Signed Coordinate Invariance, invariant lagrangians and manifolds, the time problem in quantum cosmology, quantum space time, spacetimes and antispacetimes.

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In an interesting paper, Linde formulated a model that claims to resolve the cosmological constant problem [1]. This requires the existence of two universes, each with its own set of coordinates x^μ and y^μ containing matter and gravity components that mirror each other, but with the corresponding actions having opposite signs, like spacetime and an antispacetime, as in

$$S = \int d^4x d^4y \sqrt{-g(x)}\sqrt{-\bar{g}(y)}(\frac{M_P^2}{16\pi}R(x) + L(\phi(x)) - \frac{M_P^2}{16\pi}R(y) - L((\bar{\phi}(y)) \quad (1)$$

$R(x)$ is defined in terms of a $g_{\mu\nu}$ metric, while $R(y)$ is defined in terms of a $\bar{g}_{\mu\nu}$ metric and where $L(x)$ and $L(y)$ have exactly the same functional form with respect to their corresponding mirror fields, like for a scalar field $\phi(x)$, there will be a potential $V(\phi(x))$, same with kinetic terms, etc. that are appearing in $L(x)$, while in $L(y)$ there will be corresponding field $\bar{\phi}(y)$ with a potential $V(\bar{\phi}(y))$, then in $L(y)$ the metric $\bar{g}_{\mu\nu}$ appears instead of the metric $g_{\mu\nu}$, then the theory is obviously invariant under $V \rightarrow V + constant$ [1]. This non local coexistece of a spacetime and an antispacetime was shown by Linde to have remarkable properies concerning its behavior with respect to the cosmological constant problem. As we will see , we can think of this as having regions where the measure of integration can change sign, an effect that must take place at the same time as we double the space time, to realize Linde´s ideas.

The model by Linde is however non local, so this is an aspect that is not desirable,

II. GENERAL RELATIVITY AND OTHER THEORIES USE A RIEMANNIAN VOLUME ELEMENT THAT IS NOT INVARIANT UNDER SIGNED GENERAL COORDINATE TRANSFORMATIONS

The action of GR, and other theories that use the standard Riemannian volume element $d^4x\sqrt{-g}$ is of the form,

$$S = \int d^4x\sqrt{-g}L \tag{2}$$

where L is a generally coordinate invariant lagrangian. Now notice that under a general coordinate transformation,

$$d^4x \rightarrow Jd^4x$$

, while

$$\sqrt{-g} \rightarrow \mid J \mid^{-1} \sqrt{-g}$$

where J is the jacobian of the transformation and $\mid J \mid$ is the absolute value of the transformation. Therefore $d^4x\sqrt{-g} \rightarrow \frac{J}{|J|}d^4x\sqrt{-g}$, so invariance is achieved only for $J = \mid J \mid$, that is if $J > 0$, that is signed general coordinate transformations are excluded.

One could argue that when taking the square root of the determinant of the metric one may choose the negative solution when it suit us, but this would be an arbitrary procedure if no specific rule is given to choose the positive or the negative root. We choose instead to declare that $\sqrt{-g}$ is always positive and replace it in the measure by something else whose sign is well defined.

A. Invariance of the action with non invariant lagrangian density (integrand) and compensating non invariant manifold of integration?

If conditions are optimal, the non invariance of the lagrangian density (integrand) , which includes the measure, in a signed coordinate transformation, could be compensated by the non invariance of the manifold of integration. For example, in a time reversal transformation, the integrand will change sign, but , if there no obstructions, and we can then change the limits of integrations, the exchange of the limits of integrations will involve an additional exchange of signs that can compensate for the sign change in the integrand.

If the manifold has boundaries in coordinate space, introducing coordinate transformations that change the boundaries involve compensationg terms at the boundaries, even when the transformations are infinitesimal, so at this point the invariance of the action becomes complicated and problematic, in particular in cosmology where the universe may have a beginning in time.

As we discuss in the next sections, where we will discuss a possible realization of the invariance under signed general coordinate transformations in the context of the non local Linde Universe multiplication model, without invoking a change in the manifold of integration, but rather transforming the field variables, and then in the following section a return to a local theory by the use of the modified measure formalism.

Anti space (y space time) solves the problem in Linde Model

III. INVARIANCE OF SIGNED GENERAL COORDINATE TRANSFORMATIONS IN THE NON LOCAL LINDE UNIVERSE MULTIPLICATION MODEL, OR SPACETIME ANTISPACE MODEL

The Linde non local model can offer a limited way out to obtain signed general coordinate invariance, so we can allow general coordinate transformations where the jacobian of the transformation, say in the x space , is negative, but still we do not consider the possibility that it could change from positive to negative.

As long as the jacobian is uniformly negative over all space, invariance will be achieved if the same transformation is performed both in the x and y coordinates In these cases, we do not invoke any transformation or change in the manifold of integration, which we have argued is a questionable operation.

As we will see in the next section, the use of Metric Independent Non-Riemannian Volume-Forms and Volume elements allows us to resolve this issue with no such restrictions on possible changes of signs of the jacobian of the coordinate transformation in different regions of space time and without invoking non local actions.

IV. METRIC INDEPENDENT NON-RIEMANNIAN VOLUME-FORMS AND VOLUME ELEMENTS INVARIANT UNDER SIGNED GENERAL COORDINATE TRANSFORMATIONS

One can define a metric independent measure from a totally anti symmetric tensor gauge field, for example

$$\Phi(A) = \frac{1}{3!}\varepsilon^{\mu\nu\kappa\lambda}\partial_\mu A_{\nu\kappa\lambda} \quad , \tag{3}$$

Then, under a general coordinate transformation

$$\Phi(A) \to J^{-1}\Phi(A)$$

. . Therefore $d^4x\Phi(A) \to d^4x\Phi(A)$, so invariance is achieved regardless of the sign of J.

Some ideas concerning theories using these measures

First we review our previous papers where we have considered the action of the general form involving two independent non-metric integration measure densities generalizing the model analyzed in [22] is given by

$$S = \int d^4x\, \Phi_1(A)\Big[R + L^{(1)}\Big] + \int d^4x\, \Phi_2(B)\Big[L^{(2)} + \epsilon R^2 + \frac{\Phi(H)}{\sqrt{-g}}\Big] . \quad (4)$$

Here the following definitions are used:

- The quantities $\Phi_1(A)$ and $\Phi_2(B)$ are two densities and these are independent non-metric volume-forms defined in terms of field-strengths of two auxiliary 3-index antisymmetric tensor gauge fields

$$\Phi_1(A) = \frac{1}{3!}\varepsilon^{\mu\nu\kappa\lambda}\partial_\mu A_{\nu\kappa\lambda} \quad , \quad \Phi_2(B) = \frac{1}{3!}\varepsilon^{\mu\nu\kappa\lambda}\partial_\mu B_{\nu\kappa\lambda} . \quad (5)$$

The density $\Phi(H)$ denotes the dual field strength of a third auxiliary 3-index antisymmetric tensor

$$\Phi(H) = \frac{1}{3!}\varepsilon^{\mu\nu\kappa\lambda}\partial_\mu H_{\nu\kappa\lambda} . \quad (6)$$

- The scalar curvature $R = g^{\mu\nu} R_{\mu\nu}(\Gamma)$ and the Ricci tensor $R_{\mu\nu}(\Gamma)$ are defined in the first-order (Palatini) formalism, in which the affine connection $\Gamma^{\mu}_{\nu\lambda}$ is *a priori* independent of the metric $g_{\mu\nu}$. Let us recall that $R + R^2$ gravity within the second order formalism was originally developed in [3].

- The two different Lagrangians $L^{(1,2)}$ correspond to two matter field Lagrangians

On the other hand, the variation of (4) w.r.t. auxiliary tensors $A_{\mu\nu\lambda}$, $B_{\mu\nu\lambda}$ and $H_{\mu\nu\lambda}$ becomes

$$\partial_\mu \Big[R + L^{(1)}\Big] = 0 \quad , \quad \partial_\mu \Big[L^{(2)} + \epsilon R^2 + \frac{\Phi(H)}{\sqrt{-g}}\Big] = 0 \quad , \quad \partial_\mu \Big(\frac{\Phi_2(B)}{\sqrt{-g}}\Big) = 0 \,, \tag{7}$$

whose solutions are

$$\frac{\Phi_2(B)}{\sqrt{-g}} \equiv \chi_2 = \mathrm{const} \ , \ R + L^{(1)} = -M_1 = \mathrm{const} \ , \ L^{(2)} + \epsilon R^2 + \frac{\Phi(H)}{\sqrt{-g}} = -M_2 = \mathrm{const} \,. \tag{8}$$

Here the parameters M_1 and M_2 are arbitrary dimensionful and the quantity χ_2 corresponds to an arbitrary dimensionless integration constant.

The resulting theory is called a Two Measure Theory, due to the presence of the Two measures $\Phi_1(A)$ and $\Phi_2(A)$.

But for the purpose of this paper this is two general, since we want to restrict to a theory that will give us ordinary General Relativity, and we want to keep the general coordinate invariance under signed general coordinate invariance.

For obtaining GR dynamics, we can restrict to one measure, so let us take

$$\Phi_1(A) = \Phi_2(B) = \Omega$$

also to make some contact for example with [9] , where an additional set of four fields is introduced, we express Φ in terms of four scalar fields

$$\Omega = \frac{1}{3!}\varepsilon^{\mu\nu\kappa\lambda}\varepsilon^{abcd}\partial_\mu\varphi_a\partial_\nu\varphi_b\partial_\kappa\varphi_c\partial_\lambda\varphi_d \tag{9}$$

(one has to point out that in the earlier formulations of modified measures theories we used the 4 scalar field representation for the measure, see [12] ,) The mapping of the four scalars to the coordinates x^μ may be topologically non trivial, as in [9] and this multivaluedness could be of use to obtain Linde´s Universe multiplication as well. Finally, we have to correct the equation

$$\frac{\Phi_2(B)}{\sqrt{-g}} \equiv \chi_2 = \text{const.} \tag{10}$$

for another equation the will be invariant under signed general coordinate invariant transformations, which will be

$$\frac{\Omega^2}{(-g)} \equiv \chi = K^2 = \text{const} > 0. \tag{11}$$

without loss of generality we define K to be positive. The resulting action that replaces (5) is,

$$S = \int d^4x\,\Omega\Big[R + L\Big] + \int d^4x\,\Omega^2\Big[\frac{\Phi(H)}{(-g)}\Big]\,. \tag{12}$$

the density $\Phi(H)$ remains defined eq. (6) so the integration obtained from the variation of the H gauge field is eq. (11) now. The solution of eq. (11) are

The solution of eq. (11) are

$$\frac{\Omega}{\sqrt{(-g)}} = \pm K.$$

VI. THE INVARIANT SCALAR INTEGRATION MANIFOLD AND INVARIANT LAGRANGIAN DENSITY

Notice that using the volume element converts the the integration over coordinates in the action into integration over scalar fields, since

$$\Phi d^4x = d\varphi_1 d\varphi_2 d\varphi_3 d\varphi_4$$

The integration manifold existing in the four scalar field manifold is in fact completely unaffected by any coordinate transformation taking place in the x space. The lagrangian density is also a scalar not affected by any coordinate transformation, the theory formulated in this way does not require any boundary terms if the boundaries are for example formulated in the scalar field space.

In the case of (12) for example,

$$S = \int d\varphi_1 d\varphi_2 d\varphi_3 d\varphi_4 L$$

, where

$$L = \Big[R + L\Big] + \Omega\Big[\frac{\Phi(H)}{(-g)}\Big]$$

VII. GRAVITATIONAL EQUATIONS OF MOTION

We start by considering the equation that results from the variation of the degrees of freedom that define the measure Ω, that is the scalar fields φ_a, these are,

$$A^{\mu}_{a}\partial_{\mu}(R + L + 2\Omega\frac{\Phi(H)}{(-g)}) = 0 \tag{14}$$

where

$$A^{\mu a} = \frac{1}{3!}\varepsilon^{\mu\nu\kappa\lambda}\varepsilon^{abcd}\partial_{\nu}\varphi_b\partial_{\kappa}\varphi_c\partial_{\lambda}\varphi_d \tag{15}$$

Notice that the determinant of $A^{\mu a}$ is proportional to Ω^3, so if the measure is not vanishing, the matrix $A^{\mu a}$ is non singular and therefore $\partial_{\mu}(R + L + 2\Omega\frac{\Phi(H)}{(-g)}) = 0$, so that,

$$R + L + 2\Omega\frac{\Phi(H)}{(-g)} = M = constant \tag{16}$$

The variation with respect to the metric $g^{\mu\nu}$, we obtain.

$$\Omega(R_{\mu\nu} + \frac{\partial L}{\partial g^{\mu\nu}}) + g_{\mu\nu}\Omega^2\frac{\Phi(H)}{(-g)} = 0 \tag{17}$$

solving $\Omega\frac{\Phi(H)}{(-g)}$ from (16) and inserting into (17), we obtain,

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \frac{1}{2}Mg_{\mu\nu} + \frac{\partial L}{\partial g^{\mu\nu}} - \frac{1}{2}g_{\mu\nu}L = 0 \tag{18}$$

which gives exactly the form of Einstein equation with the canonical energy momentum defined from L

$$T_{\mu\nu} = g_{\mu\nu}L - 2\frac{\partial}{\partial g^{\mu\nu}}L\ .$$

One issue that should be addressed is that of the gauge fixing in the φ_a space. Indeed, we notice that the only thing where these fields appear in the equations of motion is Ω, but this quantity is invariant under volume preserving diffeomorphisms of the fields φ_a, $\varphi'_a = \varphi'_a(\varphi_a)$ which satisfy

$$\epsilon_{a_1 a_2 a_3 a_4} \frac{\partial \varphi'_{b_1}}{\partial \varphi_{a_1}} \frac{\partial \varphi'_{b_2}}{\partial \varphi_{a_2}} \frac{\partial \varphi'_{b_3}}{\partial \varphi_{a_3}} \frac{\partial \varphi'_{b_4}}{\partial \varphi_{a_4}} = \epsilon_{b_1 b_2 b_3 b_4} \tag{20}$$

so the study of the best gauge for the φ_a fields for further comparison with the x^μ space could be a very important subject. Of course when we say that the mapping between the φ_a and the x^μ spaces, we want to exclude multi valuedness due to volume preserving diffeomorphisms of the fields φ_a, if for example different signs for Ω are associated to the same point in x^μ space, it is clear that there are at least two points in φ_a space associated to one point in x^μ space, and these two points in the φ_a are not related through a volume preserving diff. This could be an effect analogous to the Universe Multiplication of Linde.

VIII. LINDE´S UNIVERSE MULTIPLICATION AND RELATION TO A BRANE ANTI BRANE SYSTEM AND MEASURE FIELD MULTIVALUEDNESS INSTEAD OF NON LOCALITY

We can immediately see some similar features between the Linde universe multiplication as described by eq. (1) and the modified measure theory, with the measure assuming a positive or a negative value, as expressed by eq, (13), instead of the obvious non locality of the Linde approch, the modified measure approach can offer instead multi valued feature of the φ_a space with respect to the x^μ space. The double solution for the measure (13) can be valid for the same coordinate x^μ , which may correspond however to non unique values in the φ_a space. The doubling of the measure (13) has its correspondence in the signed reparametrization invariant formulation of modified measure [38] and the corresponding existence of strings and antistrings as well as branes and anti branes in such fromulation.

What your measure fields can do !

IX. TURNING MANIFOLDS WITH BOUNDARIES INTO MANIFOLDS WITHOUT BOUNDARIES, THE PROBLEM OF TIME IN QUANTUM COSMOLOGY, QUANTUM SPACE TIME, SPACE TIMES AND ANTI SPACE TIMES

As we will discuss in this section, these concepts can have interesting applications to quantum cosmology: 1) the consideration of one of the measure fields as a time, instead of the ordinary coordinate time can transform a manifold with boundaries, as a universe with an origin of time, which is difficult to handle, into a manifold which avoids these difficulties in by considering that the fundamental manifold in terms of the measure fields does not have boundaries and 2) the problem of time, associated with the coordinate time, which produces vanishing Hamiltonians is avoided. Considering instead one of the measure fields as time avoids this problem and lead us to a new formulation of a quantum space time, 3) using the measure fields as the basis to construct the quantum space time 4) defining space times and antispacetimes, this is in fact related to 1).

Indeed, concerning the first point, in most cosmological models we assume that the universe has an origin in time, this leads then to a manifold with boundaries if we think the fundamental manifold is coordinate space and time. A well known example of such a phenomena is the pair creation in QED, which we review in the next subsection.

First look at a well known example in QED

A. Pair Creation in a Strong Uniform Electric Field in QED, as an example of turning a manifold with boundary in time to a manifold without boundary in proper time

From the 'Feynman' perspective, negative energy waves propagating into the past are physically realized as the antiparticles propagating into the future. As he has shown [32] that from the classical equations of motion for a particle in an external field can be written as

$$m\frac{d^2 z^\mu}{d\tau^2} = e\frac{dz_\nu}{d\tau}F^{\mu\nu} \tag{21}$$

where τ is the proper time. If we note as Feynman has that if we allow $\tau \to -\tau$ the equation becomes

$$m\frac{d^2 z^\mu}{d\tau^2} = -e\frac{dz_\nu}{d\tau}F^{\mu\nu} \tag{22}$$

which is identical to the previous equation except that the particles charge has changed. In other words, as far as its charge is concerned, it has become the antiparticle. Thus, proper time running backward (i.e., $\tau \to -\tau$), while keeping the coordinate time unchanged. led to the particle becoming an antiparticle. Of course we can take the equivalent, but more suitable for our purposes transformation that we change the direction of coordinate time, while requiring that the proper time remains unchanged.

This would be exactly analogous to taking one of the four scalars as an analogous of the proper time and the coordinate time, which would be another entity.

particle in a field is most convenient written in terms of the Maxwell tensor $F_{\mu\nu}$ where for a constant electric $\mathbf{E}$ in the $x-$direction, $F^{01} = \; - \; E$, $F^{10} = E$ $F^0_{\;1} = E$, $F^1_{\;0} = E$. More explicitly, we have

$$F^{\mu}{}_{\nu} = \begin{bmatrix} 0 & E & 0 & 0 \\ E & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{23}$$

The equation of motion for this particle in a constant electric field is

$$m\frac{d^2x^{\mu}}{d\tau^2} = eF^{\mu}{}_{\nu}\frac{dx^{\nu}}{d\tau} \tag{24}$$

The formal solution of for $u^{\mu} = \frac{dx^{\mu}}{d\tau}$ is $u^{\mu}(\tau) = \exp[\frac{e}{m}F^{\alpha}{}_{\beta}\tau]^{\mu}_{\nu}u^{\nu}(0)$ The exponential can be expanded and we have

$$\exp[\frac{e}{m}F^{\alpha}{}_{\beta}\tau]^{\mu}_{\nu} = \delta^{\mu}_{\nu} + \frac{e}{m}\tau E\Delta^{\mu}_{\nu} + \frac{1}{2}(\frac{e}{m}\tau E\Delta^{\mu}_{\nu})^2 + \cdots \tag{25}$$

where

$$\Delta = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{26}$$

Separating even and odd power in Eq. 25 we have

$$\begin{aligned} u^0 &= \cosh\left(\frac{eE\tau}{m}\right)u^0(0) + \sinh\left(\frac{eE\tau}{m}\right)u^1(0) \\ u^1 &= \sinh\left(\frac{eE\tau}{m}\right)u^0(0) + \cosh\left(\frac{eE\tau}{m}\right)u^1(0) \end{aligned} \tag{27}$$

Integrating with respect to τ yields (where we have dropped arbitrary constants of integration)

$$\begin{aligned} x^0 &= \frac{m}{eE}\left\{\sinh\left(\frac{eE\tau}{m}\right)u^0(0)+\cosh\left(\frac{eE\tau}{m}\right)u^1(0)\right\} \\ x^1 &= \frac{m}{eE}\left\{\cosh\left(\frac{eE\tau}{m}\right)u^0(0)+\sinh\left(\frac{eE\tau}{m}\right)u^1(0)\right\} \end{aligned} \tag{28}$$

We choose the following boundary conditions $u^1(0)=0,\ \ u^0(0)=1$ which leads to

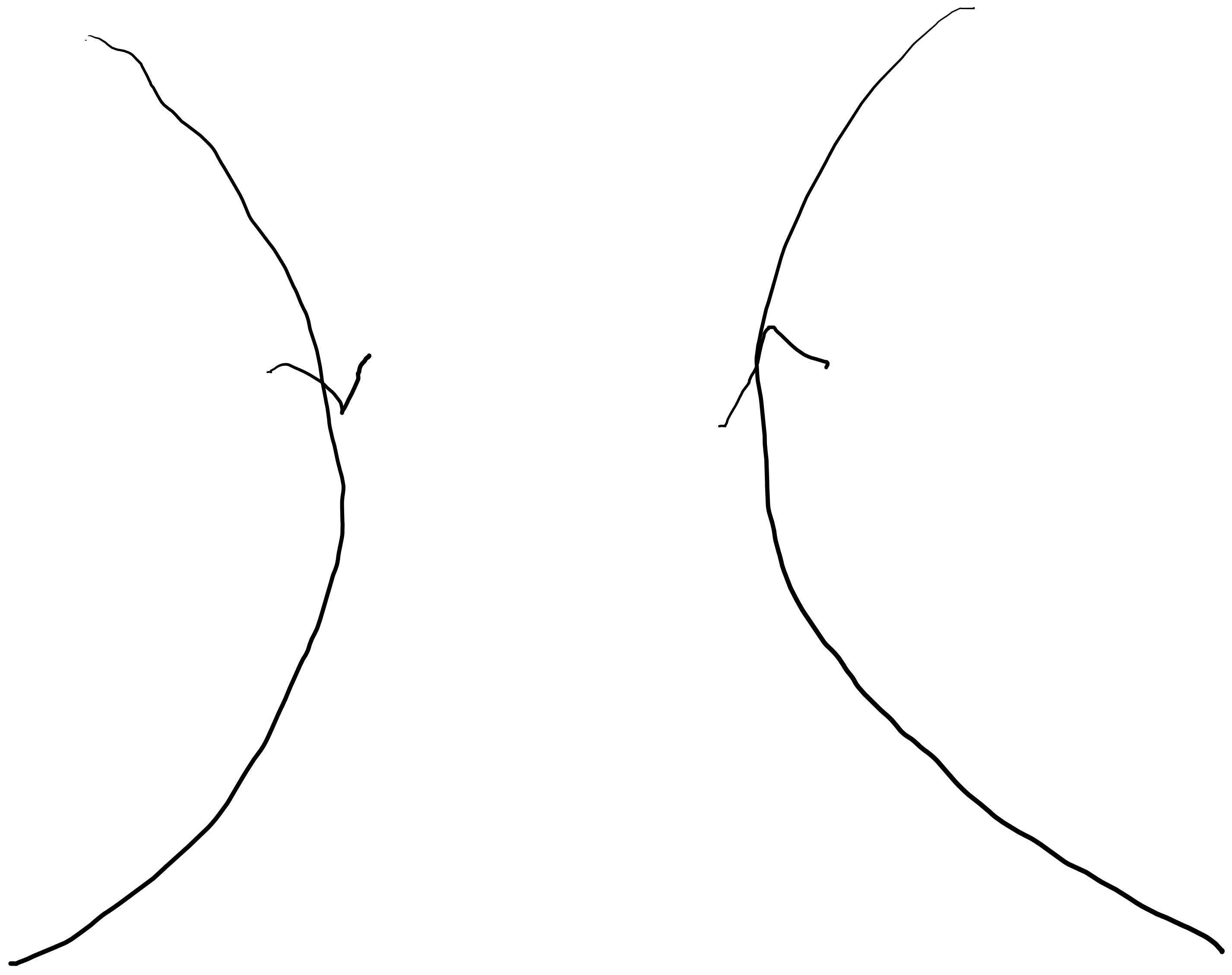
$$\begin{aligned} x^0 &= \frac{m}{eE}\sinh\left(\frac{eE\tau}{m}\right) \\ x^1 &= \frac{m}{eE}\cosh\left(\frac{eE\tau}{m}\right) \end{aligned} \tag{29}$$

and for the boundary condition $u^1(0)=0,\ \ u^0(0)=-1$ leads to

$$\begin{aligned} x^0 &= -\frac{m}{eE}\sinh\left(\frac{eE\tau}{m}\right) \\ x^1 &= -\frac{m}{eE}\cosh\left(\frac{eE\tau}{m}\right) \end{aligned} \tag{30}$$

The solution given by Eq. 29 represents a particle solution while the solution of Eq. 30 represents the anti-particle solution. Both solutions, together satisfy

$$(x^1)^2-(x^0)^2=\left(\frac{m}{eE}\right)^2 \tag{31}$$



At classical level, these solutions are distinct and one solution can not evolved into the other. Thus, a particle couldn't evolve to an anti-particle. However, the semi-classical approximation which consists of considering the classical equations of motion but with imaginary time. Then inserting $t = -it_E$ we obtain that the hyperbola of Eq. 31 becomes a circle

$$(x^1)^2 + t_E^2 = \left(\frac{m}{eE}\right)^2 \tag{32}$$

This tunneling solution can now interpolate between the anti-particle and particle solutions. In the imaginary time region, the action, $S = -iS_E$ where S_E is given by

$$S_E = \int dt_E \left\{ m\sqrt{1 + \left(\frac{dx}{dt_E}\right)^2} - eEx \right\} \tag{33}$$

Introduction the angular variable θ where $x = \frac{m}{eE}\cos\theta, \quad t_E = \frac{m}{eE}\sin\theta$ we obtain that $S_E = \pi m^2/eE$. Since the probability is given (up to pre factors) by $\exp(-S_E) = \exp(-\frac{\pi m^2}{eE})$ for $eE > 0$. Notice that the distance of the particle and antiparticle at the moment of creation is $\Delta x = \frac{2m}{eE}$ which has a physical interpretation is manifest in writing it as

$$W = eE\Delta x = 2mc^2 \tag{34}$$

where we have restored the 'c' to make its physical meaning clearer. Thus we can create a pair of particle-antiparticle in a constant electric field by performing work, W in a distance Δx equal to the the sum of the rest masses of the particles, i.e. $2mc^2$. In the case for an electric field that doesn't extend through all of space, still it must be extended enough to perform the work equal to the sum rest masses of the two particles in order to create a pair.

From the point of view of our discussion here, we see that in the pair creation process, the proper time proceeds from minus infinity to plus infinity, while for this manifold the minimum coordinate time sits in euclidean space at $t_E = -\frac{m}{eE}$, the coordinate manifold is a manifold with boundaries, while the proper time manifold does not have boundaries associated with it.

NO BOUNDARIES IN PROPER TIME!

Another problem with the use of coordinate time in cosmology that we could resolve now !

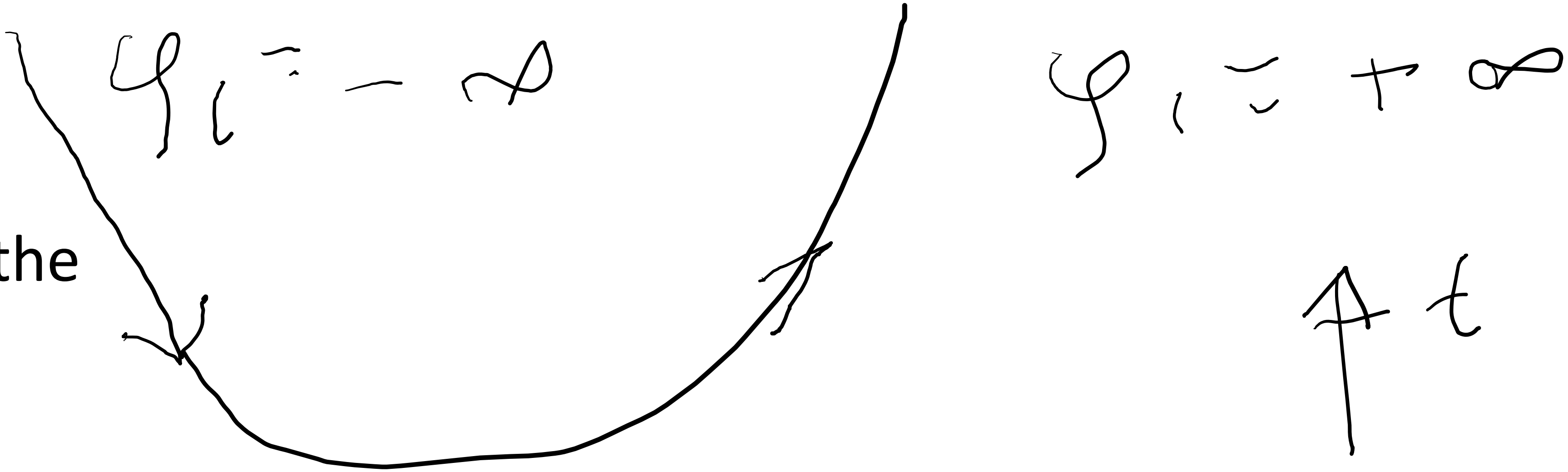
B. Resolution of The problem of Time in Quantum Cosmology using one of the measure fields as time

A related difficulty related to the use of coordinate time in quantum cosmology is that the invariance under reparametrizations in the coordinate time leads to the constraint that the Hamiltonian equals zero. In this case the analogous of rge Schroedinger equation for Quantum gravity will tell us that the wave function is time independent.

In the presence of this problem, the ¨time problem in Quantum Cosmology¨, many proposals for alternative fields to replace the cosmic time were proposed, for a review see [35].

To choose the corresponding time among the four measure fields, we can use the volume preserving diffeomorphisms (20) or the general coordinate invariance to choose three of the measure scalars as the three spacial coordinates x, y, z, the remaining scalar can be defined as the time like coordinate, which cas run in the same direction of the coordinate time or against.

Is our universe bouncing or created from nothing?



1, If one of the Measure fields is the Physical time, it is BOUNCING.

2. If the coordinate time is the physical time, then the universe is created from nothing as in Hawking picture.

I prefer 1, we live in the $(\varphi_1, \varphi_2, \varphi_3, \varphi_4)$ space, we said, means BOUNCE!

C. Quantum Space Time

By considering a gauge (of the volume preserving diffeomorphisms or the general coordinate invariance groups) where three of the coordinates (t, x, y, z), say (x, y, z) are set to be equal to three of the measure fields, we define a procedure by means of which coordinates become dynamical , since the measure fields are real dynamical variables and space is now represented by the measure fields. Notice that these measure fields have non vanishing canonically conjugated momenta, etc. so they represent the dynamical and quantized space time (for some papers and a review of quantum space time see [36]).

One can see that the for the canonically conjugate momenta for the measure fields, the system shows constraints that relate the canonically conjugate momenta with the spacial derivatives of these measure gauge fields, indeed,

$$\pi^{\varphi_a} = M\epsilon^{aa_1a_2a_3}\epsilon^{ijk}\partial_i\varphi_{a_1}\partial_j\varphi_{a_2}\partial_k\varphi_{a_3} \tag{35}$$

Showing a constraint that relates the canonically conjugate momenta of the measure fields with the spacial derivatives of these fields. The commutators could be calculated not from the Poison brackets but from the Dirac brackets [37], that are defined from the Poisson brackets of the constraints, including the gauge fixing, like the ones we have mentioned. On the mass shell M is a constant, but off the mass shell it must be defined as $M = R + L + 2\Omega\frac{\Phi(H)}{(-g)}$. The quadratic term in Ω in the action implies that the dependence of the action on the measure time derivatives is not linear, which is good because lagrangian systems linear in the time derivatives of some variable have some difficulties.

If the Dirac brackets between the measure fields can be non vanishing, even if the Poisson brackets are zero, we could obtain a non commutative space time .

DIRAC BRACKETS

At this point, the second class constraints will be labeled $\tilde{\phi}_a$. Define a matrix with entries

$$M_{ab} = \{\tilde{\phi}_a, \tilde{\phi}_b\}_{PB}.$$

In this case, the Dirac bracket of two functions on phase space, f and g, is defined as

$$\{f, g\}_{DB} = \{f, g\}_{PB} - \sum_{a,b} \{f, \tilde{\phi}_a\}_{PB} M_{ab}^{-1} \{\tilde{\phi}_b, g\}_{PB} \ ,$$

where $M^{-1}{}_{ab}$ denotes the ab entry of M's inverse matrix. Dirac proved that M *will always be invertible*.

D. Particles vs antiparticles, Strings vs anti Stings , Branes vs anti Branes and Space Time vs anti Space Time

We have reviewed already the discussion of particles vs antiparticles due to Feynman, which consisted of interpreting antiparticles as particles where the proper time moves in the opposite direction of the coordinate time.

In the case of extended objects, which also can be formulated in terms of modified measures, as we have mentioned before, and in particular, one can advance the concepts of strings vs anti strings and that of branes vs anti branes [38] . The antistrings are identified, in analogy with the Feynman picture for antiparticles when the sign of the Riemannian measure is opposite to the metric independent measure, defined in terms of world sheet scalars, so the same rule should be applied with the identification of the anti space time in comparison with the space time. To see this more clearly, in the case of a string, we can use the volume preserving diffeomorphisms to set one measure field to a spatial world sheet variable, then the remaining measure field is a time like coordinate that can go in the direction of the world sheet coordinate or the opposite, which is the difference between strings and anti strings and similar for branes and anti branes.

For a $4D$ space time (or we can say a space time filling brane), we use the volume preserving diffeomorphisms to set a gauge where three measure fields are set equal to three spatial coordinates, then the remaining measure field is the analog of the proper time in the particle case, which can run in the same direction or oposite to the coordinate time, defining a space time or an anti space time.

For the future,

One could also go beyond General Relativity like theories, and generalize the modified measure theory (4) in the following way,

$$S = \int d^4x \, \Phi_1(A) \left[R + L^{(1)} \right] + \int d^4x \, \Phi_2(B) \left[L^{(2)} + \epsilon R^2 + \Phi_2(B) \frac{\Phi(H)}{(-g)} \right] . \qquad (36)$$

- Study in more details consequences the proposed quantum space time in different gauges that connect measure fields and coordinates.
- Study in more detail universe creation with a no boundary formulation and without nasty boundary terms thanks to the coordinate invariant measuse fields manifolds, etc

THANK YOU FOR YOUR ATTENTION!!!