Fabian Wagner, 29/05/2023
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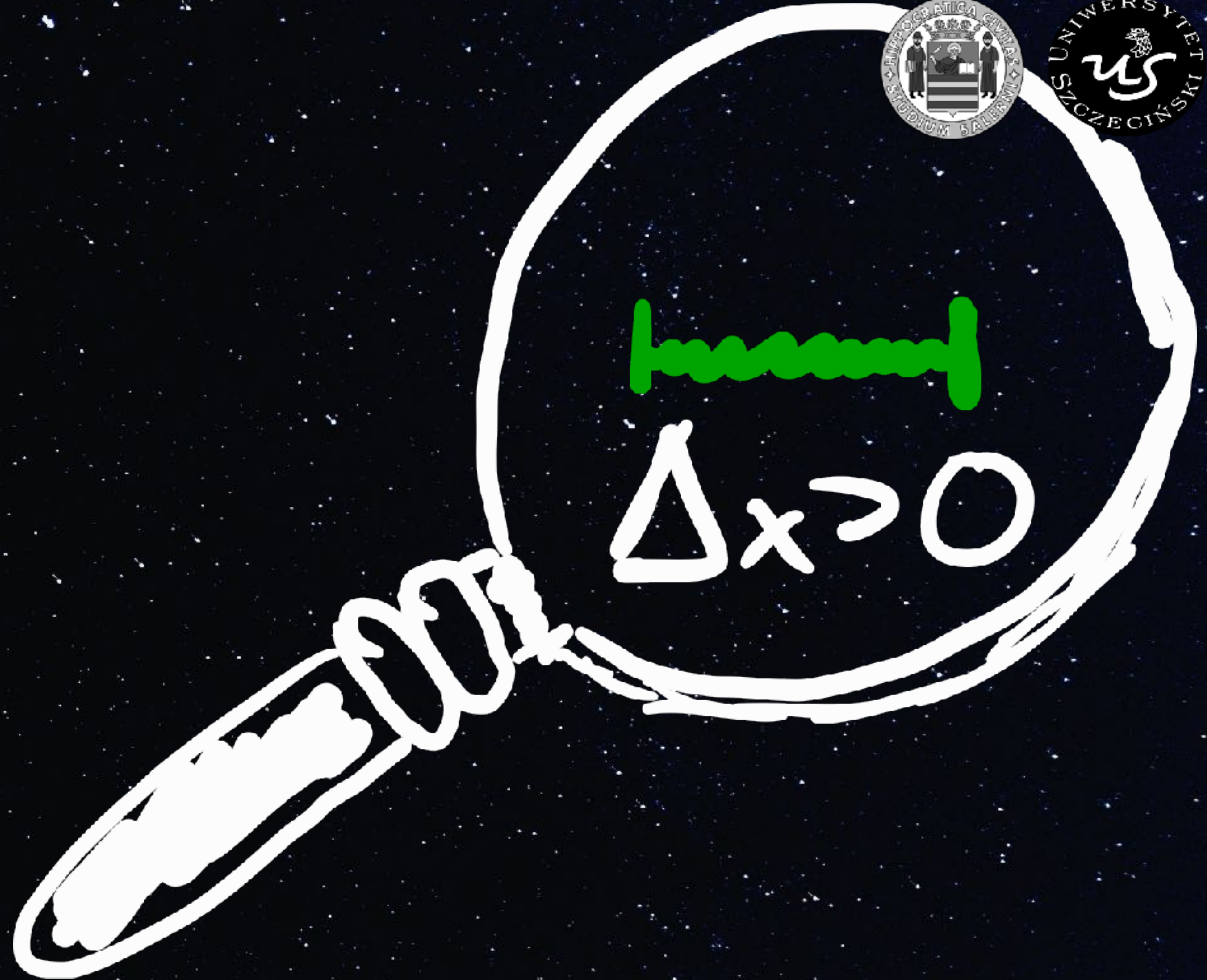
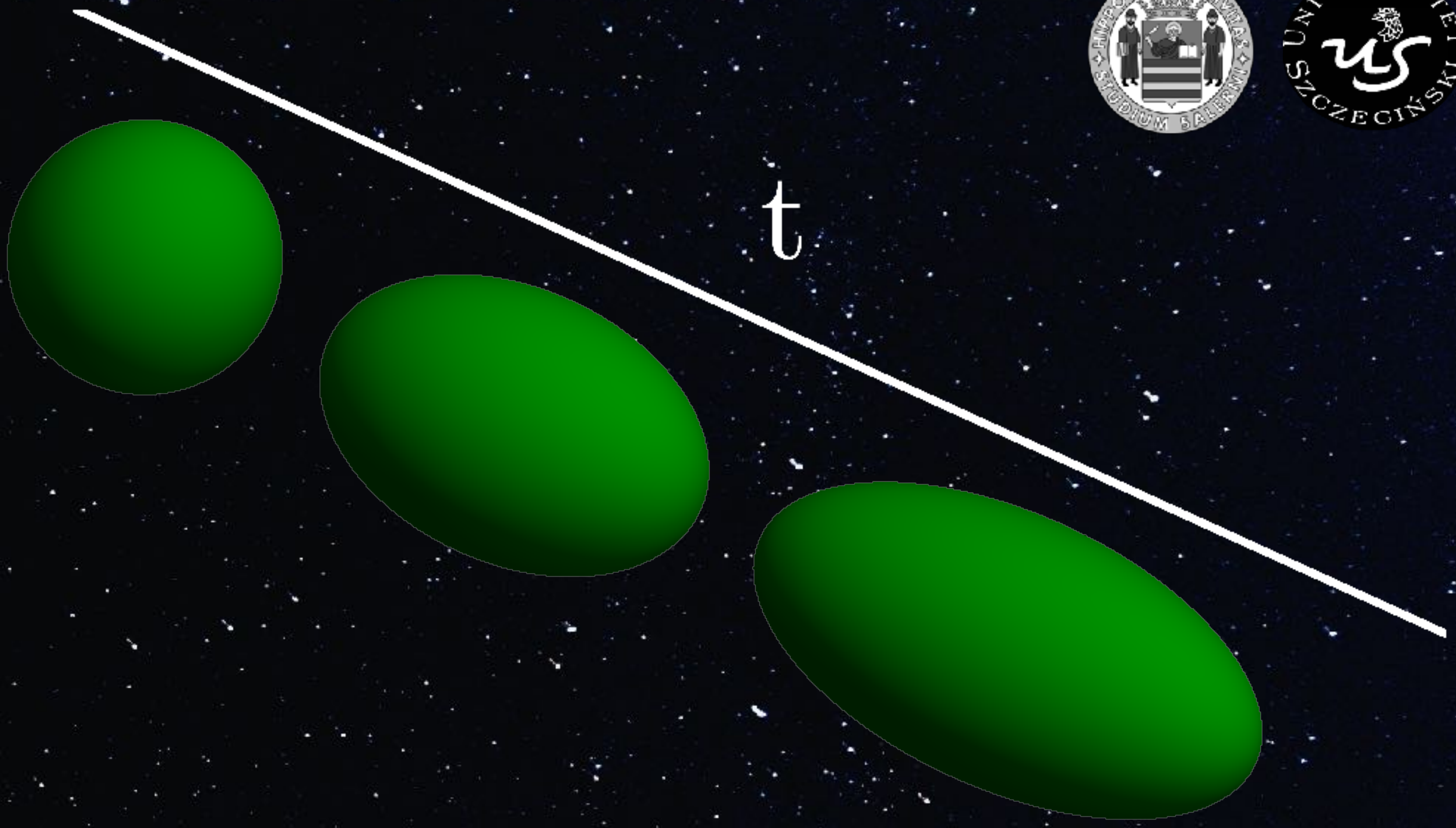
A new perspective on minimal-length quantum mechanics



What is the essence of the minimal length?



Can we make a more informed guess about minimal-length dynamics?

Overview

1. Introduction to the conventional model
2. The essence of the minimal length
3. Minimal-length compatible relativity principles
4. Conclusion

Introduction

Deformed Heisenberg algebra

$$[\hat{x}, \hat{p}] = i f(\hat{p})$$

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Robertson-Schrödinger

$$\Delta x \Delta p \geq \frac{1}{2} |\langle f \rangle|$$

Deformed Heisenberg algebra

Kempf, Mangano, Mann (1994)

$$[\hat{x},\hat{p}] = i\left(1+\ell^2\hat{p}^2\right)$$

Robertson-Schrödinger

$$\Delta x \geq \ell$$

Deformed Heisenberg algebra

Kempf, Mangano, Mann (1994)

$$[\hat{x},\hat{p}] = i(1+\ell^2\hat{p}^2)$$

Robertson-Schrödinger

$$\Delta x \geq \ell$$

GUP

Kinematics

$$[\hat{x}, \hat{p}] = i f(\hat{p})$$

Dynamics

$$\hat{H} = \frac{\hat{p}^2}{2M} + V(\hat{x})$$

Kinematics

$$[\hat{x}, \hat{k}] = i$$

$$f(\hat{p}) = \frac{\mathrm{d}\hat{\mathrm{p}}}{\mathrm{d}\hat{\mathrm{k}}}$$

Dynamics

$$\hat{H} = \frac{\hat{p}^2(\hat{k})}{2M} + V(\hat{x})$$

The essence of the minimal length

The minimal length as starting point

$$\Delta x \geq \ell$$

The minimal length as starting point

$\Delta x \geq \ell$

- Δx invariant under translations

➡ $\ell = \sqrt{\lambda_0}$ smallest eigenvalue of $\hat{x}^2$

- put system into box in k-space $-B \leq k \leq B$

$$\sqrt{\lambda_0} = \frac{\pi}{2B}$$

The minimal length as starting point

$$\Delta x \geq \ell$$

- Δx invariant under translations

➡ $\ell = \sqrt{\lambda_0}$ smallest eigenvalue of $\hat{x}^2$

- put system into box in k-space $-B \leq k \leq B$

$$B = \frac{\pi}{2\ell}$$

$$[\hat{x}, \hat{p}] = i\sqrt{1 + l^2\hat{p}^2}$$

Maggiore (1993), Fadel, Maggiore (2021)

Summary

- minimal length = cut-off in wave-number space
- $\hat{p}$ and deformed Heisenberg = additional structure

Why $\hat{H} = \dfrac{\hat{p}^2}{2M} + V(\hat{x})$?

Minimal-length compatible relativity principles

Wave-number addition

minimal length = cut-off in wave-number space

$$\hat{k}_1 \oplus \hat{k}_2 \neq \hat{k}_1 + \hat{k}_2$$

Deformed Galilean algebra

minimal length = cut-off in wave-number space

$$[\hat{G}, \hat{k}] = iMg(\hat{k})$$

$$[\hat{k}, \hat{H}_0] = 0$$

$$[\hat{G}, \hat{H}_0] = i\hat{H}_0'(\hat{k})g(\hat{k})$$

Relativity principle: Consequences

- $\hat{k}_A \oplus \hat{k}_B$ associative and commutative
- $\exists \hat{p} = p(\hat{k})$ such that

$$\hat{p}_A \oplus \hat{p}_B = \hat{p}_A + \hat{p}_B$$

- deformed Heisenberg algebra:

$$[\hat{x}, p(\hat{k})] = ip'(\hat{k}) \equiv if(\hat{p})$$

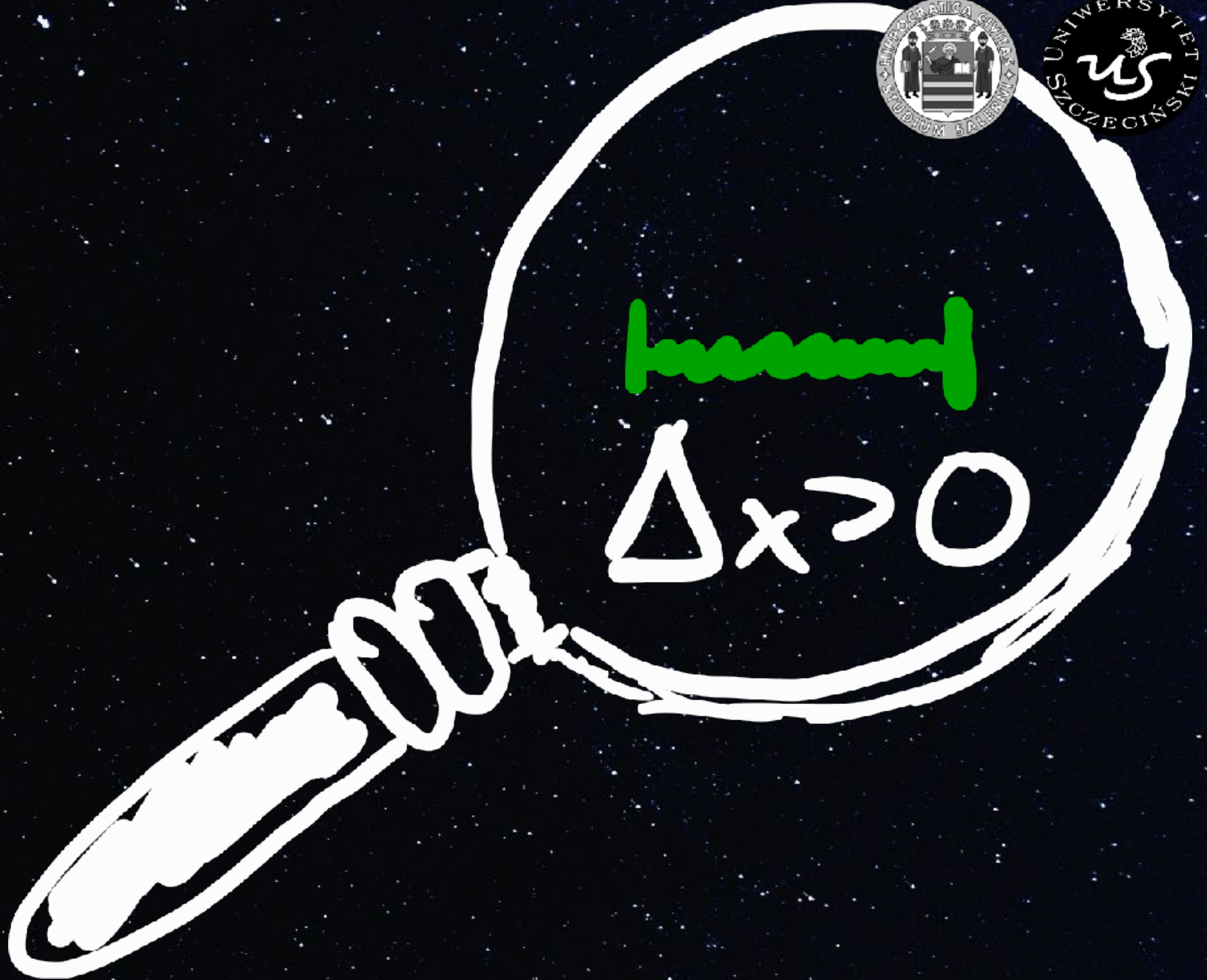
Relativity principle: Consequences

- momentum-space diffeomorphism = canonical trafo

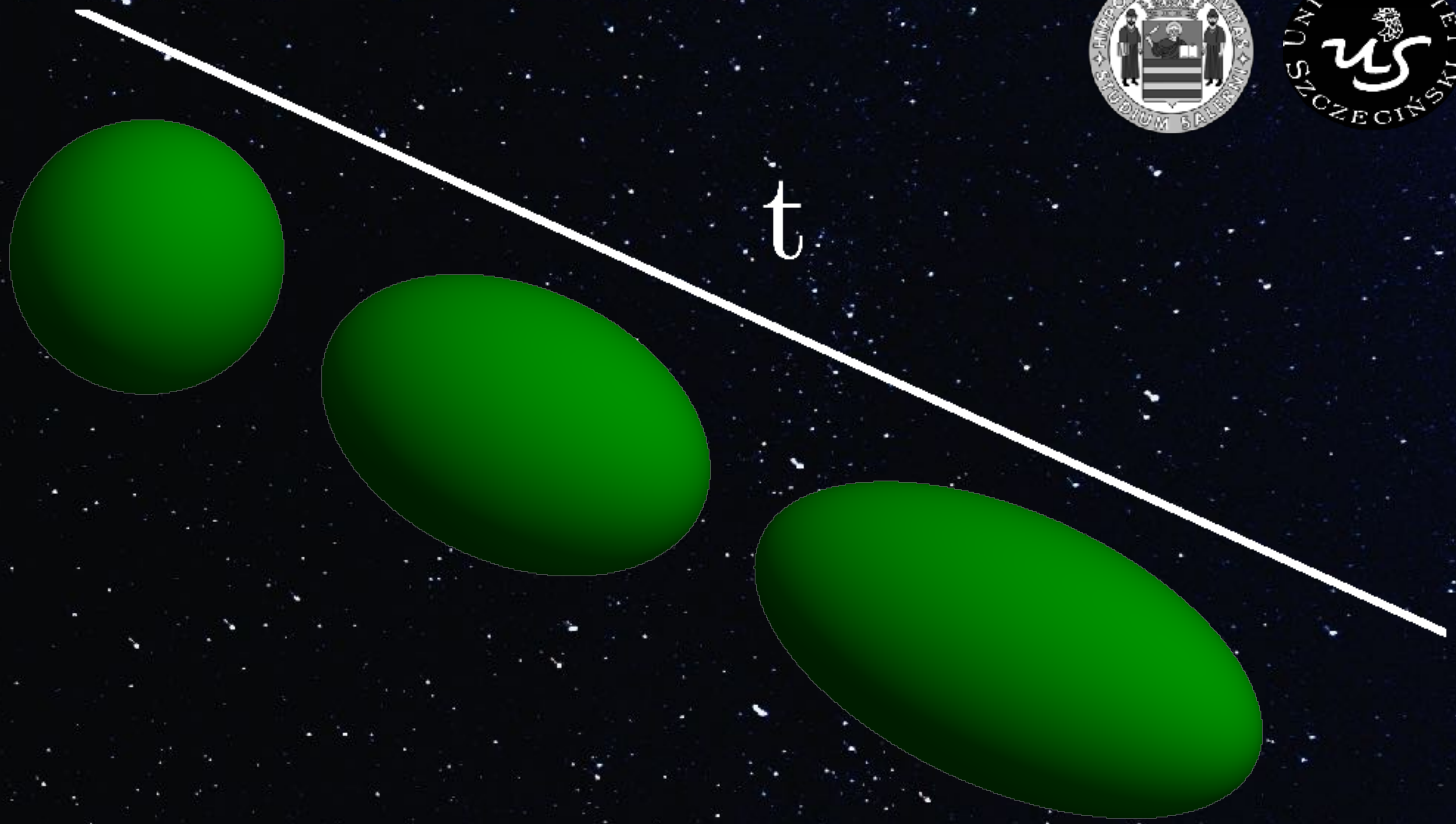
$$[\hat{X}, \hat{p}] = i$$

$$\hat{H} = \frac{\hat{p}_A^2}{2M} + \frac{\hat{p}_B^2}{2M} + V(|\hat{X}_A - \hat{X}_B|)$$

Conclusion



What is the essence of the minimal length?



Can we make a more informed guess about minimal-length dynamics?

Takeaways

- minimal length = cut-off in wave-number space
- momentum $\hat{p}$ and GUP = additional structure

➡ choice of Hamiltonian rather arbitrary

- Hamiltonian from deformed relativity principle
- momentum $\hat{p}$ and GUP emerge
- dynamics canonically related to Galilean one

➡ nontrivial evolution of positions

➡ relative-locality like effects