Constant-roll inflation in Randall-Sundrum II cosmology

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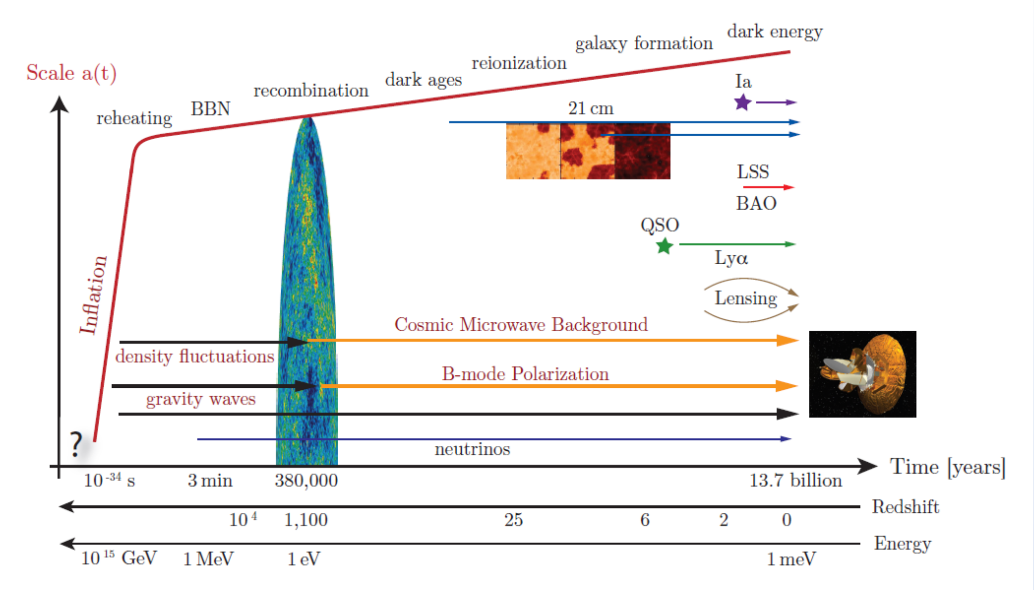
Instead of an Introduction

- A model of ***constant-roll inflation*** (CRI), where the second slow-roll parameter η remains constant, has been investigated.
- In this case the equation for the Hubble rate has an analytical solution, which describes four possible scenarios of inflation.
- The corresponding observational parameters n_s and r are determined, and their values are compared with observational data.
- The scenario when inflation is driven by a tachyon field in the framework of Randall-Sundrum II cosmology is considered and compared with the ``standard`` one.
- We reconstruct the potential which correspond to the model with constant η, and discuss an attractor behaviour in the model.
- A short, initial, excursion to the holographic RSII constant-roll inflation model was made and presented.

The brane world (and RSII) in cosmological inflation and relation to quantum gravity

- From the swampland conjectures which can be used as criteria to distinguish effective field theories (EFT) that can be UV-completed to a QG theory.

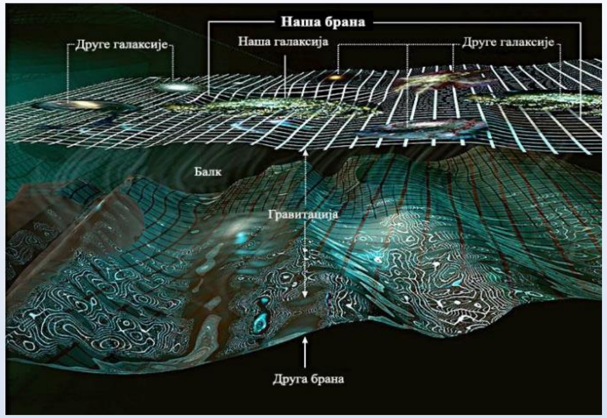
Inflation



- The ***inflation theory*** proposes a period of extremely rapid (exponential) expansion of the universe during the an early stage of evolution of the universe.
- The inflation theory predicts that during inflation (it takes about 10^{-34} s) radius of the universe increased, at least $e^{60} \approx 10^{26}$ times.
- Although inflationary cosmology has successfully complemented the Standard Model, the process of inflation, in particular its origin, is still largely unknown.
- Recent years brought us a ***lot of evidence*** from WMAP and Planck observations of the CMB
- The most important way to ***test inflationary cosmological models*** is to compare the computed and measured values of the ***observational parameters.***

Figure: *Baumann, D. TASI Lectures on Inflation. (2009), arXiv:0907.5424 [hep-th]*

Braneworld cosmology



- Braneworld universe is based on the scenario in which matter is confined on a brane moving in the higher dimensional bulk with only gravity allowed to propagate in the bulk.
- One of the simplest models - Randall-Sundrum (RS)
- RS model was originally proposed to solve the hierarchy problem (1999)
- Later it was realized that this model, as well as any similar braneworld model, may have interesting cosmological implications
- Two branes with opposite tensions are placed at some distance in 5 dimensional space
  - **RS model** – observer reside on the brane with negative tension, distance to the 2nd brane corresponds to the Netwonian gravitational constant
  - **RSII model** – observer is placed on the positive tension brane, the 2nd brane is pushed to infinity

N. Bilic, D.D. Dimitrijevic, G.S. Djordjevic, M. Milosevic, *Tachyon inflation in an AdS braneworld with back-reaction*, International Journal of Modern Physics A. 32 (2017) 1750039.

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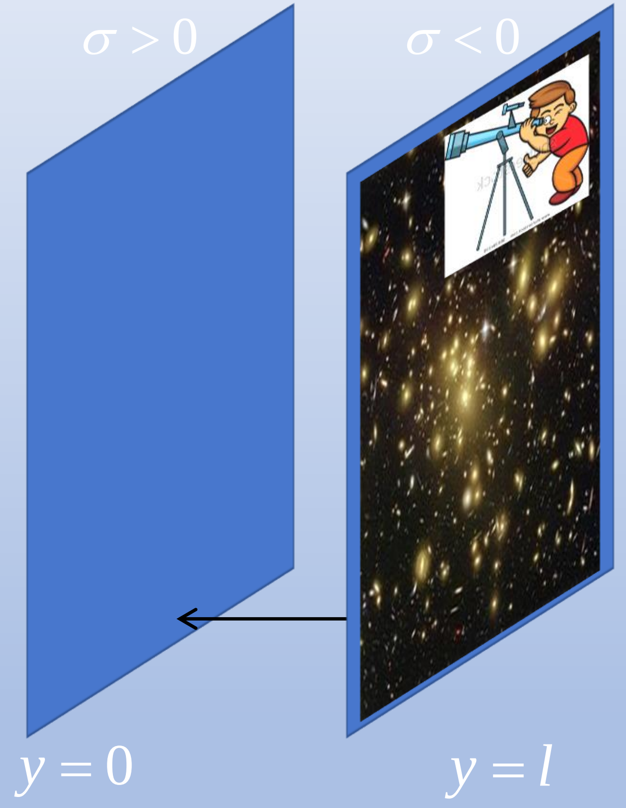
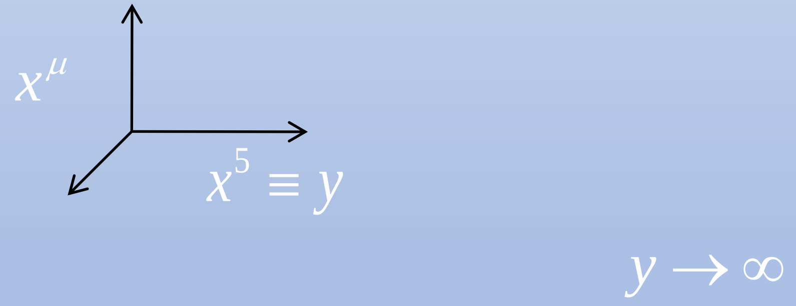
RSI Model

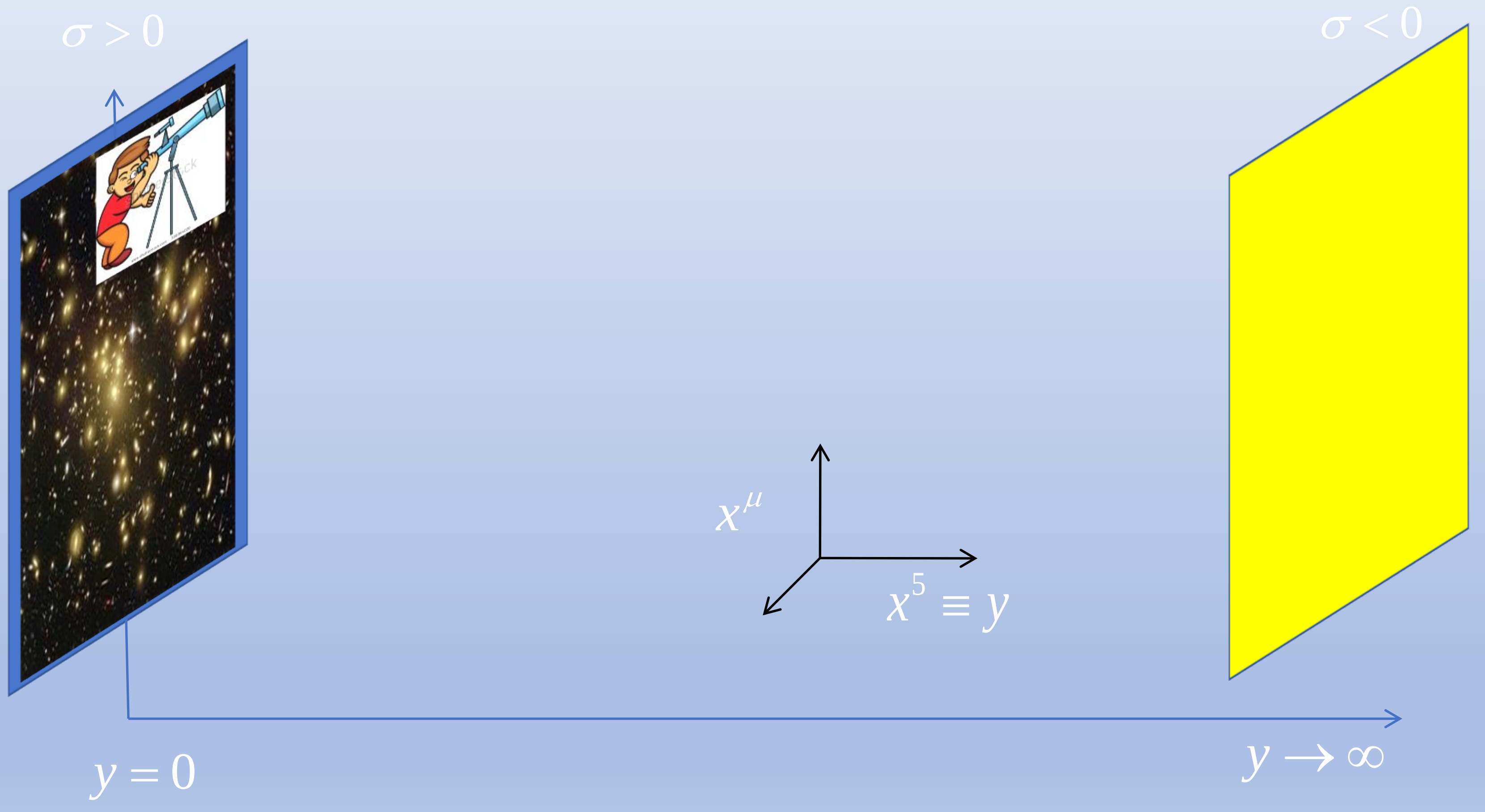


- Observers reside on the negative tension brane at $y = l$.
- The coordinate position $y = l$ of the negative tension brane
- serves as a compactification radius so that the effective
- compactification scale is $\mu_c = 1/l$.



RSII Model

- Observers reside on the positive tension brane at
- $y = 0$ and the negative tension brane is pushed off to infinity in the fifth dimension.



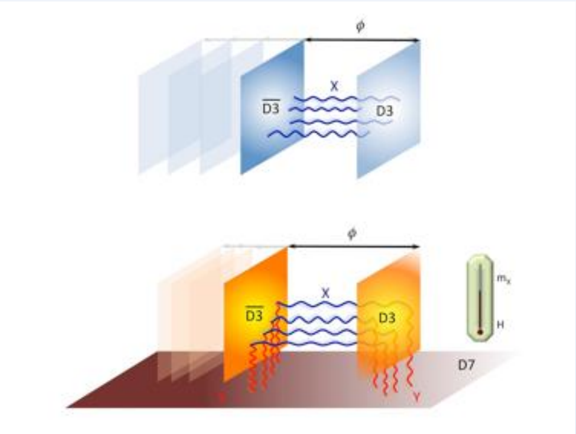
Lagrangian of a scalar field - $\mathcal{L}(X,\phi)$

- In general case – any function of a scalar field ϕ and kinetic energy $X \equiv \frac{1}{2}\partial_\mu\phi\partial_\nu\phi$.
  - Canonical field, potential $V(\phi)$
    $$\mathcal{L}(X,\phi) = BX - V(\phi),$$
  - Non-canonical models
    $$\mathcal{L}(X,\phi) = BX^n - V(\phi),$$
  - Dirac-Born-Infeld (DBI) Lagrangian
    $$\mathcal{L}(X,\phi) = -\frac{1}{f(\phi)}\sqrt{1-2f(\phi)X} - V(\phi),$$
    - Special case – tachyonic $\mathcal{L}(X,\phi) = -V(\phi)\sqrt{1-2\lambda X}$

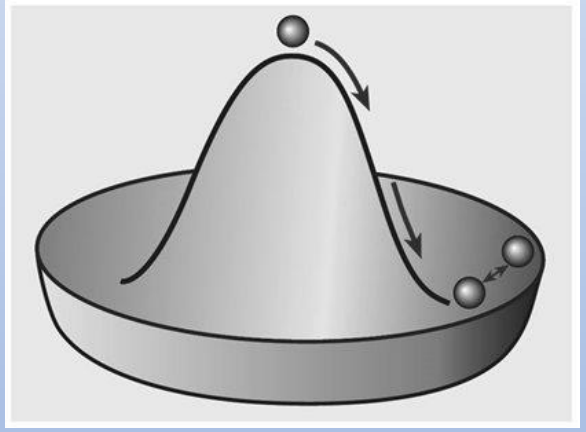
Tachyons

- Traditionally, the word tachyon was used to describe a hypothetical particle which propagates faster than light (Sommerfeld 1904 ?).
- In modern physics this meaning has been changed
  - The effective tachyonic field theory was **proposed** by A. Sen
  - **String theory**: states of quantum fields with imaginary mass (i.e. negative mass squared)
  - It **was believed**: such fields permitted propagation faster than light
  - However it **was realized** that the imaginary mass creates an instability and tachyons spontaneously decay through the process known as tachyon condensation

Tachyion Fields



- No classical interpretation of the ”imaginary mass”
  - The instability: The potential of the tachyonic field is initially at a local maximum rather than a local minimum (like a ball at the top of a hill)
  - A small perturbation - forces the field to roll down towards the local minimum.
  - Quanta are not tachyon any more, but rather an ”ordinary” particle with a positive mass.



Slow-roll parameters

- Friedmann equations in standard cosmology

$$H^2 = \frac{8\pi}{3M_4^2}\rho \qquad \dot{H} = -\frac{4\pi}{M_4^2}\,(\rho + p)$$

$$H(t) = \frac{\dot{a}(t)}{a(t)}$$ The Hubble expansion rate

- The Hubble slow-roll parameters

$$\epsilon = -\frac{\dot{H}}{H^2} \qquad \eta = -\frac{\ddot{H}}{2H\dot{H}} \qquad \eta = -\frac{\ddot{\phi}}{H\dot{\phi}}$$

$$N = \int_{t_\mathrm{i}}^{t_\mathrm{f}} H\,dt$$ e-folds number

- The horizon-flow parameters

$$\varepsilon_0 \equiv H_*/H \qquad \varepsilon_{i+1} \equiv \frac{d\ln|\varepsilon_i|}{dN}, \quad i \geq 0 \qquad \dot{\varepsilon}_i = H\varepsilon_i\varepsilon_{i+1}$$

$$\varepsilon_2 = \frac{\dot{\varepsilon}_1}{\varepsilon_1 H}$$

- The parameter η can be expressed trough ε_i

$$\eta = \varepsilon_1 - \frac{1}{2}\varepsilon_2$$

$$\varepsilon_1 = -\frac{\dot{H}}{H^2}$$

The constant-roll inflation

$$\eta = \varepsilon_1 - \frac{1}{2}\varepsilon_2 \quad \Rightarrow \quad \ddot{H} + 2\eta H\dot{H} = 0, \quad \eta = \mathrm{const}$$

- Nontrivial solutions

$$H_1(t) = -\frac{\beta}{\eta}\tan(\beta t + \gamma) \qquad H_2(t) = \frac{\beta}{\eta}\cot(\beta t + \gamma) \qquad H_3(t) = \frac{\beta}{\eta}\tanh(\beta t + \gamma) \qquad H_4(t) = \frac{\beta}{\eta}\coth(\beta t + \gamma)$$

$$\varepsilon_1(t) = \frac{\eta}{\sin^2(\beta t + \gamma)} \qquad \varepsilon_1(t) = \frac{\eta}{\cos^2(\beta t + \gamma)} \qquad \varepsilon_1(t) = -\frac{\eta}{\sinh^2(\beta t + \gamma)} \qquad \varepsilon_1(t) = \frac{\eta}{\cosh^2(\beta t + \gamma)}$$

$$\varepsilon_2(t) = 2\eta\cot^2(\beta t + \gamma) \qquad \varepsilon_2(t) = 2\eta\tan^2(\beta t + \gamma) \qquad \varepsilon_2(t) = -2\eta\coth^2(\beta t + \gamma) \qquad \varepsilon_2(t) = -2\eta\tanh^2(\beta t + \gamma)$$

$$N(t) = \frac{1}{\eta}\log\cos(\beta t + \gamma) + C \qquad N(t) = \frac{1}{\eta}\log\sin(\beta t + \gamma) + C_4 \qquad N(t) = \frac{1}{\eta}\log\cosh(\beta t + \gamma) + C_4$$

$$\eta > 0 \qquad \eta > 0 \qquad \eta < 0$$

The parameters ε_i cannot be simultaneously positive, the inflation stage never ends!

The solutions which provide a consistent inflationary model.

The constant-roll inflation

- All solutions H(θ) lead to the same function ε_1(N) and ε_2(N).

$$\varepsilon_1(N) = \frac{\eta}{1-(1-\eta)e^{2\eta(N-N_f)}}$$

$$\varepsilon_2(N) = \frac{2\eta(1-\eta)e^{2\eta(N-N_f)}}{1-(1-\eta)e^{2\eta(N-N_f)}}$$

- The observational parameters

$$n_s \simeq 1 - 2\varepsilon_{1i} - \varepsilon_{2i}$$

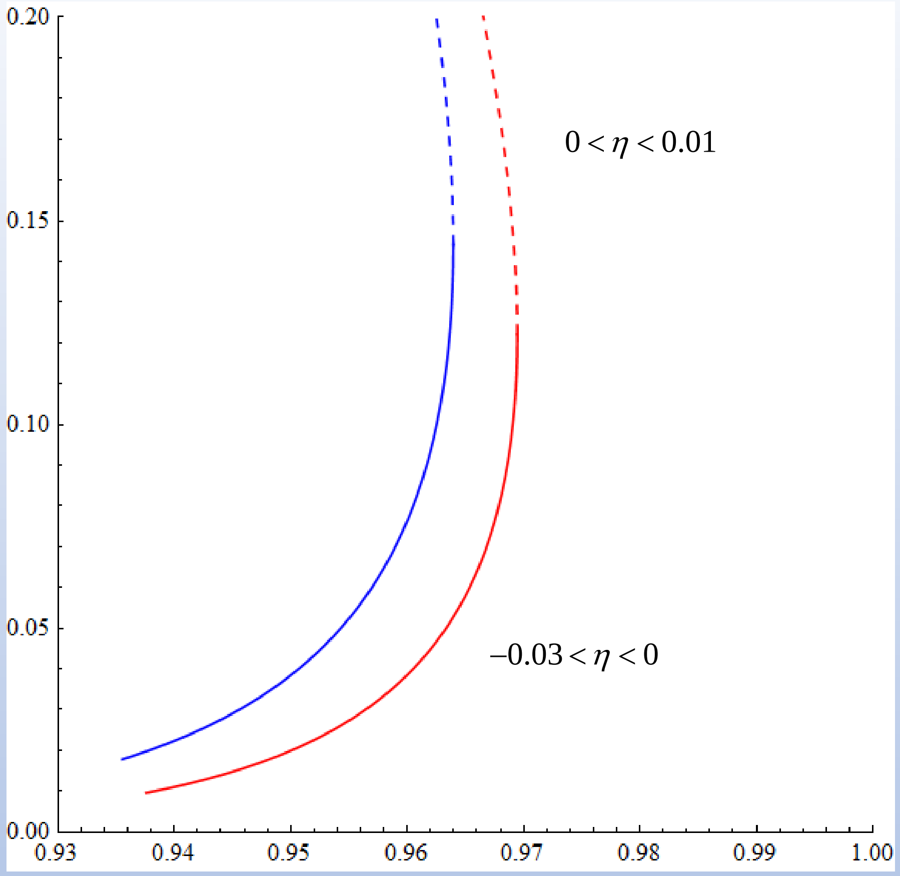
$$r \simeq 16\varepsilon_{1i}$$

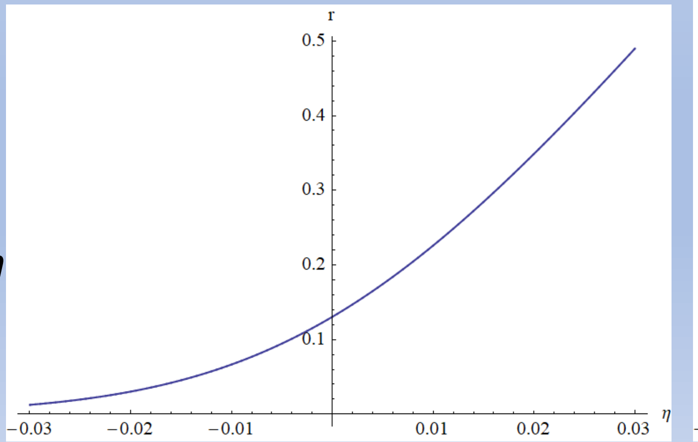
- The observational constraints from Planck 2018

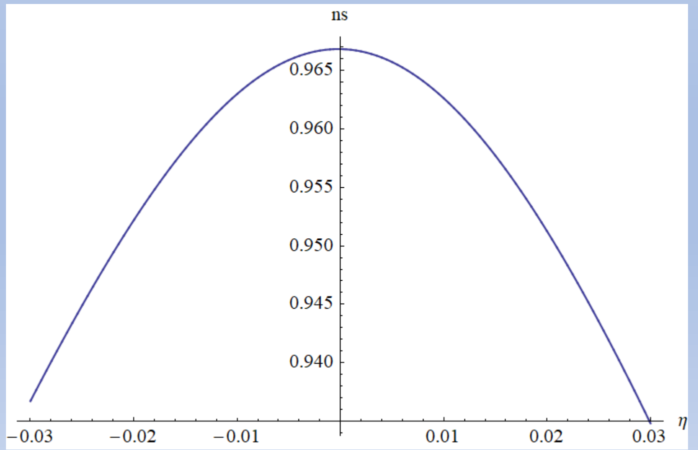
$$n_s = 0.9649 \pm 0.0042$$

$$r < 0.056$$

- A better agreement is achieved for negative and small values of the parameter η







The standard and the RSII cosmology with tachyon matter

- Assuming the geometry of the universe to be described by a five-dimensional FLRW metric

$$ds_5^2 = -dt^2 + a(t)\delta_{ij}dx^i dx^j + dx_5^2$$

- RSII cosmology

$$H^2 = \frac{8\pi}{3M_4^2}\rho(1+\frac{\rho}{2\lambda})$$

$$\dot{H} = -\frac{4\pi}{M_4^2}(1+\frac{\rho}{\lambda})(\rho+p)$$

$\rho \gg \lambda$

The energy density is larger than the tension of the brane

$$H^2 \simeq \frac{4\pi}{3M_4^2}\frac{\rho^2}{\lambda}$$

$$\dot{H} \simeq -\frac{4\pi}{M_4^2}\frac{\rho}{\lambda}(\rho+p)$$

- DBI Lagrangian (homogenius and isotropic case)

$$L = -V(\theta)\sqrt{1-\dot{\theta}^2}$$

$$p = -V\sqrt{1-\dot{\theta}^2}$$

$$\rho = \frac{V}{\sqrt{1-\dot{\theta}^2}}$$

Hamilton-Jacobi formalism

$$\dot{H} = H_{,\theta}\dot{\theta}$$

$$\dot{\theta} = -\frac{n}{3}\frac{H_{,\theta}}{H^2}$$

$n = 1$ RSII cosmology

$n = 2$ Standard cosmology

The constant-roll inflation with a tachyon field

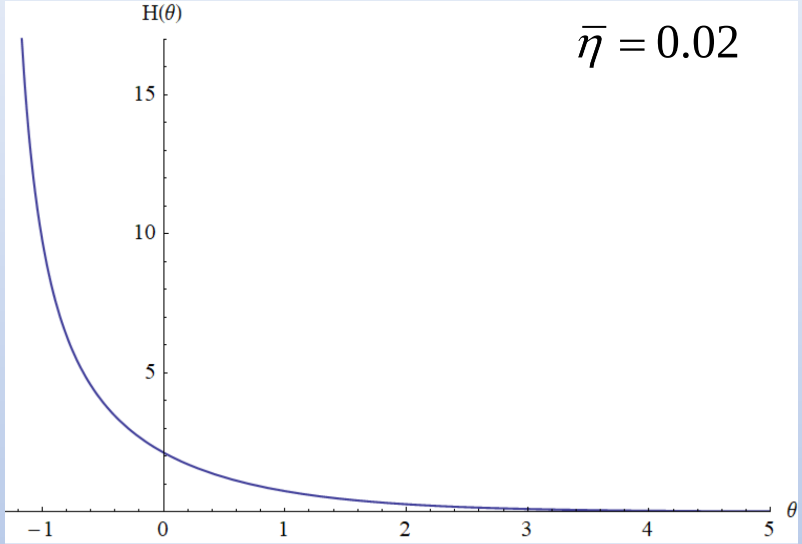
$$\ddot{H} + 2\eta H\dot{H} = 0 \quad \Rightarrow \quad H_{,\theta\theta}H - H_{,\theta}^2 - 3\frac{\eta}{n}H^4 = 0$$

$$H(\theta) = \frac{2nC_1 e^{\sqrt{C_1}(\theta+C_2)}}{e^{2\sqrt{C_1}(\theta+C_2)} - 3\bar{\eta}C_1}$$

$$\eta = \bar{\eta} / n$$

$$C_1 = 1 \quad C_2 = 0$$

$$H = \frac{2ne^{\theta}}{e^{2\theta} - 3\bar{\eta}}$$

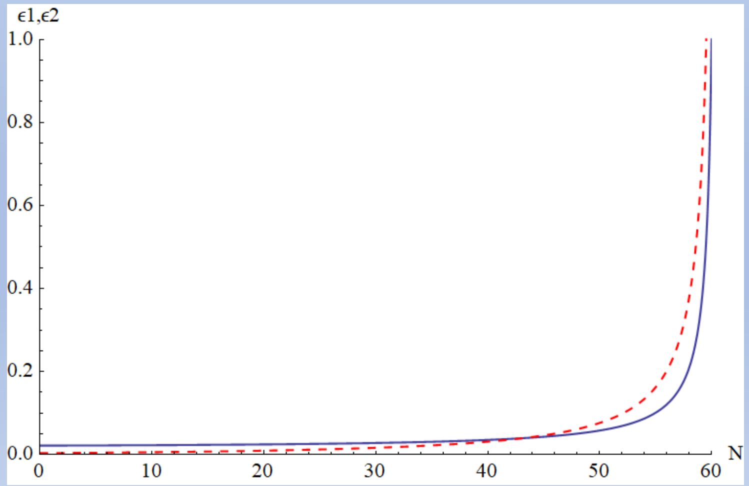


$$H(t) = -\frac{1}{\sqrt{3\bar{\eta}}}\tan(\sqrt{\bar{\eta}/3}t + 2C_3), \quad \bar{\eta} > 0$$

$$H(t) = -\frac{1}{\sqrt{3|\bar{\eta}|}}\tanh(\sqrt{|\bar{\eta}|/3}t + 2C_3), \quad \bar{\eta} < 0$$

$$a(t) \propto \left[\cos\left(\sqrt{\bar{\eta}/3}t + 2C_3\right)\right]^{\frac{1}{\bar{\eta}}}, \quad \bar{\eta} > 0$$

$$a(t) \propto \left[\cosh\left(\sqrt{|\bar{\eta}|/3}t + 2C_3\right)\right]^{-\frac{1}{\bar{\eta}}}, \quad \bar{\eta} < 0$$



The observational parameters

- The inflation parameters in the second order in the slow-roll parameters

$$n_s = 1 - 2\varepsilon_{1i} - \varepsilon_{2i} - \left(2\varepsilon_{1i}^2 + (2C' + 3 - 2\alpha)\varepsilon_{1i}\varepsilon_{2i} + C'\varepsilon_{2i}\varepsilon_{3i}\right)$$

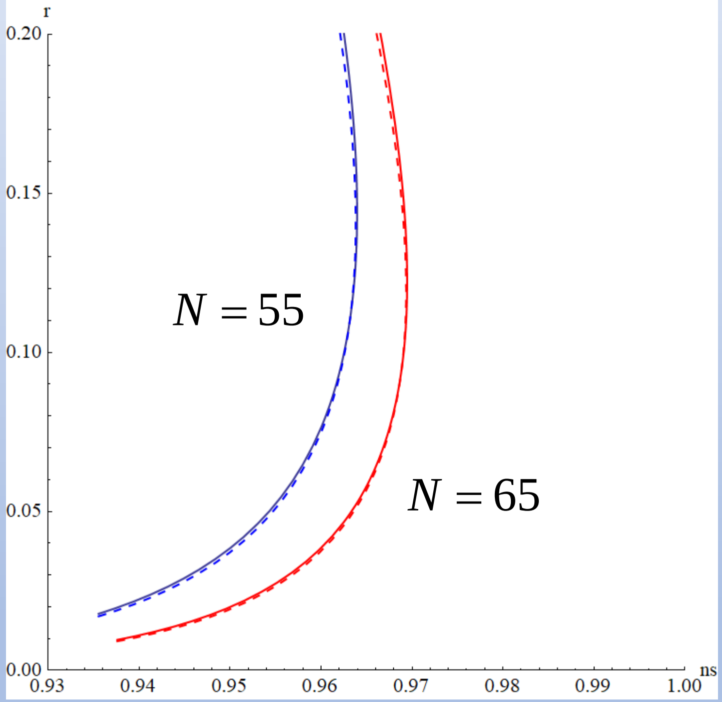
$$r = 16\varepsilon_{1i}(1 + C'\varepsilon_{2i} - 2\alpha\varepsilon_{1i})$$

$\alpha = 1/6$ Standard cosmology

$\alpha = 1/12$ RSII cosmology

$C' = -0.72$

$\varepsilon_{3i} = 2\varepsilon_{1i}$

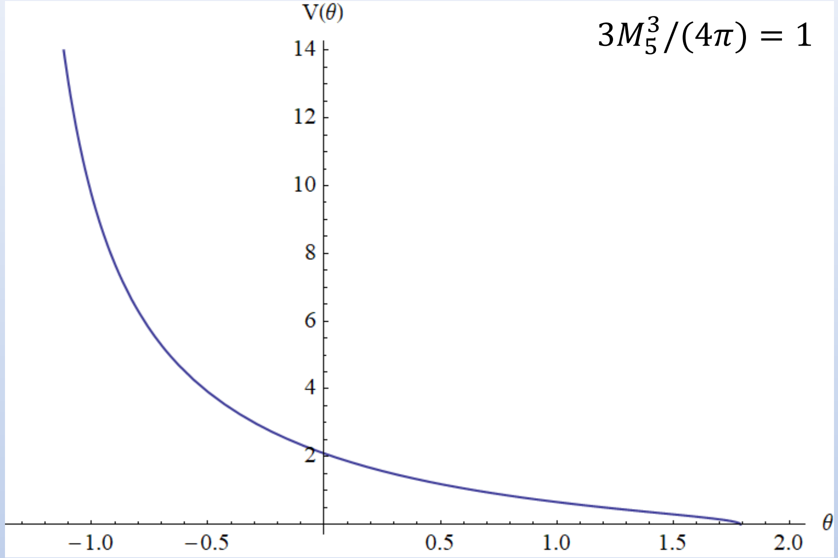


- A better agreement of analytical and observational results is evident for higher values of *N*
- The influence of the second order in the slow-roll parameters is insignificant

The attractor behavior

- The reconstructed potentials

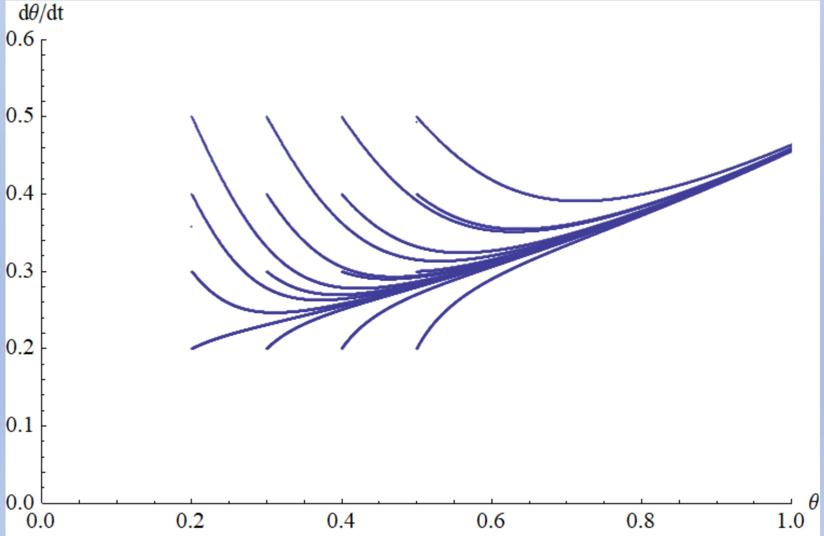
$$V = \frac{3M_5^3}{4\pi} H \sqrt{1 - \frac{1}{9}\frac{H_{,\theta}^2}{H^4}} = \frac{3M_5^3}{4\pi} \sqrt{\frac{4e^{2\theta}}{(e^{2\theta} - 3\bar{\eta})^2} - \frac{1}{9}}$$



- The results displayed in phase space show that there is a curve which attract most trajectories obtained for several initial conditions

$$0.2 \le \theta_\mathrm{i} \le 0.5 \quad 0.2 \le \dot{\theta}_\mathrm{i} \le 0.5$$

which provide that the inflationary trajectories are **attractors**.



Considering the brane world (and RSII) in cosmological inflation and relation to quantum gravity (Motivation)

• From the swampland conjectures which can be used as criteria to distinguish effective field theories (EFT) that can be UV-completed to a QG theory.
a) The first criterion requires the field range traversed by the fields to be bounded from above by a value of order one, whereas
b) the second criterion imposes a lower bound on the gradient of the potential.
The latter bound is in direct tension with inflation where the first slow-roll parameter
$\varepsilon_1 = M_p^2\, |V'/V|^2$ must be smaller than one.
Thus, some inflationary models are not compatible with these criteria, and hence can not be embedded into a consistent theory of quantum gravity. However, inflationary models in the brane-world scenario have the potential to evade the swampland constraints.
• In the brane world scenario, the Friedmann equation will contain both quadratic and linear terms, which in the high energy regime (i.e. $\rho \gg \lambda$) the linear term can be ignored. In this case, unlike the standard four-dimensional cosmology, the Hubble parameter behaves as $H \propto \rho$ rather than $H \propto \rho^{1/2}$, a novel aspect of the CRI scenario in this context.

CRI in ``holography``

- The scenario in which the brane (with an effective tachyon field) is located at the boundary of the AdS_5 space is referred as **the holographic braneworld**.
- The effective four-dimensional Einstein equations on the holographic boundary of AdS5 yields a modified Friedmann equations

$$h^2 - \frac{1}{4}h^4 = \frac{\kappa^2}{3}\ell^4\rho \qquad \dot{h}\left(1-\frac{1}{2}h^2\right) = -\frac{\kappa^2}{2}\ell^3(p+\rho)$$

where h is a dimensionless Hubble expansion rate and the fundamental coupling is related to the AdS_5 curvature radius

$$0 \le h^2 \le 2 \qquad \kappa^2 = \frac{8\pi G_N}{\ell^2}$$

- From the general condition for constant-roll inflation $\ddot{\theta} = \eta \frac{h}{\ell}\dot{\theta}$

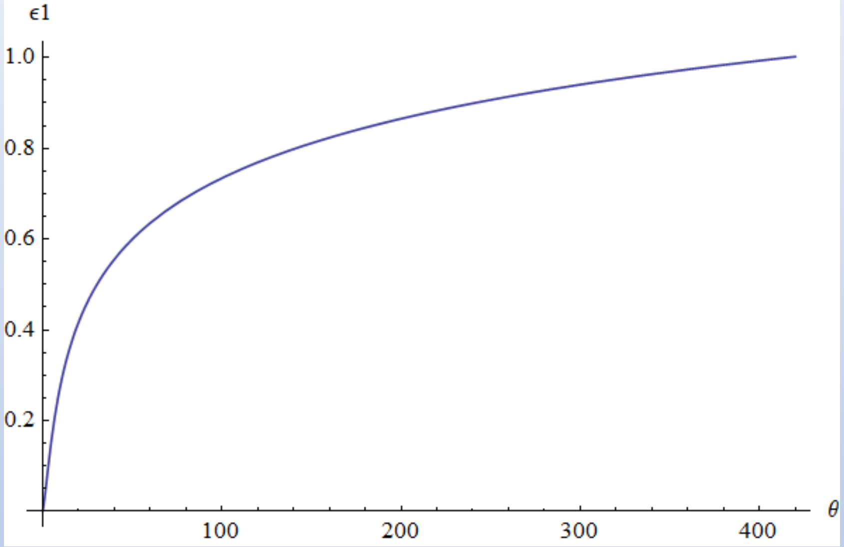
using the Hamilton-Jacobi formalism $\dot{h} = h_{,\theta}\dot{\theta}$

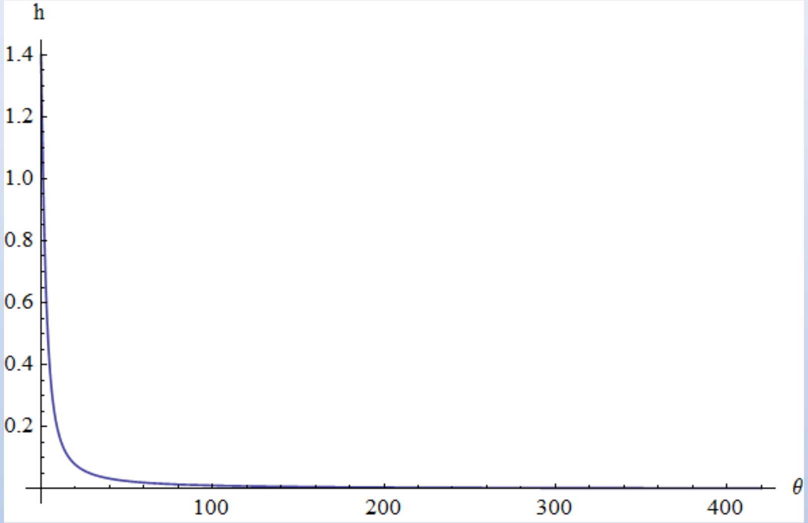
one obtains

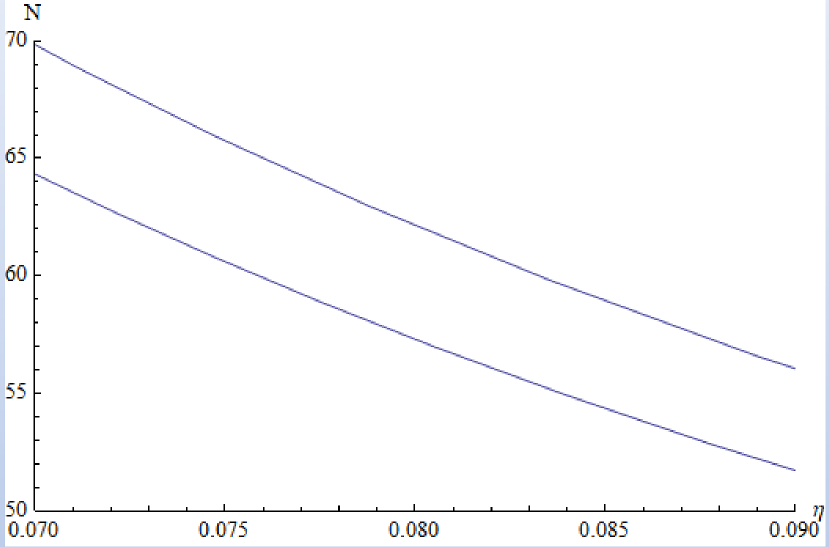
$$hh_{,\theta\theta} - 2h_{,\theta}^2\left(1+\frac{h^2}{4(1-\frac{1}{2}h^2)(1-\frac{1}{4}h^2)}\right) + \frac{3}{2\ell^2}\eta h^4 \frac{1-\frac{1}{4}h^2}{1-\frac{1}{2}h^2} = 0 \qquad V = \frac{3}{\kappa^2}h^2(1-\frac{1}{4}h^2)\sqrt{1-\frac{4\ell^2}{9}\frac{h_{,\theta}^2}{h^4}\left(\frac{1-\frac{1}{2}h^2}{1-\frac{1}{4}h^2}\right)^2} \qquad \varepsilon_2 = 2\eta - \frac{h^2}{2(1-\frac{1}{2}h^2)(1-\frac{1}{4}h^2)}\varepsilon_1$$

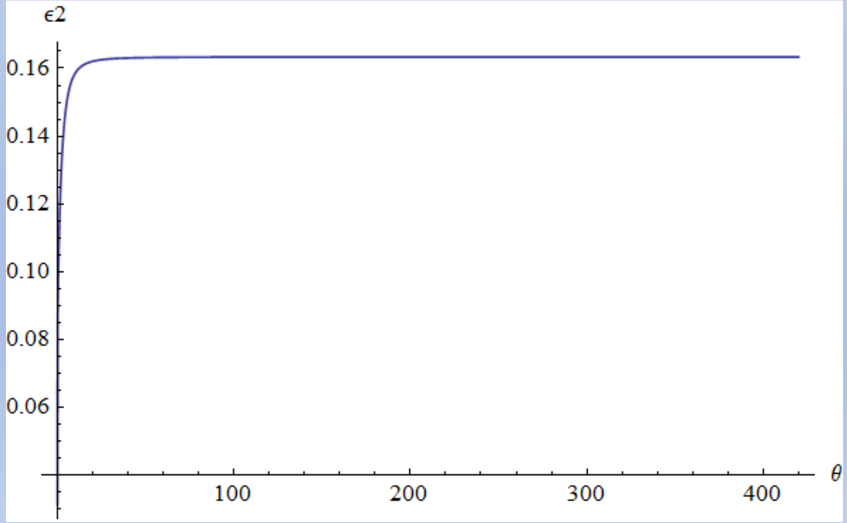
- The expressions obtained in the CRI in holography differ from those in CRI in the standard cosmology!

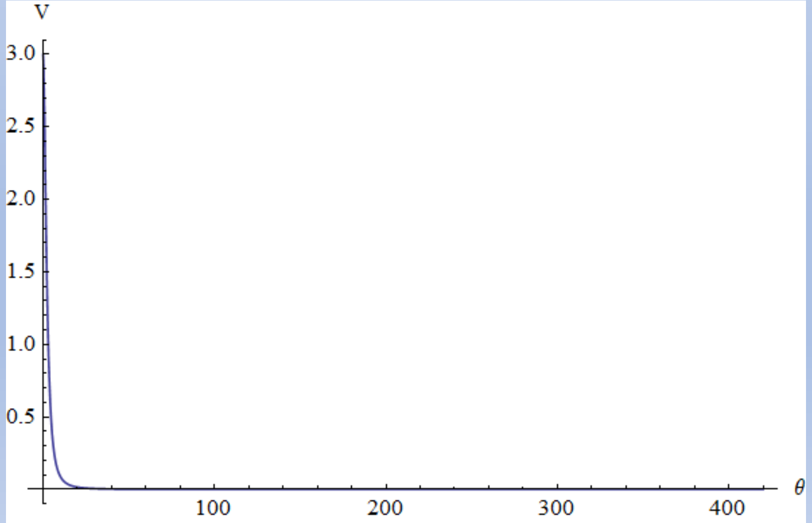
CRI in ``holography``

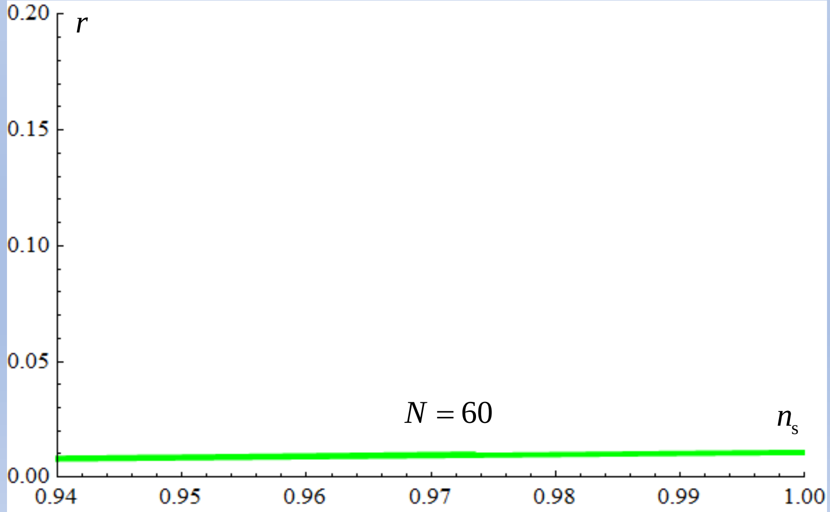












Conclusion

- We have studied the constant-roll inflation with tachyon field in RSII Cosmology, with constant slow-roll parameter η, and for fixed η.
- Its definition leads to differential equation for the Hubble expansion rate, which have the exact (4+1) solutions.
- We found Hubble slow-roll parameters (ε_1, ε_2) as a function of parameter η for all (4 nontrivial) solutions H(θ).
- It was shown show that three of four solutions H(θ) provide a consistent inflationary model. Furthermore, and as very important, all solutions lead to the **same** function $\varepsilon_1(N)$ and $\varepsilon_2(N)$.
- We calculated the values of n_s and r and compared it with the latest Planck results.
- By comparing those values with constraints from observation data we estimate the parameter η. The better agreement is achieved for negative and small value of the parameter η.
- In addition, for standard and RSII cosmology we have calculated inflation parameters in the second order in the slow-roll parameters. No significant difference was obtained for the parameters in these two cases.
- A correct attractor behaviour was found.
- The model of CRI in holographic cosmology gives a lower value for number of e-fold and closer to typical value N=60 then the tachyon CRI in standard cosmology
- The recently proposed swampland criteria is a measure for separating the consistent EFT from the inconsistent EFT. As (it is believed) Inflation occurred at the energy scale below the Planck energy and hence could be described by a low-energy effective field theory of string(brane) theory.
- Therefore, it is a natural desire, and the next step, to construct an inflationary model based on a consistent EFT, and to apply the swampland conjectures.

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Hvala na organizaciji i gostoprimstvu !!!