Cosmological evolution from modified Bekenstein entropy

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talk based on

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Corrections to the Bekenstein entropy

$$S = k_B \frac{A}{4\ell_{\mathrm{Pl}}^2}$$

logarithmic corrections have been obtained in
a number of different approaches to quantum gravity
(string theory, AdS/CFT, loop quantum gravity, GUP, …)

$$S = k_B \frac{A}{4\ell_{\mathrm{Pl}}^2} + k_B C \log\left(\frac{A}{A_0}\right) + \mathcal{O}\left(\frac{A_0}{A}\right)$$

theory-specific
constant C

$A_0 \sim \ell_{\mathrm{Pl}}^2$

see Filip Požar's talk

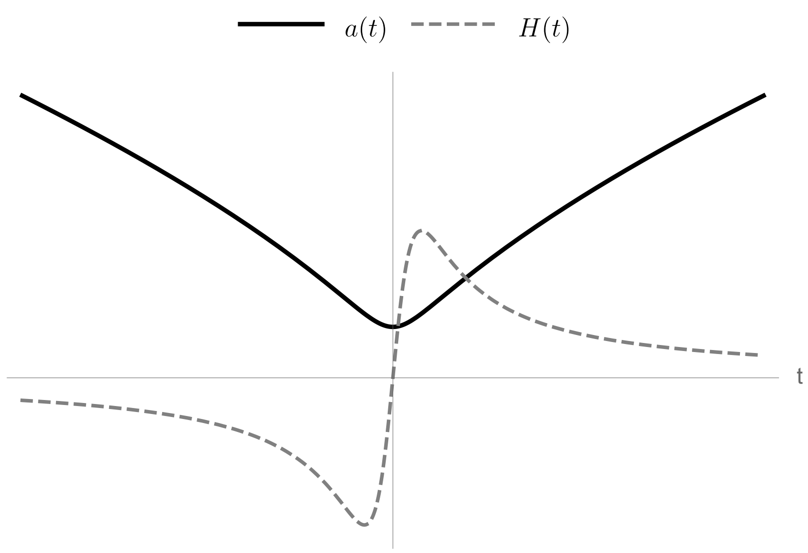
The case of loop quantum gravity

In LQG one finds a modified entropy with log corrections
[Kaul Majumdar; Meissner; Engle, Noui, Perez, Pranzetti; …]

$$S = k_{\mathrm{B}} \frac{A}{4\ell_{\mathrm{Pl}}^2} - \frac{3}{2} k_{\mathrm{B}} \log\left(\frac{A}{A_0}\right) + \mathcal{O}\left(\frac{A_0}{A}\right)$$

In the cosmological (symmetry reduced) sector, LQC predicts the resolution of the initial singularity with a bounce

At the bounce $H = 0$ and $\dot{H} > 0$ locally



Is it true in general that a theory of QG will predict both log-corrections to the entropy and a cosmological bounce?

Modified gravity from modified entropy

[Alonso-Serrano, Liška]

Gravitational dynamics is reconstructed from local equilibrium conditions, generalising Jacobson's approach on the thermodynamics of spacetime

The idea is to modify the thermodynamics of spacetime to include leading order QG effects, in order to derive an *emergent gravitational dynamics* that is able to capture general features of low-energy QG.

$$\boxed{S_{\mu\nu} - \alpha\,\kappa\, S_{\mu\lambda}S^{\lambda}_{\ \nu} + \frac{\alpha\,\kappa}{4}\left(R_{\kappa\lambda}R^{\kappa\lambda} - \frac{1}{4}R^2\right) g_{\mu\nu} = \kappa\left(T_{\mu\nu} - \frac{1}{4}T g_{\mu\nu}\right)}$$

$S_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{4}R\, g_{\mu\nu}$ $\qquad$ $\kappa \equiv 8\pi G$ $\qquad$ $\alpha \sim C$ in the entropy formula

For $\alpha \to 0$ one gets the trace-free Einstein equations (unimodular gravity)

As in unimodular gravity, energy-momentum conservation $\nabla^{\mu} T_{\mu\nu} = 0$ *does not* follow from the field equations. It is a *separate assumption*.

Also, as in unimodular gravity: the cosmological constant Λ is just an *integration constant*

Cosmological background

Spatially flat FLRW geometry

$$ds^2 = -dt^2 + a(t)^2 \delta_{ij} dx^i dx^j$$

Assume a perfect fluid as matter

$$T_{\mu\nu} = (\rho + p) u_\mu u_\nu + p\, g_{\mu\nu}$$

equation of state: $p = w\,\rho$

From the field equations we obtain just one independent equation

$$\boxed{\dot{H}(1 - \alpha\,\kappa\,\dot{H}) = -\frac{\kappa}{2}(\rho + p)}$$

If we also assume energy-momentum conservation, then we have

$$\boxed{\dot{\rho} + 3H(\rho + p) = 0}$$

Cosmological dynamics can be studied as a 2D **dynamical system**.

Care is needed to distinguish between the two cases $\alpha > 0$ and $\alpha < 0$.

Dynamical system analysis: case $\alpha > 0$

Dynamical system techniques represent a powerful tool in cosmology, with several applications to general relativity and modified gravity

[Coley; Wainwright, Ellis; etc]

The idea is to regard $\dot{H}(1-\alpha\,\kappa\,\dot{H}) = -\frac{\kappa}{2}(\rho+p)$ as a *constraint* on $\dot{H},\ \rho$.

This way, we can better deal with multiple solution branches.

This motivates introducing a new variable φ such that

$$\dot{H} = \frac{1}{2\alpha\kappa}\left(1 \pm \cosh\varphi\right)\ , \quad \rho = \frac{1}{2\alpha\kappa^2(1+w)}(\sinh\varphi)^2$$

$$\dot{\varphi} = -\frac{3}{2}(1+w)H\tanh\varphi$$

All we have to do now is study the dynamical system for H and φ

Dynamical system analysis: case $\alpha > 0$

$$\dot{H} = \frac{1}{2\alpha\kappa}\left(1 \pm \cosh\varphi\right) \qquad \dot{\varphi} = -\frac{3}{2}(1+w)H\tanh\varphi$$

First consequences

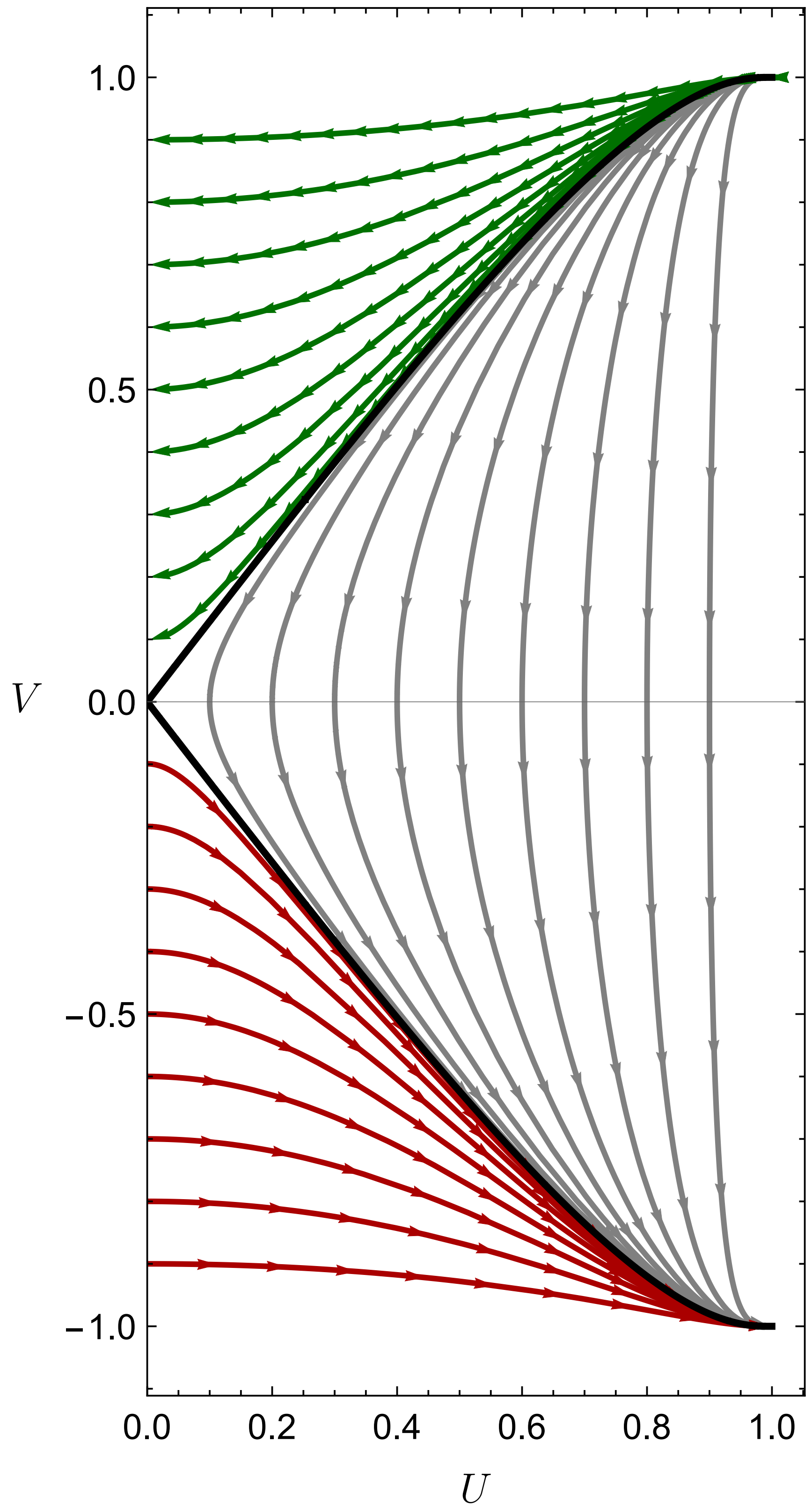
There are two branches:

- one with $\dot{H} > 0$ at all times $\Longrightarrow$ not viable
- one with $\dot{H} \leq 0$ (as in standard cosmology), which we focus on.

Note: in the latter branch there is **no bounce**,
since this would require that H has a zero, around which $\dot{H} > 0$

Nonetheless, is it still possible to obtain viable cosmological solutions in this model?

Phase portrait ($\alpha > 0$)



$$\boxed{\dot{H} = \frac{1}{2\alpha\kappa}\left(1 \pm \cosh\varphi\right) \quad , \quad \dot{\varphi} = -\frac{3}{2}(1+w)H\tanh\varphi}$$

Compactify the "phase space":

$U = \tanh\varphi$,

$V = \tanh(\sqrt{\kappa}H)$

Orbits fall in three classes:

- ever-collapsing
- re-collapsing
- ever-expanding, with a late-time de Sitter era

At early times, potentially viable orbits approach a past attractor corresponding to **power-law inflation**

$a(t) \sim t^q$, with $q = 4/(3(1+w))$

However, the slow-roll condition $\epsilon \ll 1$ and $N_{\mathrm{e-folds}} \approx 60$ can only be satisfied with fine-tuning:

$w + 1 \ll 1$ and $\alpha/(1+w) \approx \frac{3}{32\pi}$

(def. $\epsilon \equiv -\dot{H}/H^2 = 1/q$)

Dynamical system analysis: case $\alpha < 0$

In this case we use a different parametrisation

$$\dot{H} = \frac{1}{2|\alpha|\kappa}\left(\cos\varphi - 1\right) \ , \quad \rho = \frac{1}{2|\alpha|\kappa^2(1+w)}(\sin\varphi)^2$$

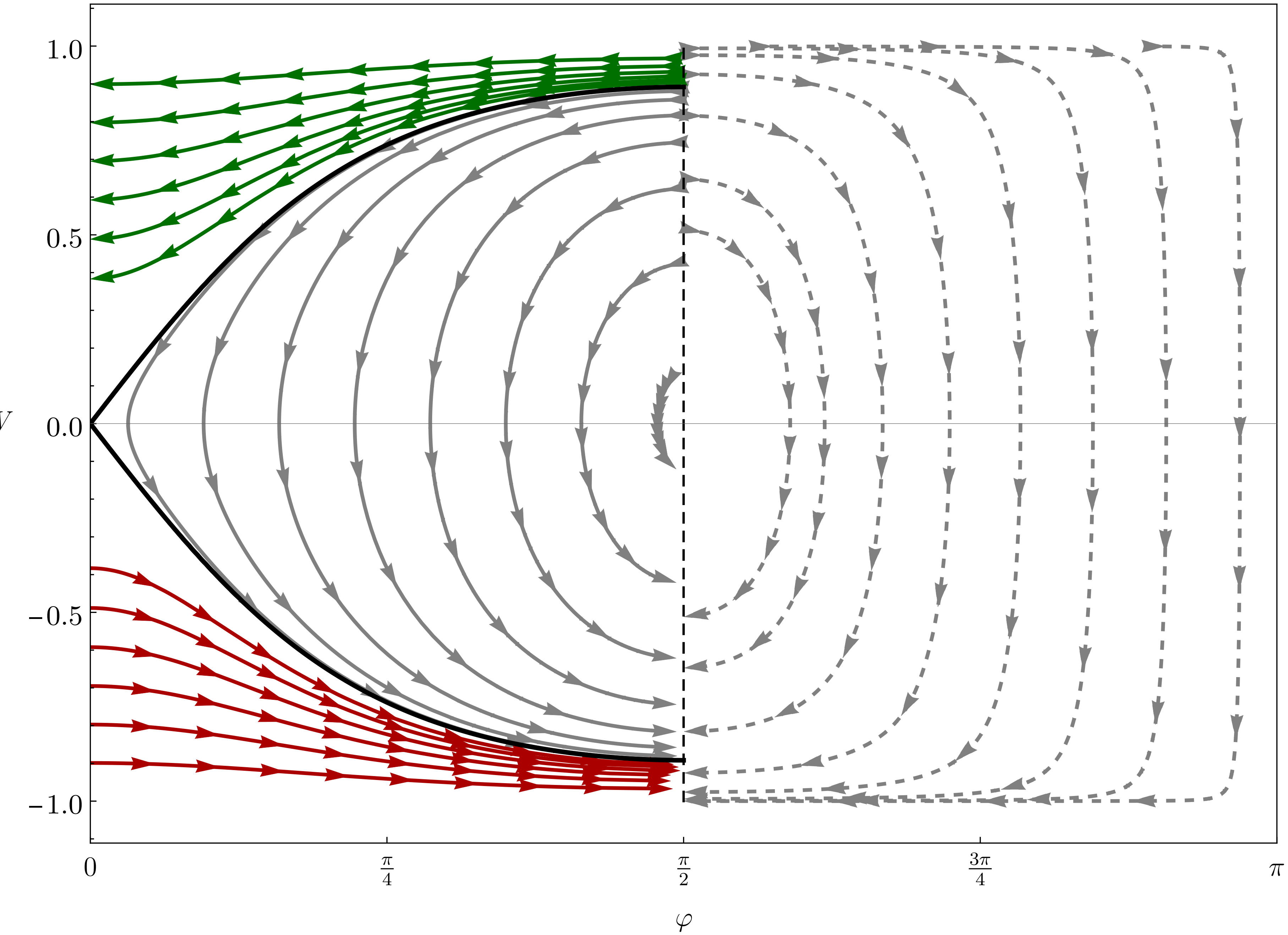
The continuity equation then gives

$$\dot{\varphi} = -\frac{3}{2}(1+w)H\tan\varphi$$

In this case there is only one branch, where $\dot{H} \leq 0$ identically

Again, $\dot{H} \leq 0$ implies that there is **no bounce**.

Phase portrait ($\alpha < 0$)



$$\dot{H} = \frac{1}{2|\alpha|\kappa}\left(\cos\varphi - 1\right) \ ,$$
$$\rho = \frac{1}{2|\alpha|\kappa^2(1+w)}(\sin\varphi)^2$$

Compactify the “phase space”:

$$W = \tanh\left(\sqrt{\kappa}H\right)$$

Again, orbits fall in three classes:

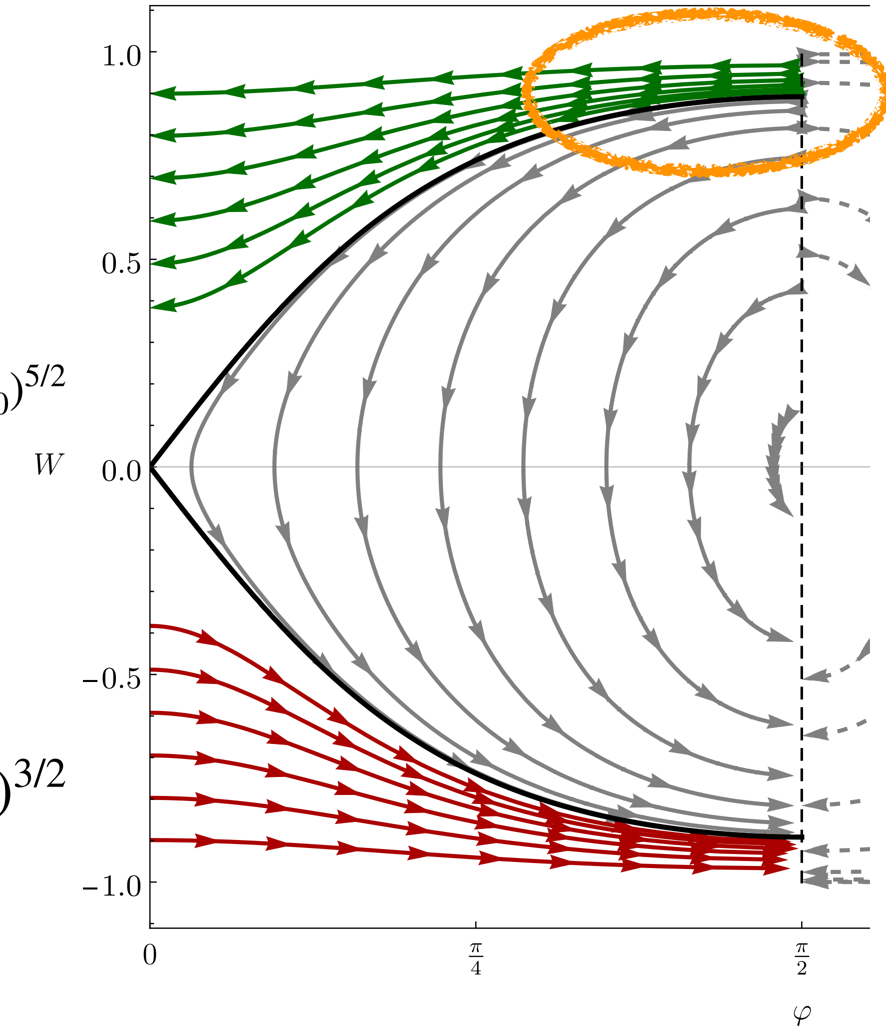
- ever-collapsing
- re-collapsing
- ever-expanding, with a late-time de Sitter era

Also in this case, potentially viable solutions approach an inflationary attractor in the past

Past attractor ($\alpha < 0$)

In the far past, the Hubble rate has the asymptotics
(the singularity is at $t = t_0$)

$$H \approx H_{\rm i} - \frac{t-t_0}{2|\alpha|\kappa} + \sqrt{\frac{(1+w)H_{\rm i}}{3}}\,\frac{(t-t_0)^{3/2}}{|\alpha|\kappa} - \frac{1}{20}\left[\frac{1}{2\alpha^2\kappa^2}\sqrt{\frac{3(w+1)}{H_{\rm i}}} + \frac{1}{|\alpha|\kappa}\left(3(w+1)H_{\rm i}\right)^{3/2}\right](t-t_0)^{5/2}$$



The slow-roll parameters read:

$$\epsilon = -\dot{H}/H^2 = \frac{1}{2|\alpha|\kappa H_{\rm i}^2}\left(1 - \sqrt{3(1+w)H_{\rm i}}(t-t_0)^{1/2} + \frac{(t-t_0)}{|\alpha|\kappa H_{\rm i}}\right) + \mathcal{O}(t-t_0)^{3/2}$$

$$\eta = \frac{\dot{\epsilon}}{H\epsilon} = -\frac{1}{2}\sqrt{\frac{3(1+w)}{H_i}}(t-t_0)^{-1/2} + \frac{1}{|\alpha|\kappa H_{\rm i}^2} - \frac{3}{2}(1+w) + \mathcal{O}(t-t_0)^{1/2}\ .$$

Therefore, the slow-roll conditions $\epsilon, |\eta| \ll 1$ cannot be satisfied simultaneously

(unless we fine-tune $w+1 \ll 1$, which however amounts to assuming inflation)

Theoretical constraints from the background dynamics

So far, we have identified potentially viable background solutions that:

- are dominated by a cosmological constant at late times ✓
- approach an inflationary attractor at early times. However, they cannot satisfy slow-roll conditions or predict the correct number of e-folds unless we fine-tune $w \approx -1$ ✗

Therefore, the model is incomplete in the early universe.

In order to be cosmologically viable we require that departures from general relativity be strongly suppressed after reheating.

$$\dot{H}(1-\alpha\,\kappa\,\dot{H}) = -\frac{\kappa}{2}(\bar{\rho}+\bar{p})$$

$$|\alpha\,\kappa\,\dot{H}| \ll 1 \qquad \Longrightarrow \qquad \boxed{|\alpha| \ll \frac{1}{2\kappa H_{\rm reh}^2} = \frac{M_{\rm Pl}^2}{2H_{\rm reh}^2}}$$

Scalar perturbations

We work in the longitudinal gauge

$$ds^2 = a(\eta)^2\Big(-(1+2\phi(\eta,x))d\eta^2 + (1-2\psi(\eta,x))\delta_{ij}dx^i dx^j\Big)$$

Assume matter with no anisotropic stress for simplicity.
As in GR this *implies* $\psi = \phi$

Perturbing the field equations, we obtain the dynamics of the gravitational potential:

$$\left(1+\frac{\alpha\kappa}{a^2}(1+c_s^2)(\mathcal{H}^2-\mathcal{H}')\right)\phi''+\left(3(1+c_s^2)\mathcal{H}+\frac{\alpha\kappa}{a^2}(1+c_s^2)\left(\mathcal{H}^3+\mathcal{H}\mathcal{H}'-\mathcal{H}''\right)\right)\phi'+$$
$$\left((1+3c_s^2)\mathcal{H}^2+2\mathcal{H}'-\frac{\alpha\kappa}{a^2}(1+c_s^2)\left(\mathcal{H}^4-5\mathcal{H}^2\mathcal{H}'+2(\mathcal{H}')^2+\mathcal{H}\mathcal{H}''\right)\right)\phi-c_s^2\Delta\phi=\frac{\kappa}{2}a^2(\delta p)_{\mathrm{nad}}\,.$$

Depending on the signs of the (time-dependent) coefficients,
there may be **instabilities**

Scalar perturbations: early-time instabilities

Let us focus on radiation domination ($a \sim \eta\ , \quad \mathcal{H} = 1/\eta$)

$$\left(1+\frac{8\alpha\kappa}{3A^2\eta^4}\right)\phi'' + \left(\frac{4}{\eta}-\frac{8\alpha\kappa}{3A^2\eta^5}\right)\phi' + \left(\frac{k^2}{3}-\frac{40\alpha\kappa}{3A^2\eta^6}\right)\phi = 0$$

$A \equiv a(\eta_i)/\eta_i$

$\boxed{\alpha < 0}$

The coefficient of ϕ'' becomes negative at

$$\eta < \eta_\star \equiv \left(\frac{8|\alpha|\kappa}{3A^2}\right)^{1/4}$$

This can be avoided if $\eta_\star \ll \eta_{\rm reh}$,

which is ensured by

$$|\alpha| \ll \frac{1}{2\kappa H_{\rm reh}^2}$$

$\boxed{\alpha > 0}$

For $\alpha > 0$, both the coefficients of ϕ and ϕ' may become negative

Requiring that the ϕ' coefficient be positive gives once again $|\alpha| \ll 1/(2\kappa H_{\rm reh}^2)$

The most stringent constraint is obtained by requiring that the ϕ coefficient be positive *for all* k modes

Scalar perturbations: early-time instabilities

$$\left(1+\frac{8\alpha\kappa}{3A^2\eta^4}\right)\phi''+\left(\frac{4}{\eta}-\frac{8\alpha\kappa}{3A^2\eta^5}\right)\phi'+\underbrace{\left(\frac{k^2}{3}-\frac{40\alpha\kappa}{3A^2\eta^6}\right)}_{>0}\phi=0$$

$A \equiv a(\eta_i)/\eta_i$

It is sufficient to impose this requirement on the largest observable modes, with $\bar{k} \simeq a_o H_o$

$$\alpha \ll \frac{M_{\rm Pl}^2}{40H_{\rm reh}^2}\bar{k}^2\eta_{\rm reh}^2 = \frac{M_{\rm Pl}^2 H_o^2}{40H_{\rm reh}^4}e^{2N}$$

With $N = \log(a_o/a_{\rm reh})$, $H_{\rm reh} \simeq (g_s^{1/2}\pi/9.5)T_{\rm reh}^2/M_{\rm Pl}$, $g_s \simeq 10^2$, $N \simeq 60$

$$\alpha \ll 10^{-72}\left(\frac{M_{\rm Pl}}{T_{\rm reh}}\right)^8 \quad \Longrightarrow$$

if $T_{\rm reh} = \mathcal{O}(10^{15}\mathrm{GeV})$ we have $\alpha \ll 10^{-45}$

The constraint on α becomes much looser for $T_{\rm reh} = \mathcal{O}(10^9\mathrm{GeV})$ or lower

Tensor perturbations

$$ds^2 = a(\eta)^2\Big(-d\eta^2 + (\delta_{ij} + h_{ij}(\eta, x))dx^i dx^j\Big)$$

h_{ij} transverse and traceless: $h^i{}_i = 0$, $\partial^i h_{ij} = 0$

$$\left(1 + \frac{\alpha\kappa}{a^2}\left(\mathscr{H}' - \mathscr{H}^2\right)\right)\left(h''_{ij} + 2\mathscr{H}h'_{ij} - \Delta h_{ij}\right) = 2\kappa a^2 \pi_{ij}$$

Effectively, this kind of modification to the propagation of tensor perturbations amounts to a modified coupling to anisotropic stress.

Summary

Logarithmic corrections to the Bekenstein entropy are expected in several different approaches to quantum gravity. The proposal in [Alonso-Serrano, Liška] is to use this as a starting point to build a modified gravity theory that is able to capture this general feature.

The field equations represent a generalization of unimodular gravity.

Self-accelerating solutions: late de Sitter era and early-time inflationary attractor. However, standard requirements of the inflationary scenario (e-folds, slow-roll) are *not satisfied*.

Constraints on α can be derived if we require that: (i) deviations from GR are small during radiation domination, (ii) scalar perturbations are well-behaved.

Conclusion: cosmologically viable solutions exist in the post-reheating stages. However, in its present form the model *does not* realise an alternative early universe scenario.

The early-time behaviour of solutions could be improved by including higher-order quantum gravity corrections.

Thanks for your attention!

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