Quasinormal modes in New General Relativity

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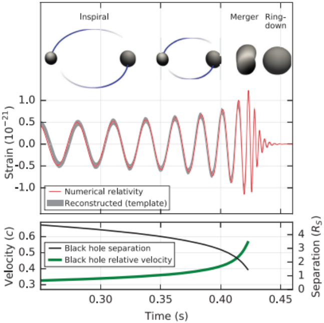
Gravitational waves and quasinormal modes

Detection of gravitational waves in 2015. [1]
Three stages for the GW signal: 1) Inspiral 2) Merger 3) Ringdown



Ringdown → Quasinormal modes: Discrete spectrum

✓ Information about the source of GWs
✓ Distinguish between black holes and other compact objects
✓ Comparison with GR and test for other gravity theories

[1] B. P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration), "Observation of Gravitational Waves from a Binary Black Hole Merger", Phys. Rev. Lett. 116, 061102 (2016)

Black hole perturbations in GR [2]

Einstein equations

$$R_{\mu\nu} = 8\pi(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T) \tag{1}$$

In vacuum:

$$R_{\mu\nu} = 0 \tag{2}$$

Schwarzschild background

$$ds^2 = -(1 - \frac{2M}{r})dt^2 + (1 - \frac{2M}{r})^{-1}dr^2 + r^2(d\theta^2 + sin^2\theta d\phi^2) \tag{3}$$

Small perturbation of the metric

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu} \tag{4}$$

Perturbed Christoffel symbols

$$\Gamma^{\rho}{}_{\mu\nu} = \bar{\Gamma}^{\rho}{}_{\mu\nu} + \delta\Gamma^{\rho}{}_{\mu\nu}, \quad \delta\Gamma^{\rho}{}_{\mu\nu} = \frac{1}{2}\bar{g}^{\kappa\alpha}(\bar{\nabla}_{\mu}h_{\nu\rho} + \bar{\nabla}_{\nu}h_{\mu\rho} - \bar{\nabla}_{\rho}h_{\mu\nu}) \tag{5}$$

Perturbed Ricci tensor

$$R_{\mu\nu} = \bar{R}_{\mu\nu} + \delta R_{\mu\nu}, \quad \boxed{\delta R_{\mu\nu} = \bar{\nabla}_{\rho}\delta\Gamma^{\rho}{}_{\nu\mu} - \bar{\nabla}_{\nu}\delta\Gamma^{\rho}{}_{\rho\mu}} \tag{6}$$

[2] T. Regge and J. A. Wheeler, "Stability of a Schwarzschild Singularity", Phys. Rev. 108, 1063–1069 (1957)

Metric perturbation $h_{\mu\nu}$

- Spherically symmetric background

  ✓ Decomposition into separate functions of (t, r) and (θ, ϕ).

  Spatial part: Harmonic time dependence $F(t, r) = F(r)e^{-i\omega t}$

  Angular part: Spherical harmonics $Y_{lm} = Y_{lm}(\theta, \phi) \quad (l = 0, 1, 2, ... - l \leq m \leq l)$

$$\nabla^2 Y_{lm} = \frac{1}{sin\theta}\frac{\partial}{\partial\theta}(sin\theta\frac{\partial Y_{lm}}{\partial\theta}) + \frac{1}{sin^2\theta}\frac{\partial^2 Y_{lm}}{\partial\phi^2} = -l(l+1)Y_{lm}, \tag{7}$$

  ✓ We can set $m = 0$.

- Invariance under spatial rotations $(\theta, \phi) \rightarrow (\pi - \theta, \pi + \phi)$

$$h_{\mu\nu} = \begin{pmatrix} S & S & V & V \\ S & S & V & V \\ V & V & T & T \\ V & V & T & T \end{pmatrix} \tag{8}$$

  Factor $(-1)^{(l+1)}$ for axial parity and $(-1)^l$ for polar parity.

- Regge-Wheeler gauge

  Axial: $\xi^\mu = (0, 0, \Lambda(t, r)\epsilon^{ab}\partial_b)Y_{lm}$, Polar: $\xi^\mu = (M_0(t, r), M_1(t, r), M_2(t, r)\gamma^{ab}\partial_b)Y_{lm}$

$$\boxed{h_{\mu\nu} = h^{(axial)}_{\mu\nu} + h^{(polar)}_{\mu\nu}} \tag{9}$$

$$h^{(axial)}_{\mu\nu} = \begin{pmatrix} 0 & 0 & -h_0(t,r)\sin^{-1}\theta\ \partial_\phi & h_0(t,r)\sin\theta\ \partial_\theta \\ 0 & 0 & -h_1(t,r)\sin^{-1}\theta\ \partial_\phi & h_1(t,r)\sin\theta\ \partial_\theta \\ \text{Sym} & \text{Sym} & h_2(t,r)\ \sin^{-1}\theta(\partial^2_{\theta\phi} - \cos\theta\ \sin^{-1}\theta\ \partial_\phi) & -h_2(t,r)\sin\theta\ W_{\theta\phi} \\ \text{Sym} & \text{Sym} & \text{Sym} & -h_2(t,r)\sin\theta\ X_{\theta\phi} \end{pmatrix} Y_{LM} \tag{10}$$

$$h^{(polar)}_{\mu\nu} = \begin{pmatrix} B(r)H_0(t,r) & H_1(t,r) & c_0(t,r)\ \partial_\theta & c_0(t,r)\partial_\phi \\ \text{Sym} & H_2(t,r)/B(r) & c_1(t,r)\ \partial_\theta & c_1(t,r)\partial_\phi \\ \text{Sym} & \text{Sym} & r^2(K(r) + G(t,r)\ \partial_{\theta\theta}) & \frac{1}{2}r^2G(t,r)X_{\theta\phi} \\ \text{Sym} & \text{Sym} & \text{Sym} & r^2 sin^2\theta(K(t,r) + G(t,r)t(\partial_{\theta\theta} - W_{\theta\phi}) \end{pmatrix}$$

Harmonic time dependence, $m = 0$, Regge-Wheeler gauge:

$$h^{(axial)}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & h_0(r)\sin\theta\ \partial_\theta \\ 0 & 0 & 0 & h_1(r)\sin\theta\ \partial_\theta \\ 0 & 0 & 0 & 0 \\ \text{Sym} & \text{Sym} & 0 & 0 \end{pmatrix} e^{-i\omega t} Y_{lm} \tag{11}$$

$$h^{(polar)}_{\mu\nu} = \begin{pmatrix} B(r)H_0(r) & H_1(r) & 0 & 0 \\ \text{Sym} & \frac{H_2(r)}{B(r)} & 0 & 0 \\ 0 & 0 & r^2K(r) & 0 \\ 0 & 0 & 0 & r^2\sin^2\theta K(r) \end{pmatrix} e^{-i\omega t} Y_{lm} \tag{12}$$

Non-trivial background field equations

Axial

$$\delta R_{23} = B\frac{dh_1}{dr} + \frac{i\omega}{B}h_0 + \frac{2M}{r^2}h_1 = 0$$

$$\delta R_{13} = \frac{i\omega}{B}\frac{dh_0}{dr} - \frac{2i\omega}{rB}h_0 + \left(\frac{l(l+1)}{r^2} - \frac{2}{r^2} - \frac{\omega^2}{B}\right)h_1 = 0$$

$$\delta R_{03} = B\frac{d^2h_0}{dr^2} + i\omega B\frac{dh_1}{dr} - \left(\frac{l(l+1)}{r^2} - \frac{4M}{r^3}\right)h_0 + \frac{2i\omega B}{r}h_1 = 0 \qquad (13)$$

Polar

$$\delta R_{01} = \frac{dK}{dr} + \frac{l(l+1)}{2i\omega r^2}H_1 + \frac{r-3M}{r^2B}K - \frac{1}{r}H = 0$$

$$\delta R_{02} = B\frac{dH_1}{dr} + \frac{2M}{r^2}H_1 + i\omega(K+H) = 0$$

$$\delta R_{12} = B\frac{dH}{dr} - B\frac{dK}{dr} + i\omega H_1 + \frac{2M}{r^2}H = 0$$

$$\delta R_{00} = B^2\frac{d^2H}{dr^2} + 2i\omega B\frac{dH_1}{dr} - \frac{2MB}{r^2}\frac{dK}{dr} + \frac{2B}{r}\frac{dH}{dr} + 2i\omega(2r-3M)r^2H_1 - 2\omega^2K - \left(\omega^2 + \frac{l(l+1)B}{r^2}\right)H = 0$$

$$\delta R_{11} = B^2\frac{d^2H}{dr^2} - 2B^2\frac{d^2K}{dr^2} + 2i\omega B\frac{dH_1}{dr} - \frac{2B(2r-3M)}{r^2}\frac{dK}{dr} + \frac{2}{r}\frac{dH}{dr} + \frac{2i\omega M}{r^2}H_1 - \left(\omega^2 - \frac{l(l+1)B}{r^2}\right)H = 0$$

$$\delta R_{22} = r^2B\frac{d^2K}{dr^2} + (4r-6M)\frac{dK}{dr} - 2rB\frac{dH}{dr} - 2i\omega rH_1 + \left(2 + \frac{\omega^2r^2}{B} - l(l+1)\right)K - 2H = 0$$

$$\delta R_{33} = \sin^2\theta\, r^2B\frac{d^2K}{dr^2} + (4r-6M)\frac{dK}{dr} - 2rB\frac{dH}{dr} - 2i\omega rH_1 + \left(2 + \frac{\omega^2r^2}{B} - l(l+1)\right)K - 2H = 0 \qquad (14)$$

Metric affine theories of gravity

Motivation: Tensions in GR
a) Expansion of the universe b) Dark matter and dark energy c) Quantum field theory

Metric $g_{\mu\nu}$ and affine connection $\Gamma^{\rho}{}_{\mu\nu}$

Curvature

$$R^{\mu}{}_{\nu\rho\sigma} = \partial_\rho \Gamma^{\mu}{}_{\nu\sigma} - \partial_\sigma \Gamma^{\mu}{}_{\nu\rho} + \Gamma^{\mu}{}_{\tau\rho}\Gamma^{\tau}{}_{\nu\sigma} - \Gamma^{\mu}{}_{\tau\sigma}\Gamma^{\tau}{}_{\nu\rho} \tag{15}$$

Torsion

$$T^{\mu}{}_{\nu\rho} = \Gamma^{\mu}{}_{\nu\rho} - \Gamma^{\mu}{}_{\rho\nu} \tag{16}$$

Non-metricity.

$$Q_{\mu\nu\rho} = \nabla_\mu g_{\nu\rho} = \partial_\mu g_{\nu\rho} - \Gamma^{\sigma}{}_{\nu\mu} g_{\sigma\rho} - \Gamma^{\sigma}{}_{\rho\mu} g_{\nu\sigma} \tag{17}$$

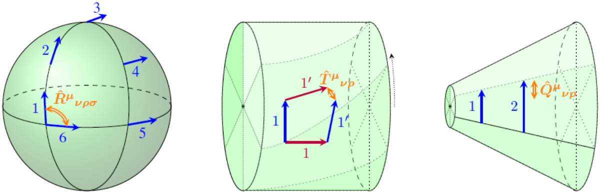


Figure: Made by Laur Järv

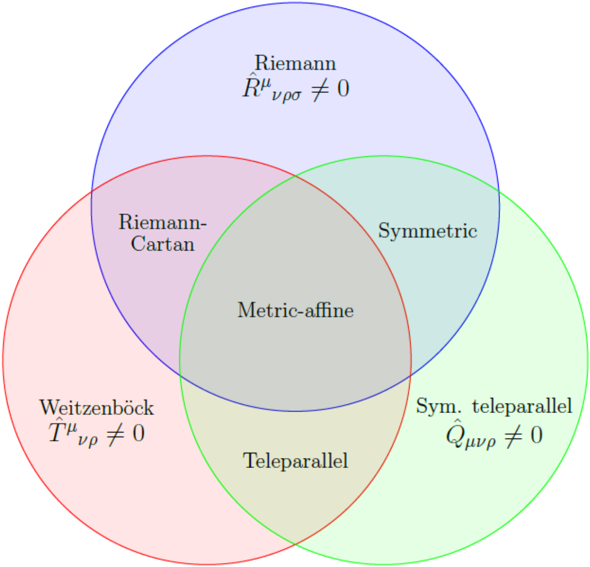


Figure: S. Bahamonde et al., "Teleparallel gravity: from theory to cosmology," Rep. Prog. Phys. 86 (2023)

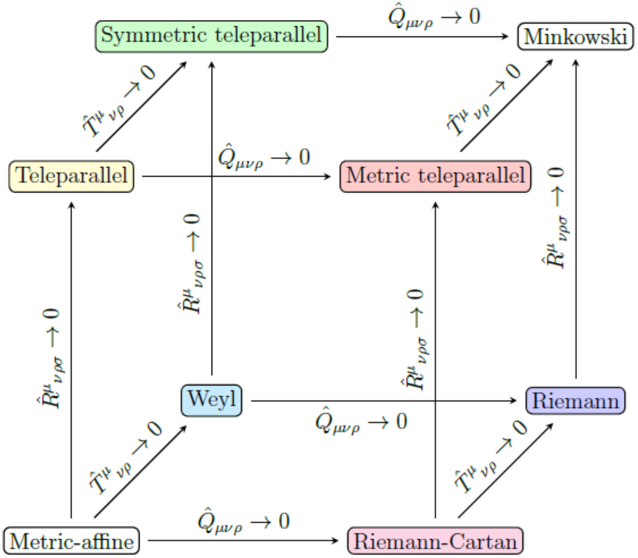


Figure: S. Bahamonde et al., "Teleparallel gravity: from theory to cosmology," Rep. Prog. Phys. 86 (2023)

Fundamentals of Metric Teleparallel Gravity

Gravity is the result of torsion.

$$T^{\mu}{}_{\nu\rho} = \Gamma^{\mu}{}_{\nu\rho} - \Gamma^{\mu}{}_{\rho\nu} \tag{18}$$

Simplest theory: Teleparallel Equivalent of GR (TEGR)

Fundamental variables: Tetrad $\theta^{a}{}_{\mu}$, inverse e_a and spin connection $\omega^{A}{}_{B\mu}$

$$\theta^{a} = \theta^{a}{}_{\mu} dx^{\mu}, \quad e_a = e_a{}^{\mu} \partial_{\mu}, \quad \omega^{A}{}_{B\mu} = \Lambda^{a}{}_{c} \partial_{\mu} \Lambda_{b}{}^{c} \tag{19}$$

$$\theta^{a}{}_{\mu} e_b{}^{\mu} = \delta^{a}_{b}, \quad \theta^{a}{}_{\mu} e_a{}^{\nu} = \delta^{\nu}_{\mu} \tag{20}$$

Metric

$$g_{\mu\nu} = \eta_{ab} \theta^{a}{}_{\mu} \theta^{b}{}_{\nu}, \quad g^{\mu\nu} = \eta^{ab} e_a{}^{\mu} e_b{}^{\nu} \tag{21}$$

Connection

$$\Gamma^{\rho}{}_{\mu\nu} = e_a{}^{\rho} (\partial_{\nu} \theta^{a}{}_{\mu} + \omega^{a}{}_{b\nu} \theta^{b}{}_{\mu}) \tag{22}$$

Weitzenbock gauge

$$\omega^{A}{}_{B\mu} = 0 \tag{23}$$

Irreducible decomposition of torsion

$$T_{\rho\mu\nu} = \frac{2}{3}(\mathfrak{t}_{\rho\mu\nu} - \mathfrak{t}_{\rho\nu\mu}) + \frac{1}{3}(g_{\rho\mu}\mathfrak{v}_{\nu} - g_{\rho\nu}\mathfrak{v}) + \epsilon_{\rho\mu\nu\kappa}\mathfrak{a}^{\kappa} \tag{24}$$

Gravitational Lagrangian

$$\mathcal{L}_G = c_v\ \mathfrak{v}^{\mu}\mathfrak{v}_{\mu} + c_a\ \mathfrak{a}^{\mu}\mathfrak{a}_{\mu} + c_t\ t^{\rho\mu\nu}t_{\rho\mu\nu} \tag{25}$$

$$\mathfrak{v}_{\mu} = T^{\rho}{}_{\rho\mu}, \quad \mathfrak{a}_{\mu} = \frac{1}{6}\epsilon_{\mu\nu\rho\sigma}T^{\nu\rho\sigma} \quad \mathfrak{t}_{\mu\nu\rho} = T_{(\mu\nu)\rho} + \frac{1}{3}(g_{\rho(\mu}\mathfrak{v}_{\nu)} - g_{\mu\nu}\mathfrak{v}_{\rho}) \tag{26}$$

1-parameter NGR: Values from TEGR and 1 free parameter

$$c_t = \frac{2}{3},\ c_v = -\frac{2}{3},\ c_a = \frac{3}{2} + \delta \tag{27}$$

Field equations

$$\begin{aligned}8\pi E_{\mu\nu} = c_a &\left(\frac{1}{2}\mathfrak{a}^{\rho}\mathfrak{a}_{(\rho}g_{\mu\nu)} - \frac{4}{9}\epsilon_{\nu\alpha\beta\gamma}\mathfrak{a}^{\alpha}\mathfrak{t}_{\mu}{}^{\beta\gamma} - \frac{2}{9}\epsilon_{\mu\nu\rho\sigma}\mathfrak{a}^{\rho}\mathfrak{v}^{\sigma} - \frac{1}{3}\epsilon_{\mu\nu\rho\sigma}\mathring{\nabla}^{\rho}\mathfrak{a}^{\sigma}\right) + \\ + c_t &\left(\frac{2}{3}\mathfrak{t}_{\alpha[\beta\gamma]}\mathfrak{t}^{\alpha\beta\gamma}g_{\mu\nu} - \frac{4}{3}\mathfrak{t}_{\mu[\rho\sigma]}\mathfrak{t}_{\nu}{}^{\rho\sigma} + 2\mathring{\nabla}^{\rho}\mathfrak{t}_{\mu[\nu\rho]} - \frac{2}{3}\mathfrak{t}_{\nu[\mu\rho]}\mathfrak{v}^{\rho} + \frac{1}{2}\epsilon_{\mu\alpha\beta\gamma}\mathfrak{a}^{\alpha}\mathfrak{t}_{\nu}{}^{\beta\gamma}\right) + \\ + c_v &\left(\frac{1}{2}\mathfrak{v}^{\rho}\mathfrak{v}_{(\rho}g_{\mu\nu)} + \frac{4}{3}\mathfrak{t}_{\mu[\rho\nu]}\mathfrak{v}^{\rho} + 2g_{\mu[\nu}\mathring{\nabla}^{\rho}\mathfrak{v}_{\rho]} - \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\mathfrak{a}^{\rho}\mathfrak{v}^{\sigma}\right).\end{aligned} \tag{28}$$

[3] K. Hayashi and T. Shirafuji, "New general relativity", Phys. Rev. D 19, 3524–3553 (1979)

Background Field Equations

Most general spherically symmetric background tetrad [4]

$$\theta^a{}_\mu = \begin{pmatrix} C_1 & C_2 & 0 & 0 \\ C_3 \sin\theta\cos\phi & C_4 \sin\theta\cos\phi & C_5\cos\theta\cos\phi - C_6\sin\phi & -(C_5\sin\phi + C_6\cos\theta\cos\phi)\sin\theta \\ C_3 sin\theta \sin\phi & C_4\sin\theta\sin\phi & C_5\cos\theta\sin\phi + C_6\cos\phi & (C_5\cos\phi - C_6\cos\theta\sin\phi)\sin\theta \\ C_3\cos\theta & C_4\cos\theta & -C_5\sin\theta & C_6\sin^2\theta \end{pmatrix} \tag{29}$$

Metric

$$g_{00} = C_3^2 - C_1^2, \quad g_{11} = C_4^2 - C_2^2, \quad g_{22} = C_5^2 + C_6^2, \quad g_{33} = (C_5^2 + C_6^2) sin^2\theta,$$
$$g_{01} = g_{10} = C_3 C_4 - C_1 C_2 \tag{30}$$

Schwarzschild metric

$$C_3 = \sqrt{C_1^2 - F^2}, \quad C_4 = \frac{|C_1|}{F^2}, \quad C_6 = \sqrt{r^2 - C_5^2}, \quad C_3 C_4 = C_1 C_2 \tag{31}$$

where $C_1, ..., C_6 = C(r)$ and

$$F = \sqrt{1 - \frac{2M}{r}} \tag{32}$$

[4] M. Hohmann et al., "Modified teleparallel theories of gravity in symmetric spacetimes", Phys. Rev. D 100, 084002 (2019)

Non-trivial background field equations

$$\bar{E}_{00} = \frac{2\delta F^2}{9r^4 C_{51}^2}(r^2 F^2 C_{52}^2 - 4rC_1 C_{52} + 4C_{51}^4) = 0$$

$$\bar{E}_{01} = -\frac{8\delta}{9r^3}\boxed{C_{52} C_3} = 0$$

$$\bar{E}_{11} = \frac{2\delta F^2}{9r^4 C_{51}^2}(rFC_{52} + 2C_{51}^2)(rFC_{52} - 2C_{51}^2) = 0$$

$$\bar{E}_{22} = \frac{2\delta}{9r^4 F^2 C_{51}^2}(rFC_{52} + 2C_{51}^2)(rFC_{52} - 2C_{51}^2) = 0$$

$$\bar{E}_{33} = \frac{2\delta}{9r^4 F^2 C_{51}^2}(rFC_{52} + 2C_{51}^2)(rFC_{52} - 2C_{51}^2)\sin^2\theta = 0$$

$$\bar{E}_{23} = -\frac{\delta}{9C_{51}}(rF^2 C_{52}' + C_{52} + \frac{F^2 C_5 C_{52}^2}{C_{51}^2} - 2C_{51}^2 C_1') + \frac{4\delta C_5 C_{51}}{9r^2} - \frac{2\delta C_1 C_{51}}{3r} - \frac{4\delta C_1 C_5 C_{52}}{9rC_{51}} + \frac{5\delta F^2 C_{52}}{9C_{51}} = 0 \quad (33)$$

where

$$C_{51} = \sqrt{r^2 - C_5^2}, \quad C_{52} = r\frac{dC_5}{dr} - C_5 \quad (34)$$

Solutions

$$\boxed{\text{Branch 1}: C_5 = kr \ (k = \text{constant}), \quad \text{Branch 2}: C_3 = 0} \quad (35)$$

Perturbations in NGR [5]

Small perturbation of the tetrad

$$\theta^a{}_\mu = \bar{\theta}^a{}_\mu + \tau^a{}_\mu \tag{36}$$

Decomposition into symmetric and antisymmetric part

$$\tau_{\mu\nu} = h_{\mu\nu} + a_{\mu\nu}, \quad h_{\mu\nu} = 2\tau_{(\mu\nu)}, \quad a_{\mu\nu} = 2\tau_{[\mu\nu]} \tag{37}$$

Perturbed connection

$$\Gamma^\rho{}_{\mu\nu} = \bar{\Gamma}^\rho{}_{\mu\nu} + \delta\Gamma^\rho{}_{\mu\nu}, \quad \delta\Gamma^\rho{}_{\mu\nu} = \bar{g}^{\rho\kappa}\bar{\nabla}_\nu\tau_{\kappa\mu} \tag{38}$$

Perturbed torsion

$$T^\rho{}_{\mu\nu} = \bar{T}^\rho{}_{\mu\nu} + \delta T^\rho{}_{\mu\nu}, \quad \delta T^\rho{}_{\mu\nu} = \bar{g}^{\rho\kappa}(\bar{\nabla}_{[\mu}h_{\nu]\kappa} - \bar{\nabla}_{[\mu}a_{\nu]\kappa}) \tag{39}$$

Perturbed vector, axial and tensor irreducible components

$$\mathfrak{v}_\mu = \bar{\mathfrak{v}}_\mu + \delta\mathfrak{v}_\mu, \quad \mathfrak{a}_\mu = \bar{\mathfrak{a}}_\mu + \delta\mathfrak{a}_\mu, \quad \mathfrak{t}_{\mu\nu\rho} = \bar{\mathfrak{t}}_{\mu\nu\rho} + \delta\mathfrak{t}_{\mu\nu\rho} \tag{40}$$

Perturbed field equations

$$E_{\mu\nu} = \bar{E}_{\mu\nu} + \delta E_{\mu\nu} = 0 \tag{41}$$

Decomposition in symmetric and antisymmetric parts

$$\delta E_{\mu\nu} = \delta E_{(\mu\nu)} + \delta E_{[\mu\nu]} \tag{42}$$

[5] Helen Asuküla, "Quasinormal modes of Schwarzschild black holes in 1-parameter New General Relativity" (2021) (Master Thesis, Institute of Physics, University of Tartu)

Tetrad perturbation $\tau_{\mu\nu}$

The symmetric part $h_{\mu\nu}$ is the same as in GR.

We use the same assumptions to find $a_{\mu\nu}$.

$$\boxed{a_{\mu\nu} = a_{\mu\nu}^{(axial)} + a_{\mu\nu}^{(polar)}} \tag{43}$$

$$a_{\mu\nu}^{(axial)} = \begin{pmatrix} 0 & 0 & 0 & a_0(r)\sin\theta\,\partial_\theta \\ 0 & 0 & 0 & a_1(r)\sin\theta\,\partial_\theta \\ 0 & 0 & 0 & -a_2(r)\sin\theta\,\partial_\theta \\ \text{ASym} & \text{ASym} & \text{ASym} & 0 \end{pmatrix} e^{-i\omega t} Y_{lm} \tag{44}$$

$$a_{\mu\nu}^{(polar)} = \begin{pmatrix} 0 & A_0(r) & A_1(r)\,\partial_\theta & 0 \\ \text{ASym} & 0 & A_2(r)\,\partial_\theta & 0 \\ \text{ASym} & \text{ASym} & 0 & 0 \\ \text{ASym} & \text{ASym} & 0 & 0 \end{pmatrix} e^{-i\omega t} Y_{lm} \tag{45}$$

Non-trivial perturbed field equations (Branch 1: $C_5 = r$)

Axial

$$\delta E_{(03)} = \delta E_{(13)} = 0 \quad \begin{cases} \delta R_{03} = 0 \\ \delta R_{13} = 0 \end{cases}$$

$$\delta E_{(23)} = \delta R_{23} = 0$$

$$\delta E_{[03]} = \delta E_{[13]} = \delta E_{[23]} = 0 \quad \rightarrow \quad a_0, a_1, a_2 = F(h_0, h_1)$$

Polar

$$\delta E_{(01)} = \delta R_{01} = 0$$
$$\delta E_{(02)} = \delta R_{02} = 0$$
$$\delta E_{(12)} = \delta R_{12} = 0$$
$$\delta E_{(00)} = \delta R_{00} = 0$$
$$\delta E_{(11)} = \delta R_{11} = 0$$
$$\delta E_{(22)} = \delta R_{22} = 0$$
$$\delta E_{(33)} = \delta R_{33} = 0$$

$$\delta E_{[01]} = -\frac{\delta c_a}{r^2}\left[\frac{dA_1}{dr} - A_0 + \frac{2C_1 - F^2 - 1}{2rF^2}A_1 + \left(i\omega - \frac{rC_{12} + C_{11}^2}{rC_{11}}\right)A_2\right] = 0$$

Summary

Perturbed field equations for a Schwarzschild background at linear order:

in GR and NGR for the simplest branch $C_5 = r$.

For the symmetric part in NGR, we obtain the same QNM as in GR.

The axial antisymmetric part in NGR is fully determined by the symmetric one. No extra modes.

For the polar antisymmetric part in NGR, we need extra constraints.

In GR and NGR for the branch $C_5 = r$, the axial and polar parts do not mix.

Future work paths

- Study of the other branch
- Non-Schwarzschild spacetimes
- f(T) gravity
- Symmetric TG, f(Q) gravity
- Higher order perturbations

References

- B. P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration), "Observation of Gravitational Waves from a Binary Black Hole Merger", Phys. Rev. Lett. 116, 061102 (2016)
- T. Regge and J. A. Wheeler, "Stability of a Schwarzschild Singularity", Phys. Rev. 108, 1063–1069 (1957).
- K. D. Kokkotas and B. G. Schmidt, "Quasi-Normal Modes of Stars and Black Holes", Living Reviews in Relativity 2, (1999)
- H.-P. Nollert, "Quasinormal modes: the characteristic 'sound' of black holes and neutron stars", Classical and Quantum Gravity 16, R159–R216 (1999)
- S. Bahamonde et al., "Teleparallel gravity: from theory to cosmology," Rep. Prog. Phys. 86 (2023)
- K. Hayashi and T. Shirafuji, "New general relativity", Phys. Rev. D 19, 3524–3553 (1979)
- H. Asuküla, "Quasinormal modes of Schwarzschild black holes in 1-parameter New General Relativity" (2021) (Master Thesis, Institute of Physics, University of Tartu)