Phenomenology of DSR-relativistic in-vacuo dispersion in FLRW spacetime

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Introduction

- In many quantum gravity models Planck scale-deformed dispersion relations are considered ($E_{QG} \sim E_{Pl} \sim 10^{19}$Gev).

- Energy dependent speed of light $\mathrm{v} \approx 1 + \eta \frac{E}{E_{Pl}}$ (where η is a dimensionless parameter).

- Difference in the time of flight of photons amplified by cosmological distances $\Delta t \approx \eta \, L \frac{\Delta E}{E_{Pl}}$.

“Tests of quantum gravity from observations of gamma-ray bursts”, Amelino-Camelia, Ellis, Mavromatos, Nanopoulos, *Nature* 393 (1998) 763-765

LIV vs DSR

• LIV

- Deformed dispersion relations but undeformed Poincaré transformations to connect inertial reference frames

→Relativistic invariance is broken → preferred reference frame.

• DSR

- Deformed dispersion relations, deformed composition laws of momenta and deformed Poincaré transformations to connect inertial reference frames in order to accomodate a new energy scale $\sim E_{Pl}$ →

The relativity principle is preserved.

“Relativity in space-times with short distance structure governed by an observer independent (Planckian) length scale”, Amelino-Camelia, *Int.J.Mod.Phys.D* 11 (2002) 35-60

1+1-D MINKOWSKI SPACETIME

Deformed dispersion relations (LIV)

- Deformed dispersion relation at first order in $\frac{1}{E_{Pl}}$:

$$m^2 = E^2 - P^2 + \frac{1}{E_{Pl}}(\alpha E^3 + \beta E P^2)$$

- Undeformed Poincaré transformations to connect inertial reference frames

$$[E, P] = 0\,; \qquad [N, E] = P; \qquad [N, P] = E$$

- Energy dependent time of flight of photons:

$$m = 0 \rightarrow v = \frac{\partial E}{\partial P} = 1 - \frac{1}{E_{Pl}} P(\alpha + \beta) \rightarrow \Delta t = \frac{1}{E_{Pl}} \mathrm{L}\, \Delta E(\alpha + \beta)$$

Deformed dispersion relations(DSR)

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$$m^2 = E^2 - P^2 + \frac{1}{E_{Pl}}(\alpha E^3 + \beta E P^2)$$

- Deformed Poincaré transformations to connect inertial reference frames

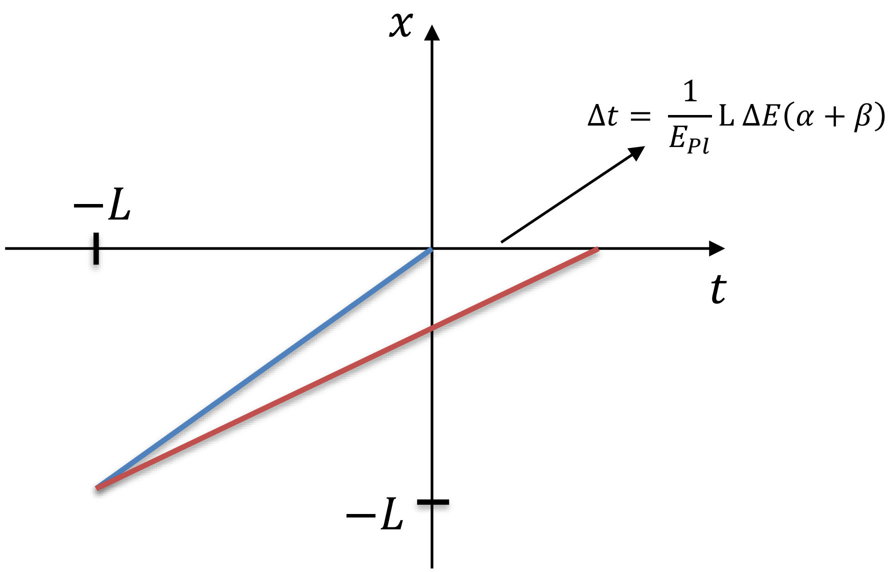
$$[E,P] = 0\,; \quad [N,E] = P - \frac{1}{E_{Pl}} EP(\alpha + \beta); \quad [N,P] = E + \frac{1}{2\,E_{Pl}} \alpha E^2 + \frac{1}{2E_{Pl}} \beta P^2$$

- Energy dependent time of flight of photons:

$$m = 0 \rightarrow \Delta t = \frac{1}{E_{Pl}} \mathrm{L}\, \Delta E(\alpha + \beta)$$

“Taming Nonlocality in Theories with Planck-Scale Deformed Lorentz Symmetry”, Amelino-Camelia,Matassa, Mercati, Rosati, *Phys.Rev.Lett.* 106 (2011) 071301

Time delay



- In blue the worldline of a soft photon for which we neglect Planck scale corrections.
- In red the worldline of an hard photon.

CURVED SPACETIME

Jacob and Piran formula in FLRW spacetime

$$\Delta t = \frac{\eta_1 \, D(z)}{E_{Pl}} \Delta E = \frac{\eta_1}{E_{Pl}} \Delta E \int_0^z \frac{d\,x}{H(x)} (1+x)$$

- This formula can be obtained

  1) assuming that the comoving distance, traveled by both particles and emitted from the same source, is the same.
  2) using the relation $E = \frac{E_0}{1+z}$ (where E_0 is the energy at source and E is the measured energy).

- ΔE is the measured energy difference .

- z is the redshift of the source and $H(z) = \mathrm{H}_0 \sqrt{\Omega_M (1+z)^3 + \Omega_\Lambda}$.

"Lorentz-violation-induced arrival delays of cosmological particles", Jacob, Piran, *JCAP* 01 (2008) 031

LIV generalization of the time delay formula in FLRW

- Symmetries in a LIV model can be broken in many ways, therefore the most general LIV formula for the time delay can contain a very large number of terms.

- An example of time delay formula that generalizes the Jacob and Piran result is:

$$\Delta t = \frac{\widetilde{D}(z)}{E_{Pl}}\Delta E = \frac{1}{E_{Pl}}\Delta E \int_0^z \frac{d\,x}{H(x)}\left(\eta_1(1+x) + \eta_2 + \eta_3(1+x)^2 + \eta_4\frac{1}{(1+x)} + \cdots\right)$$

“ Planck-scale-modified dispersion relations in FRW spacetime” , Rosati, Amelino-Camelia, Marciano, Matassa, *Phys.Rev.D* 92 (2015) 12, 124042

DSR in 1+1-D de Sitter spacetime

- The most general deformation of the de-Sitter dispersion relation at first order in $\frac{1}{E_{Pl}}$ is:

$$m^2 = E^2 - P^2 - 2HNP + \frac{1}{E_{Pl}}(\alpha\, E^3 + \beta\, EP^2 + 2\gamma\, HNEP + 4\mu\, H^2N^2E)$$

- The algebra of symmetry generators that leaves the previous dispersion relation invariant is:

- $$[E, p] = Hp - \frac{1}{E_{Pl}} HE[\, (\alpha + \gamma - \sigma)p + 4\mu\, HN]$$
- $$[N, E] = p + HN - \frac{1}{E_{Pl}} E[(\alpha + \beta - \sigma)p + HN(\alpha + \gamma - \sigma)]$$
- $$[N, p] = E + \frac{1}{2E_{Pl}}[(\alpha + 2\sigma)E^2 + \beta\, p^2 + 2\gamma\, HNp + 4\mu\, H^2N^2].$$

Deformed composition laws

- An important ingredient of DSR models concerns the conservation law of energy-momenta that must be deformed in order to be compatible with the previous deformed symmetry algebra.

- $E_{tot} = E_1 + E_2 + \frac{1}{E_{Pl}}\left((2\sigma - \beta - a - b)P_1P_2 + (c - \gamma + \sigma)H\,(N_1P_2 + P_1N_2) - \alpha\, E_1E_2 + 2(c - 2\mu)H^2N_1N_2\right)$

- $P_{tot} = P_1 + P_2 + \frac{1}{E_{Pl}}\left((\sigma - b)E_1P_2 + (\sigma - a)E_2P_1 + cH(N_1E_2 + E_1N_2)\right)$

- $N_{tot} = N_1 + N_2 + \frac{1}{E_{Pl}}(aE_1N_2 + bE_2N_1)$

Most general DSR formula for time delays in FLRW

$$\Delta t = \frac{\Delta E}{E_{Pl}} \int_0^z \frac{d\,x}{H(x)} (1+x) \left(\eta_1 + \eta_2 \left(1 - \left(1 - \frac{H(x)}{1+x} \int_0^x \frac{d\,y}{H(y)} (1+y) \right)^2 \right) + \eta_3 \left(1 - \left(1 - \frac{H(x)}{1+x} \int_0^x \frac{d\,y}{H(y)} (1+y) \right)^4 \right) \right)$$

- $\eta_1 = (\alpha + \beta)$, $\eta_2 = (-\alpha - \gamma + \sigma + 2\mu)$, $\eta_3 = -\mu$.
- ΔE is the measured energy difference.
- z is the redshift of the source and $H(z) = \mathrm{H}_0\sqrt{\Omega_M(1+z)^3 + \Omega_\Lambda}$.
- The most general DSR formula contains the Jacob e Piran term (η_1) and just two new terms allowed.

“ Planck-scale-modified dispersion relations in FRW spacetime” , Rosati, Amelino-Camelia, Marciano, Matassa, *Phys.Rev.D* 92 (2015) 12, 124042

“Amelino-Camelia, Frattulillo, Gubitosi, Rosati, Bedic, arXiv:2307.05428v1 “

SOME NOTEWORTHY SPECIAL CASES

Curvature induced scenarios

- A scenario with interesting phenomenological implications is the one where the quantum gravity effects are triggered by spacetime curvature.

- Only terms involving powers of z higher than 1 contribute to curvature-induced time-delay effects.

Considering the leading order expansion in terms of the redshift of the time delay expression, we obtain

$$\Delta t = \frac{\Delta E}{E_{Pl}} \frac{1}{H_0} \eta_1 \, z + O(z^2),$$

thus, we have to impose $\eta_1 = 0$.

“ Phenomenology of curvature induced quantum gravity effects” , Amelino-Camelia, Rosati, Bedic, *Phys.Lett.B* 820 (2021) 136595

Scenarios with undeformed addition of energy

- In DSR models a desirable feature is that the addition law of particle energies remains undeformed.

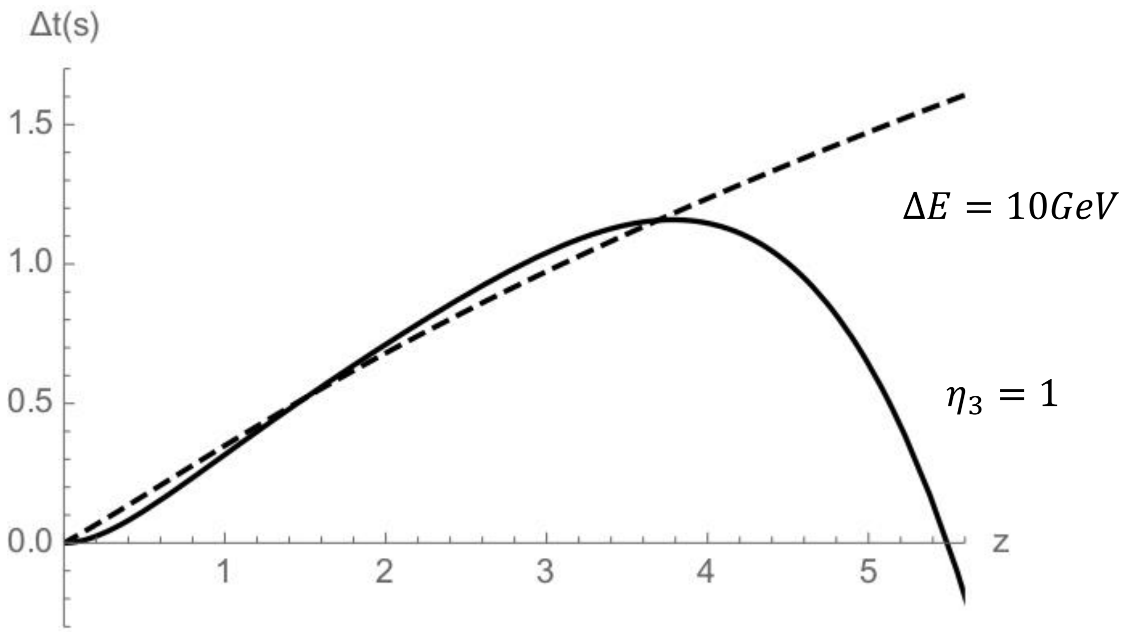
1. Preserving the linearity of addition of energies is advantageous from the point of view of the interpretation of the results.

2. Undeformed composition law of energies would correspond (in the framework of quantum groups) to a “primitive coproduct” for time-translation generators, that is necessary for having a “time-like” q-deformation of de Sitter symmetries.

- In order to have an undeformed addition of energy we have to impose $\eta_2 = 0$.

A one-parameter scenario: curvature-induced and undeformed addition of energy

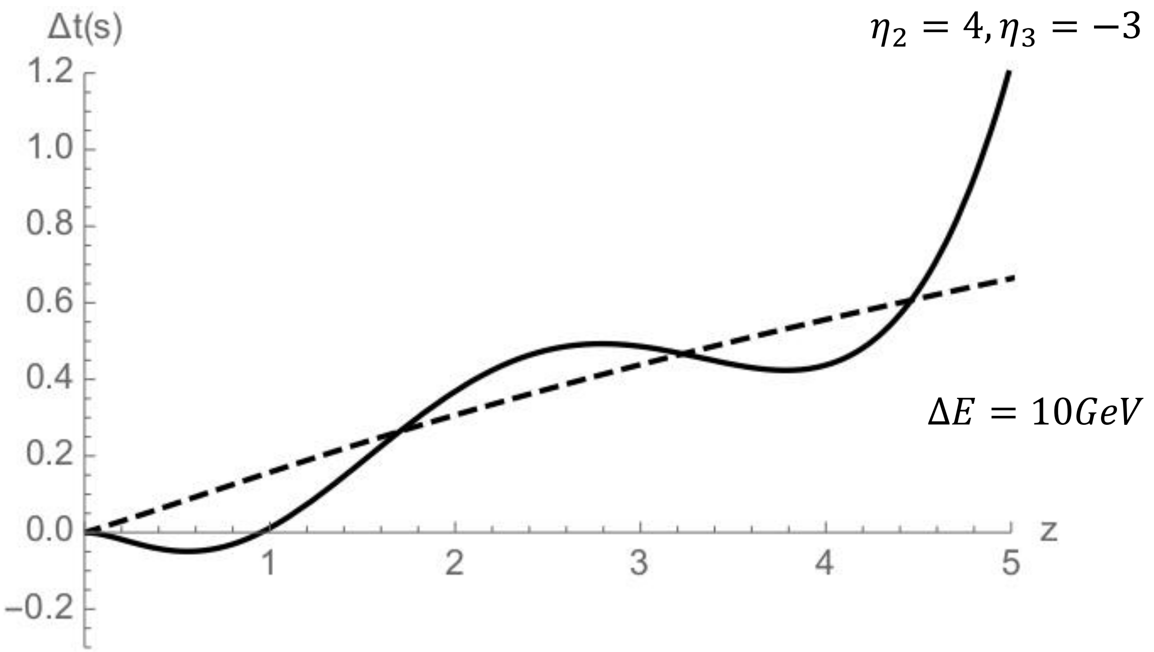
$$\Delta t = \frac{\Delta E}{E_{Pl}} \int_0^z \frac{d\,x}{H(x)} (1+x) \left(\eta_3 \left(1 - \left(1 - \frac{H(x)}{1+x} \int_0^x \frac{d\,y}{H(y)} (1+y) \right)^4 \right) \right)$$



The dashed line represents the expected time delay for the “Jacob-Piran” case and it is normalized imposing that the two lines cross at z = 1.5

Scenarios with time delay changing sign

$$\Delta t = \frac{\Delta E}{E_{Pl}} \int_0^z \frac{d\,x}{H(x)} (1+x) \left(\eta_2 \left(1 - \left(1 - \frac{H(x)}{1+x} \int_0^x \frac{d\,y}{H(y)} (1+y) \right)^2 \right) + \eta_3 \left(1 - \left(1 - \frac{H(x)}{1+x} \int_0^x \frac{d\,y}{H(y)} (1+y) \right)^4 \right) \right)$$



The dashed line represents the expected time delay for the “Jacob-Piran” case, and it is normalized imposing that the two lines cross at z = 1.7

Conclusions

- We derived the most general DSR formula for time delays in FLRW spacetimes.

- This formula contains the Jacob and Piran term and just two new terms are allowed.

- We analyzed some interesting scenarios which present deviations from the famous formula proposed by Jacob and Piran for the LIV scenario, paving the way for novel phenomenological studies.

Thank you!