Stellar Structure in Scalar-Tensor Symmetric Teleparallel Gravity

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Scalar-Tensor Symmetric Teleparallel Gravity[2]

Introduction

Vanishing curvature and torsion

$$R^{\sigma}{}_{\rho\mu\nu} = \partial_{\mu}\Gamma^{\sigma}{}_{\nu\rho} - \partial_{\nu}\Gamma^{\sigma}{}_{\mu\rho} + \Gamma^{\sigma}{}_{\mu\lambda}\Gamma^{\lambda}{}_{\nu\rho} - \Gamma^{\sigma}{}_{\mu\lambda}\Gamma^{\lambda}{}_{\nu\rho} = 0 \, , \tag{1}$$

$$T^{\sigma}{}_{\mu\nu} = \Gamma^{\sigma}{}_{\mu\nu} - \Gamma^{\sigma}{}_{\nu\mu} = 0 \, , \tag{2}$$

and nonmetricity

$$Q_{\rho\mu\nu} = \nabla_{\rho} g_{\mu\nu} = \partial_{\rho} g_{\mu\nu} - \Gamma^{\lambda}{}_{\rho\mu} g_{\lambda\nu} - \Gamma^{\lambda}{}_{\rho\nu} g_{\mu\lambda} \, , \tag{3}$$

$$Q = -\frac{1}{4} Q_{\lambda\mu\nu} Q^{\lambda\mu\nu} + \frac{1}{2} Q_{\lambda\mu\nu} Q^{\lambda\mu\nu} + \frac{1}{4} Q_{\mu} Q^{\mu} - \frac{1}{2} Q_{\mu} \bar{Q}^{\mu} \, , \quad Q_{\mu} = Q^{\nu}_{\mu\nu} \, , \quad \bar{Q}_{\mu} = Q^{\nu}_{\nu\mu} \, . \tag{4}$$

Levi-Civita Riemann tensor and Ricci scalar

$$\mathring{R}^{\sigma}{}_{\rho\mu\nu} = \mathring{\nabla}_{\nu} L^{\sigma}{}_{\mu\rho} - \mathring{\nabla}_{\mu} L^{\sigma}{}_{\nu\rho} + L^{\sigma}{}_{\nu\lambda} L^{\lambda}{}_{\mu\rho} - L^{\sigma}{}_{\mu\lambda} L^{\lambda}{}_{\nu\rho} \, , \tag{5}$$

$$L^{\sigma}{}_{\mu\nu} = \frac{1}{2}\left(Q^{\sigma}{}_{\mu\nu} - Q_{\mu}{}^{\sigma}{}_{\nu} - Q_{\nu}{}^{\sigma}{}_{\mu}\right) \, , \tag{6}$$

$$\mathring{R} = Q + \mathring{\nabla}_{\mu}\left(\bar{Q}^{\mu} - Q^{\mu}\right) \tag{7}$$

[2] Bahamonde et al. 2022

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Field Equations

The action reads

$$S = \frac{1}{2\kappa^2} \int \mathrm{d}^4 x \sqrt{-g} \left(\mathcal{A}(\Phi) Q - \mathcal{B}(\Phi) g^{\alpha\beta} \partial_\alpha \Phi \partial_\beta \Phi - 2\mathcal{V}(\Phi) \right) + S_m [g, \Psi] \ . \tag{8}$$

Metric field equation[3]

$$\begin{aligned} \kappa^2 T_{\mu\nu} =& \mathcal{A}(\Phi) \mathring{G}_{\mu\nu} + 2 \frac{\mathrm{d}\mathcal{A}(\Phi)}{\mathrm{d}\Phi} P^{\alpha}{}_{\mu\nu} \partial_\alpha \Phi - \mathcal{B}(\Phi) \partial_\mu \Phi \partial_\nu \Phi \\ & + \frac{g_{\mu\nu}}{2} \left(\mathcal{B}(\Phi) g^{\alpha\beta} \partial_\alpha \Phi \partial_\beta \Phi + 2\mathcal{V}(\Phi) \right) \ , \end{aligned} \tag{9}$$

Scalar Field Equation

$$0 = 2 \mathring{\Box} \Phi \mathcal{B}(\Phi) + \frac{\mathrm{d}\mathcal{B}}{\mathrm{d}\Phi} g^{\alpha\beta} \partial_\alpha \Phi \partial_\beta \Phi + \frac{\mathrm{d}\mathcal{A}}{\mathrm{d}\Phi} Q - 2 \frac{\mathrm{d}\mathcal{V}}{\mathrm{d}\Phi} \ , \tag{10}$$

Connection Equation

$$0 = \frac{1}{2} \frac{\mathrm{d}\mathcal{A}}{\mathrm{d}\Phi} Q_\alpha K_\mu{}^\alpha + \frac{\mathrm{d}^2\mathcal{A}}{\mathrm{d}\Phi^2} \partial_\alpha \Phi K_\mu{}^\alpha + \frac{\mathrm{d}\mathcal{A}}{\mathrm{d}\Phi} \nabla_\alpha K_\mu{}^\alpha \ . \tag{11}$$

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$$P^{\alpha}{}_{\mu\nu} = -\frac{1}{4} \left(g_{\mu\nu} \bar{Q}^\alpha + \delta^\alpha{}_\mu Q_\nu \right) - \frac{1}{4} Q^\alpha{}_{\mu\nu} + \frac{1}{4} Q_\mu{}^\alpha{}_\nu + \frac{1}{4} g_{\mu\nu} Q_\alpha \ , \quad K_\mu{}^\beta = \left[\frac{1}{2} Q_\mu g^{\alpha\beta} - \frac{1}{2} \delta^\alpha_\mu Q^\beta - Q_\mu{}^{\alpha\beta} + \delta^\alpha_\mu Q^{\gamma\beta}_\gamma \right] (\partial_\alpha \mathcal{A})$$

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Static and spherically symmetric geometry

Connection[4]

- Two solution sets for a static and spherically symmetric connection with zero curvature and torsion
- First solution set
  - Three independent components $\Gamma^{\phi}{}_{r\phi}$, $\Gamma^{t}{}_{rr}$, $\Gamma^{r}{}_{rr}$ and constraint for $(\Gamma^{\phi}{}_{r\phi})'$
  - Effectively reduces to GR

$$\frac{3}{2}\frac{\mathrm{d}\mathcal{A}}{\mathrm{d}\Phi}\Phi'(r) = 0 \,, \tag{12}$$

- Second solution set
  - Four independent components $\Gamma^{t}{}_{\theta\theta}$, $\Gamma^{t}{}_{rr}$, $\Gamma^{r}{}_{\theta\theta}$, $\Gamma^{r}{}_{rr}$ and constraints for $(\Gamma^{r}{}_{\theta\theta})'$, $(\Gamma^{t}{}_{\theta\theta})'$
  - Allows for solutions beyond GR

$$\frac{1}{2}\left[2c(k-2c)\Gamma^{t}{}_{\theta\theta} + k\right]\frac{\mathrm{d}\mathcal{A}}{\mathrm{d}\Phi}\Phi'(r) = 0 \,, \tag{13}$$

[4] D'Ambrosio et al. 2022

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Static and spherically symmetric geometry

Connection, solution set 2 branch 2

$$\Gamma^{\rho}{}_{\mu\nu}(r,\theta) = \left[\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \Gamma^{t}{}_{rr} & 0 & 0 \\ 0 & 0 & \Gamma^{t}{}_{\theta\theta} & 0 \\ 0 & 0 & 0 & \Gamma^{t}{}_{\theta\theta}\sin^2(\theta) \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \Gamma^{r}{}_{rr} & 0 & 0 \\ 0 & 0 & \Gamma^{r}{}_{\theta\theta} & 0 \\ 0 & 0 & 0 & \Gamma^{r}{}_{\theta\theta}\sin^2(\theta) \end{bmatrix} \right.$$
$$\left. \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{\Gamma^{r}{}_{\theta\theta}} & 0 \\ 0 & -\frac{1}{\Gamma^{r}{}_{\theta\theta}} & 0 & 0 \\ 0 & 0 & 0 & -\sin(\theta)\cos(\theta) \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{\Gamma^{r}{}_{\theta\theta}} \\ 0 & 0 & 0 & \cot(\theta) \\ 0 & -\frac{1}{\Gamma^{r}{}_{\theta\theta}} & \cot(\theta) & 0 \end{bmatrix} \right] \tag{14}$$

with the relations

$$\frac{\mathrm{d}\Gamma^{r}{}_{\theta\theta}}{\mathrm{d}r} = -\Gamma^{r}{}_{\theta\theta}\Gamma^{r}{}_{rr} - 1 \,, \quad \frac{\mathrm{d}\Gamma^{t}{}_{\theta\theta}}{\mathrm{d}r} = -\Gamma^{r}{}_{\theta\theta}\Gamma^{t}{}_{rr} - \frac{\Gamma^{t}{}_{\theta\theta}}{\Gamma^{r}{}_{\theta\theta}} \,. \tag{15}$$

Metric

$$\mathrm{d}s^2 = -g_{tt}(r)\mathrm{d}t^2 + g_{rr}(r)\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2 \,. \tag{16}$$

Metric field equations

$$-8\pi G\rho = \frac{\mathcal{B}\Phi'^2}{2g_{rr}} + \frac{\Phi'\left(r^2 + 2r\Gamma^r{}_{\theta\theta} + \left(\Gamma^r{}_{\theta\theta}\right)^2 g_{rr}\right)}{r^2\Gamma^r{}_{\theta\theta}g_{rr}}\frac{\mathrm{d}\mathcal{A}}{\mathrm{d}\Phi} - \frac{\left(rg'_{rr} + g_{rr}^2 - g_{rr}\right)\mathcal{A}}{r^2 g_{rr}^2} + \mathcal{V} \tag{17}$$

$$8\pi G p = -\frac{\mathcal{B}\Phi'^2}{2g_{rr}} - \frac{\Phi'\left(r^2 - \left(\Gamma^r{}_{\theta\theta}\right)^2 g_{rr}\right)}{r^2\Gamma^r{}_{\theta\theta}g_{rr}}\frac{\mathrm{d}\mathcal{A}}{\mathrm{d}\Phi} + \frac{\left(rg'_{tt} - g_{rr}g_{tt} + g_{tt}\right)\mathcal{A}}{r^2 g_{rr}g_{tt}} + \mathcal{V} \tag{18}$$

$$\begin{aligned} 8\pi G p = {} & \frac{\mathcal{B}\Phi'^2}{2g_{rr}} + \frac{\Phi'\left(r\Gamma^r{}_{\theta\theta}g'_{tt} + 2rg_{tt} + 2\Gamma^r{}_{\theta\theta}g_{tt}\right)}{2r\Gamma^r{}_{\theta\theta}g_{rr}g_{tt}}\frac{\mathrm{d}\mathcal{A}}{\mathrm{d}\Phi} + \mathcal{V} \\ & - \frac{\left(-2rg_{rr}g_{tt}g''_{tt} + rg_{rr}\left(g'_{tt}\right)^2 + rg_{tt}g'_{rr}g'_{tt} - 2g_{rr}g_{tt}g'_{tt} + 2g_{tt}^2g'_{rr}\right)\mathcal{A}}{4rg_{rr}^2g_{tt}^2} \end{aligned} \tag{19}$$

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Static and spherically symmetric field equations

Connection equation

$$
\begin{aligned}
0 =& \frac{2\left(r^2-\left(\Gamma^r{}_{\theta\theta}\right)^2 g_{rr}\right)\Phi'^2}{r^2\Gamma^r{}_{\theta\theta}g_{rr}}\frac{\mathrm{d}^2\mathcal{A}(\Phi)}{\mathrm{d}\Phi^2} \\
&+\Bigg(2r^2 g_{rr}g_{tt}\Phi'' + r^2 g_{rr}\Phi' g'_{tt} - r^2 g_{tt}\Phi' g'_{rr} + 4r g_{rr}g_{tt}\Phi' - 2\left(\Gamma^r{}_{\theta\theta}\right)^2 g_{rr}^2 g_{tt}\Phi'' \\
&\quad - \left(\Gamma^r{}_{\theta\theta}\right)^2 g_{rr}^2\Phi' g'_{tt} - \left(\Gamma^r{}_{\theta\theta}\right)^2 g_{rr}g_{tt}\Phi' g'_{rr} - 4\Gamma^r{}_{\theta\theta}g_{rr}^2 g_{tt}\Phi'\left(\Gamma^r{}_{\theta\theta}\right)'\Bigg)\frac{1}{r^2\Gamma^r{}_{\theta\theta}g_{rr}^2 g_{tt}}\frac{\mathrm{d}\mathcal{A}}{\mathrm{d}\Phi} \qquad (20)
\end{aligned}
$$

Scalar field equation

$$
0 = -2\frac{\mathrm{d}\mathcal{V}(\Phi)}{\mathrm{d}\Phi} + \frac{\Phi'^2}{g_{rr}}\frac{\mathrm{d}\mathcal{B}(\Phi)}{\mathrm{d}\Phi} + \frac{\left(2r g_{rr}g_{tt}\Phi'' + r g_{rr}\Phi' g'_{tt} - r g_{tt}\Phi' g'_{rr} + 4 g_{rr}g_{tt}\Phi'\right)\mathcal{B}(\Phi)}{r g_{rr}^2 g_{tt}} - \mathcal{Q}\frac{\mathrm{d}\mathcal{A}}{\mathrm{d}\Phi} \qquad (21)
$$

Tolman-Oppenheimer-Volkoff equation[5]

Modified Gravity Theories

- Modified gravity theory that can be written as

$$\sigma(\Psi^i)\left(\mathring{G}_{\mu\nu} - W_{\mu\nu}\right) = \kappa^2 T_{\mu\nu} \ , \tag{22}$$

$$\mathring{G}_{\mu\nu} = \kappa^2 T^{eff}_{\mu\nu} = \frac{\kappa^2}{\sigma} T_{\mu\nu} + W_{\mu\nu} \ , \tag{23}$$

where $\kappa^2 = 8\pi G$ and such that $\mathring{\nabla}_\mu T^{\mu\nu}_{eff} = 0$

- Perfect fluid

$$T_{\mu\nu} = (\rho + p)\, u_\mu u_\nu + p g_{\mu\nu} \ , \quad u_\mu u^\mu = -1 \ , \tag{24}$$

- Static and spherically symmetric geometry $u_0 = -\sqrt{g_{tt}}$.

[5] Wojnar and Velten 2016

Tolman-Oppenheimer-Volkoff equation

Modified Gravity Theories

The components of the Ricci tensor read

$$R_{tt} = -\frac{g_{tt}''}{2g_{rr}} + \frac{g_{tt}'}{4g_{rr}}\left(\frac{g_{rr}'}{g_{rr}} + \frac{g_{tt}'}{g_{tt}}\right) - \frac{g_{tt}'}{rg_{rr}} = -\frac{\kappa^2}{2\sigma}(\rho + 3p)\, g_{tt} + W_{tt} + \frac{g_{tt}W}{2}\,, \tag{25}$$

$$R_{rr} = \frac{g_{tt}''}{2g_{tt}} - \frac{g_{tt}'}{4g_{tt}}\left(\frac{g_{rr}'}{g_{rr}} + \frac{g_{tt}'}{g_{tt}}\right) - \frac{g_{rr}'}{rg_{rr}} = -\frac{\kappa^2}{2\sigma}(\rho - p)\, g_{rr} + W_{rr} - \frac{g_{rr}W}{2}\,, \tag{26}$$

$$R_{\theta\theta} = -1 - \frac{r}{2g_{rr}}\left(-\frac{g_{rr}'}{g_{rr}} + \frac{g_{tt}'}{g_{tt}}\right) + \frac{1}{g_{rr}} = -\frac{\kappa^2}{2\sigma}(\rho - p)\, r^2 + W_{\theta\theta} - \frac{r^2W}{2}\,. \tag{27}$$

Combining these equations to

$$\frac{R_{tt}}{2g_{tt}} + \frac{R_{rr}}{2g_{rr}} + \frac{R_{\theta\theta}}{r^2} \quad \rightarrow \quad g_{rr}(r) = \left(1 - \frac{2G\mathcal{M}(r)}{r}\right)^{-1}\,, \tag{28}$$

and using $\mathring{\nabla}_\mu T^{\mu\nu}_{eff} = 0$ as well as the metric component relations in (27)

$$\frac{g_{tt}'}{g_{tt}} = \frac{g_{rr} - 1}{r} + \frac{\kappa^2 r g_{rr}}{\sigma}\Pi\,, \quad \frac{g_{rr}'}{g_{rr}} = \frac{1 - g_{rr}}{r} + \frac{\kappa^2 r g_{rr}}{\sigma}Q \tag{29}$$

TOV equation

$$\left(\frac{\Pi}{\sigma}\right)' = -\frac{G\mathcal{M}}{r^2}\left(\frac{Q}{\sigma}+\frac{\Pi}{\sigma}\right)\left(1+\frac{4\pi r^3\Pi}{\sigma\mathcal{M}}\right)\left(1-\frac{2G\mathcal{M}}{r}\right)^{-1} + \frac{2}{\kappa^2 r}\left(\frac{W_{rr}}{g_{rr}}-\frac{W_{\theta\theta}}{r^2}\right) \tag{30}$$

with effective energy density and pressure

$$Q(r) := \rho(r) + \frac{\sigma(r)W_{tt}(r)}{\kappa^2 g_{tt}(r)}\,, \quad \Pi(r) := p(r) + \frac{\sigma(r)W_{rr}(r)}{\kappa^2 g_{rr}(r)}\,, \tag{31}$$

and mass

$$\frac{\mathrm{d}\mathcal{M}(r)}{\mathrm{d}r} = 4\pi r^2 \frac{Q(r)}{\sigma(r)}\,. \tag{32}$$

Tolman-Oppenheimer-Volkoff equation

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Metric field equation

$$\kappa^2 T_{\mu\nu} = \sigma(\Psi^i)\left(\mathring{G}_{\mu\nu} - W_{\mu\nu}\right) ,$$

$$\kappa^2 T_{\mu\nu} = \mathcal{A}\mathring{G}_{\mu\nu} + 2\frac{\mathrm{d}\mathcal{A}}{\mathrm{d}\Phi}P^{\alpha}{}_{\mu\nu}\partial_\alpha\Phi - \mathcal{B}\partial_\mu\Phi\partial_\nu\Phi + \frac{g_{\mu\nu}}{2}\left(\mathcal{B}g^{\alpha\beta}\partial_\alpha\Phi\partial_\beta\Phi + 2\mathcal{V}\right) , \tag{33}$$

such that $\sigma(\Psi^i) = \mathcal{A}(\Phi)$ and

$$W_{\mu\nu} = -\frac{1}{\mathcal{A}}\left[2\frac{\mathrm{d}\mathcal{A}}{\mathrm{d}\Phi}P^{\alpha}{}_{\mu\nu}\partial_\alpha\Phi - \mathcal{B}\partial_\mu\Phi\partial_\nu\Phi + \frac{g_{\mu\nu}}{2}\left(\mathcal{B}g^{\alpha\beta}\partial_\alpha\Phi\partial_\beta\Phi + 2\mathcal{V}\right)\right] . \tag{34}$$

For the connection solution, set 2 branch 2, in a static and spherically symmetric geometry

$$W_{tt}(r) = \frac{g_{tt}\Phi'}{g_{rr}\mathcal{A}}\left\{\frac{\mathcal{B}\Phi'}{2} + \frac{r^2 + 2r\Gamma^r{}_{\theta\theta} + (\Gamma^r{}_{\theta\theta})^2 g_{rr}}{r^2\Gamma^r{}_{\theta\theta}}\frac{\mathrm{d}\mathcal{A}}{\mathrm{d}\Phi}\right\} + \frac{g_{tt}\mathcal{V}}{\mathcal{A}} , \tag{35}$$

$$W_{rr}(r) = \frac{\Phi'}{\mathcal{A}}\left\{\frac{\mathcal{B}\Phi'}{2} + \frac{r^2 - (\Gamma^r{}_{\theta\theta})^2 g_{rr}}{r^2\Gamma^r{}_{\theta\theta}}\frac{\mathrm{d}\mathcal{A}}{\mathrm{d}\Phi}\right\} - \frac{g_{rr}\mathcal{V}}{\mathcal{A}} , \tag{36}$$

$$W_{\theta\theta}(r) = -\frac{r^2\Phi'}{g_{rr}\mathcal{A}}\left\{\frac{\mathcal{B}\Phi'}{2} + \left(\frac{g'_{tt}}{2g_{tt}} + \frac{1}{\Gamma^r{}_{\theta\theta}} + \frac{1}{r}\right)\frac{\mathrm{d}\mathcal{A}}{\mathrm{d}\Phi}\right\} - \frac{r^2\mathcal{V}}{\mathcal{A}} , \tag{37}$$

Tolman-Oppenheimer-Volkoff equation

Scalar-Tensor Symmetric Teleparallel Gravity

'The effective energy density and pressure read

$$\mathcal{Q}(r) = \rho(r) - \frac{\Phi'}{\kappa^2 r g_{rr}} \left\{ \frac{r\mathcal{B}\Phi'}{2} + \left(2 + \frac{r}{\Gamma^r{}_{\theta\theta}} + \frac{g_{rr}\Gamma^r{}_{\theta\theta}}{r} \right) \frac{\mathrm{d}\mathcal{A}}{\mathrm{d}\Phi} \right\} - \frac{\mathcal{V}}{\kappa^2} , \tag{38}$$

$$\Pi(r) = p(r) - \frac{\Phi'}{\kappa^2 r g_{rr}} \left\{ \frac{r\mathcal{B}\Phi'}{2} + \left(\frac{r}{\Gamma^r{}_{\theta\theta}} - \frac{g_{rr}\Gamma^r{}_{\theta\theta}}{r} \right) \frac{\mathrm{d}\mathcal{A}}{\mathrm{d}\Phi} \right\} + \frac{\mathcal{V}}{\kappa^2} , \tag{39}$$

and hence the mass equation is

$$\frac{\mathrm{d}\mathcal{M}(r)}{\mathrm{d}r} = \frac{4\pi r^2}{\mathcal{A}} \left[\rho(r) - \frac{\Phi'}{\kappa^2 r g_{rr}} \left\{ \frac{r\mathcal{B}\Phi'}{2} + \left(2 + \frac{r}{\Gamma^r{}_{\theta\theta}} + \frac{g_{rr}\Gamma^r{}_{\theta\theta}}{r} \right) \frac{\mathrm{d}\mathcal{A}}{\mathrm{d}\Phi} \right\} - \frac{\mathcal{V}}{\kappa^2} \right] . \tag{40}$$

Tolman-Oppenheimer-Volkoff equation

Scalar-Tensor Symmetric Teleparallel Gravity

$$\left(\frac{\Pi}{\sigma}\right)' = -\frac{G\mathcal{M}}{r^2}\left(\frac{Q}{\sigma}+\frac{\Pi}{\sigma}\right)\left(1+\frac{4\pi r^3\Pi}{\sigma\mathcal{M}}\right)\left(1-\frac{2G\mathcal{M}}{r}\right)^{-1} + \frac{2}{\kappa^2 r}\left(\frac{W_{rr}}{g_{rr}}-\frac{W_{\theta\theta}}{r^2}\right), \tag{41}$$

$$\begin{aligned} p' = &-\frac{G\mathcal{M}}{r^2}\left(\rho+p-\frac{\mathcal{B}\left(\Phi'\right)^2}{\kappa^2 g_{rr}}-\frac{2\Phi'\frac{d\mathcal{A}}{d\Phi}}{\kappa^2{\Gamma^r}_{\theta\theta}g_{rr}}-\frac{2\Phi'\frac{d\mathcal{A}}{d\Phi}}{\kappa^2 r g_{rr}}\right)\left(1-\frac{2G\mathcal{M}}{r}\right)^{-1} \\ &\times\left(\frac{4\pi r^3}{\mathcal{A}\mathcal{M}}\left(p-\frac{\mathcal{B}\left(\Phi'\right)^2}{2\kappa^2 g_{rr}}+\frac{\mathcal{V}}{\kappa^2}-\frac{\Phi'\frac{d\mathcal{A}}{d\Phi}}{\kappa^2{\Gamma^r}_{\theta\theta}g_{rr}}+\frac{{\Gamma^r}_{\theta\theta}\Phi'\frac{d\mathcal{A}}{d\Phi}}{\kappa^2 r^2}\right)+1\right) \\ &+\frac{1}{\kappa^2\mathcal{A}}\left(\Phi'\frac{d\mathcal{A}}{d\Phi}+\frac{r\mathcal{B}\Phi'^2}{2}+\frac{r\Phi'\frac{d\mathcal{A}}{d\Phi}}{{\Gamma^r}_{\theta\theta}}\right)\left(-\frac{\mathcal{B}\left(\Phi'\right)^2}{2g_{rr}}+\mathcal{V}-\frac{\Phi'\frac{d\mathcal{A}}{d\Phi}}{{\Gamma^r}_{\theta\theta}g_{rr}}+\frac{{\Gamma^r}_{\theta\theta}\Phi'\frac{d\mathcal{A}}{d\Phi}}{r^2}\right) \\ &+\frac{p}{\mathcal{A}}\left(\Phi'\frac{d\mathcal{A}}{d\Phi}-\frac{r\mathcal{B}\Phi'^2}{2}-\frac{r\Phi'\frac{d\mathcal{A}}{d\Phi}}{{\Gamma^r}_{\theta\theta}}\right)+\frac{\left(\Phi'\right)^2\frac{d\mathcal{A}}{d\Phi}}{\kappa^2 r\mathcal{A}}\left(\frac{d\mathcal{A}}{d\Phi}-\Phi'\right) \\ &+\frac{1}{\kappa^2}\left(\frac{\mathcal{B}\Phi'}{2r}+\Phi'\frac{d\mathcal{A}}{d\Phi}\left(\frac{1}{r{\Gamma^r}_{\theta\theta}}+\frac{1}{r^2}\right)\right)\left(\frac{1}{g_{rr}}-1\right) \end{aligned} \tag{42}$$

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