Scalar-nonmetricity cosmology in the general relativity limit

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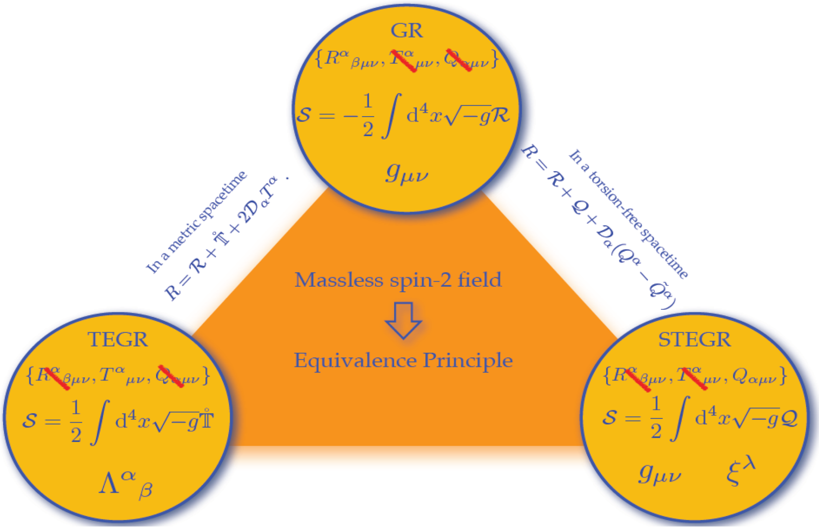
Trinity of gravity [1]



[1] Jose Beltrán Jiménez, Lavinia Heisenberg and Tomi S. Koivisto "The Geometrical Trinity of Gravity", Universe, 5(7) (2019) 173.

Geometrical Formalism

Affine connection:

$$\tilde{\Gamma}^{\lambda}{}_{\mu\nu} = \mathring{\Gamma}^{\lambda}{}_{\mu\nu} + L^{\lambda}{}_{\mu\nu}\,, \tag{1}$$

Nonmetricity tensor:

$$Q_{\rho\mu\nu} \equiv \nabla_{\rho} g_{\mu\nu} = \partial_{\rho} g_{\mu\nu} - \tilde{\Gamma}^{\beta}{}_{\mu\rho} g_{\beta\nu} - \tilde{\Gamma}^{\beta}{}_{\nu\rho} g_{\mu\beta}\,. \tag{2}$$

Disformation tensor:

$$L^{\lambda}{}_{\mu\nu} \equiv \frac{1}{2} g^{\lambda\beta}\left(-Q_{\mu\beta\nu} - Q_{\nu\beta\mu} + Q_{\beta\mu\nu}\right) = L^{\lambda}{}_{\nu\mu}\,. \tag{3}$$

Relation between GR and STEGR

$$\mathring{R} = Q + \mathring{\nabla}_{\mu}(\hat{Q}^{\mu} - Q^{\mu})\,.$$

where the nonmetricity scalar and traces are defined as [2]

$$Q \equiv -\frac{1}{4}\, Q_{\lambda\mu\nu} Q^{\lambda\mu\nu} + \frac{1}{2}\, Q_{\lambda\mu\nu} Q^{\mu\nu\lambda} + \frac{1}{4}\, Q_{\mu} Q^{\mu} - \frac{1}{2}\, Q_{\mu} \hat{Q}^{\mu}\,, \tag{4}$$

$$Q_{\mu} \equiv Q_{\mu\nu}{}^{\nu}\,, \qquad \hat{Q}_{\mu} \equiv Q_{\nu\mu}{}^{\nu}\,. \tag{5}$$

$$P^{\alpha}\mu\nu = \frac{1}{2}\frac{\partial Q}{\partial Q_{\alpha}\mu\nu}. \tag{6}$$

[2] Note that some authors define Q with the opposite overall sign,

Scalar non-metricity gravity

Scalar non-metricity action:

$$S = \frac{1}{2\kappa^2} \int \mathrm{d}^4x \sqrt{-g} \left(\mathcal{A}(\Phi) Q - \mathcal{B}(\Phi) g^{\alpha\beta} \partial_\alpha \Phi \partial_\beta \Phi - 2\mathcal{V}(\Phi) \right) + S_{\mathrm{m}} \,, \tag{7}$$

Varying w.r.t metric:

$$\mathcal{A}(\Phi) \mathring{G}_{\mu\nu} + 2 \frac{\mathrm{d}\mathcal{A}}{\mathrm{d}\Phi} P^{\lambda}{}_{\mu\nu} \partial_\lambda \Phi + \frac{1}{2} g_{\mu\nu} \left(\mathcal{B}(\Phi) g^{\alpha\beta} \partial_\alpha \Phi \partial_\beta \Phi + 2\mathcal{V}(\Phi) \right) - \mathcal{B}(\Phi) \partial_\mu \Phi \partial_\nu \Phi = \kappa^2 \mathcal{T}_{\mu\nu} \,, \tag{8}$$

Varying w.r.t connection:

$$\left(\frac{1}{2} Q_\beta + \nabla_\beta \right) \left[\partial_\alpha \mathcal{A} \left(\frac{1}{2} Q_\mu g^{\alpha\beta} - \frac{1}{2} \delta^\alpha_\mu Q^\beta - Q_\mu{}^{\alpha\beta} + \delta^\alpha_\mu Q_\gamma{}^{\gamma\beta} \right) \right] = 0, \tag{9}$$

Varying w.r.t scalar field:

$$2\mathcal{B} \mathring{\Box} \Phi + \frac{\mathrm{d}\mathcal{B}}{\mathrm{d}\Phi} g^{\alpha\beta} \partial_\alpha \Phi \partial_\beta \Phi + \frac{\mathrm{d}\mathcal{A}}{\mathrm{d}\Phi} Q - 2 \frac{\mathrm{d}\mathcal{V}}{\mathrm{d}\Phi} = 0. \tag{10}$$

The continuity equation of the matter fields [3]

$$\mathring{\nabla}_\mu \mathcal{T}^\mu{}_\nu = 0.$$

[3] L. Järv, M. Rünkla, M. Saal, and O. Vilson, Phys. Rev. D, 97 (2018) no. 12, 124025, DOI: https://doi.org/10.1103/PhysRevD.97.124025.

Spatially homogeneous and isotropic:

$$ds^2 = -dt^2 + a(t)^2\left(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi\right) \tag{11}$$

Branch of connection

Spatially flat FLRW symmetry is satisfied by three sets of connections, introducing an extra function γ. [4]

$$\Gamma^{\rho}{}_{\mu\nu} = \left[\begin{bmatrix}\gamma(t) & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}\ \begin{bmatrix}0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & -r & 0\\ 0 & 0 & 0 & -r\sin^2\theta\end{bmatrix}\ \begin{bmatrix}0 & 0 & 0 & 0\\ 0 & 0 & \frac{1}{r} & 0\\ 0 & \frac{1}{r} & 0 & 0\\ 0 & 0 & 0 & -\sin\theta\cos\theta\end{bmatrix}\ \begin{bmatrix}0 & 0 & 0 & 0\\ 0 & 0 & 0 & \frac{1}{r}\\ 0 & 0 & 0 & \cot\theta\\ 0 & \frac{1}{r} & \cot\theta & 0\end{bmatrix}\right] \tag{12}$$

$$\Gamma^{\rho}{}_{\mu\nu} = \left[\begin{bmatrix}\gamma(t) + \frac{\dot{\gamma}(t)}{\gamma(t)} & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}\ \begin{bmatrix}0 & \gamma(t) & 0 & 0\\ \gamma(t) & 0 & 0 & 0\\ 0 & 0 & -r & 0\\ 0 & 0 & 0 & -r\sin^2\theta\end{bmatrix}\ \begin{bmatrix}0 & 0 & \gamma(t) & 0\\ 0 & 0 & \frac{1}{r} & 0\\ \gamma(t) & \frac{1}{r} & 0 & 0\\ 0 & 0 & 0 & -\sin\theta\cos\theta\end{bmatrix}\ \begin{bmatrix}0 & 0 & 0 & \gamma(t)\\ 0 & 0 & 0 & \frac{1}{r}\\ 0 & 0 & 0 & \cot\theta\\ \gamma(t) & \frac{1}{r} & \cot\theta & 0\end{bmatrix}\right] \tag{13}$$

$$\Gamma^{\rho}{}_{\mu\nu} = \left[\begin{bmatrix}-\frac{\dot{\gamma}(t)}{\gamma(t)} & 0 & 0 & 0\\ 0 & \gamma(t) & 0 & 0\\ 0 & 0 & r^2\gamma(t) & 0\\ 0 & 0 & 0 & r^2\gamma(t)\sin^2\theta\end{bmatrix}\ \begin{bmatrix}0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & -r & 0\\ 0 & 0 & 0 & -r\sin^2\theta\end{bmatrix}\ \begin{bmatrix}0 & 0 & 0 & 0\\ 0 & 0 & \frac{1}{r} & 0\\ 0 & \frac{1}{r} & 0 & 0\\ 0 & 0 & 0 & -\sin\theta\cos\theta\end{bmatrix}\ \begin{bmatrix}0 & 0 & 0 & 0\\ 0 & 0 & 0 & \frac{1}{r}\\ 0 & 0 & 0 & \cot\theta\\ 0 & \frac{1}{r} & \cot\theta & 0\end{bmatrix}\right] \tag{14}$$

[4] M. Hohmann, Phys. Rev. D 104 (2021) 12, DOI: https://doi.org/10.1103/PhysRevD.104.124077

Objective

• Under which conditions these cosmological spacetime configurations with radiation, dust matter, and potential content relax to the limit of general relativity where the variation of the gravitational constant ceases and the system evolves close to general relativity?

• Does the extra free function γ in the connection bring a new degree of freedom?

• Does this extra function play any role in the context of dark matter or dark energy?

• Our analysis encompassed all possible cases to investigate the stability of the solution associated with each branch in the radiation-dominated, dust matter, and potential-dominated eras.

Connection set I

Metric equation:

$$6H^2\mathcal{A}(\Phi) - \dot{\Phi}^2\mathcal{B}(\Phi) - 2\mathcal{V}(\Phi) = 2\kappa^2\rho\,,$$
$$-4H\dot{\Phi}\mathcal{A}'(\Phi) - \left(6H^2 + 4\dot{H}\right)\mathcal{A}(\Phi) - \dot{\Phi}^2\mathcal{B}(\Phi) + 2\mathcal{V}(\Phi) = 2\kappa^2\mathrm{w}\rho\,.$$

Scalar field equation

$$-6H^2\mathcal{A}'(\Phi) - (6H\dot{\Phi} + 2\ddot{\Phi})\mathcal{B}(\Phi) - \dot{\Phi}^2\mathcal{B}'(\Phi) - 2\mathcal{V}'(\Phi) = 0$$

Non-metricity scalar:

$$Q = -6H^2 \tag{15}$$

Compare with GR equation:

$$8\pi G_N = \frac{\kappa^2}{\mathcal{A}(\Phi_*)}, \qquad \Lambda = \frac{\mathcal{V}(\Phi_*)}{\mathcal{A}(\Phi_*)}$$

Connection set II

Metric equation

$$3\dot{\Phi}\gamma\mathcal{A}'(\Phi) + 6H^2\mathcal{A}(\Phi) - \dot{\Phi}^2\mathcal{B}(\Phi) - 2\mathcal{V}(\Phi) = 2\kappa^2\rho\,,$$

$$(3\dot{\Phi}\gamma - 4H\dot{\Phi})\mathcal{A}'(\Phi) - \left(6H^2 + 4\dot{H}\right)\mathcal{A}(\Phi) - \dot{\Phi}^2\mathcal{B}(\Phi) + 2\mathcal{V}(\Phi) = 2\kappa^2 \mathrm{w}\rho\,.$$

Connection equation

$$3\gamma\left(\ddot{\Phi}\mathcal{A}'(\Phi) + 3H\dot{\Phi}\mathcal{A}'(\Phi) + \dot{\Phi}^2\mathcal{A}''(\Phi)\right) = 0\,,$$

Scalar field equation

$$(-6H^2 + 9H\gamma + 3\dot{\gamma})\mathcal{A}'(\Phi) - (6H\dot{\Phi} + 2\ddot{\Phi})\mathcal{B}(\Phi) - \dot{\Phi}^2\mathcal{B}'(\Phi) - 2\mathcal{V}'(\Phi) = 0$$

Non-metricity scalar:

$$Q = -6H^2 + 9H\bar{\gamma} + 3\dot{\bar{\gamma}} \tag{16}$$

The connection equation does not provide dynamics for the independent connection function γ but rather restrains the scalar field dynamics to:

$$\ddot{\Phi} = -3H\dot{\Phi} - \frac{\mathcal{A}''(\Phi)}{\mathcal{A}'(\Phi)}\dot{\Phi}^2 \,. \tag{17}$$

Connection set III

Metric equation

$$6H^2\mathcal{A}(\Phi) - 3\bar{\gamma}\dot{\Phi}\mathcal{A}'(\Phi) - \dot{\Phi}^2\mathcal{B}(\Phi) - 2\mathcal{V}(\Phi) = 2\kappa^2\rho$$

$$(\bar{\gamma}\,\dot{\Phi} - 4H\dot{\Phi})\mathcal{A}'(\Phi) - \left(6H^2 + 4\dot{H}\right)\mathcal{A}(\Phi) - \dot{\Phi}^2\mathcal{B}(\Phi) + 2\mathcal{V}(\Phi) = 2\kappa^2\mathrm{w}\rho$$

Connection equation

$$-6\dot{\bar{\gamma}}\,\dot{\Phi}\mathcal{A}'(\Phi) - 3\bar{\gamma}\left(\ddot{\Phi}\mathcal{A}'(\Phi) + 5H\,\dot{\Phi}\,\mathcal{A}'(\Phi) + \dot{\Phi}^2\mathcal{A}''(\Phi)\right) = 0$$

Connection set III

Scalar field equation

$$(-6H^2 + 9H\bar{\gamma} + 3\dot{\bar{\gamma}})\mathcal{A}'(\Phi) - (6H\dot{\Phi} + 2\ddot{\Phi})\mathcal{B}(\Phi) - \dot{\Phi}^2\mathcal{B}'(\Phi) - 2\mathcal{V}'(\Phi) = 0$$

Non-metricity scalar:

$$Q = -6H^2 + 9H\gamma + 3\dot{\gamma} \tag{18}$$

Expand around GR

Evolution of small perturbation around GR limit:

$$\phi(t) = \phi_* + x(t)\,, \qquad H(t) = H_*(t) + h(t)\,, \qquad \gamma(t) = \gamma_*(t) + g(t)\,, \qquad \rho(t) = \rho_*(t) + r(t)\,, \tag{19}$$

In matter dominated case:

$$H_*(t) = \frac{2}{3(t-t_s)}\,, \qquad \rho_*(t) = \frac{4(1+f_*)}{3\kappa^2(t-t_s)^2}\,, \qquad V_* = 0\,, \tag{20}$$

In relativistic matter:

$$H_*(t) = \frac{1}{2(t-t_s)}\,, \qquad \rho_*(t) = \frac{3(1+f_*)}{4\kappa^2(t-t_s)^2}\,, \qquad V_* = 0\,, \tag{21}$$

In potential dominated case:

$$H_*(t) = \sqrt{\frac{V_*}{3(1+f_*)}}\,, \qquad \rho_*(t) = 0\,, \qquad V_* = \mathrm{const.} \neq 0\,. \tag{22}$$

From the first order small scalar field equation of connection set I:

$$\ddot{x} = -3H_*\dot{x}(t) - 3H_*^2 f_*'' x(t) - V_*'' x(t) \quad (23)$$

The background scalar field equation for connection sets II and III are:

$$\left(3\gamma_*'(t) + 9H_*(t)\gamma_*(t) - 6H_*^2\right) f_*' - 2V_*' = 0. \quad (24)$$

Cases	Matter-Domination	Radiation-Domination	Potential-Domination
$f'_* = 0$, $V'_* = 0$, $V''_* = 0$ v_* is imaginary	$x(t) \sim t^{-\frac{1}{4}}$, $h(t) \sim t^{-2}$, $r(t) \sim t^{-3}$	$x(t) \sim t^{-\frac{1}{2}}$, $h(t) \sim t^{-2}$, $r(t) \sim t^{-3}$	$h(t) \sim e^{-3H_* t}$, $r(t) \sim e^{-3H_* t}$, $x(t) \sim e^{\frac{-3H_* t}{2}}$
$f'_* = 0$, $V'_* = 0$, $V''_* = 0$ v_* is real	$x(t) \sim t^{-\frac{1}{2}(1-v_*)}$, $h(t) \sim t^{-2}$, $r(t) \sim t^{-3}$	$x(t) \sim t^{-\frac{1}{4}(1-v_*)}$, $h(t) \sim t^{-2}$, $r(t) \sim t^{-3}$	$h(t) \sim e^{-3H_* t}$, $r(t) \sim e^{-3H_* t}$, $x(t) \sim e^{\frac{-3H_* t}{2}(1-v_*)}$
$f'_* = 0$, $V'_* = 0$, $V''_* < 0$	$x(t) \sim t^{-1} e^{t}$, $h(t) \sim t^{-2}$, $r(t) \sim t^{-3}$	$x(t) \sim t^{\frac{-3}{4}} e^{t}$, $h(t) \sim t^{-2}$, $r(t) \sim t^{-3}$	$\gamma_*(t) \sim e^{-3H_* t}$, $x(t) \sim e^{-3H_* t}$, $h(t) \sim e^{-3H_* t}$, $g(t) \sim te^{-3H_* t}$, $r(t) \sim e^{-3H_* t}$
$f'_* \neq 0$, $V'_* = 0$, $V''_* > 0$	$x(t) \sim t^{-1}$, $h(t) \sim t^{-2}$, $r(t) \sim t^{-3}$	$x(t) \sim t^{\frac{-3}{4}}$, $h(t) \sim t^{-2}$, $r(t) \sim t^{-3}$	$\gamma_*(t) \sim e^{-3H_* t}$, $x(t) \sim e^{-3H_* t}$, $h(t) \sim e^{-3H_* t}$, $g(t) \sim te^{-3H_* t}$, $r(t) \sim e^{-3H_* t}$

Balanced case

Scalar field at a value which satisfies $V_* f'_* = -V'_*(1 + f_*)$. Then the solution from the leading order [5] :

$$r(t) \sim e^{-3H_* t} \tag{25}$$

From the remaining perturbed equations evolve as:

$$h(t) \sim x(t) \sim \begin{cases} e^{-\frac{3H_* t}{2}}, & v_* \text{ imaginary} \\ e^{-\frac{3H_* t}{2}(1-v_*)}, & v_* \text{ real} \end{cases} , \quad v_* = \sqrt{1 - \frac{4f''_*}{3} - \frac{4(1+f_*)V''_*}{3V_*} - \frac{8f'_* V'_*}{V_*}} \tag{26}$$

This configuration is stable if

$$V_* f''_* + (1 + f_*)V'' + 2f'_* V'_* > 0$$

[5] Laur Järv and Alexey Toporensky, Phys. Rev. D 93 (2016) 024051, DOI:https://doi.org/10.1103/PhysRevD.93.024051

Cases	Matter-Domination	Radiation-Domination	Potential-Domination
$f'_* \neq 0, V'_* \neq 0$	$\gamma_*(t) \sim t, x(t) \sim t^{-1}, h(t) \sim t^0, g(t) \sim t^2, r(t) \sim t^{-1}$	$\gamma_*(t) \sim t, x(t) \sim t^{-\frac{7}{2}}, h(t) \sim t^{\frac{1}{2}}, g(t) \sim t^{\frac{5}{2}}, r(t) \sim t^{-\frac{1}{2}}$	$\gamma_*(t) \sim e^{-3H_*t}, x(t) \sim e^{-3H_*t}, h(t) \sim e^{-3H_*t}, g(t) \sim te^{-3H_*t}, r(t) \sim e^{-3H_*t}$
$f'_* \neq 0, V'_* = 0$	$\gamma_*(t) \sim t^{-1}, x(t) \sim t^{-1}, h(t) \sim t^{-2}, g(t) \sim t^0, r(t) \sim t^{-3}$	$\gamma_*(t) \sim t^{-1}, x(t) \sim t^{-\frac{1}{2}}, h(t) \sim t^{-2}\ln t, g(t) \sim t^{\frac{1}{2}}, r(t) \sim t^{-3}\ln t$	$\gamma_*(t) \sim e^{-3H_*t}, x(t) \sim e^{-3H_*t}, h(t) \sim e^{-3H_*t}, g(t) \sim te^{-3H_*t}, r(t) \sim e^{-3H_*t}$
$f'_* \neq 0, V'_* = V''_* = 0$	$\gamma_*(t) \sim t^{-1}, x(t) \sim t^{-1}, h(t) \sim t^{-2}, g(t) \sim t^{-2}, r(t) \sim t^{-3}$	$\gamma_*(t) \sim t^{-1}, x(t) \sim t^{-\frac{1}{2}}, h(t) \sim t^{-2}\ln t, g(t) \sim t^{-\frac{3}{2}}, r(t) \sim t^{-3}\ln t$	$h(t) \sim e^{-3H_*t}, r(t) \sim e^{-3H_*t}, x(t) \sim e^{\frac{-3H_*t}{2}}, \gamma_*(t) \sim e^{-3H_*t}, g(t) \sim \gamma_*(t) \sim te^{-3H_*t}$
$f'_* = 0, V'_* = 0$	$\gamma_*(t) \sim t,\ x(t) \sim t^{-\frac{1}{2}}, h(t) \sim t^{-2}, g(t) \sim t^{\frac{1}{2}}, r(t) \sim t^{-3}$	$h(t) \sim t^{-2}, r(t) \sim t^{-3}, x(t) \sim t^{\frac{-1}{4}}, \gamma_*(t) \sim t, g(t) \sim t^{\frac{3}{4}}$	$h(t) \sim e^{-3H_*t}, r(t) \sim e^{-3H_*t}, x(t) \sim e^{\frac{-3H_*t}{2}}$
$f'_* = 0, V'_* = V''_* = 0$	$\gamma_*(t) \sim t^{-1}, x(t) \sim t^{-\frac{1}{2}}, h(t) \sim t^{-2}, g(t) \sim t^{-\frac{3}{2}}, r(t) \sim t^{-3}$	$\gamma_*(t) \sim t^{-1}, h(t) \sim t^{-2}, r(t) \sim t^{-3}, x(t) \sim t^{\frac{-1}{4}}, g(t) \sim t^0$	$h(t) \sim e^{-3H_*t}, r(t) \sim e^{-3H_*t}, x(t) \sim e^{\frac{-3H_*t}{2}}$

Sudden singularity

In the case $f'_* = 0$ and $V'_* = 0$, the leading order solution from the connection equation

$$x(t) = \pm\sqrt{\frac{c_6}{t} + c_7}\,, \quad \dot{x}(t) = \mp \frac{c_6}{2t^2\sqrt{\frac{c_6}{t} + c_7}}\,.$$

$$c_6 = -2\dot{x}_0\, x_0\, t_0^2\,, \qquad c_7 = x_0^2 + 2\dot{x}_0\, x_0\, t_0\,, \tag{27}$$

At finite time $t_* = -\frac{c_6}{c_7}$. If c_6 and c_7 have opposite signs.

$$\frac{t_0}{t_*} = -\frac{c_7\, t_0}{c_6} = 1 + \frac{x_0}{2\dot{x}_0\, t_0} \tag{28}$$

Case-1: If x_0 and $\dot{x}_0$ are of the same sign, then $t_* < t_0$.

Case-2: If x_0 and $\dot{x}_0$ are of opposite signs and $|\dot{x}_0| > \frac{|x_0|}{2t_0}$, then $t_* > t_0$.

Case-3: If x_0 and $\dot{x}_0$ are of opposite signs, c_6 and c_7 are of same sign, then $|\dot{x}_0| < \frac{|x_0|}{2t_0}$

Finally, $\boxed{x_0 = -2\dot{x}_0\, t_0}$, when $c_7 = 0$.

Results

- In the connection set-I case the extra function γ does not appear in the field equations and is left undetermined. In the connection set II and III, it adds a new degree of freedom and affects the dynamics of the system.

- Comparing the metric equations, the extra function γ would have $\mathrm{w} = 1$ in set-II and $\mathrm{w} = -\frac{1}{3}$ in set-III, and thus can not play the role of dark energy or dark matter.

- In the connection set-I case, standard cosmological evolution is stable if the model functions possess a value ϕ_* which either satisfies $V'_* = 0, V''_* > 0, f'_* = 0$ or $f'_* = 0, f''_* > 0, V'_* = 0, V''_* = 0$.

- In connection set-II case, the standard cosmological evolution is stable only if $f'_* \neq 0$ and the potential is constant.

- In the connection set-III case, work is in progress.

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- In connection set-II case, the standard cosmological evolution is stable only if $f'_* \neq 0$ and the potential is constant.
- In the connection set-III case, work is in progress.

Thanks for your attention!