Greybody Factors of Spin-1/2 Particles in Schwarzschild Acoustic Black Hole Spacetime

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Universe **2023**, *9*, 108. https://doi.org/10.3390/universe9020108

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1. Introduction
2. SABH Spacetime
3. Dirac Equation
4. Rarita–Schwinger Equation
5. GFs of SABH via FERMION EMISSION
6. Conclusions

Sonic BHs

Phonon, sonic, acoustic or analogue BHs, are objects that are formed in certain types of fluids and exhibit some of the same characteristics as true BHs.
These objects were first proposed by Unruh in 1981, who demonstrated that the flow of a fluid through a converging nozzle could create an analogue of a BH horizon.

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Published: 15 August 2016

Observation of quantum Hawking radiation and its entanglement in an analogue black hole

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Nature Physics **12**, 959–965 (2016) | Cite this article

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Perspective | Published: 17 December 2018

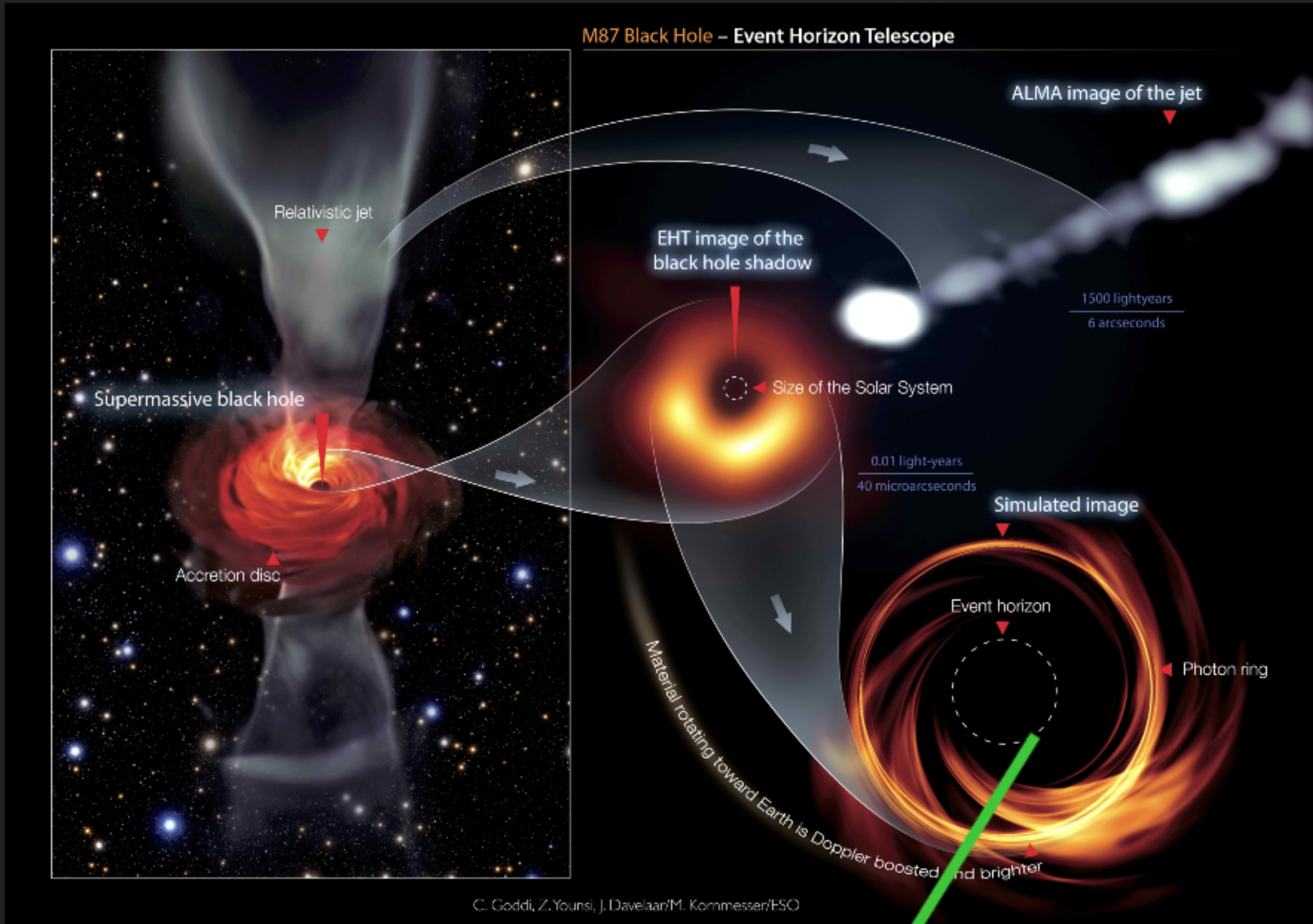
Analogue black-hole horizons

Carlos Barceló

Nature Physics **15**, 210–213 (2019) | Cite this article

Some gravitational phenomena are difficult or even impossible to observe in real spacetime. Laboratory analogues of black-hole horizons offer new perspectives on field t might help our understanding of gravitation.

These are black hole analogues—systems which mimic some of the properties of black holes. A sonic black hole can be made in a fluid which flows faster than its speed of sound. When this happens, sound can no longer escape this rapidly flowing region (it just gets swept away), just like light can't escape from the event horizon or a regular black hole.


M87 Black Hole – Event Horizon Telescope
ALMA image of the jet
Relativistic jet
EHT image of the
black hole shadow
1500 lightyears
6 arcseconds
Size of the Solar System
Supermassive black hole
0.01 light-years
40 microarcseconds
Simulated image
Accretion disc
Event horizon
Photon ring
Material rotating toward Earth is Doppler boosted and brighter
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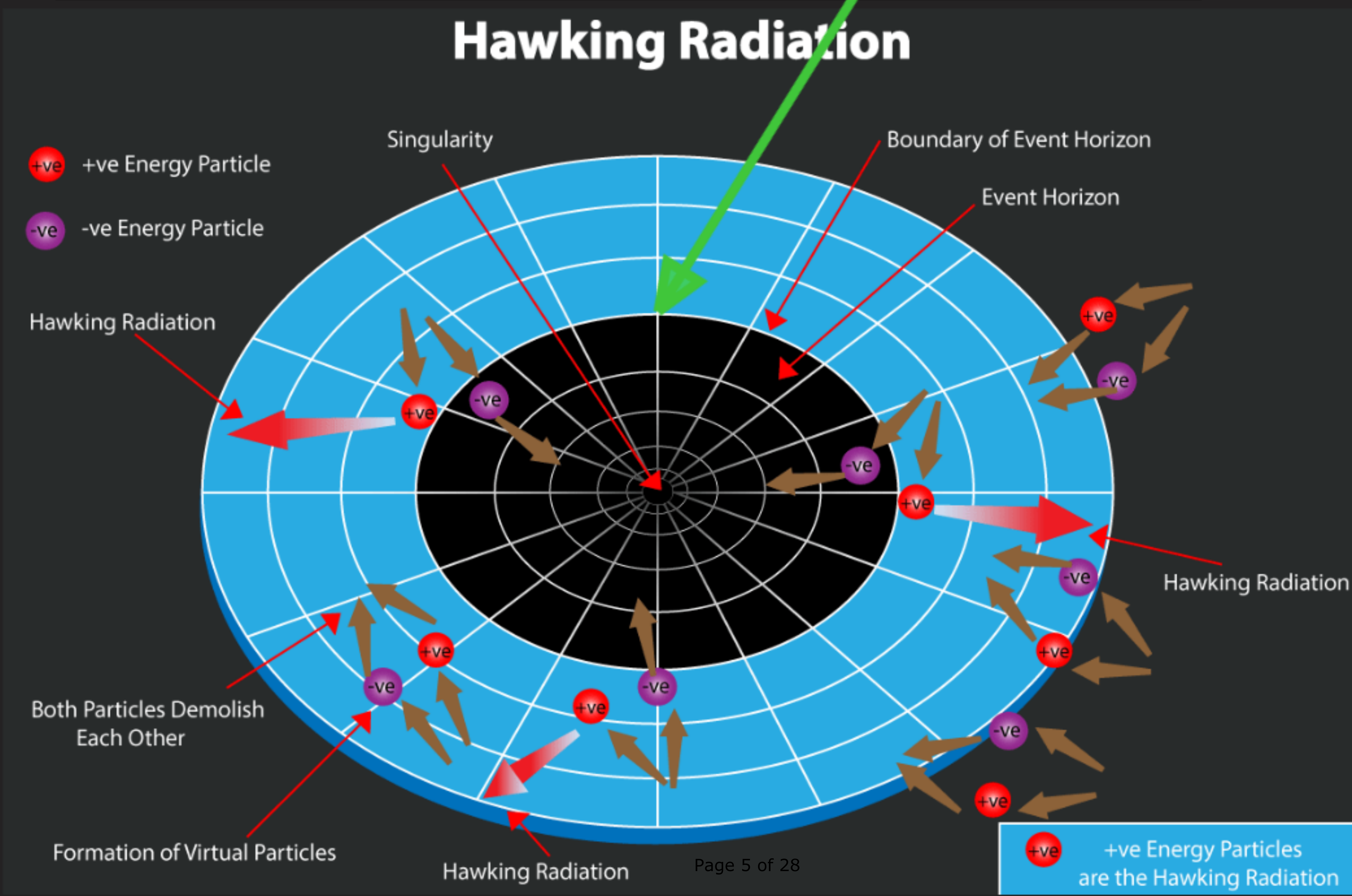
Hawking Radiation
+ve +ve Energy Particle
-ve -ve Energy Particle
Singularity
Boundary of Event Horizon
Event Horizon
Hawking Radiation
Hawking Radiation
Both Particles Demolish
Each Other
Formation of Virtual Particles
Hawking Radiation
+ve +ve Energy Particles
are the Hawking Radiation

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Published: 01 October 2015

Jacob Bekenstein

Quantum gravity pioneer

Jonathan Oppenheim

Nature Physics **11**, 805 (2015) | Cite this article

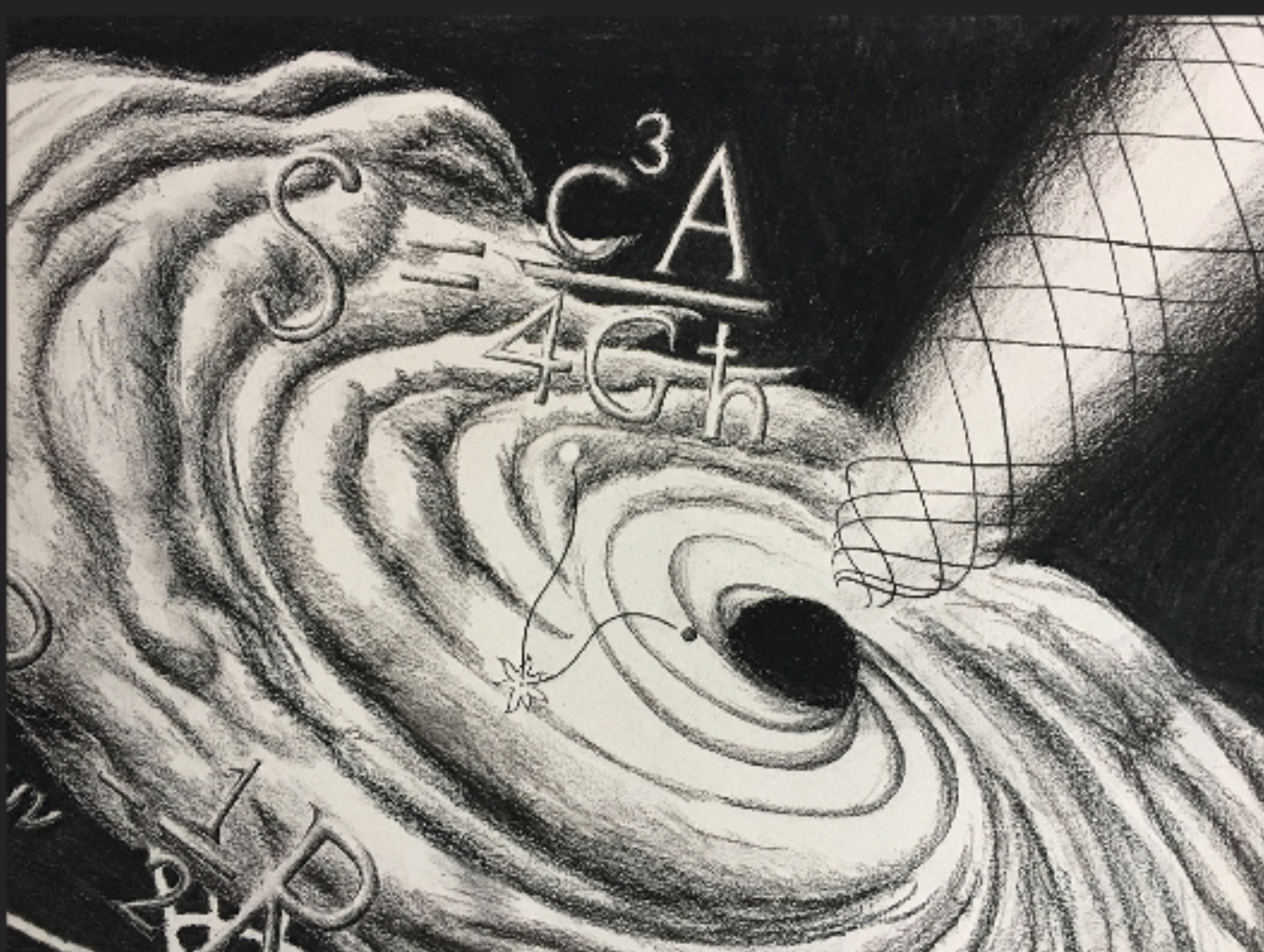
4997 Accesses | **2** Citations | **17** Altmetric | Metrics

The New York Times

Jacob Bekenstein, Physicist Who Revolutionized Theory of Black Holes, Dies at 68

Give this article

Jacob Bekenstein, third from right, who won the 2012 Wolf Prize in physics, often a forerunner for a Nobel, with recipients in other fields at ceremonies at Israel's Parliament in Jerusalem. Menahem Kahana/Agence France-Presse — Getty Images



The Soviet theoretical physicist Matvei Petrovich Bronstein (1906-1938), a pioneer of quantum gravity research whose work remains largely unknown in the west.

In 1935, when both quantum mechanics and Albert Einstein's general theory of relativity were young, a little-known Soviet physicist named Matvei Bronstein, just 28 himself, made the first detailed study of the problem of reconciling the two in a quantum theory of gravity. This "possible theory of the world as a whole," as Bronstein called it, would supplant Einstein's classical description of gravity, which casts it as curves in the space-time continuum, and rewrite it in the same quantum language as the rest of physics.

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Decoherence of black holes by Hawking radiation

Jean-Guy Demers and Claus Kiefer
Phys. Rev. D **53**, 7050 – Published 15 June 1996

ABSTRACT

We discuss in detail the semiclassical approximation for the CGHS model of two-dimensional dilatonic black holes. This is achieved by a formal expansion of the full Wheeler-DeWitt equation and the momentum constraint in powers of the gravitational constant. In highest order, the classical CGHS solution is recovered. The next order yields a functional Schrödinger equation for quantum fields propagating on this background. We show explicitly how the Hawking radiation is recovered from this equation. Although described by a pure quantum state, the expectation value of the number operator exhibits a Planckian distribution with respect to the Hawking temperature. We then show how this Hawking radiation can lead to the decoherence of black hole superpositions. The cases of a superposition of a black hole with a white hole, as well as of a black hole with no hole, are treated explicitly. © *1996 The American Physical Society.*

Classical and Quantum Gravity

LETTER TO THE EDITOR

Hawking radiation from decoherence

Claus Kiefer[1]

Published 2 November 2001 • Published under licence by IOP Publishing Ltd
Classical and Quantum Gravity, Volume 18, Number 22
Citation Claus Kiefer 2001 *Class. Quantum Grav.* **18** L151
DOI 10.1088/0264-9381/18/22/101

nature communications

Article | Open Access | Published: 04 March 2019

Decoherence of black hole superpositions by Hawking radiation

Andrew Arrasmith, Andreas Albrecht & Wojciech H. Zurek

Nature Communications **10**, Article number: 1024 (2019) | Cite this article

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Phys. Rev. D, 85 (2012)

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The action for the SABH solution is given in the relativistic Gross–Pitaevskii theory as follows

$$S = \int d^4x\sqrt{-g}(|\partial_\mu\varphi|^2 + m^2|\varphi|^2 - \frac{b}{2}|\varphi|^4),$$

Acoustic black hole in Schwarzschild spacetime: Quasinormal modes, analogous Hawking radiation, and shadows

Hong Guo, Hang Liu, Xiao-Mei Kuang, and Bin Wang
Phys. Rev. D **102**, 124019 – Published 7 December 2020

where φ is a complex scalar field as order parameter; b is a constant, and m^2 is a temperature dependent parameter assumed as $m^2 \sim (T - T_c)$ [55]. The equation of motion for φ is reduced as

$$\Box\varphi + m^2\varphi - b|\varphi|^2\varphi = 0. \quad (2)$$

Guo et al derived the SABH line-element as follows

$$ds^2 = \sqrt{3}c_s^2\left[-F(r)dt^2 + \frac{dr^2}{F(r)} + r^2(d\vartheta^2 + sin^2\vartheta d\phi^2))\right],$$

in which c_s^2 denotes the sound velocity, which can be set as $c_s^2 = 1/\sqrt{3}$ without loss of generality. The metric function $F(r)$ of SABH is given by

$$F(r) = \left(1 - \frac{2M}{r}\right)\left[1 - \xi\frac{2M}{r}\left(1 - \frac{2M}{r}\right)\right],$$

where ξ is a positive tuning parameter. One can immediately observe that metric reduces to the Schwarzschild BH as $\xi \to 0$.

at Schwarzschild coordinate radius r. It can be set as $v_r \sim \sqrt{2M\xi/r}$ in which $\xi > 0$ is required to guarantee the velocity is real. Note that recalling $c_s^2 = \frac{b\rho_0}{2}$ and rescaling $m^2 \to \frac{m^2}{2c_s^2}$, as well as $v^\mu v_\mu \to \frac{v^\mu v_\mu}{2c_s^2}$, Eq. (5) could give us the relation $v_\mu v^\mu = m^2 - 1$. As addressed in [30], one can work at the critical temperature of GP theory such that m^2 vanishes, and then one has $v_\mu v^\mu = -1$. Note that to fulfill the relation $v_\mu v^\mu = -1$, the time component of the velocity in this case can be worked out as $v_t = \sqrt{f(r) + \frac{2M\xi}{r} f(r)^2}$.

More Details about the Solution

$$\Box\varphi + m^2\varphi - b|\varphi|^2\varphi = 0. \tag{2}$$

One could fix a static background spacetime

$$ds_{\rm bg}^2 = g_{tt}dt^2 + g_{rr}dr^2 + g_{\vartheta\vartheta}d\vartheta^2 + g_{\phi\phi}d\phi^2 \tag{3}$$

and set the scalar field as $\varphi = \sqrt{\rho(\vec{x},t)}e^{i\theta(\vec{x},t)}$. In the fixed spacetime, one could assume the background solution of the scalar field as (ρ_0, θ_0), then consider the fluctuations around (ρ_0, θ_0) as

$$\rho = \rho_0 + \rho_1 \quad \text{and} \quad \theta = \theta_0 + \theta_1. \tag{4}$$

By substituting (3)–(4) into the Klein–Gordon equation (2) and considering the long-wavelength limit, one can extract two equations. One is the leading order for the background scalar field

$$b\rho_0 = m^2 - g_{\mu\nu}\partial_\mu\theta_0\partial_\nu\theta_0 = m^2 - v_\mu v^\mu, \tag{5}$$

where in the second equality we have defined $v_0 = -\partial_t\theta_0$, $v_i = \partial_i\theta_0$ $(i = r, \vartheta, \phi)$. The other is a relativistic equation governing the propagation of the phase fluctuation

$$\frac{1}{\sqrt{-\mathcal{G}}}\partial_\mu(\sqrt{-\mathcal{G}}\mathcal{G}^{\mu\nu}\partial_\nu\theta_1) = 0. \tag{6}$$

From the above fluctuation equation, one can extract and derive the effective metric $\mathcal{G}_{\mu\nu}$ as

$$\mathcal{G}_{\mu\nu} = \frac{c_s}{\sqrt{c_s^2 - v_\mu v^\mu}}\begin{pmatrix} g_{tt}(c_s^2 - v_i v^i) & \vdots & -v_i v_t \\ \cdots\cdots & \cdot & \cdots\cdots \\ -v_i v_t & \vdots & g_{ii}(c_s^2 - v_\mu v^\mu)\delta^{ij} + v_i v_j \end{pmatrix} \tag{7}$$

with $c_s^2 \equiv \frac{b\rho_0}{2}$. It is obvious that the metric $\mathcal{G}_{\mu\nu}$ encodes both the information of the background spacetime $ds_{\rm bg}$ and the background four velocity of the fluid v_μ.

Following [30], we consider $v_t \neq 0, v_r \neq 0, v_a = 0 (a = \vartheta, \phi), g_{tt}g_{rr} = -1$ and the coordinate transformation $dt \to dt - \frac{v_t v_r}{g_{tt}(c_s^2 - v_r v^r)}dr$. Then the line element of a static acoustic black hole in the background spacetime metric can be reformed from (7) as

$$ds^2 = c_s\sqrt{c_s^2 - v_\mu v^\mu}\left[\frac{c_s^2 - v_r v^r}{c_s^2 - v_\mu v^\mu}g_{tt}dt^2 + \frac{c_s^2}{c_s^2 - v_r v^r}g_{rr}dr^2 + g_{\vartheta\vartheta}d\vartheta^2 + g_{\phi\phi}d\phi^2\right]. \tag{8}$$

We shall focus on the Schwarzschild background spacetime

$$\begin{aligned} ds_{\rm bg}^2 &= g_{tt}dt^2 + g_{rr}dr^2 + g_{\vartheta\vartheta}d\vartheta^2 + g_{\phi\phi}d\phi^2 \\ &= -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2(d\vartheta^2 + \sin^2\vartheta d\phi^2), \end{aligned} \tag{9}$$

with $\mathcal{F}(r) = \left(1 - \frac{2M}{r}\right)\left[1 - \xi\frac{2M}{r}\left(1 - \frac{2M}{r}\right)\right]$, (11)

which is the acoustic black hole metric in Schwarzschild background. Here $\xi > 0$ is defined as the tuning parameter and its regime for the existence of acoustic black holes will

$$T_H = \frac{\kappa}{2\pi} = \frac{1}{4\pi} \lim_{r \to r_+} \frac{\partial_r g_{tt}}{\sqrt{g_{tt} g_{rr}}},$$

$$T_{H+} = \frac{\xi\left(\xi^{3/2}\sqrt{\xi-4} + \xi^2 - 3\sqrt{\xi}\sqrt{\xi-4} - 5\xi + 4\right)}{M\pi(\xi + \sqrt{\xi}\sqrt{\xi-4})^4}.$$

$$S_{BH} = \pi M^2(\xi + \sqrt{\xi^2 - 4\xi})^2.$$

$$dM = T_{H+} dS_{BH}$$

$$\xi \geq 4$$

it guarantees that the entropy remains real positive.

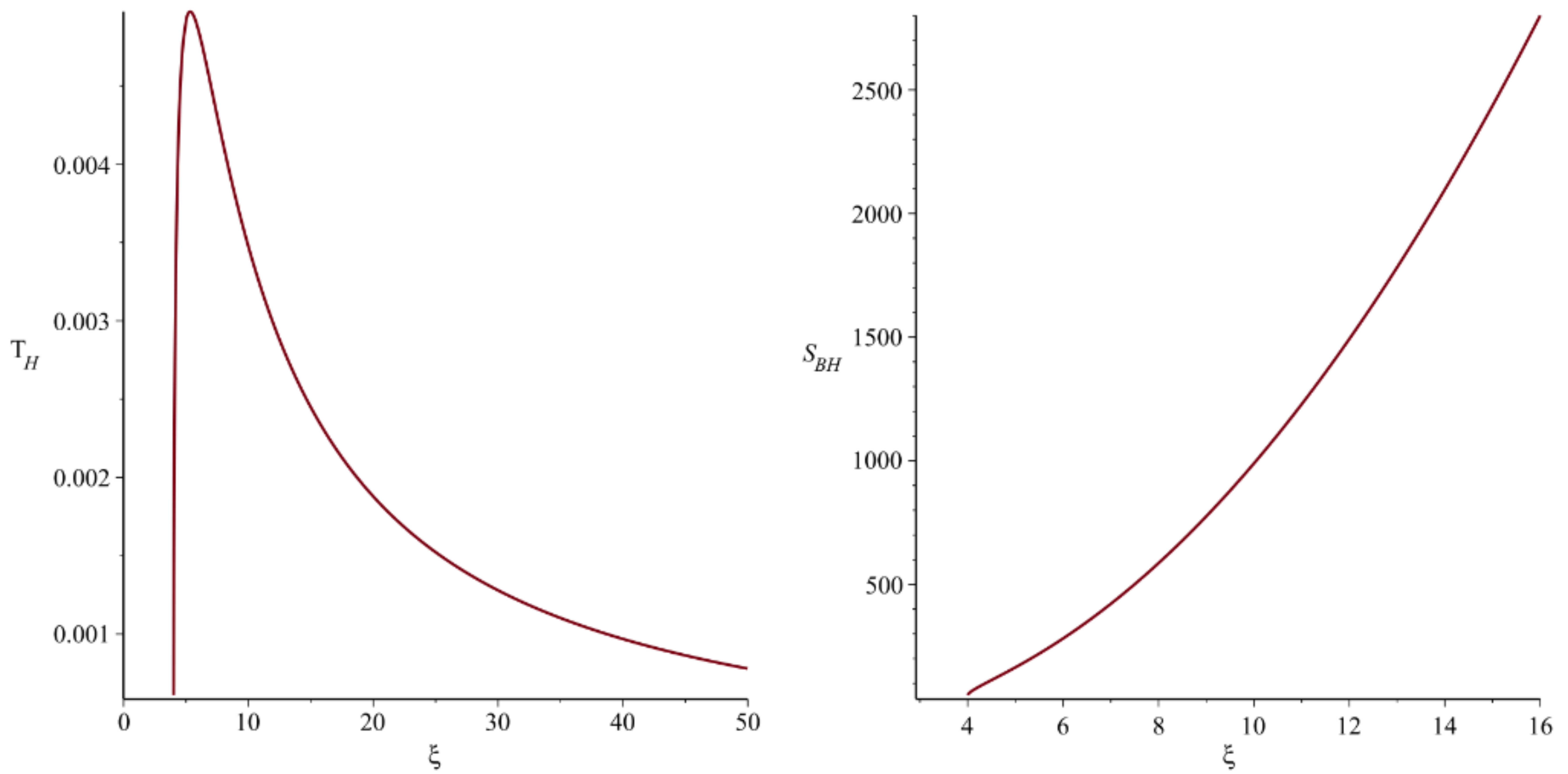


Figure 1. Graphs of Hawking temperature vs ξ (**left**) and Bekenstein–Hawking entropy vs ξ (**right**) for the SABH spacetime.

$$F(r) = \left(1 - \frac{2M}{r}\right)\left[1 - \xi\frac{2M}{r}\left(1 - \frac{2M}{r}\right)\right],$$

There are three different solutions for $F(r) = 0$: $r_{bh} = 2M$ and $r_{ac_\pm} = \left(\xi \pm \sqrt{\xi^2 - 4\xi}\right)M$, in which r_{bh} represents the optical and $r_{ac_\pm}$ represent the outer (+) and inner (−) acoustic event horizons.

To make the analysis in the existence of the acoustic event horizons region, we shall consider $\xi \geq 4$.

For $\xi = 4$, the acoustic BH becomes extremal with the clashed horizons of $r_{ac_-} = r_{ac_+} = 4$M.

Moreover, if $\xi \to \infty$, then $r_{ac_+} \to \infty$, which means that there is no way for the sound to leave the spacetime.

In the case of $\xi \geq 4$, the spacetime has four regimes:

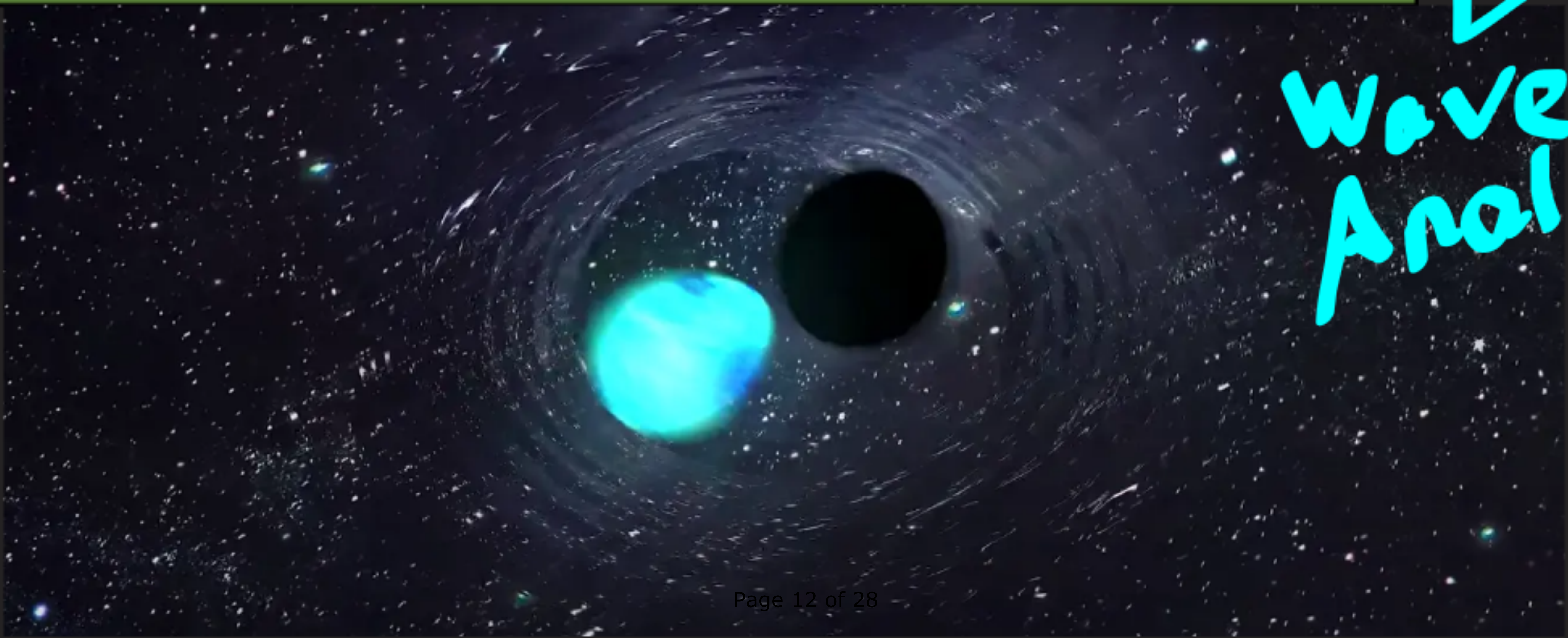
(1) $r < r_{bh}$ represents the inside of the BH; → No light, no sound

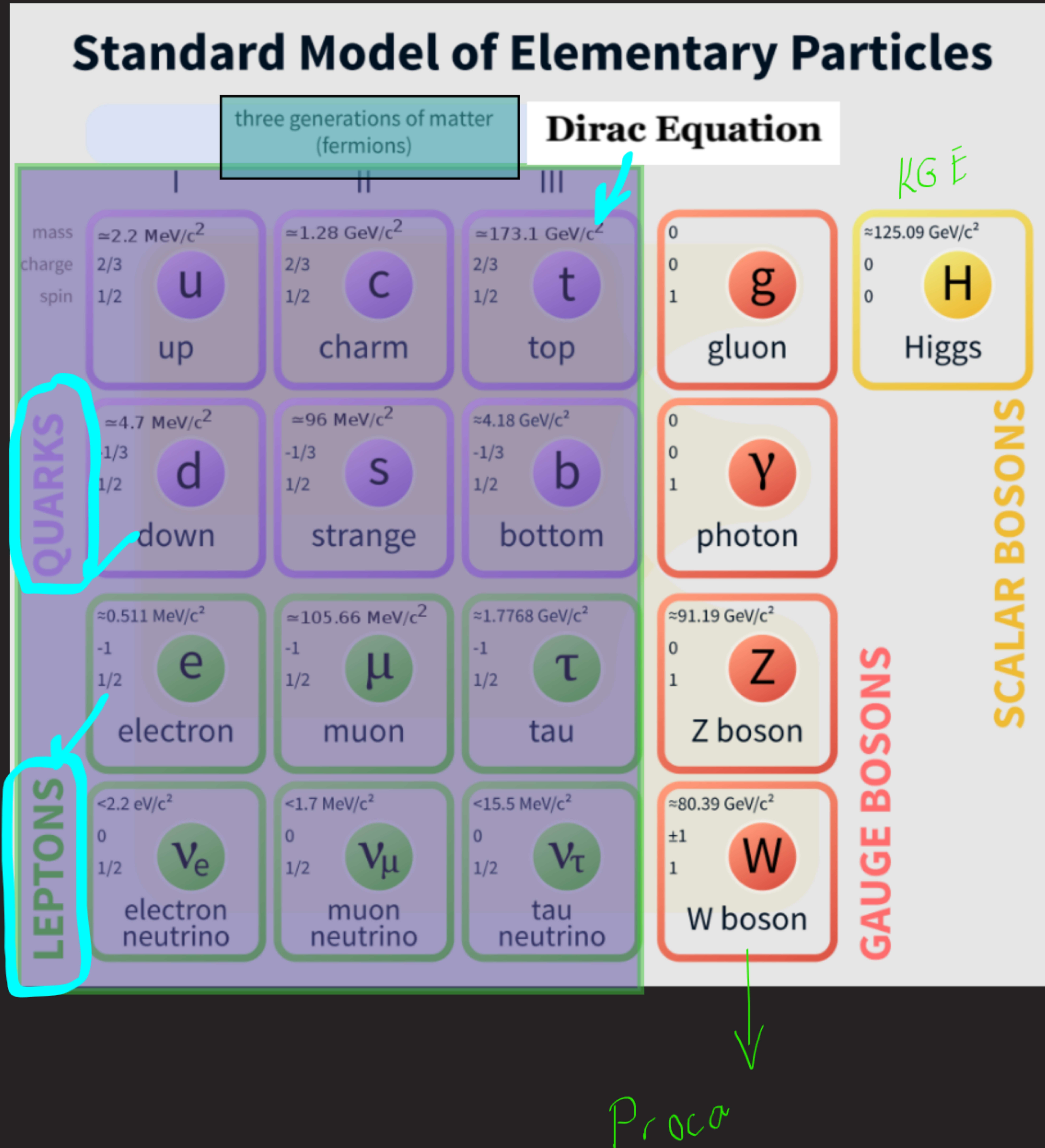
(2) $r_{bh} < r < r_{ac_-}$ and light, no sound

(3) $r_{ac_-} < r < r_{ac_+}$, which both (2) and (3) regimes mean that the sound cannot escape the BH but light can; and light, no sound

(4) in the regime of $r > r_{ac_+}$, both light and sound could escape from the BH. ✓

Wave Analysis




Standard Model of Elementary Particles
three generations of matter (fermions)
Dirac Equation
KGE
I
II
III
mass
charge
spin
≃2.2 MeV/c²
2/3
1/2
u
up
≃1.28 GeV/c²
2/3
1/2
c
charm
≃173.1 GeV/c²
2/3
1/2
t
top
0
0
1
g
gluon
≃125.09 GeV/c²
0
0
H
Higgs
≃4.7 MeV/c²
-1/3
1/2
d
down
≃96 MeV/c²
-1/3
1/2
s
strange
≃4.18 GeV/c²
-1/3
1/2
b
bottom
0
0
1
γ
photon
≃0.511 MeV/c²
-1
1/2
e
electron
≃105.66 MeV/c²
-1
1/2
μ
muon
≃1.7768 GeV/c²
-1
1/2
τ
tau
≃91.19 GeV/c²
0
1
Z
Z boson
<2.2 eV/c²
0
1/2
νe
electron neutrino
<1.7 MeV/c²
0
1/2
νμ
muon neutrino
<15.5 MeV/c²
0
1/2
ντ
tau neutrino
≃80.39 GeV/c²
±1
1
W
W boson
QUARKS
LEPTONS
GAUGE BOSONS
SCALAR BOSONS
Proca

Dirac Equation

The Dirac equation is a fundamental equation in quantum mechanics that describes the behavior of spin-$\frac{1}{2}$ particles, such as electrons. It is written as a first-order partial differential equation that describes the relationship between the wave function of a particle and its energy and momentum. In four-dimensional curved spacetime, the Dirac equation:

$$\gamma^{\alpha} e^{\mu}_{\alpha} (\partial_{\mu} + \Gamma_{\mu}) \Psi = 0,$$

where γ^{α} and $\Gamma_{\mu} = \frac{1}{8}[\gamma^{\alpha}, \gamma^{\beta}] e^{\nu}_{\alpha} e_{b\nu;\mu}$ represent the Dirac matrix and spin connection, respectively, and e^{μ}_{α} indicates the inverse of the tetrad e^{α}_{μ} which is defined as

$$e^{\alpha}_{\mu} = diag(\sqrt{F}, \frac{1}{\sqrt{F}}, r, rsin\theta).$$

In Dirac representation, the four contravariant gamma matrices are

$$\gamma^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \quad \gamma^1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix},$$

$$\gamma^2 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix}, \quad \gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}.$$

Therefore,

$$-\frac{\gamma_0}{\sqrt{F}} \frac{\partial \Psi}{\partial t} + \sqrt{F} \gamma_1 \left(\frac{\partial}{\partial r} + \frac{1}{r} + \frac{1}{4F} \frac{dF}{dr} \right) \Psi + \frac{\gamma_2}{r} \left(\frac{\partial}{\partial \theta} + \frac{1}{2} cot\theta \right) \Psi + \frac{\gamma_3}{rsin\theta} \frac{\partial \Psi}{\partial \varphi} = 0.$$

By considering the Dirac field as

$$\Psi = F^{-1/4} \Phi,$$

$$-\frac{\gamma_0}{\sqrt{F}} \frac{\partial \Phi}{\partial t} + \sqrt{F} \gamma_1 \left(\frac{\partial}{\partial r} + \frac{1}{r} \right) \Phi + \frac{\gamma_2}{r} \left(\frac{\partial}{\partial \theta} + \frac{1}{2} cot\theta \right) \Phi + \frac{\gamma_3}{rsin\theta} \frac{\partial \Phi}{\partial \varphi} = 0.$$

ansatz $$\Phi = \begin{pmatrix} \frac{iG(r)}{r}\phi^{\pm}_{jm}(\theta,\varphi) \\ \frac{H(r)}{r}\phi^{\mp}_{jm}(\theta,\varphi) \end{pmatrix} e^{-i\omega t}$$

in which

$$\Phi^{+}_{jm} = \begin{pmatrix} \sqrt{\frac{j+m}{2j}} Y_l^{m-1/2} \\ \sqrt{\frac{j-m}{2j}} Y_l^{m+1/2} \end{pmatrix}, \qquad (j = l + \frac{1}{2}),$$

and

$$\Phi^{-}_{jm} = \begin{pmatrix} \sqrt{\frac{j+1-m}{2j+2}} Y_l^{m-1/2} \\ -\sqrt{\frac{j+1+m}{2j+2}} Y_l^{m+1/2} \end{pmatrix}. \qquad (j = l - \frac{1}{2})$$

$$-\frac{\gamma_0}{\sqrt{F}}\frac{\partial \Phi}{\partial t} + \sqrt{F}\gamma_1\left(\frac{\partial}{\partial r} + \frac{1}{r}\right)\Phi + \frac{\gamma_2}{r}\left(\frac{\partial}{\partial \theta} + \frac{1}{2}cot\theta\right)\Phi + \frac{\gamma_3}{r sin\theta}\frac{\partial \Phi}{\partial \varphi} = 0.$$

After decoupling the equations, one can obtain

tortoise coordinate $dr_* = \frac{dr}{F}$

$$\frac{d^2 H}{dr_*^2} + (\omega^2 - V_1)H = 0,$$

$$\frac{d^2 G}{dr_*^2} + (\omega^2 - V_2)G = 0,$$

where

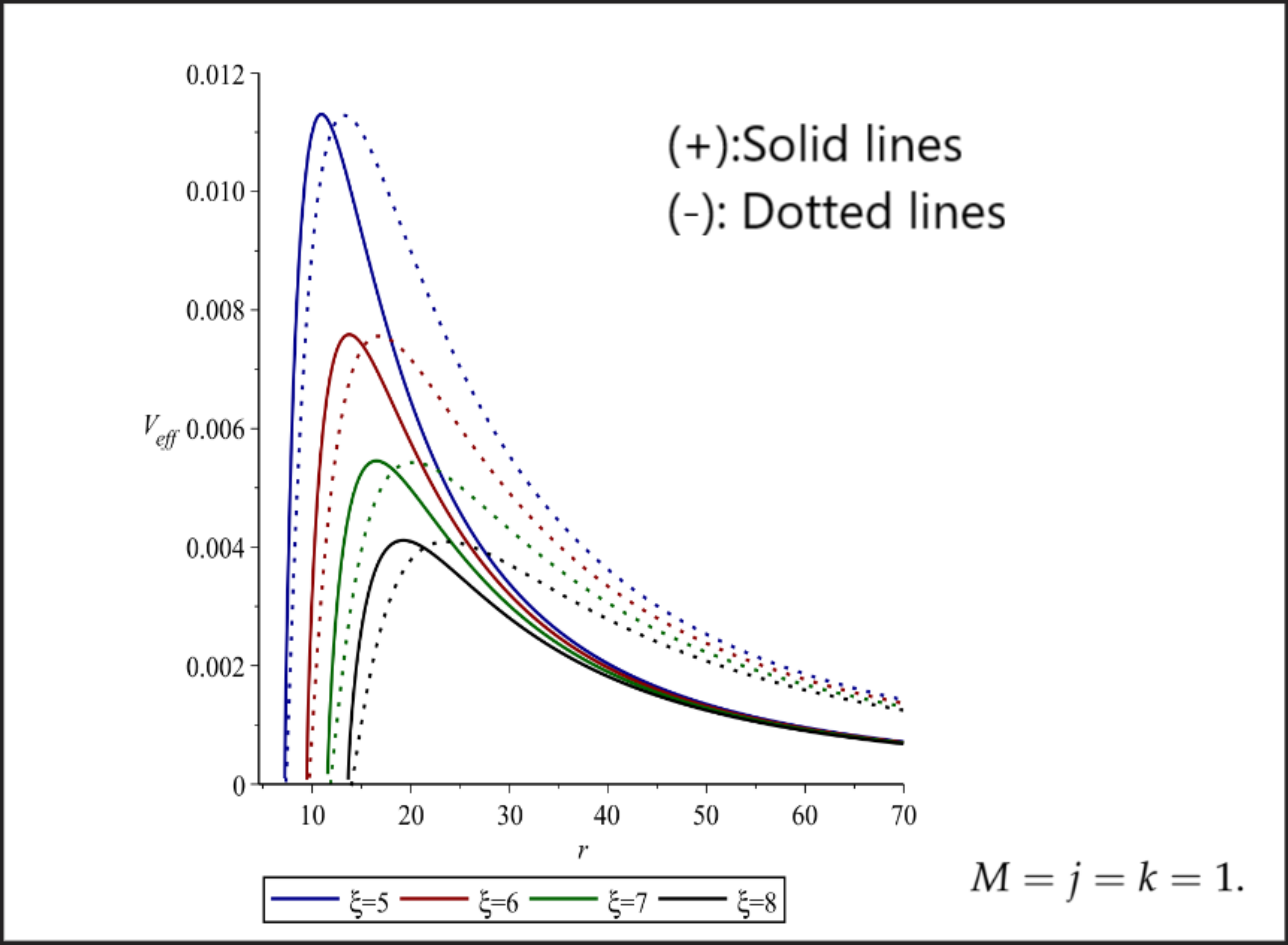
$$V_1 = \frac{\sqrt{F}|k|}{r^2}\left(|k|\sqrt{F} + \frac{r}{2}\frac{df}{dr} - f\right), \qquad (k = j + \frac{1}{2}, j = l + \frac{1}{2}),$$

+y

$$V_2 = \frac{\sqrt{F}|k|}{r^2}\left(|k|\sqrt{F} - \frac{r}{2}\frac{df}{dr} + f\right), \qquad (k = -(j + \frac{1}{2}), j = l - \frac{1}{2}).$$

-y

$$f(r) = 1 - \frac{2M}{r}$$



Thus, the effective potentials of the fermionic waves having spin-$\frac{1}{2}$ and moving in the SABH geometry are found as

$$V_{eff} = \frac{k^2 A}{r^2}\left(1 \pm \frac{1}{\sqrt{A}}\left(\frac{df(r)}{dr}(\frac{r}{2} - 2M\xi f(r)) + f(r)(\frac{3M\xi}{r}f(r) - 1)\right)\right),$$

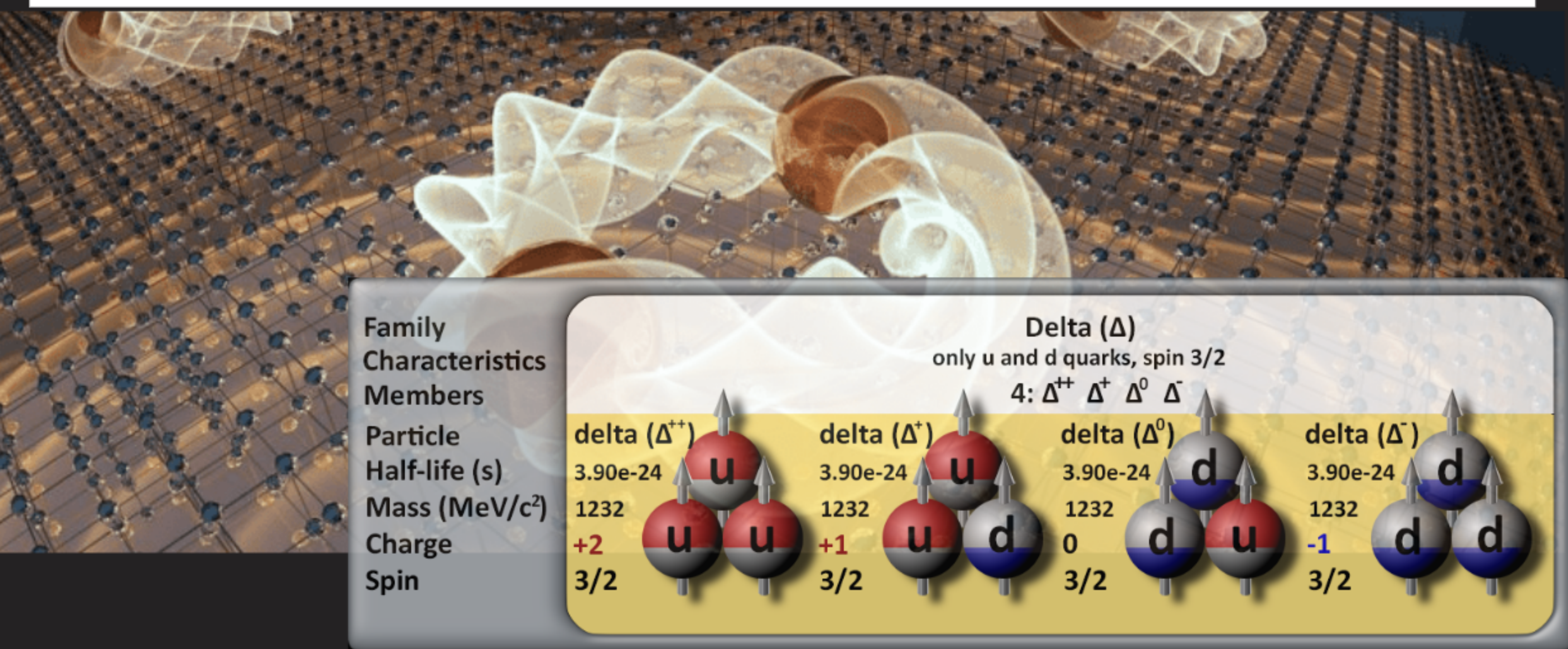
where $A = \sqrt{f(r) - \frac{2M\xi}{r}f^2(r)}$ and positive and negative signs are conjugated with spin signs. The behaviors of the effective potentials are depicted in the Figure for various ξ parameters.

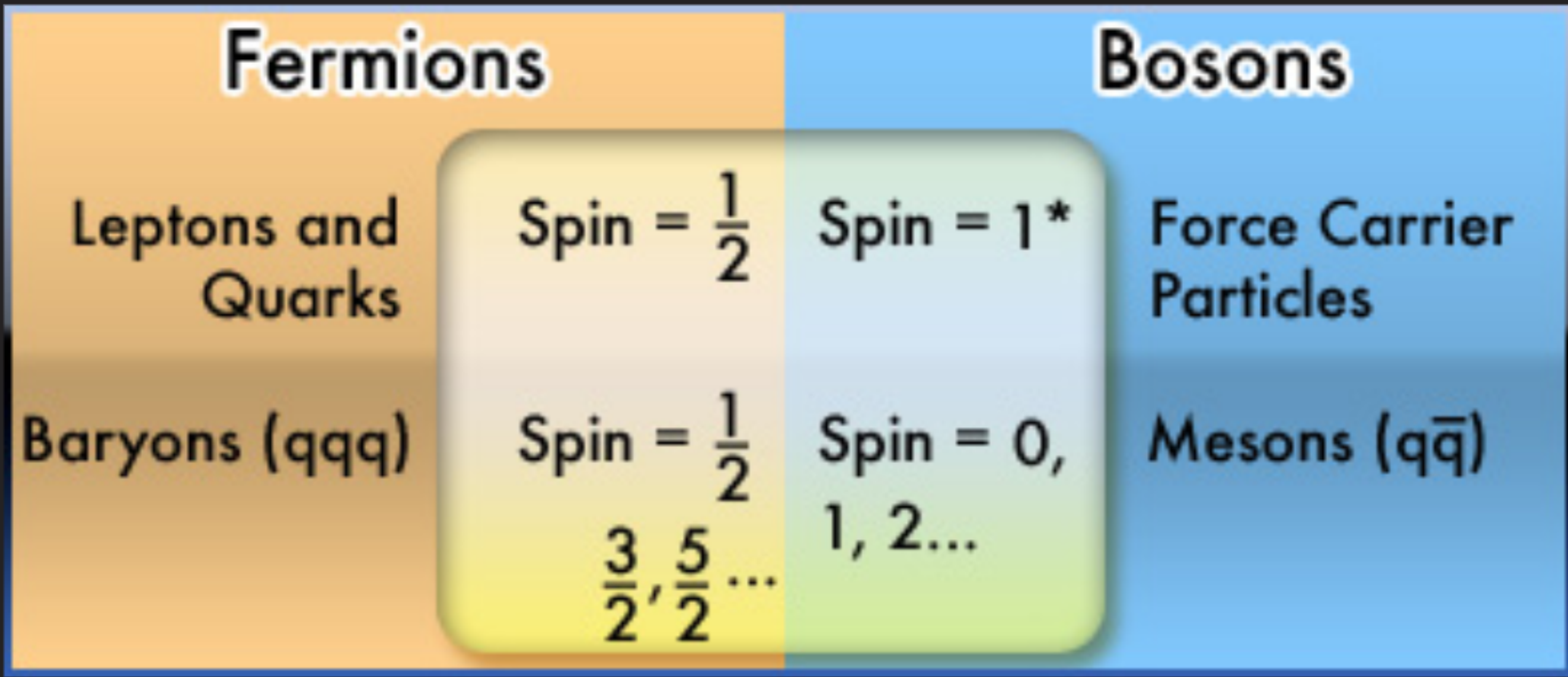
Rarita–Schwinger Equation

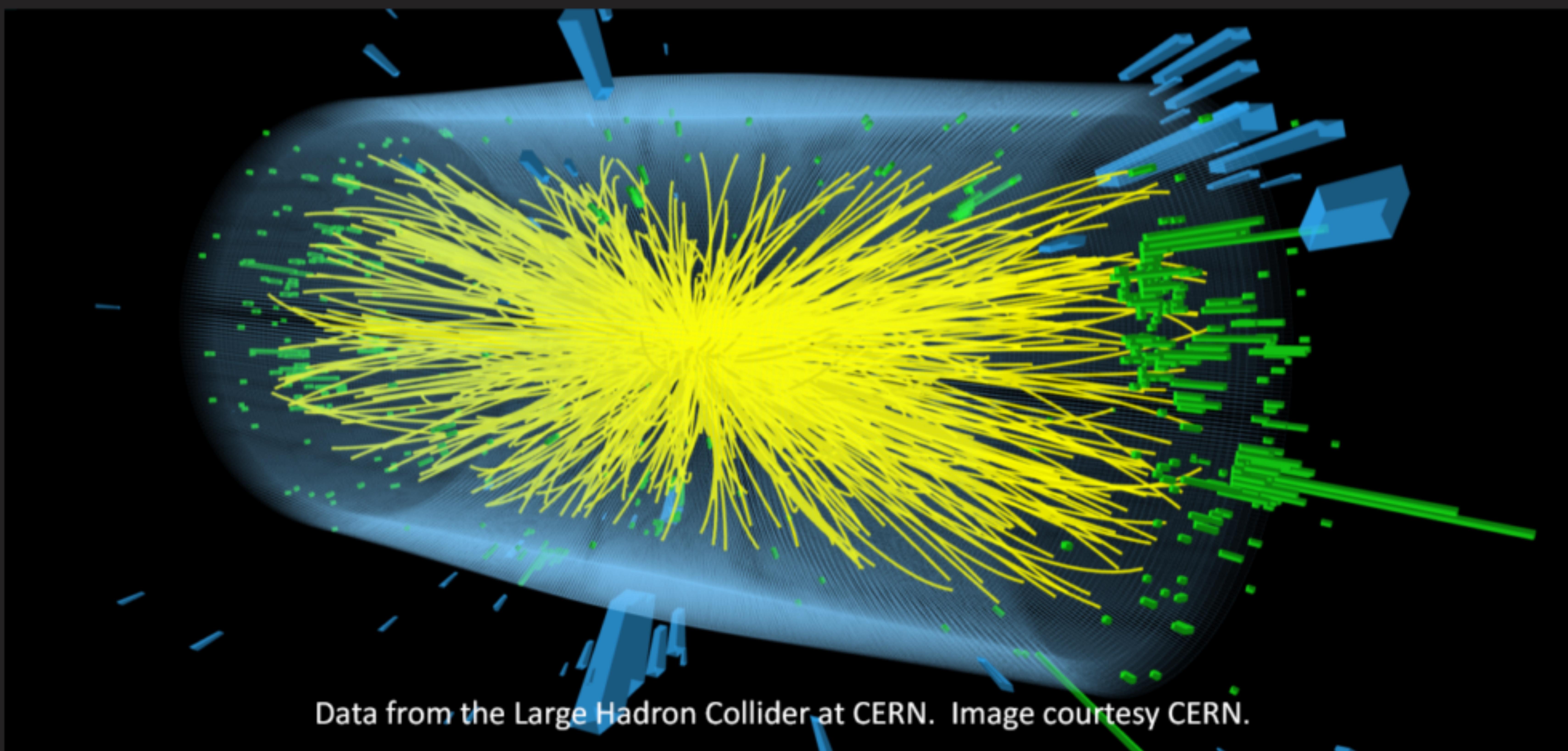
- **The Rarita-Schwinger equation is a partial differential equation that describes the behavior of spin- $\frac{3}{2}$ fields in a four-dimensional curved spacetime; spin- $\frac{3}{2}$ fields, also known as Rarita-Schwinger fields, are fields that are characterized by having spin- $\frac{3}{2}$.**

- **The Rarita-Schwinger equation was first derived by W. Rarita and J. Schwinger in 1941, and it has since been an important tool for the study of high-spin particles and their interactions with other fields.**

- **It is used in the study of supersymmetric theories, where spin-3/2 fields appear as superpartners of spin- 1/2 fields.**







Data from the Large Hadron Collider at CERN. Image courtesy CERN.

Rarita–Schwinger equation

$$\gamma^{\mu\nu\alpha}\tilde{D}_\nu\psi_\alpha = 0,$$

where ψ_α indicates the spin-3/2 field, and $\gamma^{\mu\nu\alpha}$ → the antisymmetric of Dirac matrices as

$$\gamma^{\mu\nu\alpha} = \gamma^\mu\gamma^\nu\gamma^\alpha - \gamma^\mu g^{\nu\alpha} + \gamma^\nu g^{\mu\alpha} - \gamma^\alpha g^{\mu\nu}.$$

$\tilde{D}$ is the super-covariant derivative, which is defined for four-dimensional spacetime as

$$\tilde{D}_\mu = \nabla_\mu + \frac{1}{4}\gamma_\rho F^\rho_\mu + \frac{i}{8}\gamma_{\mu\rho\sigma}F^{\rho\sigma}.$$

0 0

The covariant derivative for the spinor-vector field is

$$\nabla_\mu\psi_\nu = \partial_\mu\psi_\nu - \Gamma^\rho_{\mu\nu}\psi_\rho + \omega_\mu\psi_\nu\,,$$

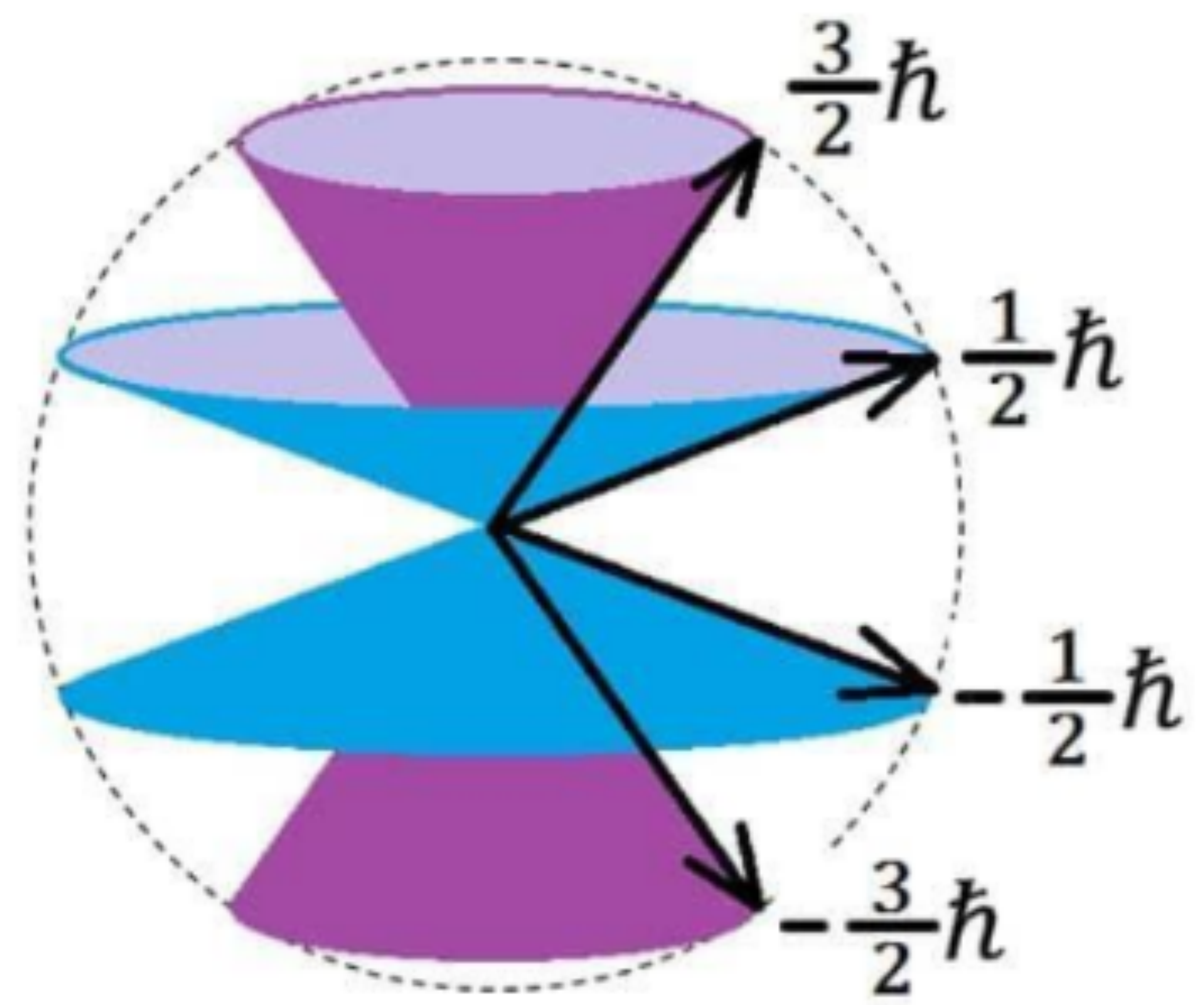
where

$$\omega_\mu = \frac{1}{2}\omega_{\mu ab}\Sigma^{ab} \quad , \quad \Sigma^{ab} = \frac{1}{4}[\gamma^a, \gamma^b]\,.$$

Spin connections

$$\omega_{\mu ab} = e^\alpha_a\left(\partial_\mu e_{\alpha b} - \Gamma^\rho_{\mu\alpha}e_{\rho b}\right).$$

A spin 3/2 particle has four possible spin states



Recall

At this stage, our concentration will be on the non-TT eigenfunctions [x]; therefore, the radial and temporal wave functions are given as

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$$\psi_r = \phi_r \otimes \bar{\psi}_{(\lambda)}, \qquad \psi_t = \phi_t \otimes \bar{\psi}_{(\lambda)},$$

in which $\bar{\psi}_{(\lambda)}$ represents an eigenspinor with an eigenvalue of $i\bar{\lambda}$ where $\bar{\lambda} = j + 1/2$ and $j = 3/2, 5/2, 7/2, \ldots$. Moreover, the angular wave function is determined by

$$\psi_{\theta_i} = \phi_\theta^{(1)} \otimes \bar{\nabla}_{\theta_i} \bar{\psi}_{(\lambda)} + \phi_\theta^{(2)} \otimes \bar{\gamma}_{\theta_i} \bar{\psi}_{(\lambda)},$$

where $\phi_\theta^{(1)}$ and $\phi_\theta^{(2)}$ depend on r and t.

By utilizing the Weyl gauge $(\phi_t = 0)$ one can obtain the following gauge invariant variable

$$\Phi = -\left(\frac{\sqrt{F}}{2} i\sigma^3\right)\phi_\theta^{(1)} + \phi_\theta^{(2)},$$

Details

Quasinormal modes and absorption probabilities of spin-3/2 fields in D-dimensional Reissner-Nordström black hole spacetimes

C.-H. Chen, H. T. Cho, A. S. Cornell, G. Harmsen, and X. Ngcobo
Phys. Rev. D **97**, 024038 – Published 25 January 2018

which can be rewritten as

$$\Phi = \begin{pmatrix} \phi_1 e^{-i\omega t} \\ \phi_2 e^{-i\omega t} \end{pmatrix}.$$

Parameters ϕ_1 and ϕ_2 are radially dependent:

$$\phi_1 = \frac{F - \bar{\lambda}^2}{B_1 F^{1/4}} \tilde{\phi}_1, \qquad \phi_2 = \frac{F - \bar{\lambda}^2}{B_2 F^{1/4}} \tilde{\phi}_2.$$

where $B_1 = \sqrt{F} - \bar{\lambda}$ and $B_2 = \sqrt{F} + \bar{\lambda}$. Now, with aid of the tortoise coordinate, we obtain a set of one-dimensional Schrödinger-like wave equations

$$-\frac{d^2}{dr_*^2}\tilde{\phi}_1 + V_1 \tilde{\phi}_1 = \omega^2 \tilde{\phi}_1,$$

$$-\frac{d^2}{dr_*^2}\tilde{\phi}_2 + V_2 \tilde{\phi}_2 = \omega^2 \tilde{\phi}_2,$$

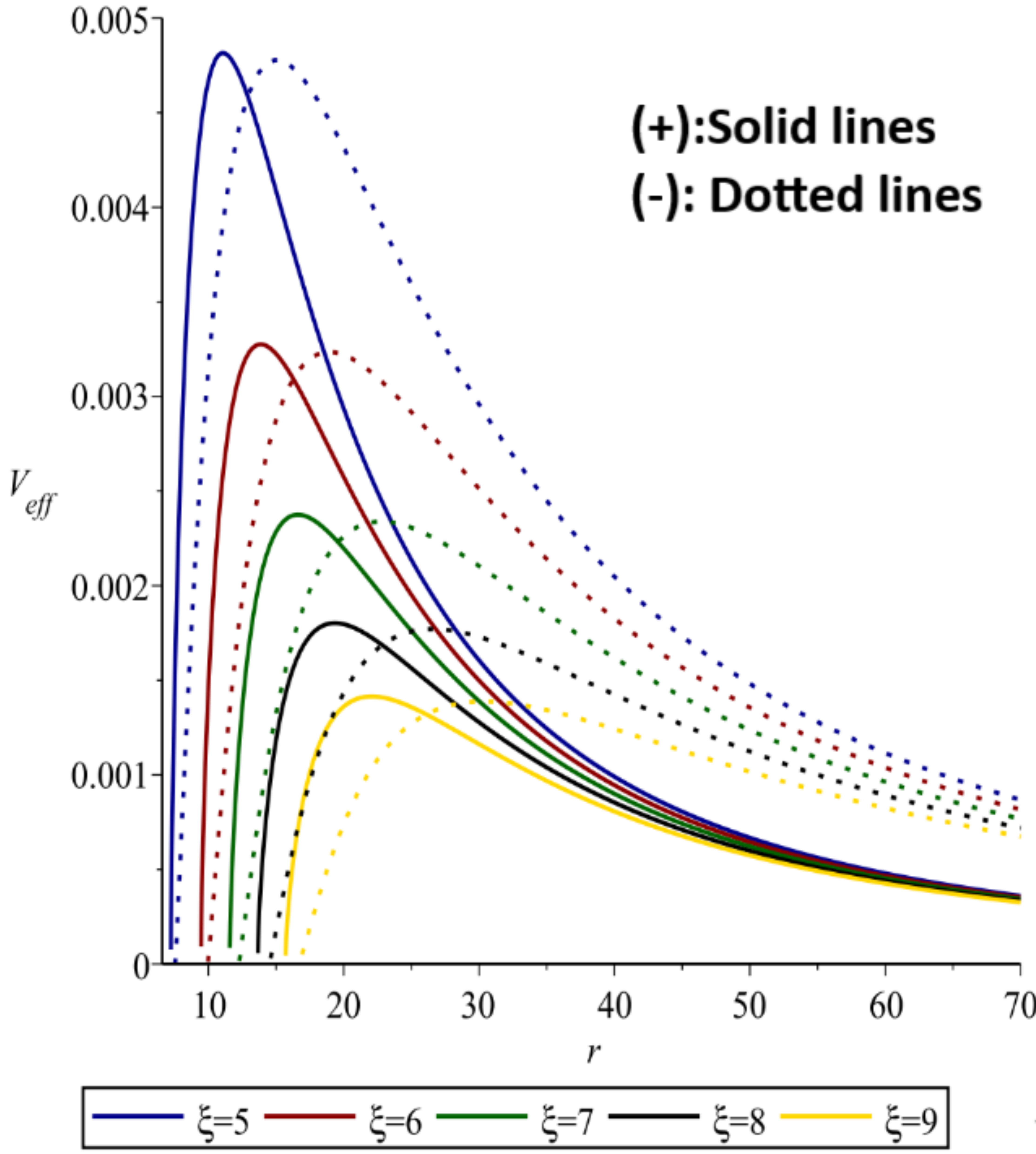
$$V_{1,2} = \pm F(r)\frac{dW}{dr} + W^2,$$

$$W = \frac{\bar{\lambda}\sqrt{F}}{r}\left(\frac{\bar{\lambda}^2 - 1}{\bar{\lambda}^2 - F}\right).$$

Therefore, the explicit forms of the effective potentials belonging to the SABH spacetime for spin-$\frac{3}{2}$ fermions are written as

$$V_{1,2} = F(r)\frac{\bar{\lambda}(1-\bar{\lambda}^2)}{r^2(F-\bar{\lambda}^2)^2}\left[\pm\left(\frac{rF'-2F}{2\sqrt{F}}\right)(F-\bar{\lambda}^2) \mp r\sqrt{F}F' + \bar{\lambda}(1-\bar{\lambda}^2)\right].$$

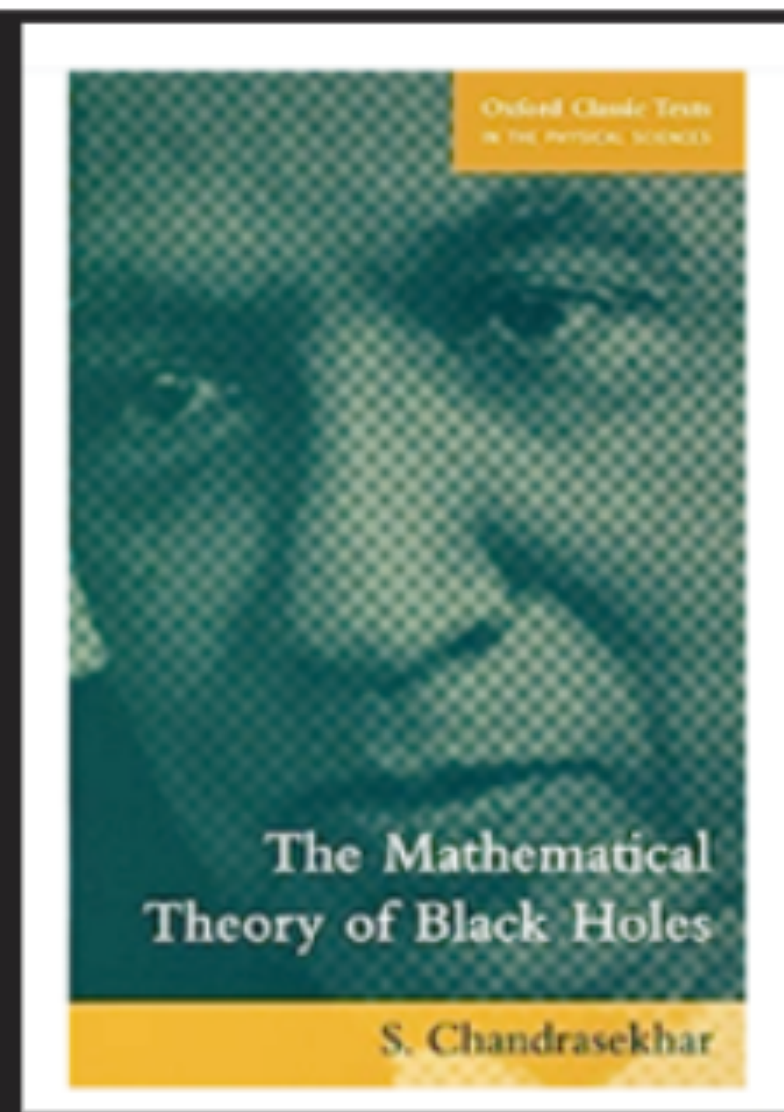
a prime symbol indicates a derivative with respect to r.



$M = j = 1.$

GFs of SABH via FERMION EMISSION

GFs for BHs are a measure of how much the spectrum of radiation emitted by a BH deviates from that of a perfect black body. The general semi-analytic bounds for the GFs are given by (and see also Chandrasekhar's famous monograph for the details).



$$\sigma(\omega) \geq \sec h^2\left[\int_{r_h}^{+\infty} \frac{V_{eff}}{2\omega F(r)} dr\right].$$

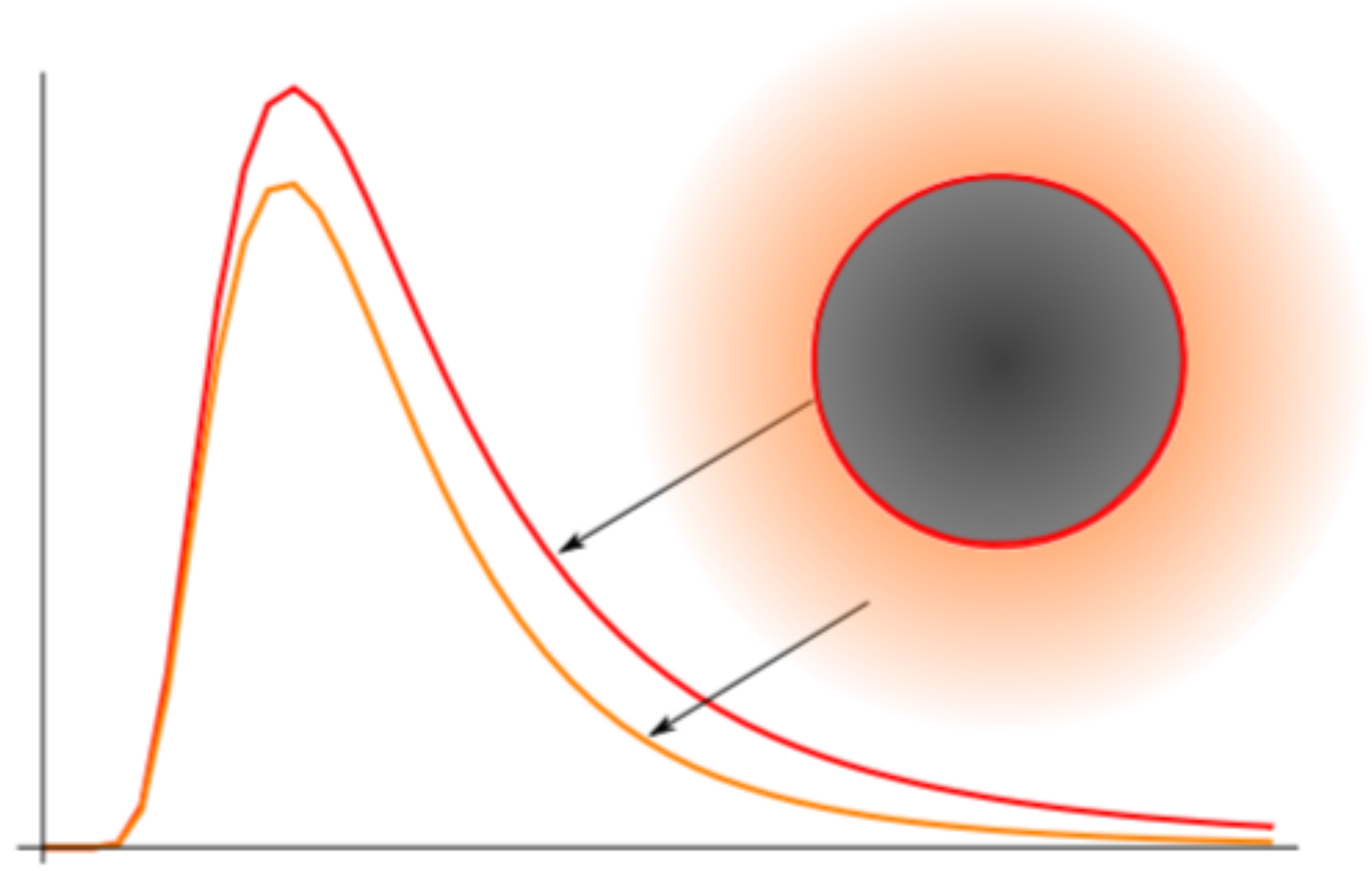


Figure: The purely thermal radiation emitted at the horizon (red) gets modified (orange) by the black hole geometry.

Spin-$\frac{1}{2}$ Fermions

Substituting the effective potential derived from Dirac equations into Equation (*), we obtain

$$\sigma(\omega) \geq \sec h^2\left[\frac{1}{2\omega}\int_{r_h}^{+\infty}\frac{|k|}{r^2}\left(|k| \pm \left(\frac{r}{2\sqrt{F}}\frac{dF(r)}{dr} - \sqrt{F(r)}\right)\right)dr\right].$$

After evaluating integral, the GFs of spin-$\frac{1}{2}$ fermions are found out to be

$$\sigma_l(\omega) \geq \sec h^2\left[\frac{|k|}{2\omega}\left(\frac{(k-1)}{r_h} + \frac{M(1+\xi)}{r_h^2} + \frac{2M^2}{3r_h^3}(1+\xi^2-\frac{9}{2}\xi) + \frac{M^3}{2r_h^4}(\xi^3-5\xi^2+3\xi+1)\right)\right].$$

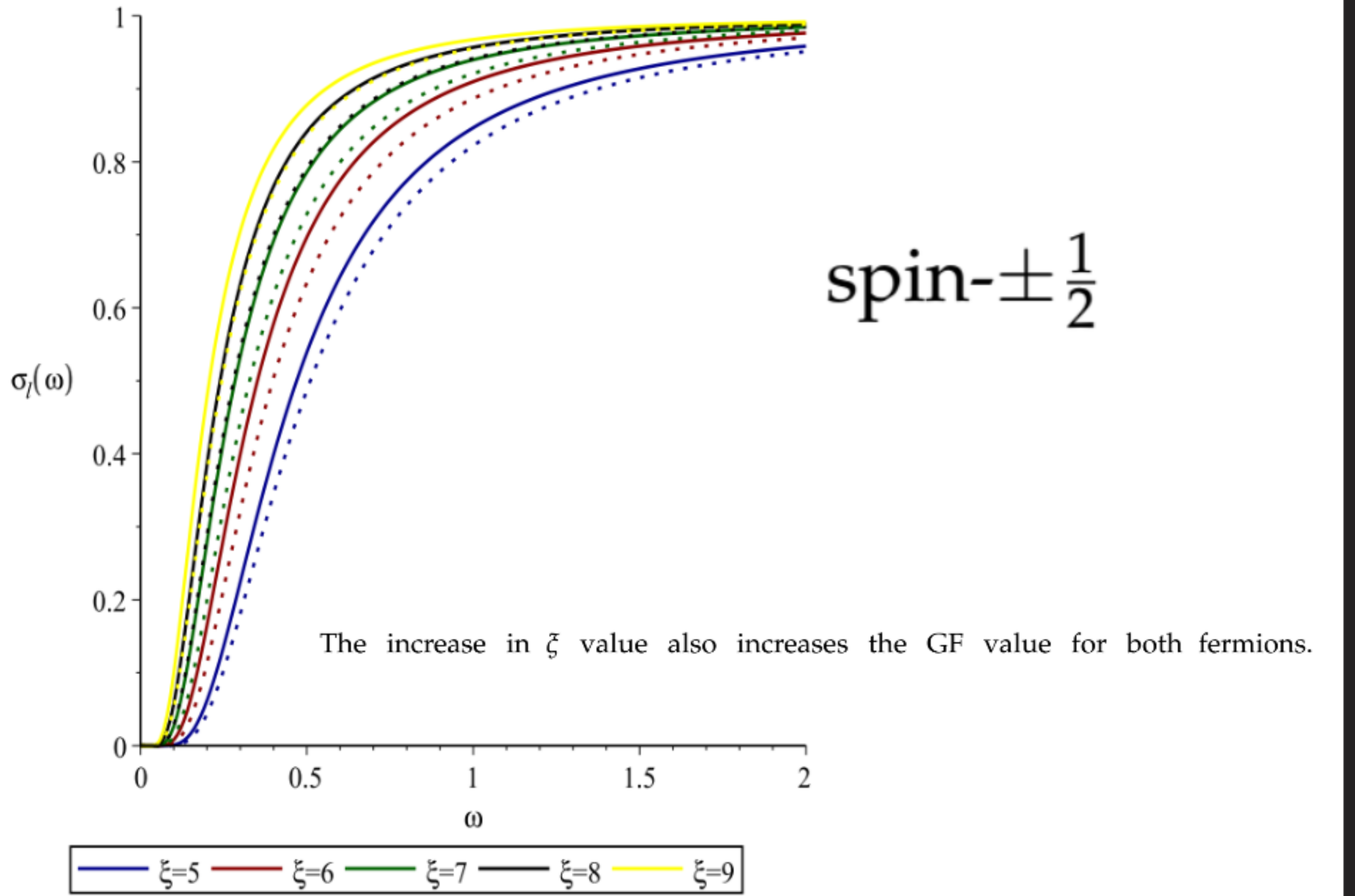


Figure. $\sigma_l(\omega)$ versus ω graph for the spin-$\frac{1}{2}$ fermions. While the solid lines stand for spin-$\frac{+1}{2}$ fermions, the dotted ones represent spin-$\frac{-1}{2}$ fermions. The physical parameters are chosen as $M = 1$ and $k = 2.5$.

Spin-$\frac{3}{2}$ Fermions

GFs of SABH via the spin-$\frac{3}{2}$ fermions

$$\sigma(\omega) \geq \sec h^2\left[\frac{1}{2\omega}\int_{r_h}^{+\infty}\bar{\lambda}(1-\bar{\lambda}^2)\left(\pm\frac{rF'-2F}{2r^2\sqrt{F}(F-\bar{\lambda}^2)}+\frac{\mp r\sqrt{F}F'+\bar{\lambda}(1-\bar{\lambda}^2)}{r^2(F-\bar{\lambda}^2)^2}\right)dr\right].$$

$$\bar{\lambda} = j + 1/2$$

$$\sigma(\omega) \geq \sec h^2\left[\frac{\bar{\lambda}(1-\bar{\lambda}^2)}{2\omega}\left(\frac{\bar{\lambda}(1-\bar{\lambda}^2)-1}{r_h(1-\bar{\lambda}^2)^2}+\frac{(1+\xi)}{2r_h^2(1-\bar{\lambda}^2)^2}(1-2\bar{\lambda}^2+\frac{4\bar{\lambda}}{1-\bar{\lambda}^2})+\frac{1}{3r_h^3(1-\lambda^2)}\times\right.\right.$$
$$\left.\left.\left(\frac{3}{2}(1+\xi)^2-12\xi-\frac{4(1+\xi)^2+16\xi}{\bar{\lambda}^2-1}-\frac{2(1+\xi)^2(\bar{\lambda}^2-3)}{(\bar{\lambda}^2-1)^2}+\frac{4\bar{\lambda}}{(1-\bar{\lambda}^2)^2(4\lambda^2\xi+3\xi^2+2\xi+3)}\right)\right)\right].$$

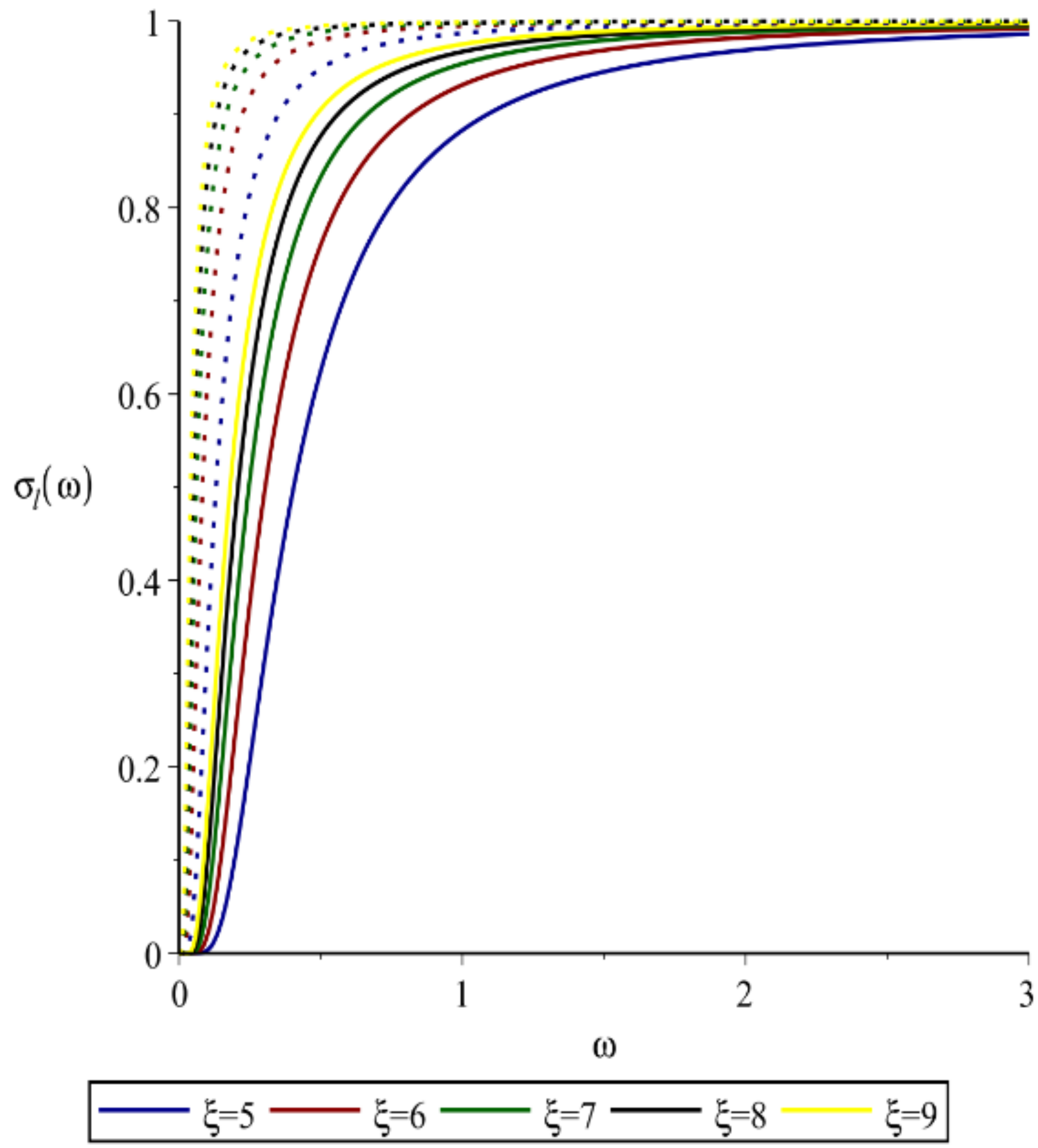


Figure. $\sigma_l(\omega)$ versus ω graph for the spin-$\frac{3}{2}$ fermions. While the solid lines stand for spin-$\frac{+3}{2}$ fermions, the dotted ones represent spin-$\frac{-3}{2}$ fermions; and also $\bar{\lambda} = 2$.

As can be seen from Figure , GFs increase with the ξ parameter and vice versa Moreover, it was observed that GFs of spin-$\frac{-3}{2}$ fermions are higher than the spin-$\frac{+3}{2}$ fermions. Therefore, one can conclude that thermal emission of Rarita–Schwinger fermions from the SABH separates the particles of different spin into separate beams. So, SABH spacetime acts as a device similar to the famous experiment that is about how electrons are measured in a Stern–Gerlach magnetic field device, which splits up and down spinned beams.

PHYSICAL REVIEW LETTERS

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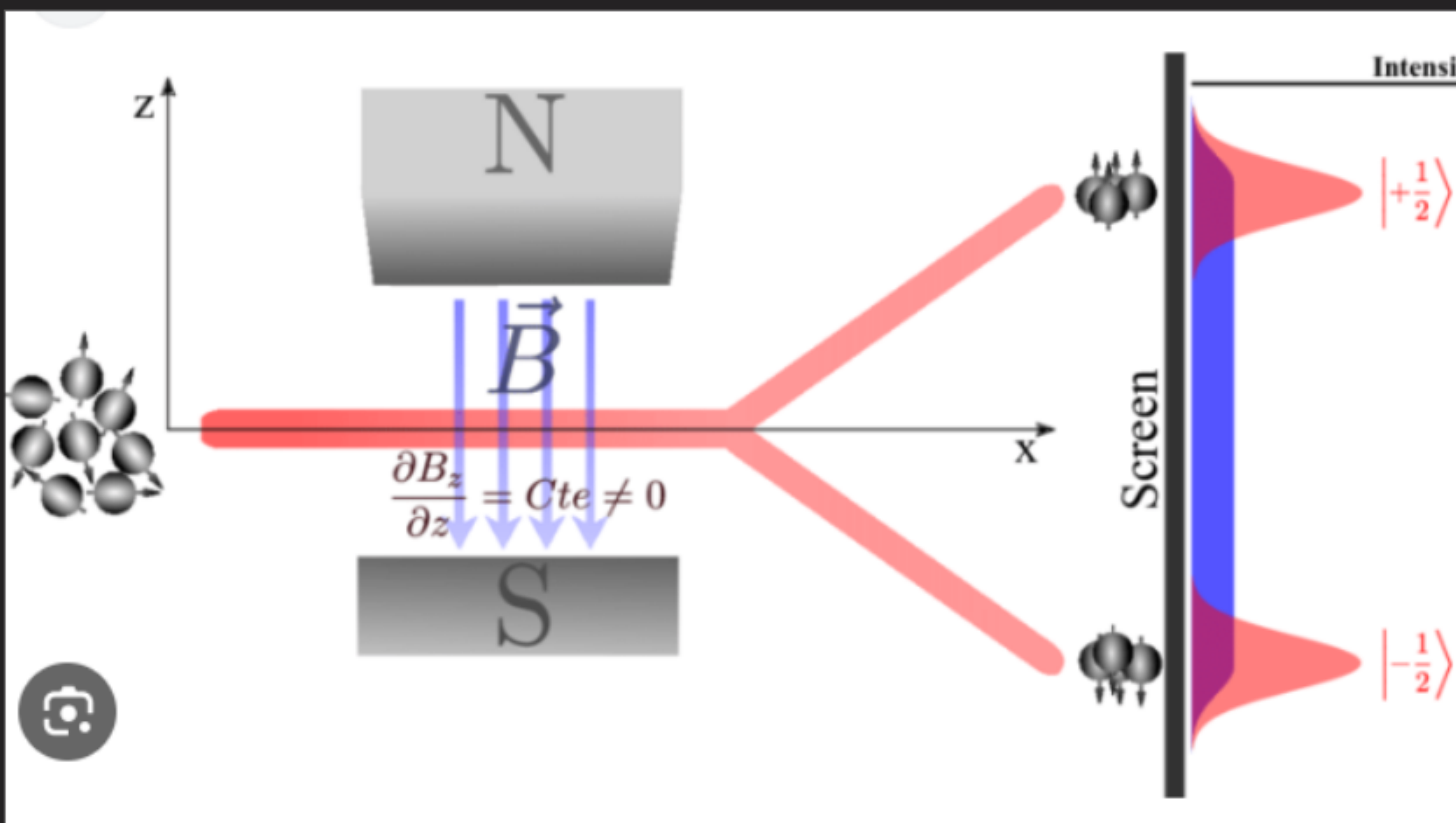
Stern-Gerlach Effect for Electron Beams

H. Batelaan, T. J. Gay, and J. J. Schwendiman
Phys. Rev. Lett. **79**, 4517 – Published 8 December 1997



3: Sketch of Stern-Gerlach experiment: A beam of silver

Conclusions

We obtained analytical GFs for the covariant massless Dirac and Rarita-Schwinger equations in the SABH spacetime.

The angular part of the solutions is given in terms of the spherical harmonic functions, while the radial equations are reduced to one-dimensional Schrödinger-like wave equations.

We studied the thermal radiation (HR) spectrum for massless fermions in the vicinity to the exterior event horizon. Namely, we obtained the quasi-spectrum of greybody spectrum for massless fermions having spin- 1/2 and spin- 3/2 propagating in the SABH spacetime.

We employed the method of semi-analytic bounds for the GFs.

We examined the behaviors of the obtained effective potentials and showed that ξ parameter modifies both the effective potentials and therefore the GFs.

* We showed that with the increased value of the ξ parameter, the GFs increase as well. This means that higher acoustic values of SABH will result in a higher probability of detecting HR.

Moreover, it was shown that the thermal emission of Rarita–Schwinger fermions from a SABH result in the separation of fermions with different spin into distinct thermal radiations.

Therefore, we presented some analytical results that might be compared with data to be detected in future.

Finally, it is important to note that previous observations have confirmed the existence of HR in an analog BH [*]

Published: 15 August 2016

Observation of quantum Hawking radiation and its entanglement in an analogue black hole

Jeff Steinhauer

Nature Physics **12**, 959–965 (2016) | Cite this article

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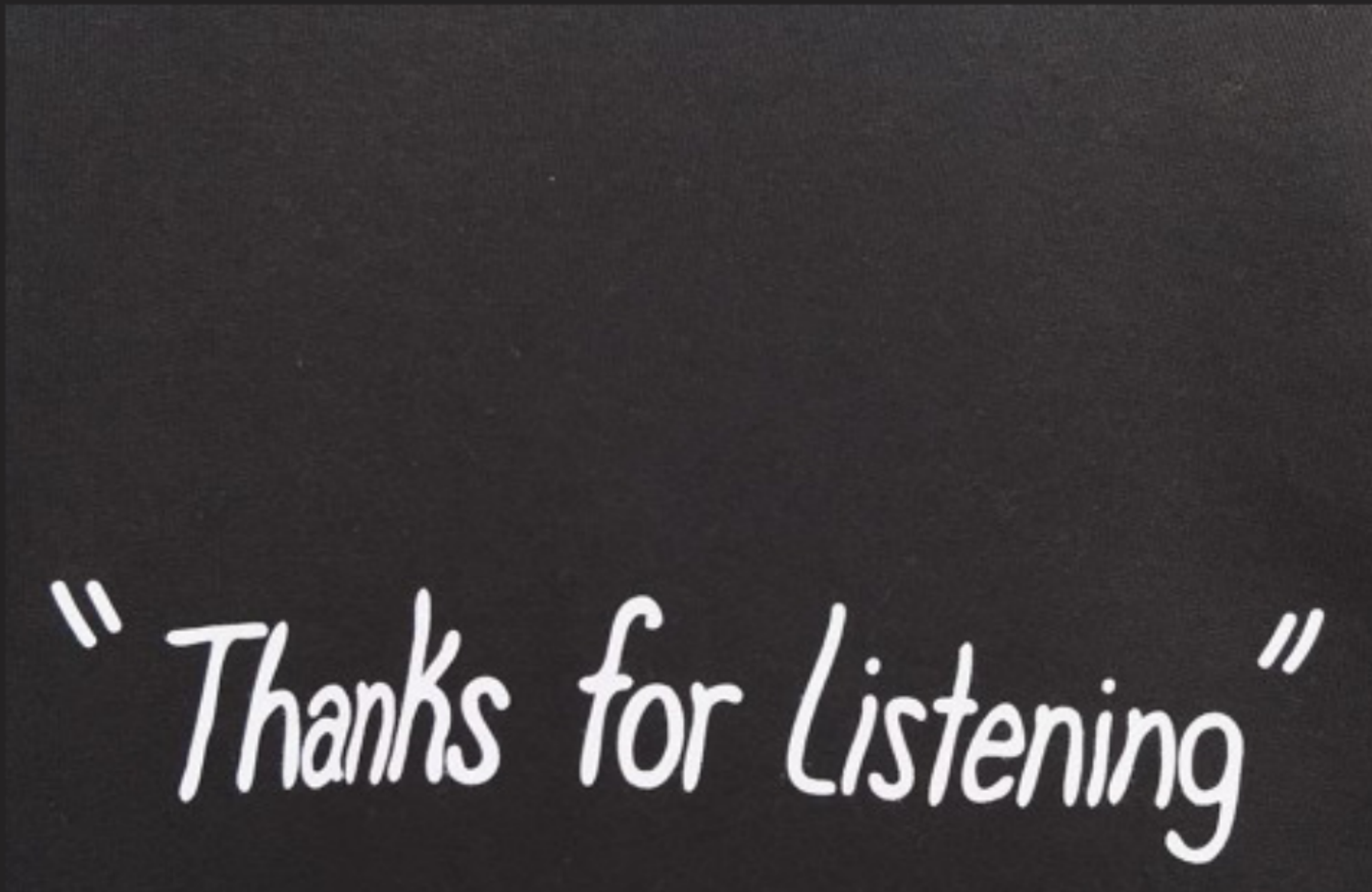
Abstract

We observe spontaneous Hawking radiation, stimulated by quantum vacuum fluctuations, emanating from an analogue black hole in an atomic Bose–Einstein condensate. Correlations


"Thanks for Listening"


Any
Questions?