Finsler (α,β)-Gravitational Waves and their Observational Signature

SJORS HEEFER – CA18108 4TH ANNUAL CONFERENCE – RIJEKA 2023

Based on joint work with Andrea Fuster - **arXiv:2302.08334**

Outline

I. Finsler gravity
II. Exact solutions
III. Linearization → gravitational waves (GWs)
IV. Observational signature of such GWs
V. Outlook

Finsler Gravity

Finsler gravity = GR formulated in the language of Finsler geometry

$$\mathrm{d}s^2 = g_{\mu\nu}(x)\mathrm{d}x^\mu \mathrm{d}x^\nu \quad \rightarrow \quad \mathrm{d}s = F(x, \mathrm{d}x)$$ ‘Finsler metric’

Vacuum field equation [1]

$$\mathrm{Ric} - \frac{F^2}{3} g^{\mu\nu} R_{\mu\nu} - \frac{F^2}{3} g^{\mu\nu} \left(\bar{\partial}_\mu \dot{S}_\nu - S_\mu S_\nu + \nabla_{\delta_\mu} S_\nu \right) = 0$$

[1] Pfeifer & Wohlfarth, Phys.Rev. D85 (2012)

Exact Solutions

Building blocks

$$\begin{cases} \alpha = \sqrt{|A|} \quad \text{with} \quad A = a_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu \ \text{ vacuum solution to EFEs} \\ \beta = b_\mu \mathrm{d}x^\mu \quad \text{1-form that is covariantly constant,} \quad \nabla_\mu b_\nu = 0 \end{cases}$$

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Prime example: α = *pp-waves* = $\sqrt{|-2\mathrm{d}u\,(\mathrm{d}v + H(u,x,y)\,\mathrm{d}u) + \mathrm{d}x^2 + \mathrm{d}y^2|}$

β = defining c.c. null 1-form = $\mathrm{d}u$

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Theorem. Any Finsler metric constructed from such α and β is a vacuum solution to Finsler gravity.

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(α,β)-metric

$$\mathrm{d}s = \alpha\phi(\beta/\alpha)$$

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Randers metric

Prime example: $\mathrm{d}s = \alpha + \beta, \quad \phi(s) = 1 + s$

Linearization in 2 Ways

1. Linearize in **departure from flatness** → pp-wave becomes *standard linearized GW*:

$$\begin{cases} \alpha = \sqrt{|-\mathrm{d}t^2 + (1+f_+(t-z))\mathrm{d}x^2 + (1-f_+(t-z))\mathrm{d}y^2 + 2f_\times(t-z)\mathrm{d}x\,\mathrm{d}y + \mathrm{d}z^2|} \\ \beta = \dfrac{\lambda}{\sqrt{2}}(\mathrm{d}t - \mathrm{d}z) \end{cases}$$

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2. Linearize in **departure from GR** (i.e. 'size' of 1-form, λ) → Generic (α,β)-metric becomes *Randers metric*

$$\mathrm{d}s = \alpha\phi\left(\frac{\beta}{\alpha}\right) \approx \alpha\left(\phi(0) + \phi'(0)\frac{\beta}{\alpha}\right) = \alpha\phi(0) + \phi'(0)\beta = \tilde{\alpha} + \tilde{\beta}$$

Finsler Gravitational Waves

Conclusion:

To 1st order, vacuum solutions are given by Randers metrics $\mathrm{d}s = \alpha + \beta$ with

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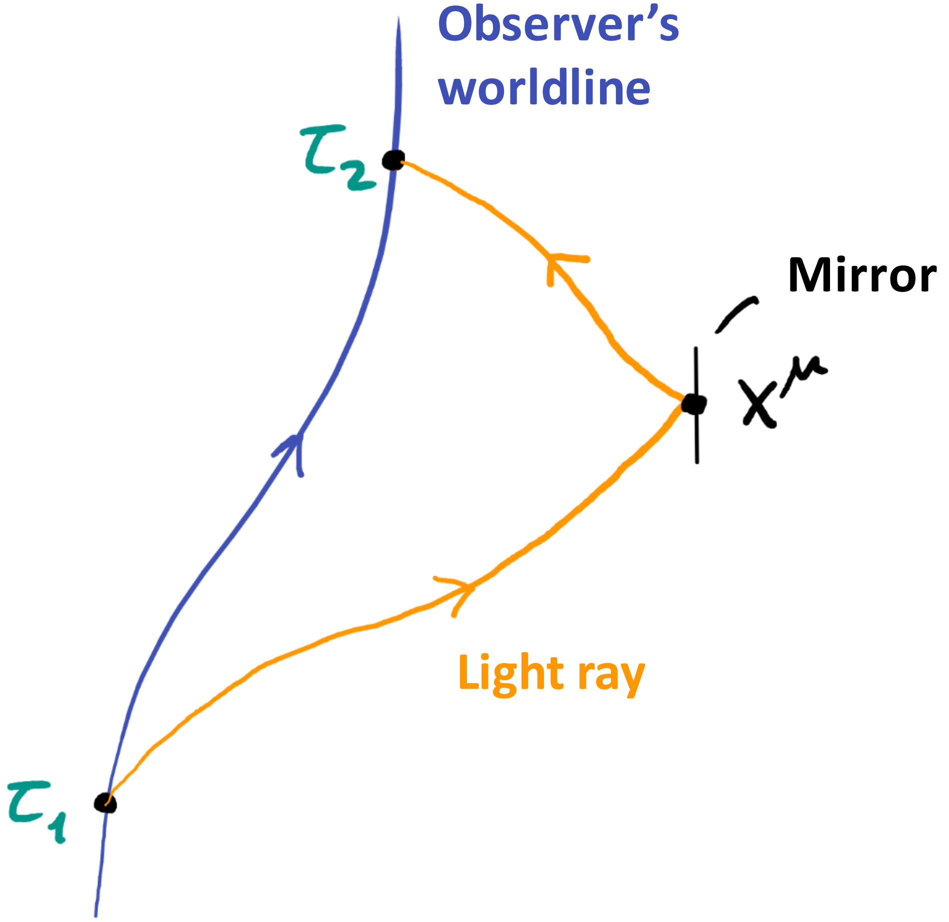
Observational signature…?

Radar Distance

Radar distance = ½ time between emission and reception of a reflected light ray

$$R(x) = \frac{\Delta\tau}{2} = \frac{\tau_2 - \tau_1}{2}$$



GR radar distance [2]:

$$R = \Delta\ell + \left(\frac{\Delta x^2 - \Delta y^2}{4\Delta\ell}\right)\bar{f}_+ + \left(\frac{\Delta x \Delta y}{2\Delta\ell}\right)\bar{f}_\times$$

$$(\Delta\ell)^2 = \delta_{ij}\Delta x^i \Delta x^j$$

$\bar{f} \sim f$ averaged over worldline

[2] M. Rakhmanov, CQG 26 (2009)

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However, define the **measurable quantities**:

ΔX = radar distance in x-direction - *in the absence of the GW*

ΔY = radar distance in y-direction - *in the absence of the GW*

ΔZ = radar distance in z-direction - *in the absence of the GW*

ΔL = radar distance in direction along interferometer arm - *in the absence of the GW*

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$\Rightarrow$ **Identical to GWs in GR!**

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Conclusion & Outlook

- The Finslerian GWs are **indistinguishable** from standard GWs in GR **with the same waveform**.
- However, Finsler effects might affect the **generation** of GWs

  → Different waveforms
- Also, different types of Finslerian gravitational waves probably exist.

Thank you!

Randers
$\mathrm{d}s = \alpha + \beta$

→

Modified Randers
$\mathrm{d}s = \mathrm{sign}(A)\alpha + |\beta|$

where
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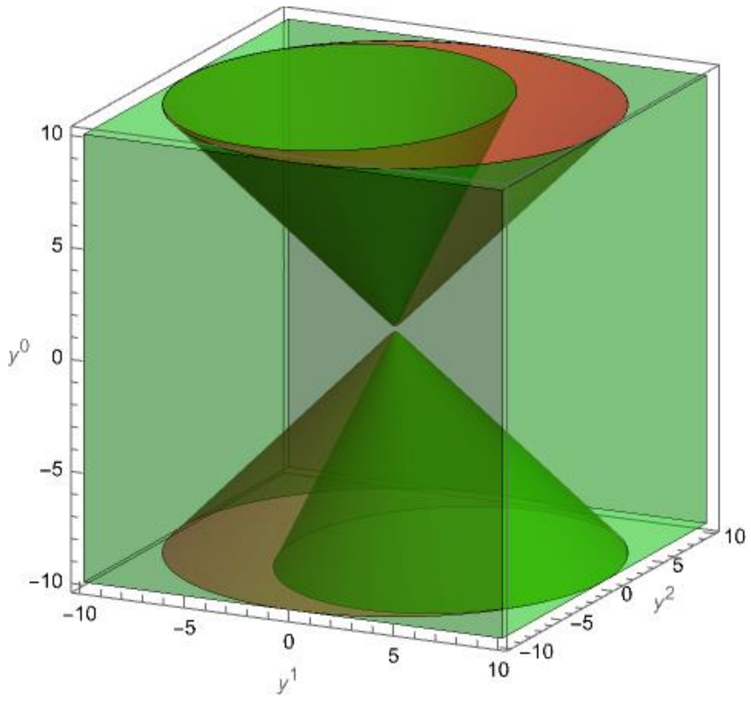
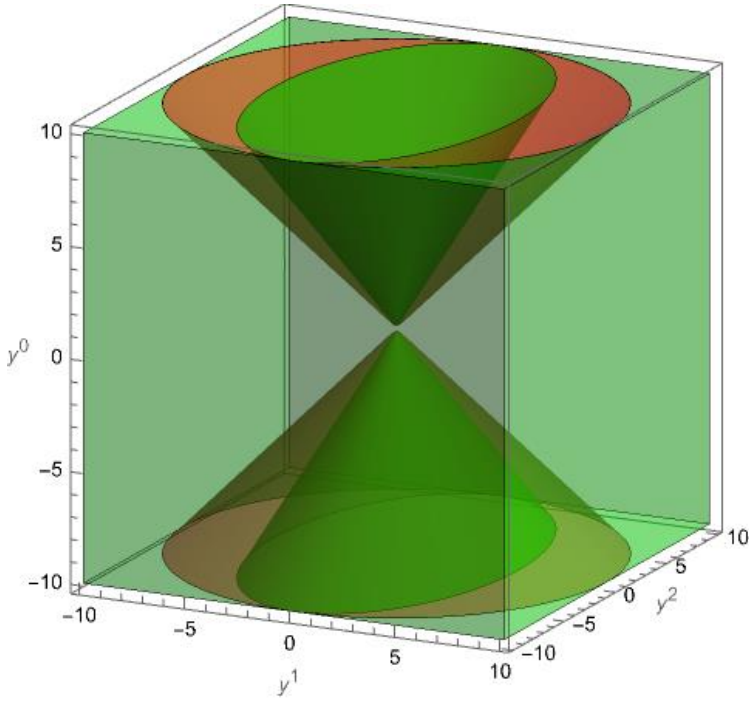
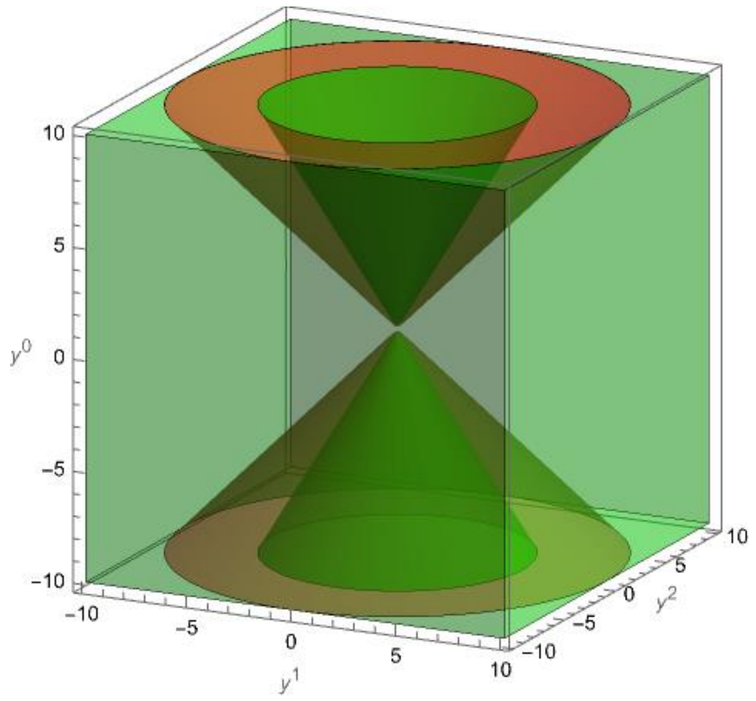


Lightlike β



Spacelike β



Timelike β

Green = Lorentzian signature Red = positive definite signature

See also N. Voicu *et al*, Universe 9 (2023) for *standard* Randers lightcones

References

M. Rakhmanov, On the round-trip time for a photon propagating in the field of a plane gravitational wave, Classical and Quantum Gravity 26, 155010 (2009).

C. Pfeifer and M. N. R. Wohlfarth, Finsler geometric extension of Einstein gravity, Phys.Rev. D85, 064009 (2012).