Implications of Palatini Gravity for Inflation and Beyond

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Minimal Inflation

- Inflation solves the **horizon** and **flatness** problems.
- When treated quantum-mechanically, it can also provide a mechanism for the generation of the perturbations that have resulted in the anisotropies observed in the CMB.

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{R}{2} - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right] , \qquad (M_{\mathrm{Pl}}^2 \equiv 1)$$

Friedmann equations:

$$\left(\frac{\dot{a}}{a}\right)^2 \equiv H^2 = \frac{1}{3}\left[\frac{\dot{\phi}^2}{2} + V\right]$$

$$\dot{H} = -\frac{1}{2}\dot{\phi}^2$$

Klein-Gordon equation:

$$\ddot{\phi} + 3H\dot{\phi} + V' = 0$$

Slow-roll Approximation (HSRPs)

Slow-roll approximation:

$$V(\phi) \gg \dot{\phi}^2\,, \qquad |\ddot{\phi}| \ll |3H\dot{\phi}|,\ |V'|$$

First **Hubble slow-roll parameter** (HSRP)

$$\epsilon_H = -\frac{\dot{H}}{H^2} = \frac{3\dot{\phi}^2}{\dot{\phi}^2 + 2V}\,, \quad \frac{\ddot{a}}{a} = H^2(1-\epsilon_H)$$

Inflation ends **exactly** when $\epsilon_H = 1$.

Second HSRP

$$\eta_H = -\frac{\ddot{\phi}}{H\dot{\phi}}$$

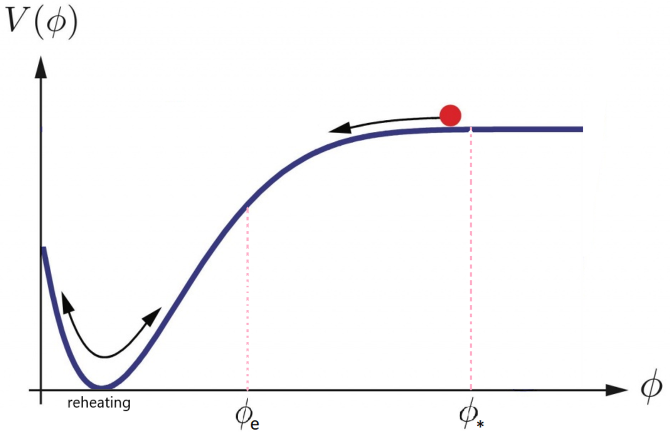
Friedmann and Klein-Gordon equations become

$$H^2 \approx \frac{1}{3}V(\phi)\,, \quad \dot{\phi} \approx -\frac{V'}{3H}\,.$$

Slow-roll Approximation (PSRPs)

The shape of the potential is encoded in the **potential slow-roll parameters**

$$\epsilon_V = \frac{1}{2}\left(\frac{V'}{V}\right)^2 , \qquad \eta_V = \frac{V''}{V}$$



Number of e-folds and Inflationary Observables

The scalar curvature power spectrum is observed to have a power-law form

$$\mathcal{P}_\zeta(k) = A_s \left(\frac{k}{k_*}\right)^{n_s-1} , \quad A_s = \frac{1}{24\pi^2}\frac{V(\phi_*)}{\epsilon_V(\phi_*)} \simeq 2.1\times 10^{-9} \;\; @ \; k_* = 0.05\,\mathrm{Mpc}^{-1}$$

The spectral tilt is

$$n_s - 1 \equiv \frac{\mathrm{d}\ln\mathcal{P}_\zeta(k)}{\mathrm{d}\ln k} \simeq -6\epsilon_V + 2\eta_V \,,$$

Tensor power spectrum

$$\mathcal{P}_T = 8\left(\frac{H}{2\pi}\right)^2 \simeq \frac{2V}{3\pi^2}$$

The tensor-to-scalar ratio is

$$r \equiv \frac{\mathcal{P}_T}{\mathcal{P}_\zeta} \simeq 16\epsilon_V$$

Number of e-folds

$$N(\phi) = \int_t^{t_{\rm end}} H\mathrm{d}t = \int_{\phi_{\rm end}}^{\phi} \frac{\mathrm{d}\phi}{\sqrt{2\epsilon_H}} \approx \int_{\phi_{\rm end}}^{\phi} \frac{\mathrm{d}\phi}{\sqrt{2\epsilon_V}} \sim 50-60$$

Inflationary Observables up to 3rd Order in Slow Roll

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Frame-dependence of higher-order inflationary observables in scalar-tensor theories

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In the context of scalar-tensor theories of gravity we compute the third-order corrected spectral indices in the slow-roll approximation. The calculation is carried out by employing the Green's function method for scalar and tensor perturbations in both the Einstein and Jordan frames. Then, using the interrelations between the Hubble slow-roll parameters in the two frames we find that the frames are equivalent up to third order. Since the Hubble slow-roll parameters are related to the potential slow-roll parameters, we express the observables in terms of the latter which are manifestly invariant. Nevertheless, the same inflaton excursion leads to different predictions in the two frames since the definition of the number of *e*-folds differs. To illustrate this effect we consider a nonminimal inflationary model and find that the difference in the predictions grows with the nonminimal coupling, and it can actually be larger than the difference between the first and third order results for the observables. Finally, we demonstrate the effect of various end-of-inflation conditions on the observables. These effects will become important for the analyses of inflationary models in view of the improved sensitivity of future experiments.

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Inflationary Observables up to 3rd Order in the PSRPs

Scalar spectral index:

$$
\begin{aligned}
n_s = 1 - 6\epsilon_V + 2\eta_V &+ \left(24\alpha - \frac{10}{3}\right)\epsilon_V^2 - (16\alpha + 2)\,\epsilon_V\eta_V + \frac{2}{3}\eta_V^2 + \left(2\alpha + \frac{2}{3}\right)\zeta_V^2 \\
&- \left(90\alpha^2 - \frac{104}{3}\alpha + \frac{3734}{9} - \frac{87\pi^2}{2}\right)\epsilon_V^3 + \left(90\alpha^2 + \frac{4}{3}\alpha + \frac{1190}{3} - \frac{87\pi^2}{2}\right)\epsilon_V^2\eta_V \\
&- \left(16\alpha^2 + 12\alpha + \frac{742}{9} - \frac{28\pi^2}{3}\right)\epsilon_V\eta_V^2 - \left(12\alpha^2 + 4\alpha + \frac{98}{3} - 4\pi^2\right)\epsilon_V\zeta_V^2 \\
&+ \left(\alpha^2 + \frac{8}{3}\alpha + \frac{28}{3} - \frac{13\pi^2}{2}\right)\eta_V\zeta_V^2 + \frac{4}{9}\eta_V^3 + \left(\alpha^2 + \frac{2}{3}\alpha + \frac{2}{9} - \frac{\pi^2}{12}\right)\rho_V^3
\end{aligned}
$$

Tensor-to-scalar ratio:

$$
\begin{aligned}
r = 16\epsilon_V \Bigg[1 &- \left(4\alpha + \frac{4}{3}\right)\epsilon_V + \left(2\alpha + \frac{2}{3}\right)\eta_V + \left(16\alpha^2 + \frac{28}{3}\alpha + \frac{356}{9} - \frac{14\pi^2}{3}\right)\epsilon_V^2 \\
&- \left(14\alpha^2 + 10\alpha + \frac{88}{3} - \frac{7\pi^2}{2}\right)\epsilon_V\eta_V + \left(2\alpha^2 + 2\alpha + \frac{41}{9} - \frac{\pi^2}{2}\right)\eta_V^2 \\
&+ \left(\alpha^2 + \frac{2}{3}\alpha + \frac{2}{9} - \frac{\pi^2}{12}\right)\zeta_V^2\Bigg]
\end{aligned}
$$

Chaotic Inflation

The potential is given by

$$V(\phi) = \lambda_n \phi^n \,.$$

The first two PSRPs are easily computed to be

$$\epsilon_V = \frac{1}{2}\left(\frac{V'}{V}\right)^2 = \frac{n^2}{2}\frac{1}{\phi^2}\,, \qquad \eta_V = \frac{V''}{V} = n\,(n-1)\,\frac{1}{\phi^2}\,.$$

In the 1st order SR approximation, inflation ends when $\epsilon_H \simeq \epsilon_V = 1$, ergo $\phi_{\rm end} = n/\sqrt{2}$.
Number of e-folds

$$N(\phi_*) = \int_{\phi_{\rm end}}^{\phi_*} \frac{{\rm d}\phi}{\sqrt{2\epsilon_V}} = \frac{\phi_*^2}{2n} - \frac{n}{4} \quad \to \quad \phi_*^2 = 2nN_* + \frac{n^2}{2}$$

Then

$$n_s = 1 - \frac{2n+4}{4N_*+1}\,, \qquad r = \frac{16n}{4N_*+1}$$

Let us now consider $N_* = 60$ and take the quadratic potential $V = \frac{1}{2}m^2\phi^2$. We find

$$n_s \simeq 0.97\,, \qquad r \simeq 0.13\,.$$

Similarly, for the quartic potential $V = \frac{1}{4}\lambda\phi^4$ we find

$$n_s \simeq 0.95\,, \qquad r \simeq 0.26\,.$$

Starobinsky Inflation

A simple extension of the Einstein-Hilbert action (Starobinsky, 1980):

$$S_{\text{Star.}} = \frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}\left(R+\alpha R^2\right) \,, \qquad M_{\text{Pl}}^2 \equiv 1 \,,$$

which belongs to the general class of $F(R)$ theories

$$S_F = \frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}\,F(R) \quad \rightarrow \quad S[g_{\mu\nu},\chi] = \frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}\,\left[F'(\chi)(R-\chi)+F(\chi)\right]$$

After a Weyl rescaling of the metric $g_{\mu\nu}$ and a field redefinition

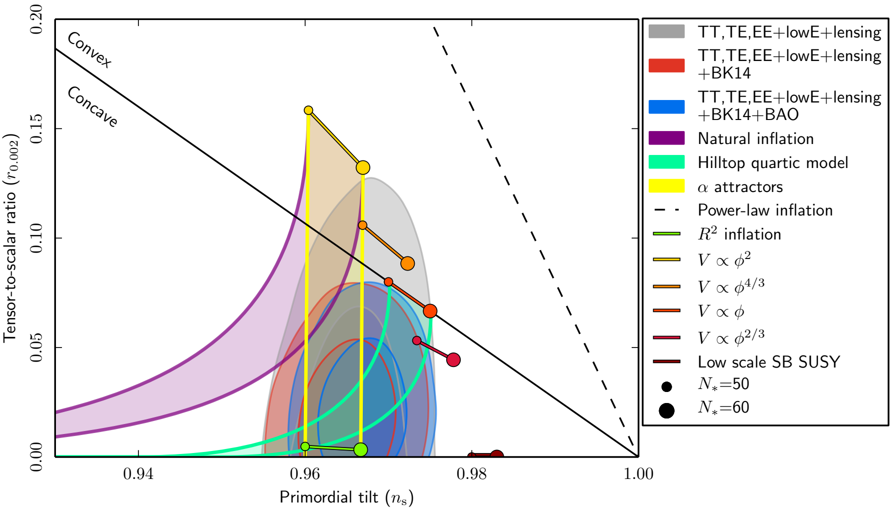
$$S[g_{\mu\nu},\varphi] = \frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}R - \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + V(\varphi)\right] \,,$$

where $V = \frac{1}{2}\frac{\chi F'(\chi)-F(\chi)}{F'(\chi)^2}$, $F'(\chi) = \exp\left(\sqrt{\frac{2}{3}}\varphi\right)$, $\varphi = \sqrt{\frac{3}{2}}\ln F'(\chi)$

For the $\left(R+\alpha R^2\right)$ model, $V(\varphi) = \frac{1}{8\alpha}\left[1-\exp\left(-\sqrt{\frac{2}{3}}\varphi\right)\right]^2$, we find for $N_* = 60$

$$n_s = 1-\frac{2}{N_*} = 0.9667 \,, \qquad r = \frac{12}{N_*^2} = 0.0033$$

Planck 2018 Results



1807.06211, 2110.00483: $n_s = 0.9649 \pm 0.0042$ and $r < 0.036$

Metric vs. Palatini

- In **metric formulation**, the metric is the only dynamical degree of freedom and the connection is always the Levi-Civita

$$S = \int \mathrm{d}^4x \sqrt{-g} \left(\frac{1+\xi\phi^2}{2} g^{\mu\nu} R_{\mu\nu}\left(g, \partial g, \partial^2 g\right) - \frac{1}{2} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi - V(\phi) \right)$$

- In **Palatini formulation**, both the metric and the connection are independent dynamical degrees of freedom

$$S = \int \mathrm{d}^4x \sqrt{-g} \left(\frac{1+\xi\phi^2}{2} g^{\mu\nu} R_{\mu\nu}\left(\Gamma, \partial \Gamma\right) - \frac{1}{2} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi - V(\phi) \right)$$

Scalar-Tensor Gravity: Metric vs. Palatini

$$S_J = \int \mathrm{d}^4x\sqrt{-g}\left(\frac{1}{2}A(\phi)g^{\mu\nu}R_{\mu\nu}(\Gamma) - \frac{1}{2}B(\phi)g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - V(\phi)\right)$$

Variation with respect to Γ gives

$$\Gamma^\lambda_{\alpha\beta} = \left\{ {\lambda \atop \alpha\beta} \right\} + (1-\kappa)\left[\delta^\lambda_\alpha\partial_\beta\omega(\phi) + \delta^\lambda_\beta\partial_\alpha\omega(\phi) - g_{\alpha\beta}\partial^\lambda\omega(\phi)\right] , \quad \omega(\phi) = \ln\sqrt{A(\phi)}$$

where $\kappa = 1$ in metric and $\kappa = 0$ in Palatini. Performing a Weyl transformation

$$\bar{g}_{\mu\nu} \equiv A(\phi)g_{\mu\nu} \quad \rightarrow \quad \sqrt{-g} = A^{-2}\sqrt{-\bar{g}}\,, \quad R = A\left(1 - \kappa \times 6\,A^{1/2}\bar{\nabla}^\mu\bar{\nabla}_\mu A^{-1/2}\right)\bar{R}\,,$$

the action becomes

$$S_E = \int \mathrm{d}^4x\sqrt{-\bar{g}}\left(\frac{1}{2}\bar{R} - \frac{1}{2}\left(\frac{B}{A} + \kappa \times \frac{3}{2}\frac{(A_{,\phi})^2}{A^2}\right)\bar{\nabla}_\mu\phi\bar{\nabla}^\mu\phi - \frac{V(\phi)}{A^2}\right)$$

Field redefinition $\phi = \phi(\chi)$ to make it canonical

$$\frac{\mathrm{d}\phi}{\mathrm{d}\chi} = \sqrt{\frac{A^2}{AB + \kappa \times \frac{3}{2}(A_{,\phi})^2}}\,.$$

Final action

$$S_\mathrm{E} = \int \mathrm{d}^4x\sqrt{-g}\left(\frac{1}{2}\bar{R} - \frac{1}{2}\bar{\nabla}_\mu\chi\bar{\nabla}^\mu\chi - U(\chi)\right), \quad U(\chi) = \frac{V(\phi(\chi))}{A^2(\phi(\chi))}$$

Higgs Inflation: Metric vs. Palatini

We consider the Higgs-like inflationary potential

$$V(\phi) = \frac{\lambda}{4}\phi^4 \,, \qquad A(\phi) = 1 + \xi\phi^2 \,, \qquad B(\phi) = 1$$

Canonical field redefinition gives

$$\phi(\chi) \simeq \frac{1}{\sqrt{\xi}} \exp\left(\sqrt{\frac{1}{6}}\chi\right) \quad (\text{Metric})\,, \qquad \phi(\chi) = \frac{1}{\sqrt{\xi}} \sinh(\sqrt{\xi}\chi) \quad (\text{Palatini})$$

The Einstein-frame potential in terms of χ can be expressed as

$$U(\chi) \simeq \frac{\lambda}{4\xi^2}\left(1 - \exp\left(-\sqrt{\frac{2}{3}}\,\chi\right)\right)^2 \,, \quad (\text{Metric})\,,$$

$$U(\chi) = \frac{\lambda}{4\xi^2} \tanh^4\left(\sqrt{\xi}\chi\right)\,, \qquad (\text{Palatini})$$

The observables are calculated to be

$$n_s \simeq 1 - \frac{2}{N_*} + \frac{3}{2N_*^2}\,, \qquad r \simeq \frac{12}{N_*^2}\,, \qquad A_s \simeq \frac{\lambda N_*^2}{72\pi^2\xi^2} \quad (\text{Metric})\,,$$

$$n_s \simeq 1 - \frac{2}{N_*} - \frac{3}{8\xi N_*^2}\,, \qquad r \simeq \frac{2}{\xi N_*^2}\,, \qquad A_s \simeq \frac{\lambda N_*^2}{12\pi^2\xi} \quad (\text{Palatini})\,.$$

When Metric = Palatini

For $\xi \lesssim 0.0047$ metric and Palatini are experimentally indistinguishable.

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β-function reconstruction of Palatini inflationary attractors

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Abstract. Attractor inflation is a particularly robust framework for developing inflationary models that are insensitive to the details of the potential. Such models are most often considered in the metric formulation of gravity. However, non-minimal models may not necessarily maintain their attractor nature in the Palatini formalism where the connection is independent of the metric. In this work, we employ the β-function formalism to classify the strong coupling limit of inflationary models in both the metric and the Palatini approaches. Furthermore, we determine the range of values for the non-minimal coupling that lead to theories being observationally indistinguishable in metric and Palatini within current accuracy. Finally, we reconstruct the Jordan frame potential for ξ-attractors by imposing an explicit form for the β-function, demonstrating the effect that the choice of metric or Palatini has on the inflationary observables of the theory.

Keywords: inflation, modified gravity, particle physics - cosmology connection

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When Metric and Palatini yield the same observables

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Equivalence of inflationary models between the metric and Palatini formulation of scalar-tensor theories

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With a scalar field nonminimally coupled to curvature, the underlying geometry and variational principle of gravity—metric or Palatini—becomes important and makes a difference, as the field dynamics and observational predictions generally depend on this choice. In the present paper, we describe a classification principle which encompasses both metric and Palatini models of inflation, employing the fact that inflationary observables can be neatly expressed in terms of certain quantities which remain invariant under conformal transformations and scalar field redefinitions. This allows us to elucidate the specific conditions when a model yields equivalent phenomenology in the metric and Palatini formalisms and also to outline a method how to systematically construct different models in both formulations that produce the same observables.

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n_s and r are the same if $A(\phi)B(\phi) \propto \left(A'(\phi)\right)^2$ and $V(\phi) \propto A(\phi)^2 \left(\ln \dfrac{A(\phi)}{A_0}\right)^2$

Palatini Inflation in Models with an R^2 Term

Journal of Cosmology and Astroparticle Physics
An IOP and SISSA journal

Palatini inflation in models with an R^2 term

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Abstract. The Starobinsky model, considered in the framework of the Palatini formalism, in contrast to the metric formulation, does not provide us with a model for inflation, due to the absence of a propagating scalar degree of freedom that can play the role of the inflaton. In the present article we study the Palatini formulation of the Starobinsky model coupled, in general nonminimally, to scalar fields and analyze its inflationary behavior. We consider scalars, minimally or nonminimally coupled to the Starobinsky model, such as a quadratic model, the induced gravity model or the standard Higgs-like inflation model and analyze the corresponding modifications favorable to inflation. In addition we examine the case of a classically scale-invariant model driven by the Coleman-Weinberg mechanism. In the slow-roll approximation, we analyze the inflationary predictions of these models and compare them to the latest constraints from the Planck collaboration. In all cases, we find that the effect of the R^2 term is to lower the value of the tensor-to-scalar ratio.

Keywords: inflation, modified gravity, gravity, alternatives to inflation

ArXiv ePrint: 1810.10418

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Rescuing quartic and natural inflation in the Palatini formalism

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Abstract. When considered in the Palatini formalism, the Starobinsky model does not provide us with a mechanism for inflation due to the absence of a propagating scalar degree of freedom. By (non)-minimally coupling scalar fields to the Starobinsky model in the Palatini formalism we can in principle describe the inflationary epoch. In this article, we focus on the minimally coupled quartic and natural inflation models. Both theories are excluded in their simplest realization since they predict values for the inflationary observables that are outside the limits set by the Planck data. However, with the addition of the R^2 term and the use of the Palatini formalism, we show that these models can be rendered viable.

Keywords: inflation, modified gravity

ArXiv ePrint: 1812.00847

JCAP03(2019)005

Palatini inflation in models with an R^2 term

In 1810.10418, 1812.00847 & 2006.09124 we considered (see also Enckell et al.: 1810.05536)

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{\alpha}{2}R^2 + \frac{1}{2}A(\phi)R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right], \quad R = g^{\mu\nu}R^{\rho}{}_{\mu\rho\nu}(\Gamma, \partial\Gamma)$$

Introducing an auxiliary scalar $\chi \equiv 2\alpha R$ and Weyl rescaling $g_{\mu\nu} \to \Omega^2 g_{\mu\nu} = [\chi + A(\phi)]g_{\mu\nu}$

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}\frac{1}{\chi + A(\phi)}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \hat{V}(\phi,\chi)\right],$$

with

$$\hat{V}(\phi,\chi) = \frac{1}{[\chi + A(\phi)]^2}\left[V(\phi) + \frac{\chi^2}{8\alpha}\right].$$

- No kinetic term has been generated for the field χ (scalaron in metric formalism)
- EOM of χ reduces to a constraint
- ϕ is the only propagating scalar DOF $\to$ inflaton

Palatini inflation in models with an R^2 term

Varying the action with respect to χ:

$$\delta_\chi S = 0 \to \chi = \frac{8\alpha V(\phi) + 2\alpha A(\phi)\,(\partial\phi)^2}{A(\phi) - 2\alpha\,(\partial\phi)^2}\,.$$

Substituting back

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}K(\phi)\,(\partial\phi)^2 + \frac{1}{4}L(\phi)\,(\partial\phi)^4 - \frac{\bar{U}}{1+8\alpha\bar{U}}\right]\,, \quad \bar{U}(\phi) \equiv \frac{V(\phi)}{[A(\phi)]^2}$$

with

$$K(\phi) \equiv \frac{1}{A(1+8\alpha\bar{U})}\,, \qquad L(\phi) = \frac{2\alpha}{A^2(1+8\alpha\bar{U})}\,.$$

Using $\left(\frac{\mathrm{d}\phi}{\mathrm{d}\zeta}\right)^2 = A(1+8\alpha\bar{U})$, we arrive at

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}\,(\partial\zeta)^2 + \frac{\alpha}{2}\,(1+8\alpha\bar{U}(\zeta))\,(\partial\zeta)^4 - U(\zeta)\right]\,, \quad U \equiv \frac{\bar{U}}{1+8\alpha\bar{U}}\,.$$

- Regardless of the shape of V, the R^2 term decreases the height of the effective potential
- For large values of ϕ tends to a plateau $M_{\mathrm{P}}^4/8\alpha$
- The rate of change of the field is also modified

Slow-roll

In a flat FRW background

$$3H^2 = \frac{1}{2}[1 + 3\alpha(1 + 8\alpha\bar{U})\dot{\zeta}^2]\dot{\zeta}^2 + U$$

$$0 = [1 + 6\alpha(1 + 8\alpha\bar{U})\dot{\zeta}^2]\ddot{\zeta} + 3[1 + 2\alpha(1 + 8\alpha\bar{U})\dot{\zeta}^2]H\dot{\zeta} + 12\alpha^2\dot{\zeta}^4\bar{U}' + U'$$

Inflation takes place when

$$\epsilon_H \equiv -\frac{\dot{H}}{H^2} = \frac{\dot{\zeta}^2}{2H^2}\left[1 + 2\alpha\left(1 + 8\alpha\bar{U}\right)\dot{\zeta}^2\right] < 1\,.$$

First order expressions for observables:

$$24\pi^2 A_s = \frac{U}{\epsilon_U} = \frac{\bar{U}}{\epsilon_{\bar{U}}}\,, \qquad n_s = 1 - 6\epsilon_U + 2\eta_U = 1 - 6\epsilon_{\bar{U}} + 2\eta_{\bar{U}}\,,$$

Tensor-to-scalar ratio

$$r = 16\epsilon_U = \frac{\bar{r}}{1 + 8\alpha\bar{U}} = \frac{\bar{r}}{1 + 12\pi^2 A_s\bar{r}\alpha}$$

Sensitivity of PICO will be approximately $\delta_r \approx 10^{-4}$.

Requiring near-future detectability means $\alpha < 4 \times 10^{10}$.

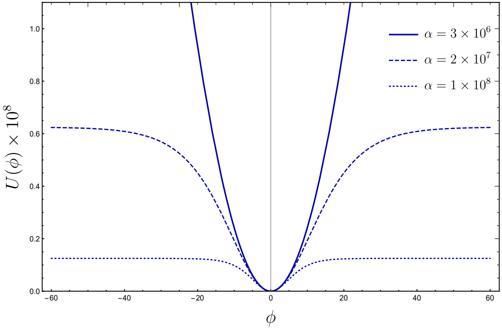
Example: Quadratic Inflation

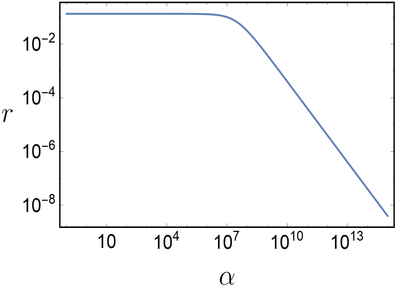
Consider the minimal quadratic potential $V(\phi) = \frac{1}{2}m^2\phi^2$ with $A(\phi) = 0$, (2102.02712)
The field redefinition and effective Einstein potential become

$$\chi = \frac{\sinh^{-1}(2m\sqrt{\alpha}\phi)}{2m\sqrt{\alpha}}, \qquad U = \frac{\tanh^2(2m\sqrt{\alpha}\chi)}{8\alpha}$$

and the observables

$$r = \frac{8}{N_* + 16\alpha m^2 N_*^2}, \qquad n_s \simeq 1 - \frac{2}{N_*}, \qquad A_s \simeq \frac{m^2 N_*^2}{6\pi^2}$$





Tachyonic Preheating

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Tachyonic preheating in Palatini R^2 inflation

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Abstract. We study preheating in the Palatini formalism with a quadratic inflaton potential and an added αR^2 term. In such models, the oscillating inflaton field repeatedly returns to the plateau of the Einstein frame potential, on which the tachyonic instability fragments the inflaton condensate within less than an e-fold. We find that tachyonic preheating takes place when $\alpha \gtrsim 10^{13}$ and that the energy density of the fragmented field grows with the rate $\Gamma/H \approx 0.011 \times \alpha^{0.31}$. The model extends the family of plateau models with similar preheating behaviour. Although it contains non-canonical quartic kinetic terms in the Einstein frame, we show that, in the first approximation, these can be neglected during both preheating and inflation.

Keywords: inflation, modified gravity, cosmological perturbation theory, particle physics - cosmology connection

ArXiv ePrint: 2102.02712

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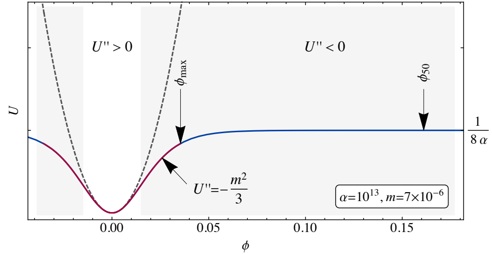
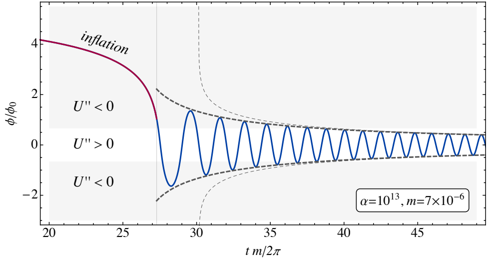
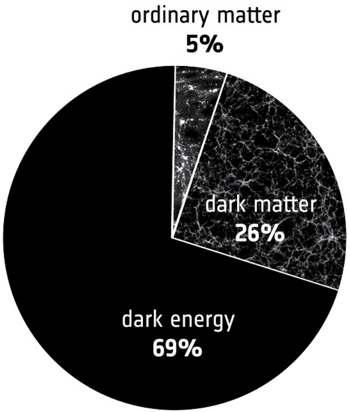
Tachyonic Preheating

- For $\alpha \gtrsim 10^{13}$ the inflaton returns to the plateau repeatedly during preheating.
- The tachyonic instability fragments the inflaton condensate within less than an e-fold.

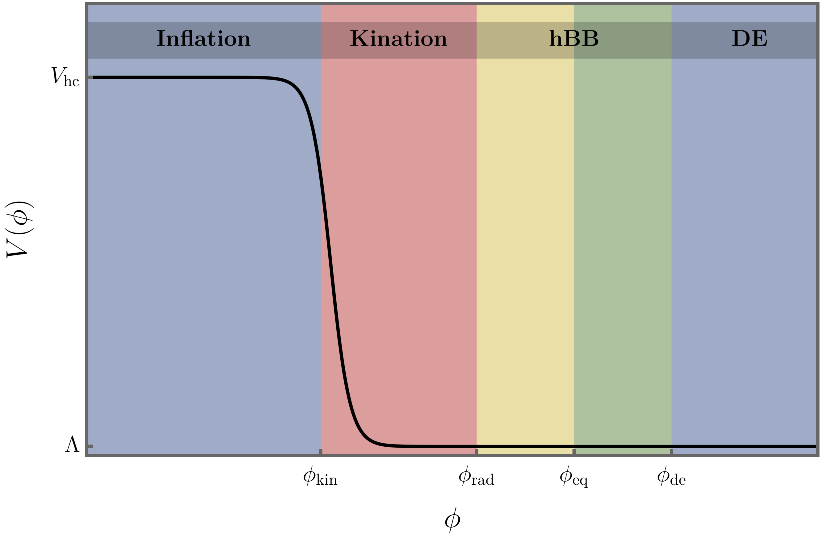




Cosmic Energy Budget (Credit: ESA)

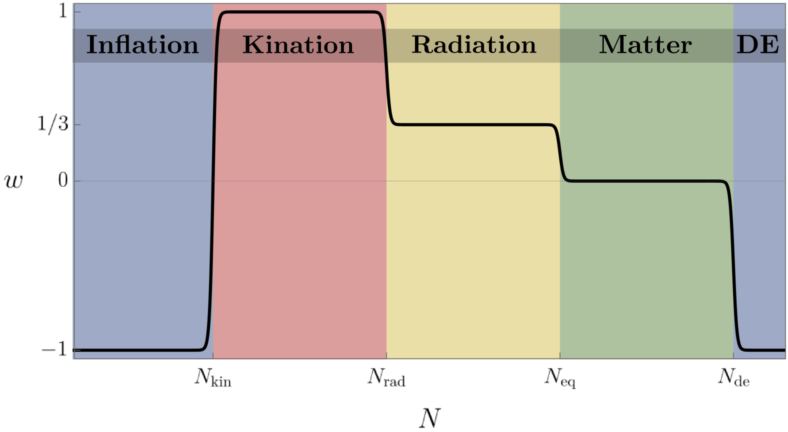


Quintessential Inflation Potential (Credit: 2112.11948)

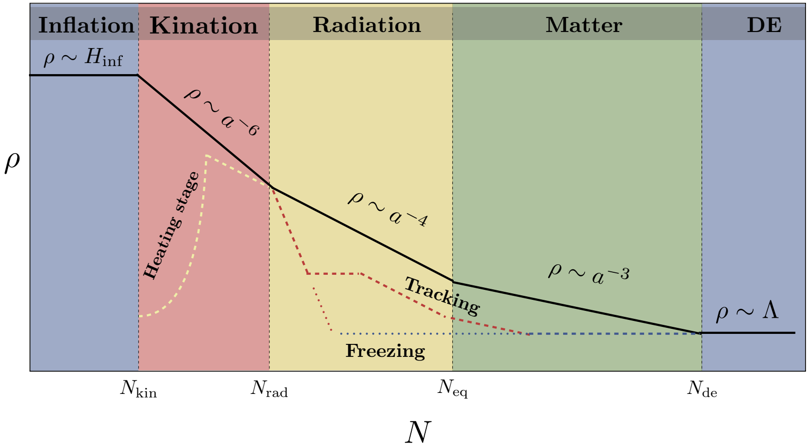


Equation of State Parameter (Credit: 2112.11948)

$$w = \frac{p}{\rho} = \frac{\dot{\phi}^2/2 - V}{\dot{\phi}^2/2 + V}$$



Energy Density (Credit: 2112.11948)



Palatini R^2 Quintessential Inflation

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Palatini R^2 quintessential inflation

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Abstract. We construct a model of quintessential inflation in Palatini R^2 gravity employing a scalar field with a simple exponential potential and coupled to gravity with a running non-minimal coupling. At early times, the field acts as the inflaton, while later on it becomes the current dark energy. Combining the scalar sector with an ideal fluid, we study the cosmological evolution of the model from inflation all the way to dark energy domination. We interpret the results in the Einstein frame, where a coupling emerges between the fluid and the field, feeding energy from the former to the latter during the matter-dominated era. We perform a numerical scan over the parameter space and find points that align with observations for both the inflationary CMB data and the late-time behaviour. The final dark energy density emerges from an interplay between the model parameters, without requiring the extreme fine-tuning of the cosmological constant in ΛCDM.

Keywords: dark energy theory, Gauss-Bonnet-Lovelock-Horndeski-Palatini etc gravity theories, inflation, modified gravity

ArXiv ePrint: 2206.14117

Palatini R^2 Quintessential Inflation

Consider the action in the Palatini formalism

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{M_{\mathrm{Pl}}^2}{2}F(\varphi, R) - \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - V(\varphi)\right] + S_{\mathrm{m}}[g_{\mu\nu}, \psi] .$$

The function $F(\varphi, R)$ takes the form

$$F(\varphi, R) = \left(1 + \frac{\xi}{M_{\mathrm{Pl}}^2}\varphi^2\right)R + \frac{\alpha}{2M_{\mathrm{Pl}}^2}R^2 , \qquad \xi(\varphi) = \xi_*\left[1 + \beta\ln\left(\frac{\varphi^2}{\mu^2}\right)\right] ,$$

with $\xi_* > 0$ and $\beta < 0$ constants, and μ an arbitrary reference scale.
The real scalar field φ is governed by an exponential potential

$$V(\varphi) = M^4 e^{-\kappa\varphi/M_{\mathrm{Pl}}} .$$

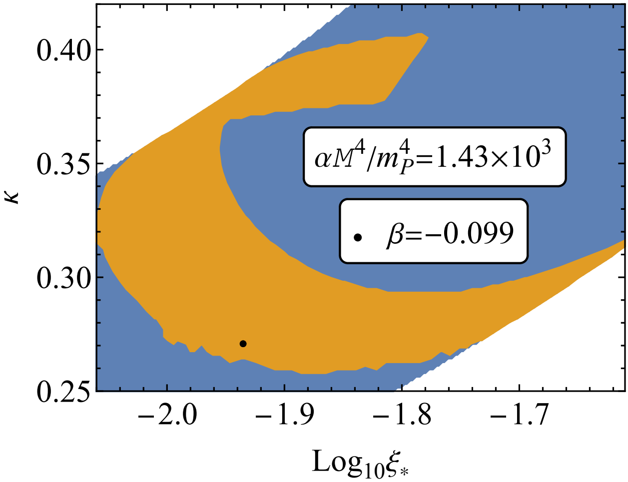
In the Einstein frame it becomes

$$U(\phi) = \frac{M_{\mathrm{Pl}}^4 M^4 e^{-\kappa\varphi(\phi)/M_{\mathrm{Pl}}}}{(M_{\mathrm{Pl}}^2 + \xi\varphi(\phi)^2)^2 + 4\alpha M^4 e^{-\kappa\varphi(\phi)/\mu}} .$$

Constraints to be satisfied

- The scalar spectral index is equal to the central value obtained by Planck, $n_s = 0.9649$.
- The tensor-to-scalar ratio is within the latest observational bounds, *i.e.*, $r < 0.036$.
- The value of the running of the scalar spectral index is within the 2σ bounds obtained by Planck, *i.e.*, $-0.0179 < \alpha_s < 0.0089$.
- The initial energy density of radiation at the end of inflation, amounts to a small perturbation of the system, *i.e.*, $\Omega_r^{\text{end}} < 0.1$.
- The initial energy density of radiation at the end of inflation is larger than the energy density corresponding to gravitational reheating, *i.e.*, $\bar{\rho}(t_{\text{end}}) > 2.25 \times 10^{-2} (\bar{H}^{\text{end}})^4$.
- The energy density ratio of the field, corresponding now to dark energy, is equal to the central value obtained by Planck of its value today, *i.e.*, $\Omega_\phi^0 = 0.6889$.
- The temperature of the universe at the onset of radiation domination is above $T_{\text{BBN}} \simeq 0.1\text{MeV}$.
- The barotropic parameter of the field is within the latest bounds, $w_\phi^0 < -0.95$.
- The running of the barotropic parameter of the field in the CPL parametrization is within the latest bounds, *i.e.*, $-0.55 < w_a^0 < 0.03$.
- The energy density of the field at present is within one order of magnitude from the central value obtained by Planck, $\bar{\rho}_{\text{DE}}^{\text{Planck}} = 7.26 \times 10^{-121} M_{\text{Pl}}^4$.
- We finally take into account the bound on the density parameter of gravitational waves coming from BBN constraints, $20\, \Omega_{\text{GW}}^{\text{end}} < \Omega_r^{\text{end}}$.

Successful parameter space



Hyperkination

Observable Gravitational Waves from Hyperkination in Palatini Gravity and Beyond

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ABSTRACT: We consider cosmology with an inflaton scalar field with an additional quartic kinetic term. Such a theory can be motivated by Palatini $R + R^2$ modified gravity. Assuming a runaway inflaton potential, we take the Universe to become dominated by the kinetic energy density of the scalar field after inflation. Initially, the leading kinetic term is quartic and we call the corresponding period hyperkination. Subsequently, the usual quadratic kinetic term takes over and we have regular kination, until reheating. We study, both analytically and numerically, the spectrum of primordial gravitational waves generated during inflation and re-entering the horizon during the subsequent eras. We demonstrate that the spectrum is flat for modes re-entering during radiation domination and hyperkination and linear in frequency for modes re-entering during kination: kinetic domination boosts the spectrum, but hyperkination truncates its peak. As a result, the effects of the kinetic period can be extended to observable frequencies without generating excessive gravitational waves, which could otherwise destabilise the process of Big Bang Nucleosynthesis. We show that there is ample parameter space for the primordial gravitational waves to be observable in the near future. If observed, the amplitude and 'knee' of the spectrum will provide valuable insights into the background theory.

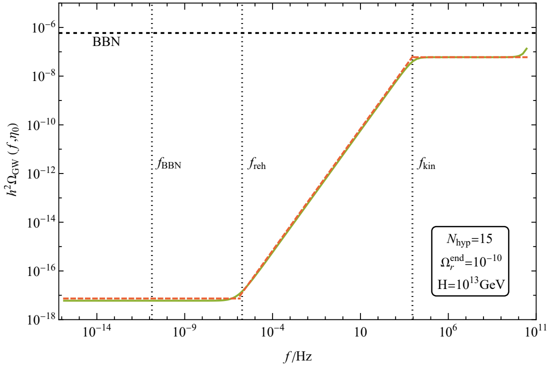
Hyperkination

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{m_{\mathrm{P}}^2}{2}R - \frac{1}{2}(\partial\phi)^2 + \frac{\alpha}{4}(\partial\phi)^4 - U\right]$$

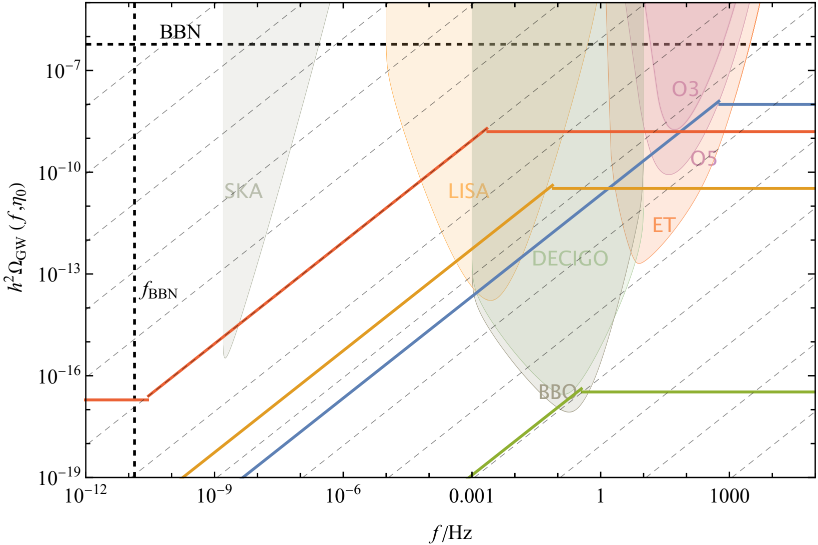
- Once inflation ends, the potential drops to zero and the field's velocity increases as the potential energy is transformed into kinetic energy.
- In our models of interest, the post-inflationary potential is of the runaway type---that is, flat and low---and the field keeps rolling onward under kinetic domination.
- If the quartic kinetic terms dominate, this phase starts with hyperkination, transitioning into standard kination later.
- To reheat the Universe, we assume a small amount of radiation is produced at the end of inflation e.g. through Ricci reheating.
- During hyperkination, the radiation energy density dilutes as fast as that of the field, $\rho_{\mathrm{r},\phi} \propto a^{-4}$, so radiation stays subdominant.
- However, when standard kination starts, the field energy density dilutes faster, $\rho_\phi \propto a^{-6}$, and the radiation fraction grows until it overtakes the field. The Universe reheats and radiation domination starts.
- Afterwards, the Universe follows the standard ΛCDM expansion history

Gravitational Wave Spectrum

$$\Omega_{\rm GW}(f,\eta_0) = \begin{cases} \dfrac{\Omega_{\rm r}^0}{96}\left(\dfrac{H}{m_{\rm P}}\right)^2 , & f < f_{\rm reh} , \\ \left(\dfrac{\Omega_{\rm r}^0}{\Omega_{\rm r}^{\rm end}}\right)^{3/4} \dfrac{H^{3/2}}{6\pi H_0^{1/2} m_{\rm P}^2} \left(\dfrac{1+\sqrt{1+36\alpha H^2/m_{\rm P}^2}}{2}\right)^{\frac{1}{2}} f , & f_{\rm reh} < f < f_{\rm kin} , \\ \dfrac{\Omega_{\rm r}^0}{12\pi^2 \Omega_{\rm r}^{\rm end}}\left(\dfrac{H}{m_{\rm P}}\right)^2 , & f_{\rm kin} < f < f_{\rm end} , \end{cases}$$



Observability of Gravitational Waves



Summary and Conclusions

- $V(\phi) + \alpha R^2$ in **metric** leads to two-field inflation
- $V(\phi) + \alpha R^2$ in **Palatini** leads to single-field inflation
- The effective potential is asymptotically flat and has a lower value
- For large α the value of r becomes much smaller
- The values of A_s and n_s are unaffected (Enckell et al.: 1810.05536)
- Flattening of potential can be used to construct successful models of quintessential inflation
- Quartic kinetic term leads to a phase of hyperkination
- Hyperkination produces a flat spectrum which truncates the linear spectrum arising from kination
- We obtain a boosted primordial GW signal with unique characteristics.

Thank you! ☺