A QUANTUM-SPACETIME MODEL WITH KINEMATIC IR/UV MIXING AND ITS COLD-ATOM PHENOMENOLOGY

Giuseppe Fabiano

University of Naples "Federico II"

COST CA18108 Fourth Annual Conference (Rijeka)

In collaboration with G. Amelino-Camelia, D. Frattulillo, F. Mercati

Quantum spacetime

- Motivated by several quantum gravity proposals, it is an effective description of spacetime at "small" distances

$$[x^\mu, x^\nu] \neq 0$$

- The commutator is proportional to some UV scale. Generally, relativistic symmetries are deformed.

- Novel physics effects present not only in the UV, but also in the IR

IR/UV mixing

- Mechanism first appeared in studies of non-commutative field theories on Moyal non-commutative space-time

$$[x^\mu, x^\nu] = \theta^{\mu\nu}$$

- In ϕ^4 theories in $d = 4$, corrections to the propagator yield contributions of the type

$$\int_0^\Lambda dk \cos\left(\frac{1}{2}k\tilde{p}\right)\frac{k^3}{k^2+m^2} \qquad \tilde{p}_\mu = \theta_{\mu\nu}p^\nu$$

Minwalla,Raamsdonk,Seiberg JHEP 02 (2000)

- In the $\Lambda \gg |\tilde{p}|^{-1}$ limit,

$$\int_0^\Lambda dk \cos\left(\frac{1}{2}k\tilde{p}\right)\frac{k^3}{k^2+m^2} \simeq \frac{1}{2}\left(\frac{2}{|\tilde{p}|^2}\right) - \frac{1}{2}m^2 \ln\left(1+\frac{\left(\frac{2}{|\tilde{p}|}\right)^2}{m^2}\right)$$

- The UV cutoff scale introduces a dependence of the integral on $\frac{1}{|\tilde{p}|}$, which yields divergence when $|\tilde{p}| \to 0$, in the IR regime. Example of **dynamical** IR/UV mixing.

κ-lightlike Minkowski spacetime

- The coordinate noncommutativity is expressed by

$$[x^0, x^1] = i\ell(x^1 - x^0) \qquad [x^0, x^i] = i\ell x^i \qquad [x^1, x^i] = i\ell x^i \qquad i = 2,3$$

- Deformation of relativistic symmetries described by a Hopf Algebra.

- Ordinary spatial isotropy is spoiled. The model is still relativistic, but with a deformed notion of spatial isotropy (interpretation?)

Blaut,Daszkiewicz,Kowalski-Glikman – Mod.Phys.Lett. A 18 (2003)

κ-lightlike Minkowski kinematics

- The coproduct inspired composition laws, at first order in ℓ, are given by

$$(p \oplus k)_0 = p_0 + k_0 - \frac{\ell}{2}(p_0 + p_1)(k_0 + k_1)$$

$$(p \oplus k)_1 = p_1 + k_1 + \frac{\ell}{2}(p_0 + p_1)(k_0 + k_1)$$

$$(p \oplus k)_i = p_i + k_i(1 - \ell(p_0 + p_1))$$

- The mass-shell relation inspired by the Casimir element, at first order reads

$$m^2 = (p_0 - p_1)^2 \left(1 + \frac{\ell(p_0 + p_1)}{2}\right) - (\mathrm{p}_2^2 + \mathrm{p}_3^2)\left(1 + \ell(p_0 + p_1)\right)$$

Kinematical IR/UV mixing

- On-shell relation in the limit $p \ll m$

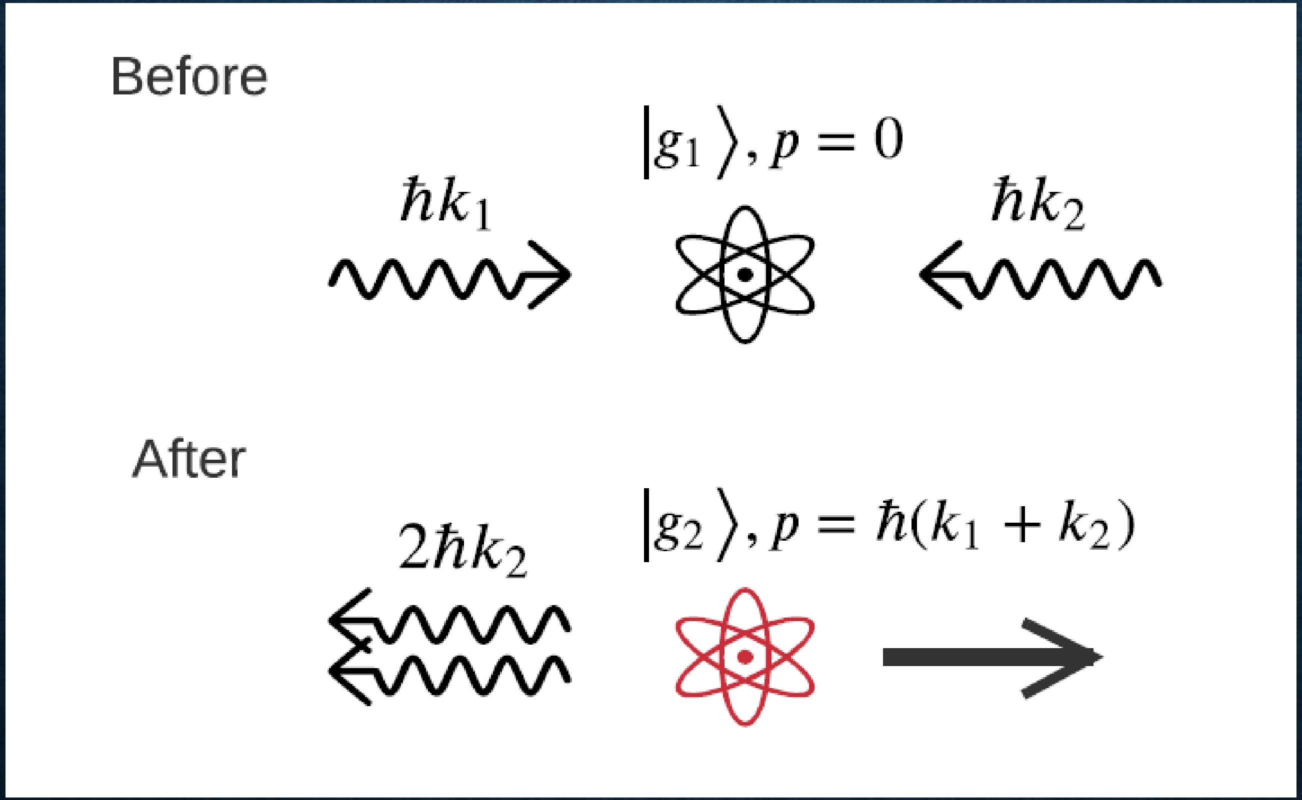
$$p_0 = m + \frac{\vec{p}^2}{2m} - \frac{\ell}{4}(\boldsymbol{m p_1} - p_2^2 - p_3^2) - \frac{p_1}{2m}(p_1^2 + 3p_2^2 + 3p_3^2)$$

- The UV scale ℓ entails a correction to the energy-momentum dispersion relation which has a term $\propto \boldsymbol{\ell m p_1}$ that is dominant in the IR regime, when $p_1 \to 0$.

- This is an example of **kinematical** IR/UV mixing since it does not depend on any dynamical model based on quantum-spacetime corrections

Cold atoms phenomenology

- Recoil of an atom in a two photon Raman transition: in cold-atom experiments it is possible to impart momentum to an atom by absorption of a photon of frequency ν and stimulated emission of a photon of frequency ν'.



Cold atoms phenomenology

- By employing standard energy-momentum conservation, and taking into account the resonance frequency for the atom, one can estimate the ratio $\frac{h}{m}$

$$\frac{\Delta\nu}{2\nu_*\left(\nu_* + \frac{p}{h}\right)} = \frac{h}{m}$$

with $\Delta\nu = \nu - \nu'$ and p is the modulus of the atom's initial momentum

- Laser frequencies are well controlled in lab experiments, leading to very precise determinations of the ratio $\frac{h}{m}$.

- Opportunity to implement quantum spacetime inspired corrections to the $\frac{h}{m}$ formula.

Cold atoms phenomenology: deformation

- Working at first order in ℓ allows for a simple parametrization of the corrections

$$\frac{\Delta\nu}{2\nu_* \left(\nu_* + \frac{p}{h}\right)}(1 + \ell\alpha) = \frac{h}{m}$$

- α is an unknown function possibly depending on the mass of the atoms, the laser frequencies and the atom's initial momentum.

- We model the process as an interaction where in the initial state we have an atom and the photon that is to be absorbed and in the final state we have the accelerated atom and the emitted photon

Results

- Leading order corrections depending on the ordering in the interaction process

$$A + \gamma \to A' + \gamma' \qquad \ell\alpha \approx -\frac{3}{4}(\ell m)\left(\frac{m}{p + h\nu_*}\right)\cos(\phi)\sin(\theta)$$

$$\gamma + A \to \gamma' + A' \qquad \ell\alpha \approx \frac{1}{4}(\ell m)\left(\frac{m}{p + h\nu_*}\right)\cos(\phi)\sin(\theta)$$

$$\gamma + A \to A' + \gamma' \qquad \ell\alpha \approx -\frac{1}{4}(\ell m)\left(\frac{m}{p + h\nu_*}\right)\cos(\phi)\sin(\theta)$$

$$A + \gamma \to \gamma' + A' \qquad \ell\alpha \approx -\frac{1}{4}(\ell m)\left(\frac{m}{p + h\nu_*}\right)\cos(\phi)\sin(\theta)$$

Amplification

- Correction terms of the type

$$\ell\alpha_i \approx k_i(\ell m)\left(\frac{m}{p + h\nu_*}\right)\cos(\phi)\sin(\theta)$$

- Amplification factor stemming from kinematical IR/UV mixing term in the dispersion relation

- First DSR result with such an amplification factor

Amelino-Camelia,Lammerzahl,Mercati,Tino, Phys. Rev. Lett.103 (2009)

Arzano, Kowalski-Glikman, EPL 90 (2010)

Interpretation

- We have no well-established dynamical framework to weigh the various channels of the interaction in the total correction
- The correction depends on the direction of the atom
- Average over channels and angles

$$\ell\langle\alpha\rangle \approx \frac{1}{16\pi}\int_{S^2} d\Omega \sum_i \alpha_i = 0$$

- Variance

$$\ell\Delta\alpha \approx \frac{1}{16\pi}\int_{S^2} d\Omega \sum_i \alpha_i^2 = \frac{\ell}{4}\frac{m^2}{p + h\nu_*}$$

Conclusions

- Encouragement in developing models with IR/UV mixing for phenomenological opportunities in table-top experiments

- Possibility of obtaining independent constraints on the deformation scale, by probing both IR and UV phenomena

THANKS FOR THE ATTENTION