction depend on* $\zeta \equiv \frac{\alpha}{M^n} = q^n \frac{\alpha}{m_{\rm p}^n} \ll 1 \qquad q = \frac{m_p}{M} \ll 1$

*The exterior space-time can be approximated by the Kerr metric*

*For the* ***secondary****, consider the skeletonized approach*

$$S_{\rm p} = -\int m(\varphi) ds = -\int m(\varphi) \sqrt{g_{\mu\nu} \frac{dy_{\rm p}^{\mu}}{d\lambda} \frac{dy_{\rm p}^{\nu}}{d\lambda}} d\lambda$$

*Eardley, ApJ 196 L59-62 (1975)*
*Damour & EF, CGQ 9, 9 (1992)*

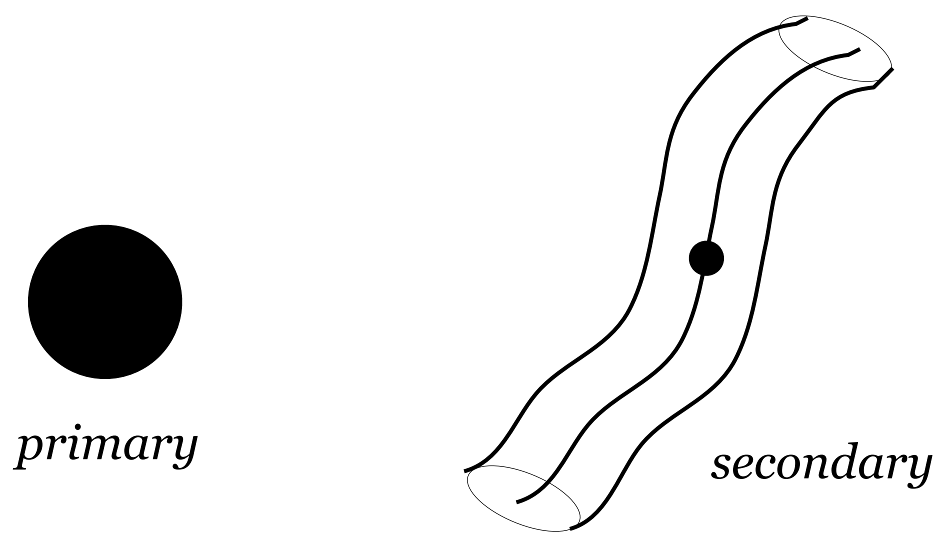
- *Extended body treated as point particle*
- $m(\varphi)$ *scalar function*

# The Setup

*Scale decoupling for the motion of the EMRI secondary*

$l \ll L$

*skeletonized body* — *exterior space-time*




*The orbital motion can be studied with perturbation theory in* $q \ll 1$

- *GR modifications affect the motion of the particle but not the background*
- *The scalar field is a perturbation of a constant value* $\varphi = \varphi_0 + \varphi_1$

# *The field's equations*

$(\zeta \sim q^n)$

*In our units* $[S_0] = (mass)^2 \quad [S_c] = (\mathrm{mass})^{2-n} \longrightarrow S_c \sim M^{-n} S_0$

$$G_{\mu\nu} = T^{\mathrm{scal}}_{\mu\nu} + \alpha T^c_{\mu\nu} + T^{\mathrm{p}}_{\mu\nu}$$

$$G_{\mu\nu} = \frac{1}{2}\partial_\mu\varphi\partial_\nu\varphi - \frac{1}{4}g_{\mu\nu}(\partial\varphi)^2 + -\alpha\frac{16\pi}{\sqrt{-g}}\frac{\delta S_c}{\delta g^{\mu\nu}} + \int m(\varphi)\frac{\delta^{(4)}(x-y_p(\lambda))}{\sqrt{-g}}\frac{dy_p^\alpha}{d\lambda}\frac{dy_p^\beta}{d\lambda}d\lambda$$

*quadratic in* $\varphi$

$$\alpha T^c_{\mu\nu} \sim \zeta^2 G_{\mu\nu} \sim q^{2n} G_{\mu\nu}$$

$$\Box\varphi + \frac{8\pi\alpha}{\sqrt{-g}}\frac{\delta S_c}{\delta\varphi} = 16\pi\int m'(\varphi)\frac{\delta^{(4)}(x-y_p(\lambda))}{\sqrt{-g}}d\lambda$$

$$\alpha\frac{\delta S_c}{\delta\varphi} \sim \zeta\Box\varphi \ll \Box\varphi$$

$m(\varphi), m'(\varphi)$ *evaluated at the value of the exterior scalar field*

# Almost as in GR

*From the scalar field equation inside the world tube, but far way to be weak field. In the body's frame*

$$\varphi = \varphi_0 + \frac{m_{\rm p}\, d}{\tilde{r}} + O\left(\frac{m_{\rm p}^2}{\tilde{r}^2}\right)$$

**scalar charge**

- *Matching with the scalar field equation outside the world tube*

$$m(\varphi_0) = m_p \qquad \frac{m'(\varphi_0)}{m_p} = -\frac{d}{4}$$

*Change in the EMRI dynamics universally captured by the scalar charge*

$$G_{\mu\nu} = T^{\rm p}_{\mu\nu} = 8\pi m_{\rm p} \int \frac{\delta^{(4)}(x - y_p(\lambda))}{\sqrt{-g}} \frac{dy_p^\alpha}{d\lambda} \frac{dy_p^\beta}{d\lambda} d\lambda$$

$$\Box\varphi = -4\pi d\, m_{\rm p} \int \frac{\delta^{(4)}(x - y_p(\lambda))}{\sqrt{-g}} d\lambda$$

# The perturbation scheme

*EMRI small mass ratio naturally leads to use relativistic perturbation theory to describe their evolution*

- *Consider linear perturbations of a Schwarzschild background induced by the small body (same for Kerr)*

*grav-sector*

$$g_{\alpha\beta} = g^0_{\alpha\beta} + h_{\alpha\beta}$$

*scal-sector*

$$\varphi = \varphi_0 + \varphi_1$$

- *Decompose* $h_{\alpha\beta}$ *and* $\varphi_1$ *in tensor and scalar spherical harmonics*

  - *For the scalar field*

$$\varphi_1(t,r,\theta,\phi) = \sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell}\frac{\delta\varphi_{\ell m}(t,r)}{r}Y_{\ell m}(\theta,\phi)$$

- *Go to the Fourier space, replace into the field's equation and solve for* $\delta\varphi_{\ell m}$

# The perturbation scheme

*For the gravitational sector*

$$h_{\alpha\beta} = h^{\rm pol}_{\alpha\beta} + h^{\rm ax}_{\alpha\beta}$$

$(-1)^{\ell}$ $(-1)^{\ell+1}$

$$\mathbf{h} = \sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell} \mathcal{A}^{(0)}_{\ell m}\mathbf{a}^{(0)}_{\ell m} + \mathcal{A}^{(1)}_{\ell m}\mathbf{a}^{(1)}_{\ell m} + \mathcal{A}_{\ell m}\mathbf{a}_{\ell m} + \mathcal{B}^{(0)}_{\ell m}\mathbf{b}^{(0)}_{\ell m} + \mathcal{B}_{\ell m}\mathbf{b}_{\ell m} + \mathcal{Q}^{(0)}_{\ell m}\mathbf{c}^{(0)}_{\ell m} + \mathcal{Q}_{\ell m}\mathbf{c}_{\ell m}$$
$$+ \mathcal{D}_{\ell m}\mathbf{d}_{\ell m} + \mathcal{G}_{\ell m}\mathbf{g}_{\ell m} + \mathcal{F}_{\ell m}\mathbf{f}_{\ell m}$$

$$\mathbf{b}_{\ell m} = \frac{n_\ell r}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & Y^{\ell m}_{,\theta} & Y^{\ell m}_{,\phi} \\ 0 & Y^{\ell m}_{,\theta} & 0 & 0 \\ 0 & Y^{\ell m}_{,\phi} & 0 & 0 \end{pmatrix}$$

- *7 **polar** components + 3 **axial** harmonics*
  - *For a spherically symmetric background the 2 families decouple*
- *In the Regge-Wheeler-Zerilli gauge the components reduce to **1** axial and **1** polar functions*

*Regge & Wheeler, PRD 108, 1063 (1957)*
*Zerilli, PRD 2, 2141 (1970)*

# The wave equations

## We have 3 master equations for 3 perturbations

$$e^{-\lambda} = 1 - 2M/r$$
$$\Lambda = \ell(\ell+1)/2 - 1$$

**GR**

$$\frac{d^2 R_{\ell m}}{dr_\star^2} + \left[\omega^2 - e^{-\lambda}\left(\frac{\ell(\ell+1)}{r^2} - \frac{6M}{r^3}\right)\right] R_{\ell m} = J_{\rm ax}$$

**Regge-Wheeler**

$$\frac{d^2 Z_{\ell m}}{dr_\star^2} + \left[\omega^2 - \frac{18M^3 + 18M^2 r\Lambda + 6Mr^2\Lambda^2 + 2r^3\Lambda^2(1+\Lambda)}{r^3(3M + r\Lambda)}\right] Z_{\ell m} = J_{\rm pol}$$

**Zerilli**

$$\frac{d^2 \delta\varphi_{\ell m}}{dr_\star^2} + \left[\omega^2 - e^{-\lambda}\left(\frac{\ell(\ell+1)}{r^2} + \frac{2M}{r^3}\right)\right] \delta\varphi_{\ell m} = J_\varphi$$

**Scalar field**

- *For circular equatorial orbits*

$$\omega_p = (M/r_p^3)^{1/2}$$

$$J_\varphi = -d\, m_{\rm p} \frac{4\pi P_{\ell m}(\frac{\pi}{2})}{r^{3/2} e^\lambda} \sqrt{r - 3M}\,\delta(r - r_{\rm p})\delta(\omega - m\omega_{\rm p})$$

*Overall scale sets by the charge*

*orbit's radius*

# *The GW energy flux*

*The full solutions at infinity/horizon are needed to compute the emitted gravitational wave fluxes*

$$\dot{E}^{\pm}_{\rm grav} = \frac{1}{64\pi}\sum_{\ell=2}^{\infty}\sum_{m=-\ell}^{\ell}\frac{(\ell+2)!}{(\ell-2)!}(\omega^2|Z^{\pm}_{\ell m}|^2 + 4|R^{\pm}_{\ell m}|^2) \qquad \dot{E}^{\pm}_{\rm scal} = \frac{1}{32\pi}\sum_{\ell=1}^{\infty}\sum_{m=-\ell}^{\ell}\omega^2|\delta\varphi^{\pm}_{\ell m}|^2$$

- *The total contribution*

$$\dot{E} = \dot{E}^{+}_{\rm grav} + \dot{E}^{-}_{\rm grav} + \dot{E}^{+}_{\rm scal} + \dot{E}^{-}_{\rm scal} = \dot{E}_{\rm GR} + \delta\dot{E}_d$$

- *The binary accelerates due to the extra leakage of energy given by the scalar field channel*

- $\delta\dot{E}_d$ *enters at the* ***same*** *order in* ***q*** *as the GR leading dissipative contribution*

# How much dephasing?

*Once we have the total flux emitted by the binary we can determine its adiabatic evolution*

- *For the orbital phase*

$$\frac{dr}{dt} = -\dot{E}\frac{dr}{dE_{\rm orb}} \quad , \quad \frac{d\Phi}{dt} = \omega_p = \pm\frac{M^{1/2}}{r^{3/2} \pm \chi M^{3/2}}$$

- *The total phase can be written as*

$$\Phi_d(t) \sim \Phi_{\rm GR}(t) + \delta\Phi_d(t)$$

- *Both contributions are of the same order* $\mathcal{O}(1/q)$
- *The term* $q\delta\Phi_d(t)$ *depends only on the scalar charge*

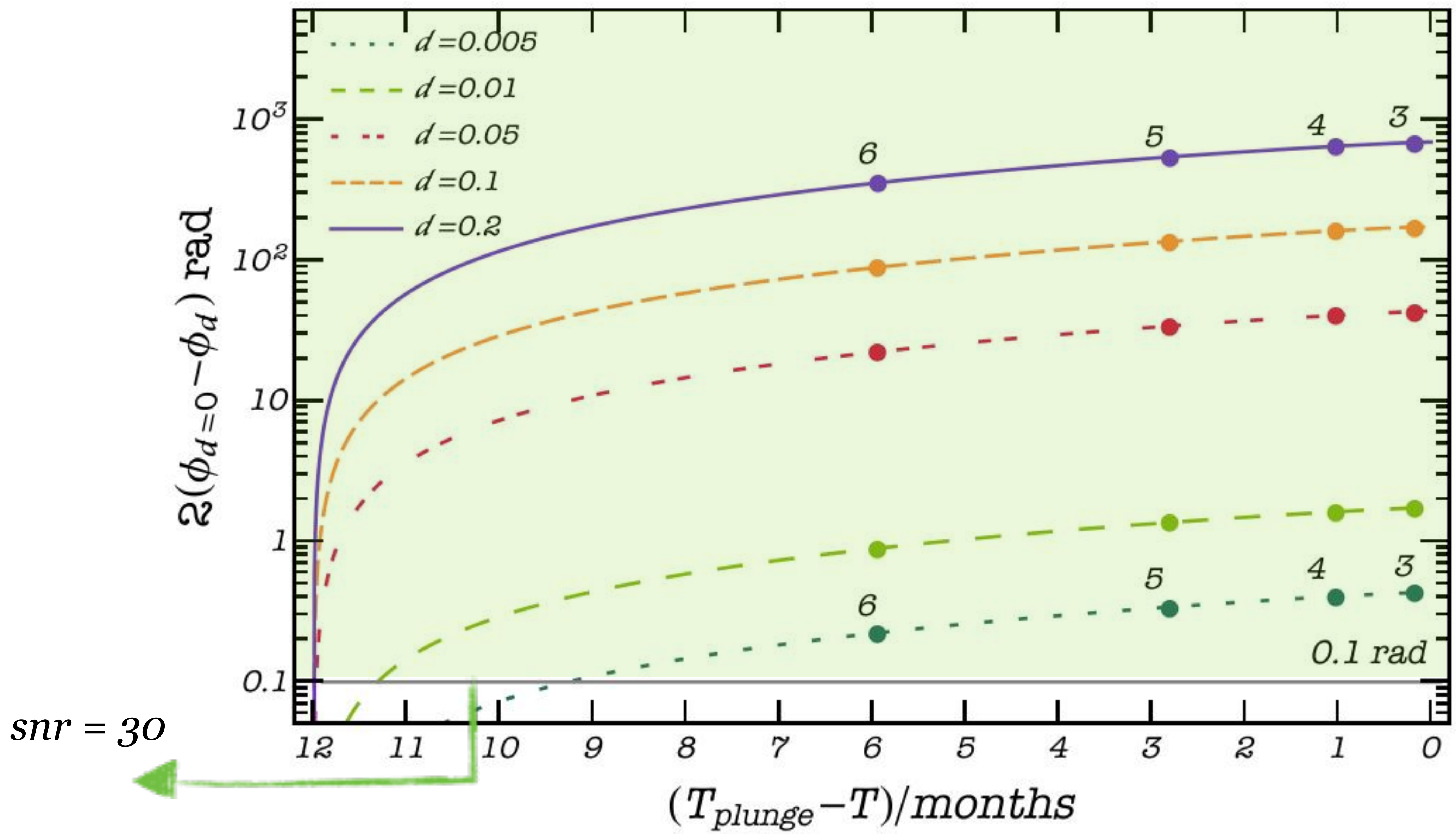
*A first assessment of the charge impact is given by studying the* ***dephasing*** *induced on the orbital phase*

$$\Phi_{d=0}(t) - \Phi_d(t)$$

# How much dephasing?

*Difference between GR - GR**d** phase evolution during the inspiral (12 months the plunge)*

$(M, m_p) = (10^6, 10)M_\odot \quad \chi = 0.9$




- *Potentially able to observe changes induced by scalar charges* $d \sim 0.005$

# The Waveform

## *The recipe to generate EMRI waveforms*

- *Compute the total energy flux emitted by the binary* $\dot{E} = \dot{E}_{\rm GR} + \delta\dot{E}_d$
- *The flux drives the binary orbital evolution*

$$\frac{dr(t)}{dt} = -\dot{E}\frac{dr}{dE_{\rm orb}} \quad , \quad \frac{d\Phi(t)}{dt} = \frac{M^{\frac{1}{2}}}{r_p^{3/2}}$$

- *Build the GW polarizations* $h_+[r(t), \Phi(t)] \quad , \quad h_\times[r(t), \Phi(t)]$
- *Given the source localization, construct the strain*

$$h(t) = \frac{\sqrt{3}}{2}[h_+F_+(\theta,\phi,\psi) + h_\times F_\times(\theta,\phi,\psi)]$$

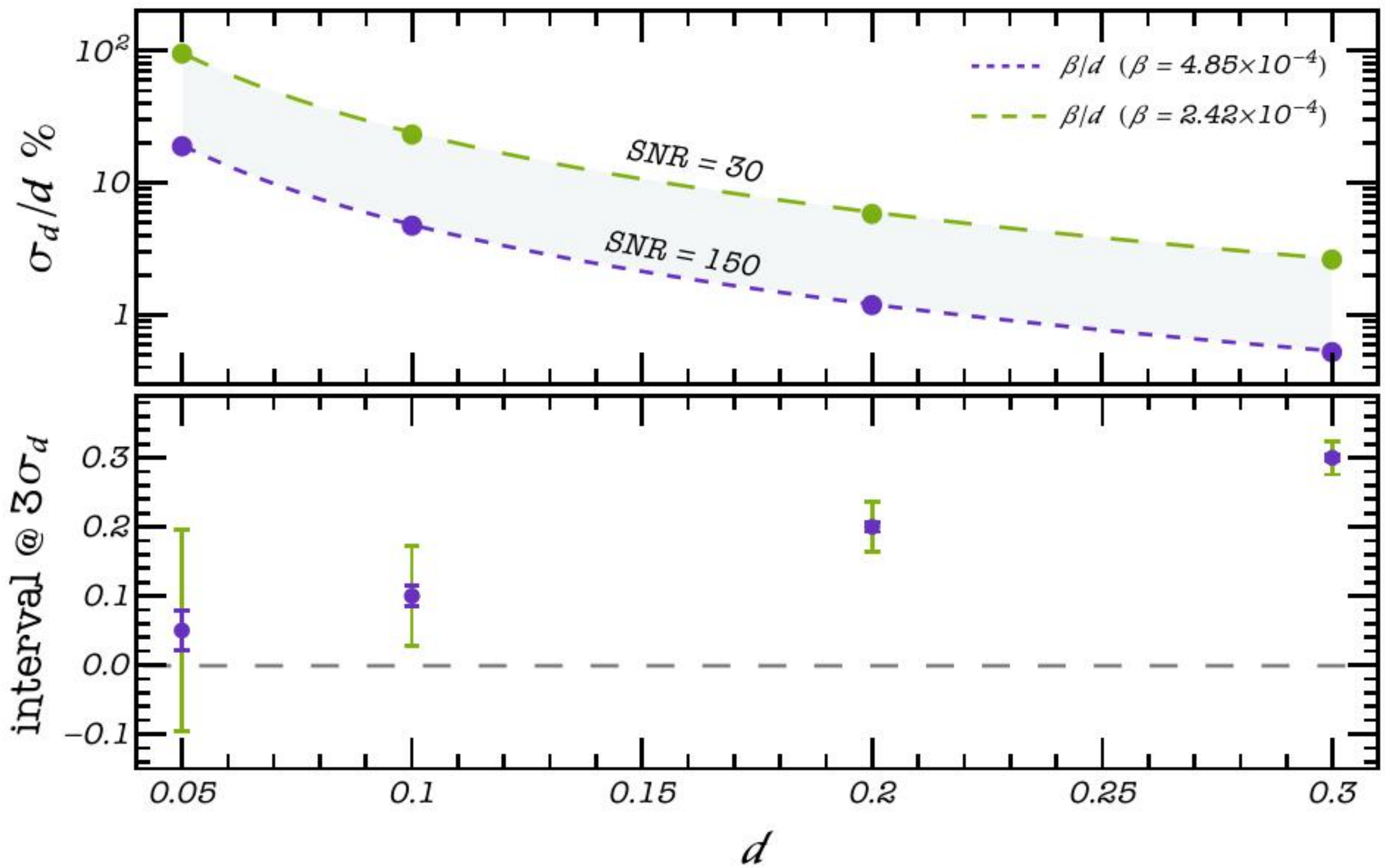
*Everything as in GR but* $\delta\dot{E}_d$*, that only depends on the scalar charge*

- *Universal family of waveforms to be tested against GR*

# Forecast on LISA bounds

*Constraints on the scalar charge for prototype EMRIs with SNR = (30,150)*

*A.M. +, Nature Astronomy 6, 4 464-470 (2022)*




- *LISA potentially able to measure **d** with % accuracy and better*
- *LISA potentially able to constrain **d** ~ 0.05 to be inconsistent with zero @ 3- $\sigma$*