Parametric resonances in domain walls and cosmic strings

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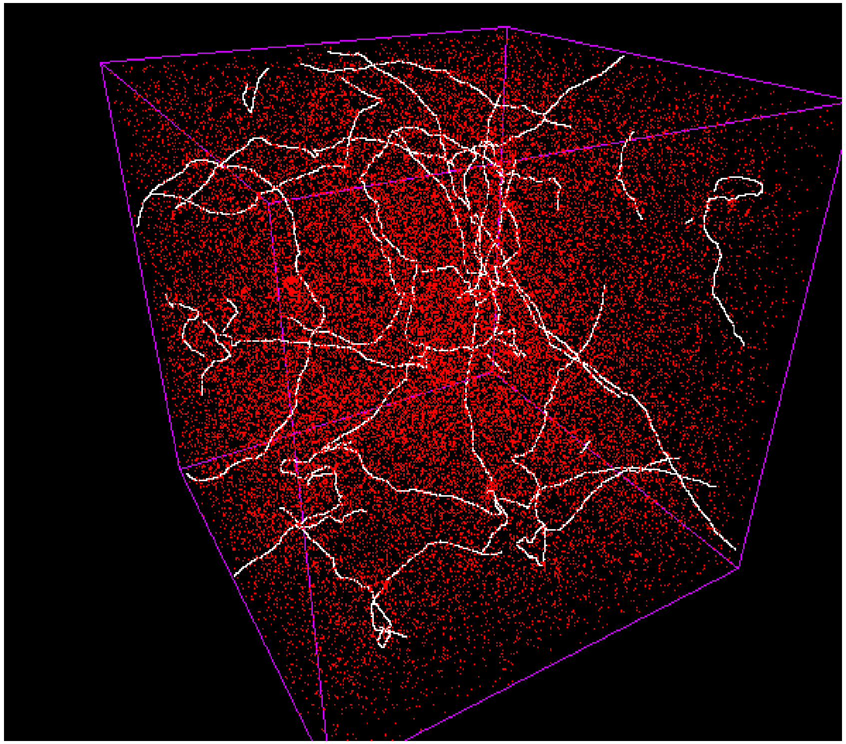
In collaboration with Jose J. Blanco-Pillado, Jon Urrestilla and Jose M. Queiruga

CONTENTS

1. MOTIVATION

To understand the discrepancies between Field Theory (FT) and Nambu-Goto (NG) simulations of **cosmic strings**.



FT simulations: equations of motion coming from FT action. Computationally challenging.

NG simulations: infinitely thin strings, equations of motion coming from NG action. Less costly computationally.

Strings in large scale FT simulations do not behave as the NG action predicts.

→ Different predictions regarding gravitational wave emission.

1. MOTIVATION

Possible explanation: strings in FT simulations are endowed with extra energy that may alter their expected (NG) dynamics.

Extra energy can be stored in the cores of the strings for long periods of time in the form of **bound states**.

Does this extra energy play a relevant role in the dynamics of strings?

→ Domain walls in 2+1 dimensions (*domain wall strings*)

→ Global strings in 3+1 dimensions

2. INTERNAL EXCITATIONS OF DOMAIN WALL STRINGS

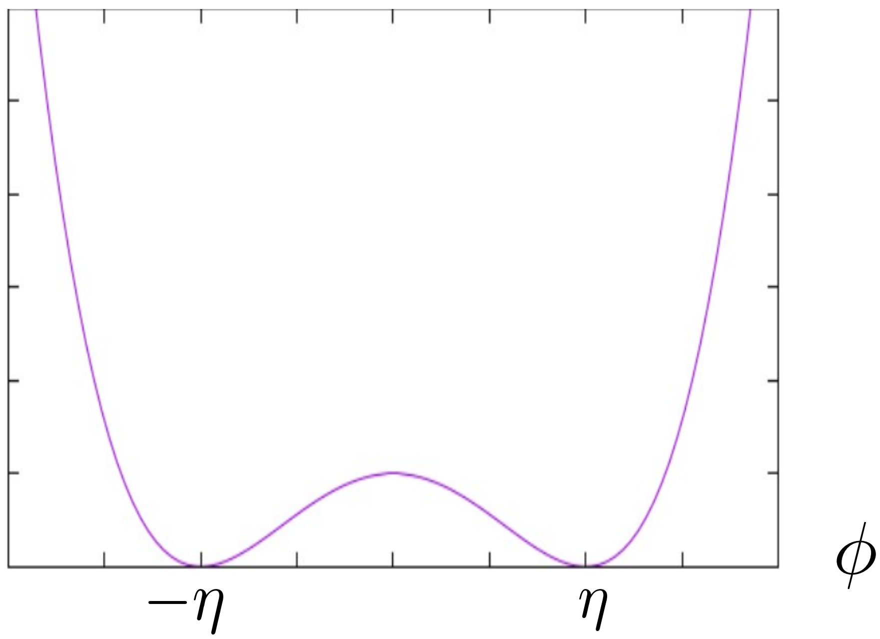
2.1. Static solution and spectrum of perturbations

Real scalar field in 2+1 dimensions:

$$S = \int d^3x \left[\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right]$$

$$\boxed{\ddot{\phi} - \nabla^2 \phi + \lambda \left(\phi^2 - \eta^2 \right) \phi = 0}$$

$$V(\phi) = \frac{\lambda}{4} \left(\phi^2 - \eta^2 \right)^2$$



Mass of small fluctuations about the vacuum: $m = \sqrt{2\lambda}\eta$

2. INTERNAL EXCITATIONS OF DOMAIN WALL STRINGS
2.1. Static solution and spectrum of perturbations

$$\ddot{\phi} - \nabla^2\phi + \lambda\left(\phi^2 - \eta^2\right)\phi = 0$$

Spatially homogeneous solutions:

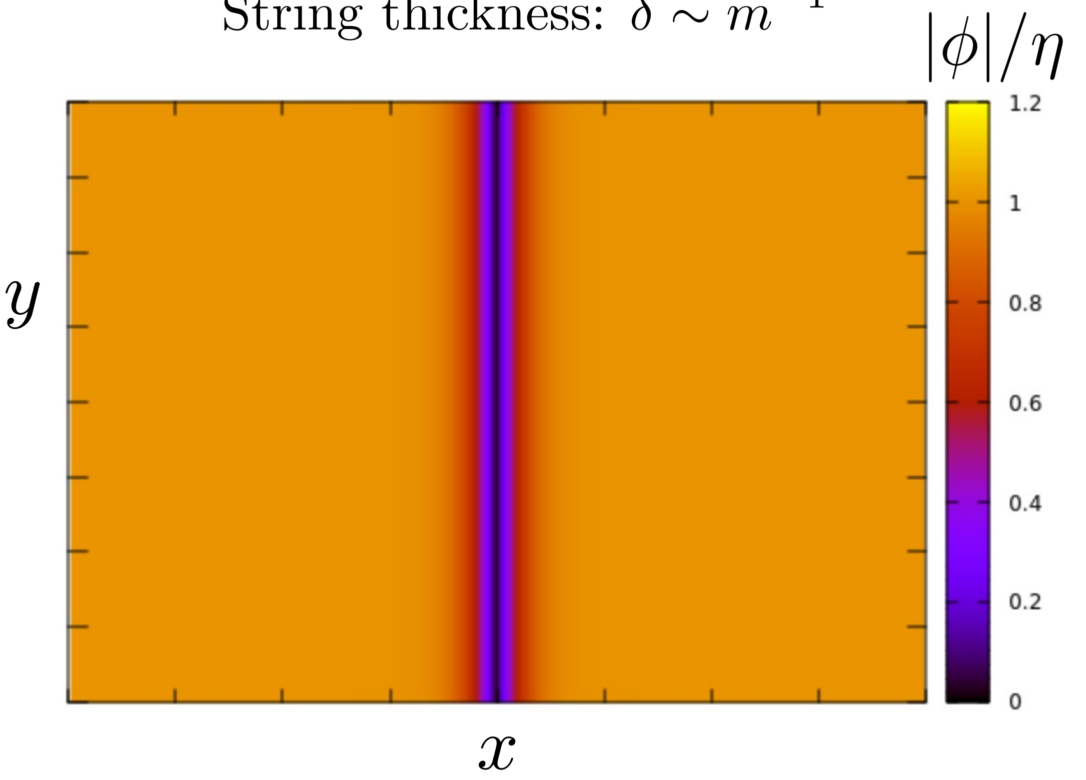
$$\phi = \pm\eta \ \ (\text{vacuum})$$

$$\phi = 0 \ \ (\text{unstable})$$

Static domain wall string solution:

$$\phi_K(x) = \eta \tanh\left(\frac{mx}{2}\right)$$

String thickness: $\delta \sim m^{-1}$



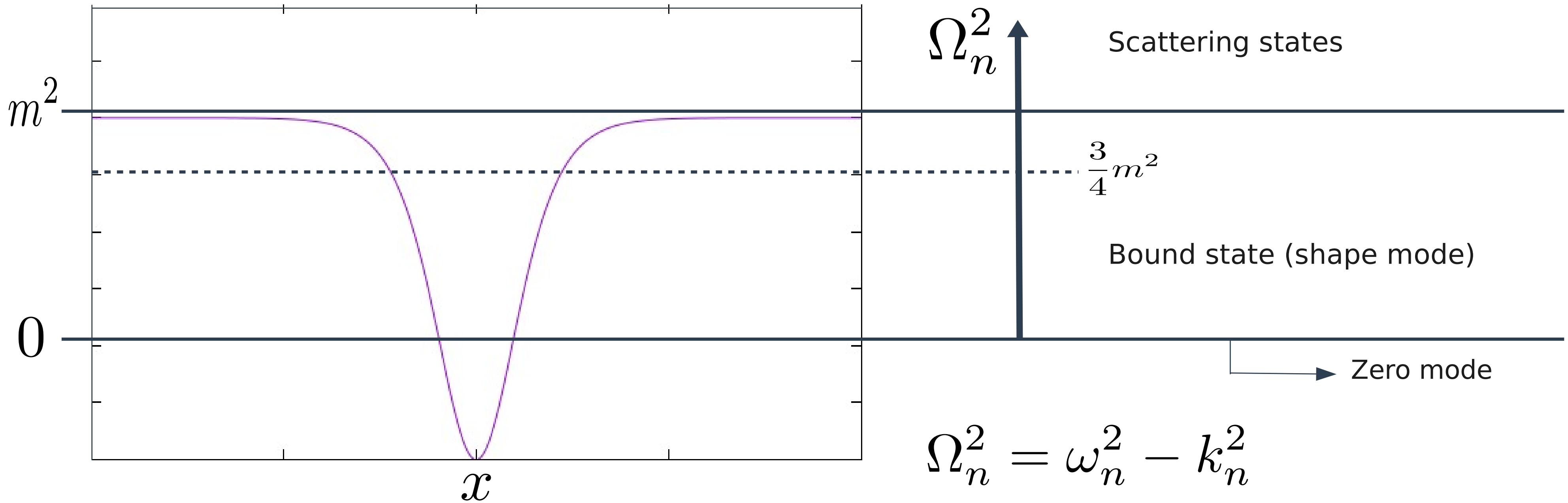
2. INTERNAL EXCITATIONS OF DOMAIN WALL STRINGS

2.1. Static solution and spectrum of perturbations

Spectrum of perturbations around the static domain wall solution:

$$\phi\left(t,x,y\right)=\phi_K\left(x\right)+\psi\left(t,x,y\right) \xrightarrow{\text{Eq. of motion}} \ddot{\psi}-\nabla^2\psi+\underline{\lambda\left[3\phi_K^2-\eta^2\right]}\psi=0$$

$$-f_n''\left(x\right)+\underline{U\left(x\right)}f_n\left(x\right)=\Omega_n^2 f_n\left(x\right) \longleftarrow \psi\left(t,x,y\right)\propto e^{i\left(k_n y-\omega_n t\right)}f_n\left(x\right)$$



$$\Omega_n^2=\omega_n^2-k_n^2$$

2. INTERNAL EXCITATIONS OF DOMAIN WALL STRINGS

2.1. Static solution and spectrum of perturbations

Zero mode. It corresponds to rigid translations of the string.

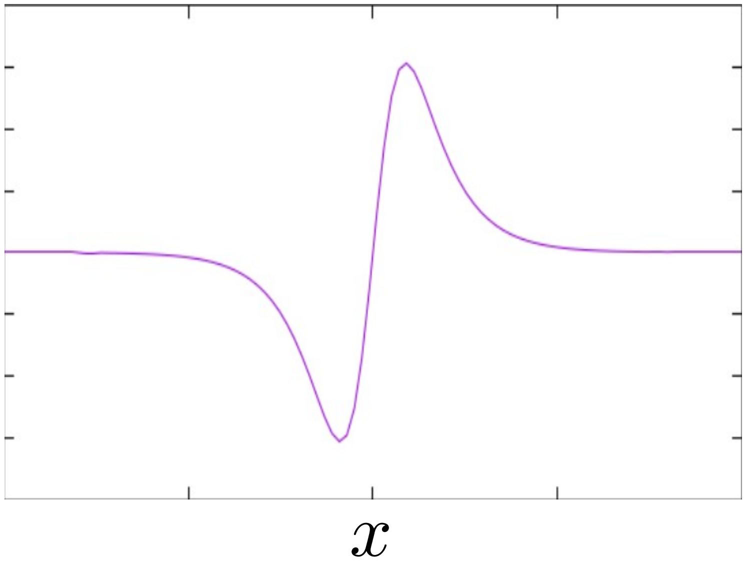
$$f_0\left(x\right) = \sqrt{\frac{3m}{8}}\cosh^{-2}\left(\frac{mx}{2}\right) \quad , \quad \Omega_0 = 0$$

Scattering modes. Travelling waves in the asymptotic limit (radiation).

$$f_p\left(x\right) = e^{ipx}\left[3\tanh^2\left(\frac{mx}{2}\right) - 1 - \frac{4p^2}{m^2} - i\frac{6p}{m}\tanh\left(\frac{mx}{2}\right)\right] \quad , \quad \Omega_p = \sqrt{m^2+p^2}$$

Shape mode. Deformation of the core of the string.

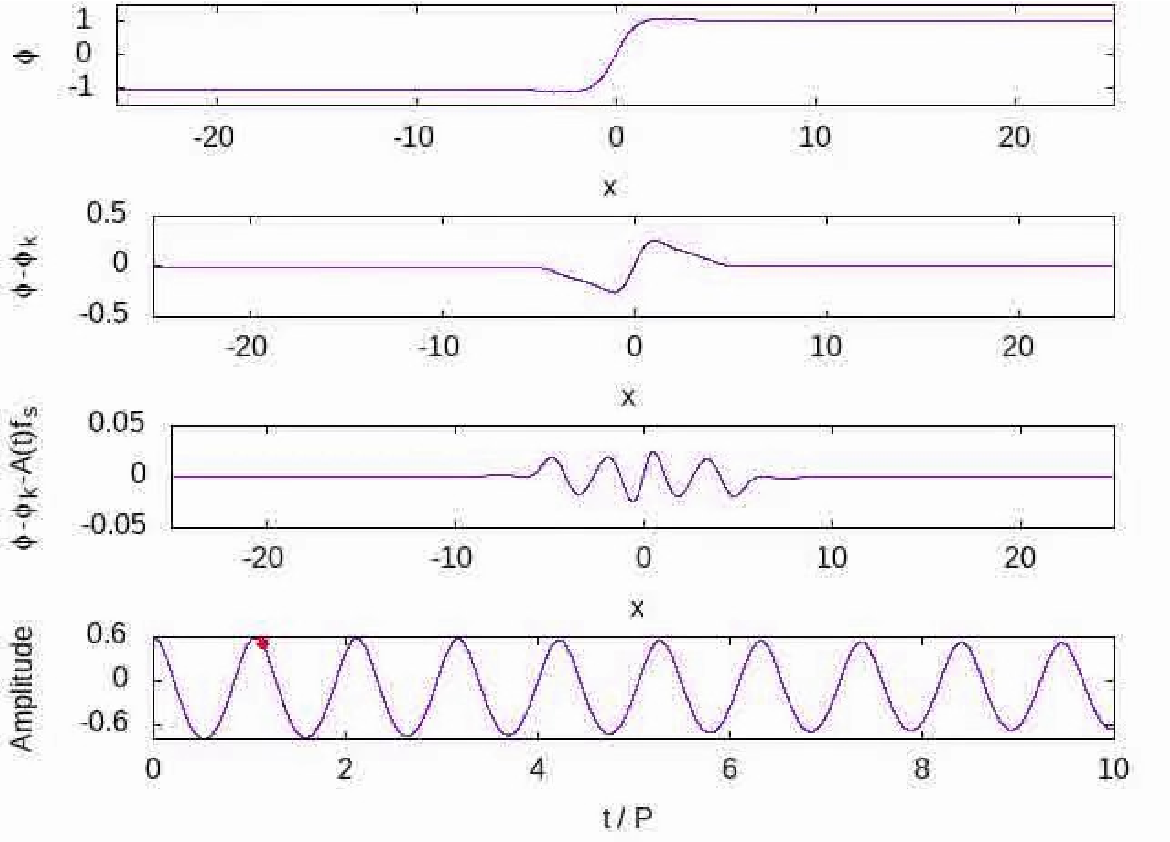
$$f_1\left(x\right) = \sqrt{\frac{3m}{4}}\sinh\left(\frac{mx}{2}\right)\cosh^{-2}\left(\frac{mx}{2}\right) \quad , \quad \Omega_1 = \frac{\sqrt{3}}{2}m$$



Due to the non-linearities, it couples to the scattering modes and its energy is slowly radiated away.

2. INTERNAL EXCITATIONS OF DOMAIN WALL STRINGS

2.2. Radiation from shape mode excitations



2. INTERNAL EXCITATIONS OF DOMAIN WALL STRINGS

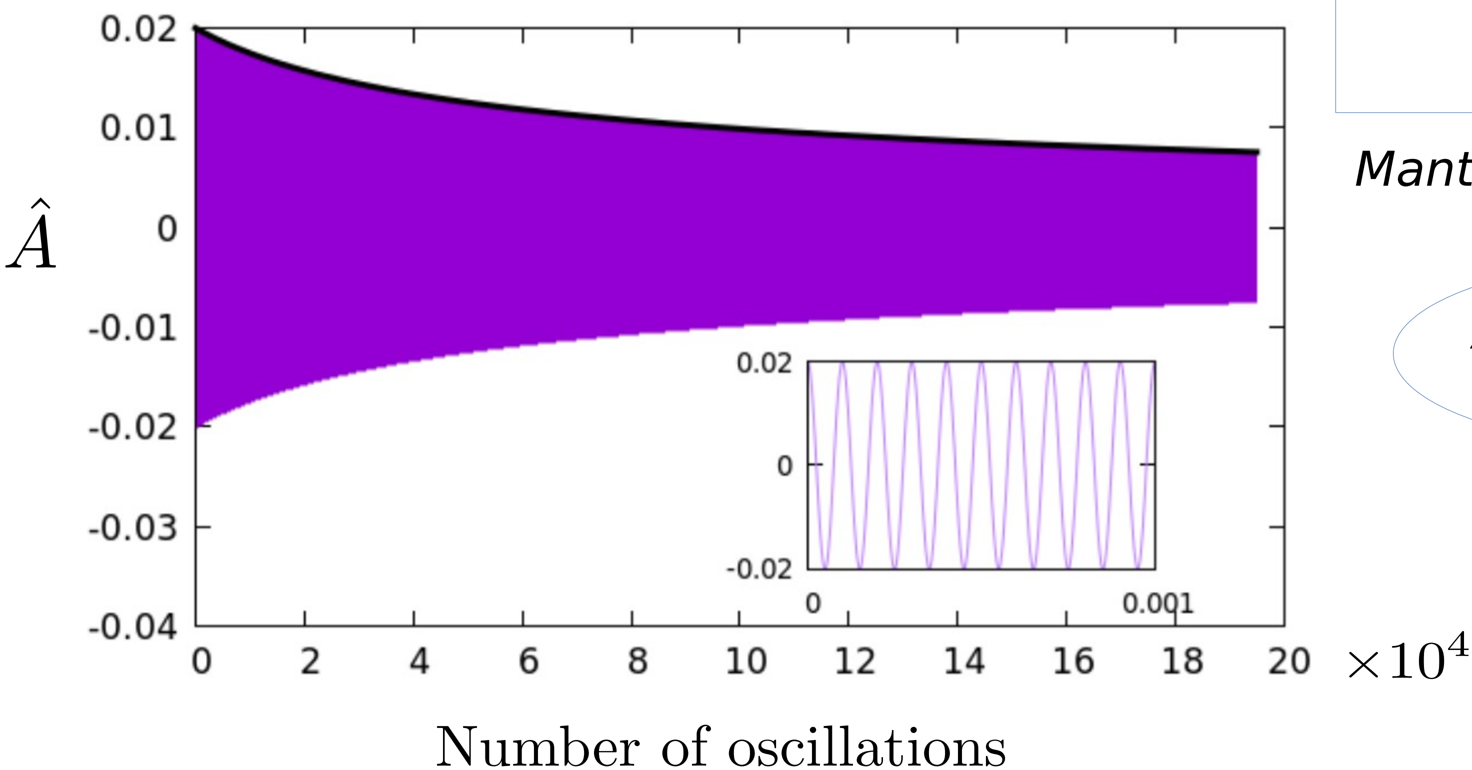
2.2. Radiation from shape mode excitations

$$\phi(t,x) = \phi_K(x) + \hat{A}(t) f_1(x) \cos(\Omega_1 t) + R(t,x)\,, \quad R(t,x): \text{ radiation field}$$

Energy stored in the shape mode: $\Delta E \propto \hat{A}^2$

$$\hat{A}(t) = \frac{1}{\sqrt{\hat{A}(0)^{-2} + \alpha t}}$$

Manton and Merabet, 1997

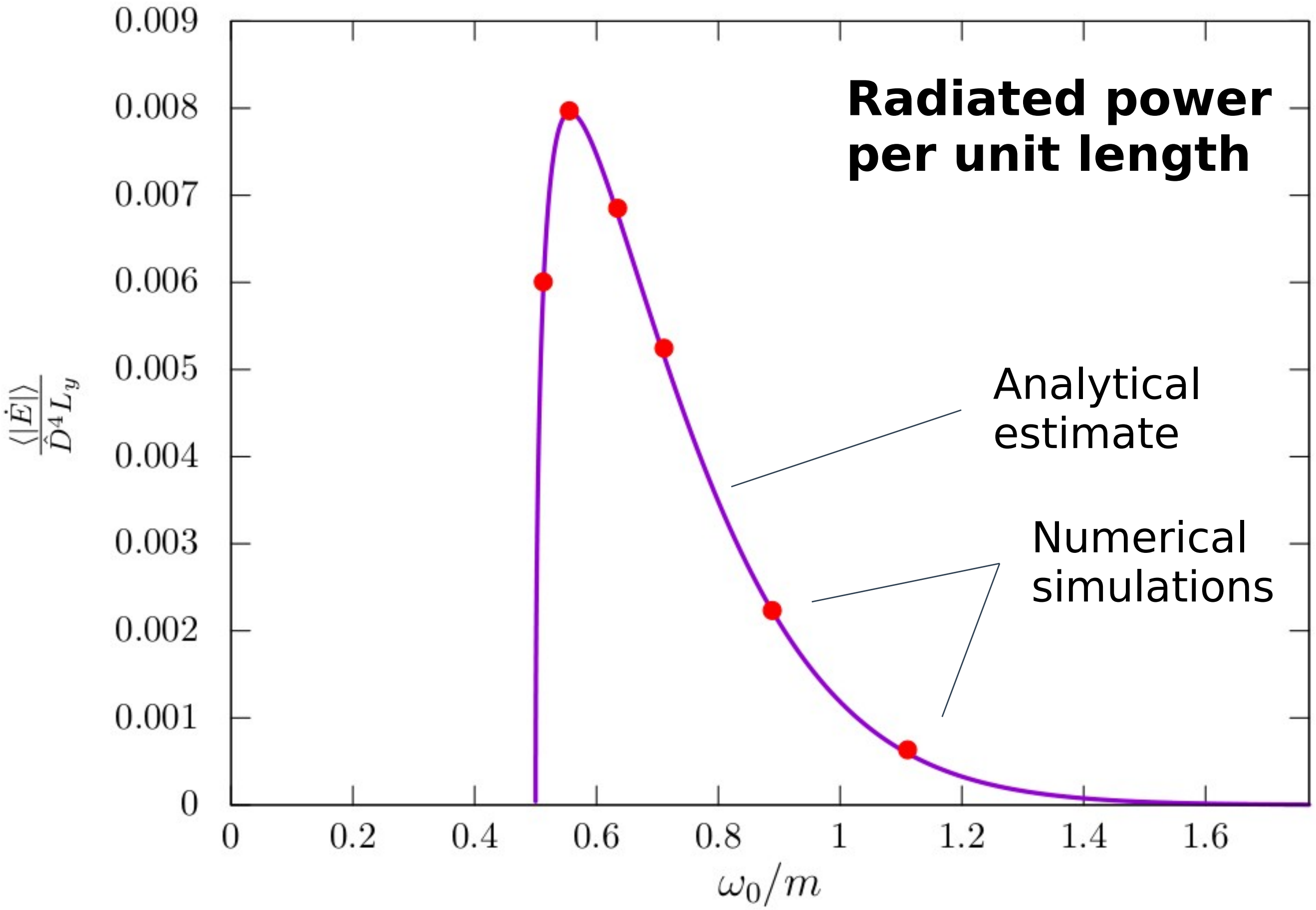


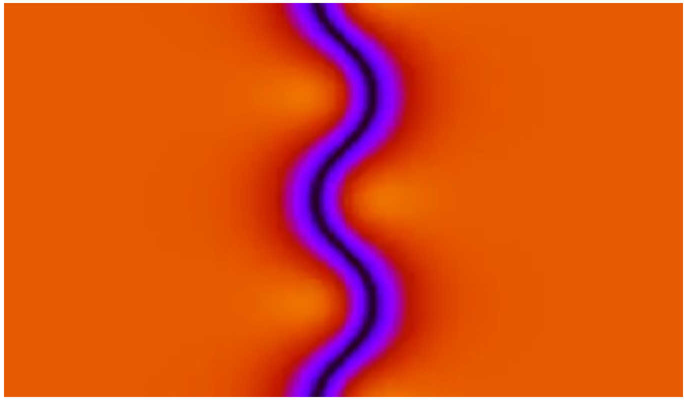
$$\tau \gg \delta \sim m^{-1}$$

$\tau:$ decay time scale

2.3. Radiation from zero mode excitations

The zero modes also couple to the scattering states.



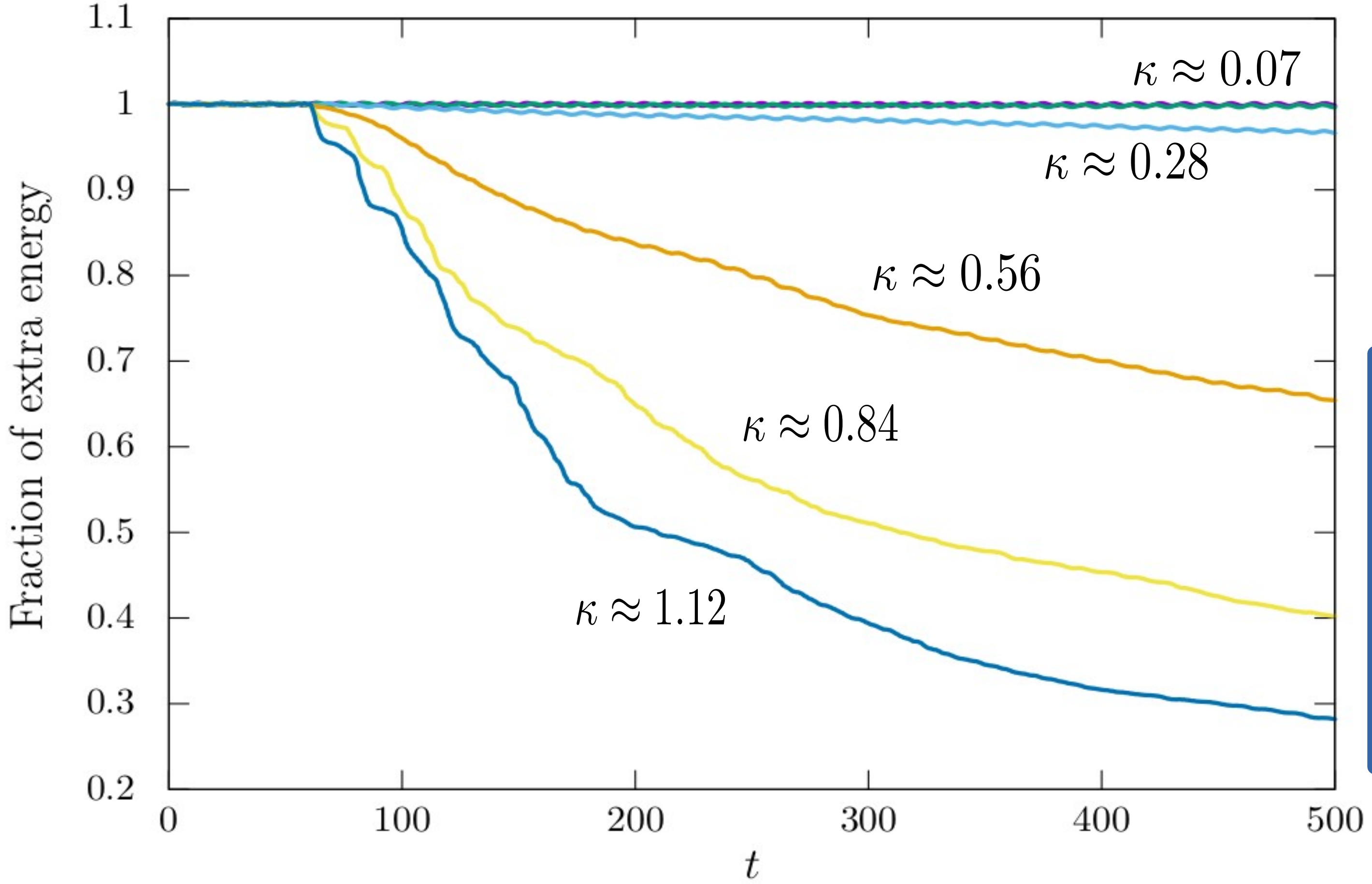


The source for radiation oscillates with angular frequency $\omega_r = 2\omega_0$

No radiation for $\omega_0 < m/2$

2.3. Radiation from zero mode excitations

The string can also radiate if it is highly curved.



$$\kappa = \delta / R$$

String thickness

Radius of curvature

Processes of energy loss:

1) Coupling to the scattering states (perturbative radiation)

2) High curvature (non-perturbative radiation)

3. DYNAMICS OF BARE DOMAIN WALL STRINGS

3.1. The Nambu-Goto action

Coordinates of the string : $X^{\rho}\left(\xi^0, \xi^1\right)$

ξ^0 and ξ^1 are the string worldsheet coordinates

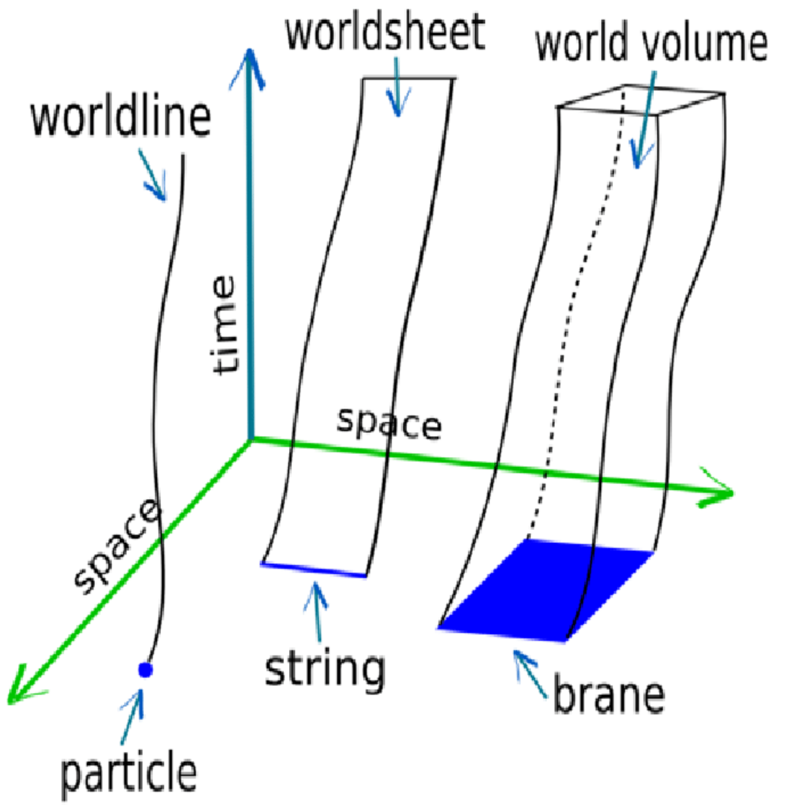
$$\boxed{S_{NG} = -\mu \int d^2\xi \sqrt{-\gamma}}$$

$$\gamma_{ab} = g_{\alpha\beta}\partial_a X^{\alpha} \partial_b X^{\beta}$$

Worldsheet metric

$$\mu = \int T^{00} dx = \left(\frac{m}{3\lambda}\right) m^2$$

Energy per unit length



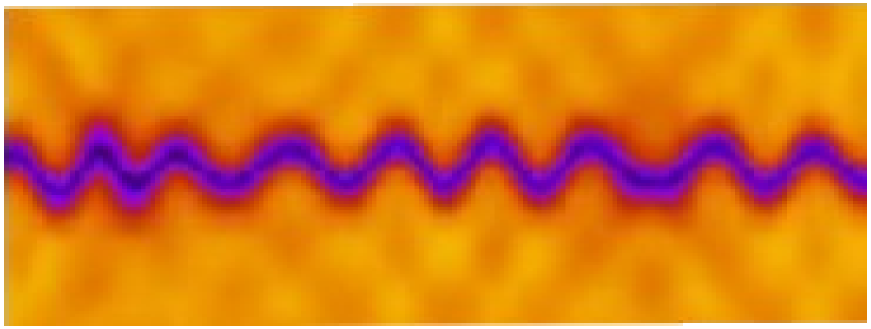
3. DYNAMICS OF BARE DOMAIN WALL STRINGS

3.1. The Nambu-Goto action

$$S_{NG} = -\mu \int d^2\xi \sqrt{-\gamma}$$

$$\partial_a \left(\sqrt{-\gamma} \gamma^{ab} \partial_b X^\rho \right) = 0$$

$$\partial_t \left(\frac{\dot{\psi}}{\sqrt{1 + \psi'^2 - \dot{\psi}^2}} \right) - \partial_y \left(\frac{\psi'}{\sqrt{1 + \psi'^2 - \dot{\psi}^2}} \right) = 0$$

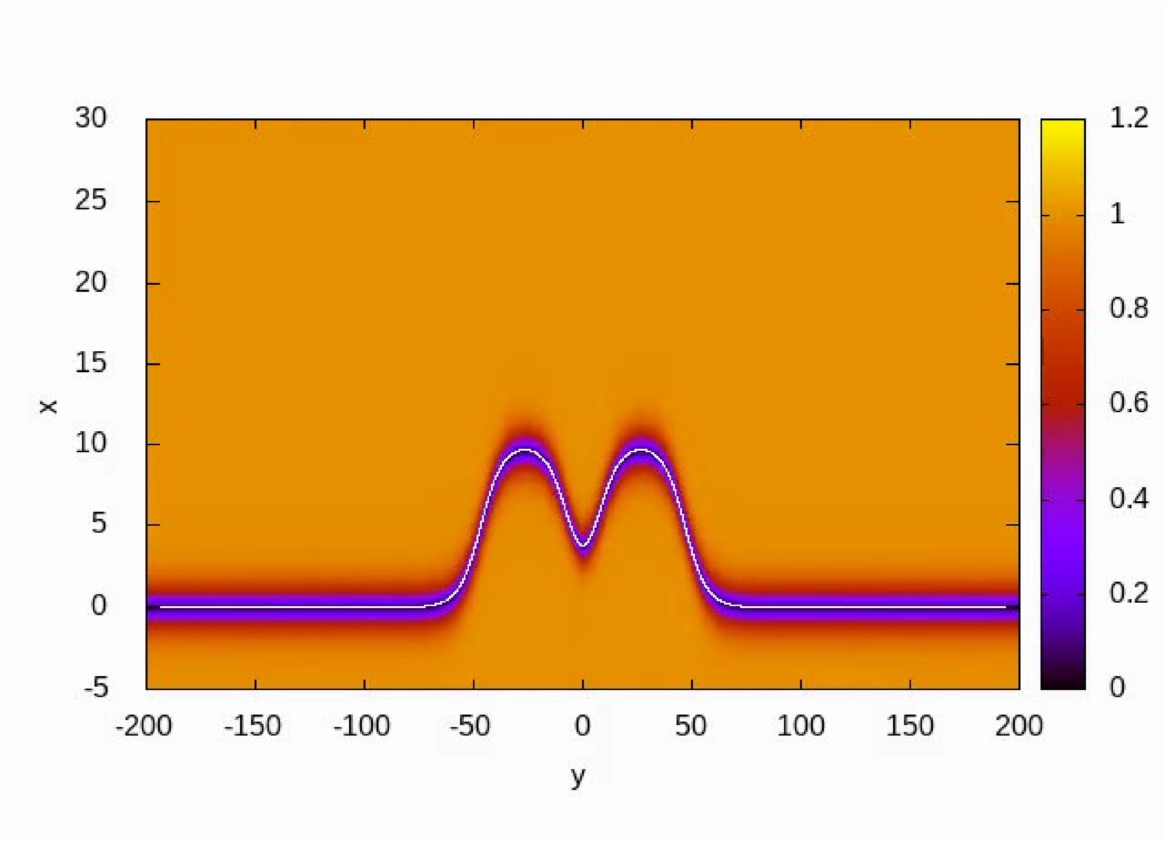


$X^0 = t$ $\quad X^1 = \psi(t, y)$ $\quad X^2 = y$

The NG equation of motion should be valid in the absence of shape mode and high curvature regions.

3. DYNAMICS OF BARE DOMAIN WALL STRINGS

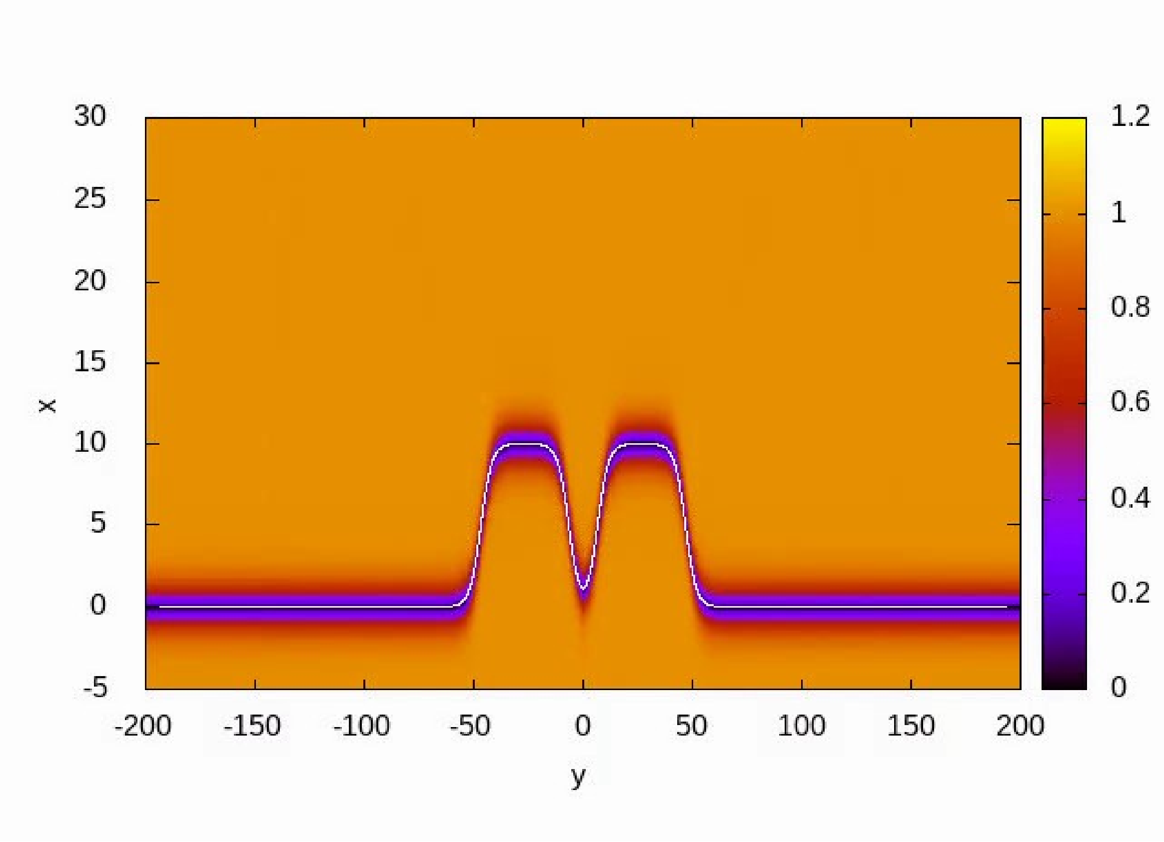
3.1. The Nambu-Goto action



$|\phi|/\eta$

In white: Nambu-Goto prediction

3.1. The Nambu-Goto action



$|\phi|/\eta$

In white: Nambu-Goto prediction

4.1. Parametric resonances in field theory simulations

The shape mode alters the equation of state of the string.

For $\phi(t,x,y) = \phi_K(x) + \theta(t) f_1(x)$, with $\theta(t) \approx \hat{A}\cos(\Omega_1 t)$,

$$\mu_A \approx \mu \left(1 + \frac{9}{8\sqrt{2}} \frac{\sqrt{\lambda}}{\eta} \hat{A}^2\right) \qquad \tau_A \approx -\mu \left[1 + \frac{9}{8\sqrt{2}} \frac{\sqrt{\lambda}}{\eta} \hat{A}^2 \cos(2\Omega_1 t)\right]$$

Oscillating tension $\rightarrow$ Amplification of transverse perturbations

Rayleigh, 1887

4. DYNAMICS OF EXCITED DOMAIN WALL STRINGS

4.1. Parametric resonances in field theory simulations

$$S = \int d^2\xi \sqrt{-\gamma}\left[-\mu + \frac{1}{2}\gamma^{ab}\partial_a\theta\partial_b\theta - V(\theta)\right]$$

$$\partial_a\left[\sqrt{-\gamma}\left(\mu\gamma^{ab} + T^{ab}\right)\partial_b X^\rho\right] = 0$$

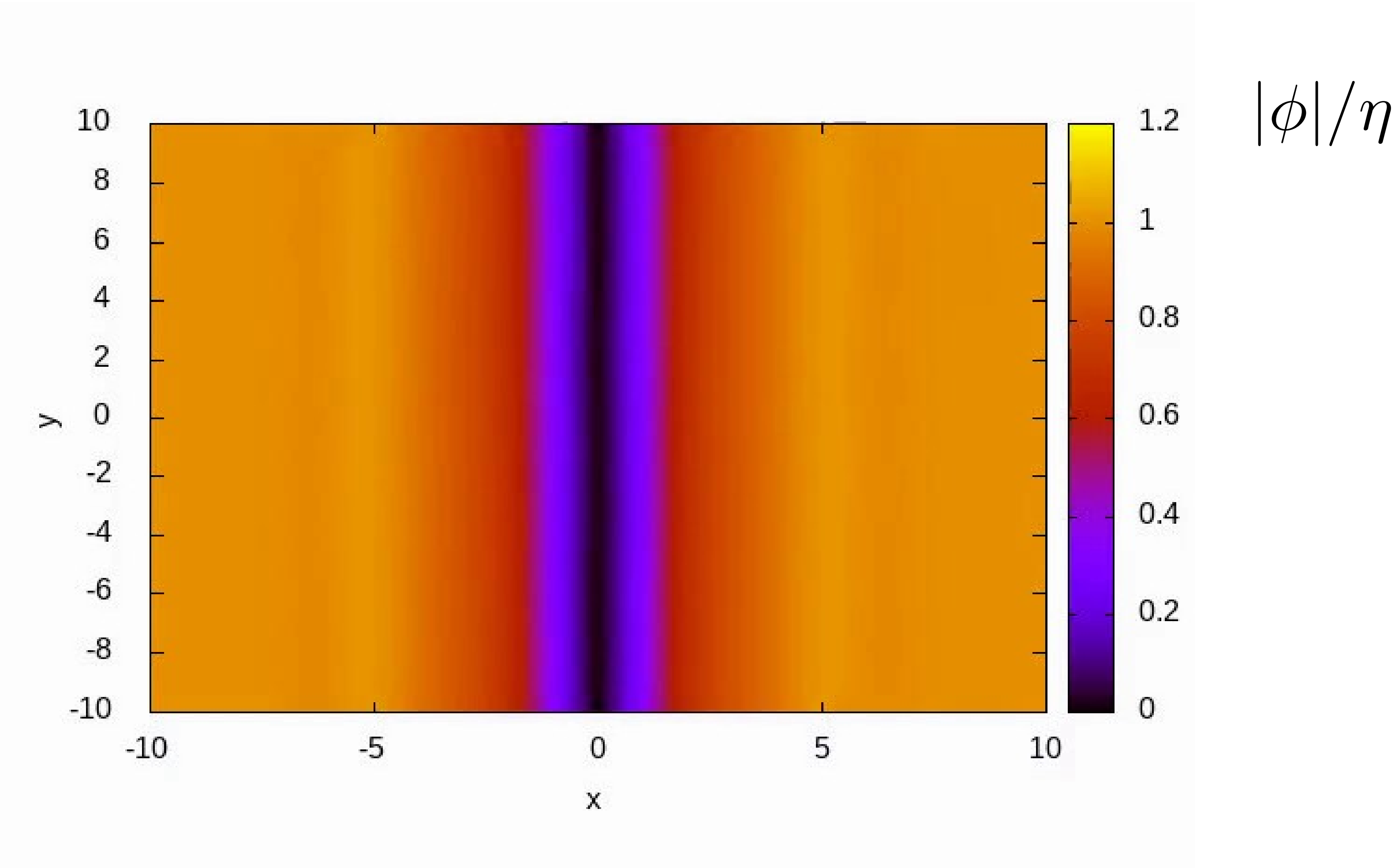
T^{ab}: energy-momentum tensor of θ

$$X^1 = \psi(t, y) = D(t)\cos(\omega_0 y)$$

$$\ddot{D} + \omega_0^2\left(1 - \frac{\dot{\theta}^2}{\mu}\right)D = 0$$

Mathieu equation

Amplification of wiggles of frequency $\omega_0 = \Omega_1$


10
8
6
4
2
0
-2
-4
-6
-8
-10
y
-10
-5
0
5
10
x
1.2
1
0.8
0.6
0.4
0.2
0
$|\phi|/\eta$

4. DYNAMICS OF EXCITED DOMAIN WALL STRINGS

4.1. Parametric resonances in field theory simulations

Instability for $\omega_0 = \Omega_1/2$

$$\phi\left(t,x,y\right)=\phi_K\left(x\right)+A\left(t\right)f_1\left(x\right)+D\left(t\right)f_0\left(x\right)\cos\left(\omega_0 y\right)$$

$$\mathcal{L} \Rightarrow L\left(A,\dot{A},D,\dot{D}\right)$$

$$\ddot{A}(t)+\frac{3}{2}\left[1+C_1D^2(t)\right]A(t)+6C_2A^2(t)+\frac{3C_1}{2}A^3(t)+\frac{3C_2}{2}D^2(t)=0\,,$$

$$\ddot{D}(t)+\left[\omega_0^2+6C_2A(t)+3C_1A^2(t)\right]D(t)+\frac{9C_1}{4}D^3(t)=0\,,$$

4. DYNAMICS OF EXCITED DOMAIN WALL STRINGS

4.1. Parametric resonances in field theory simulations

While D is small:

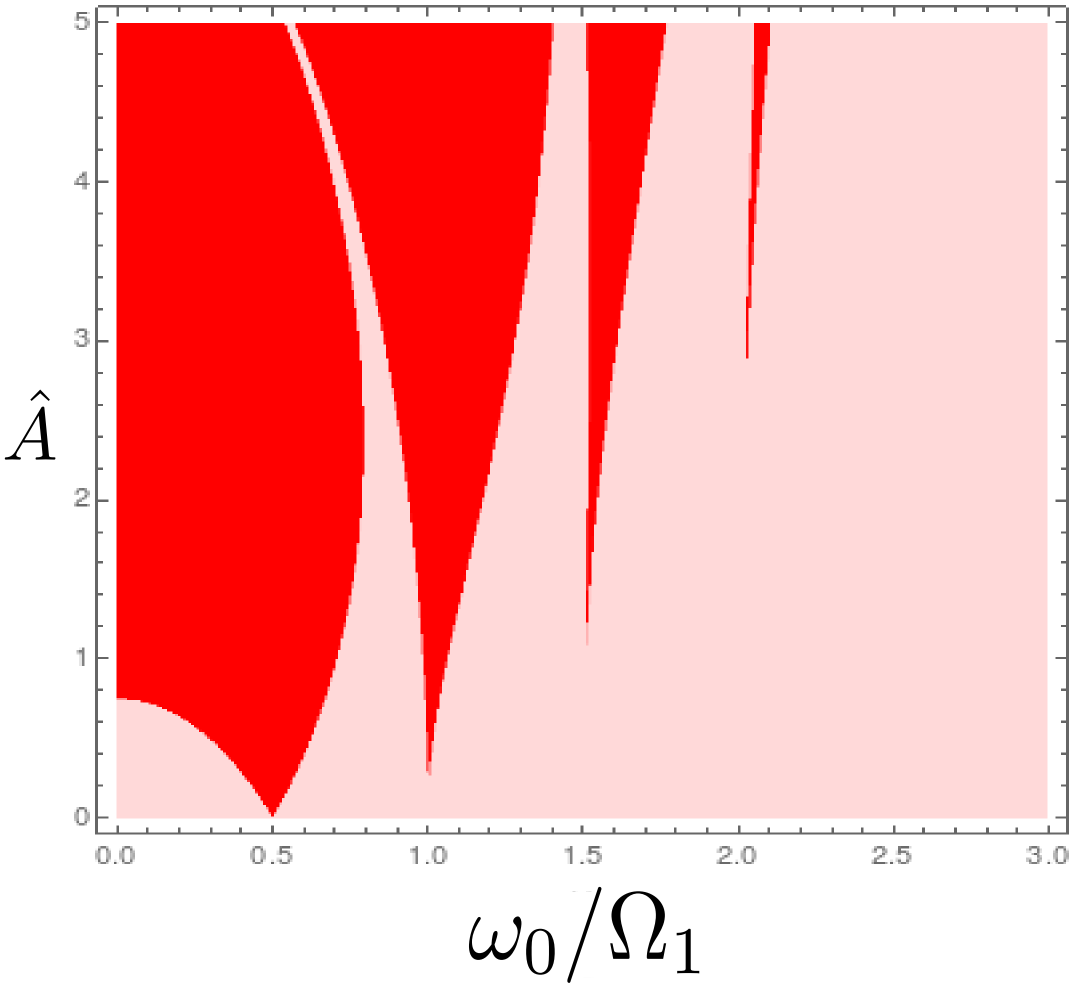
$$\ddot{A} + \frac{3}{2}A = 0 \rightarrow A = \hat{A}\cos(\Omega_1 t)$$

$$\boxed{\ddot{D} + \left(\omega_0^2 + 6C_2 A\right) D = 0}$$

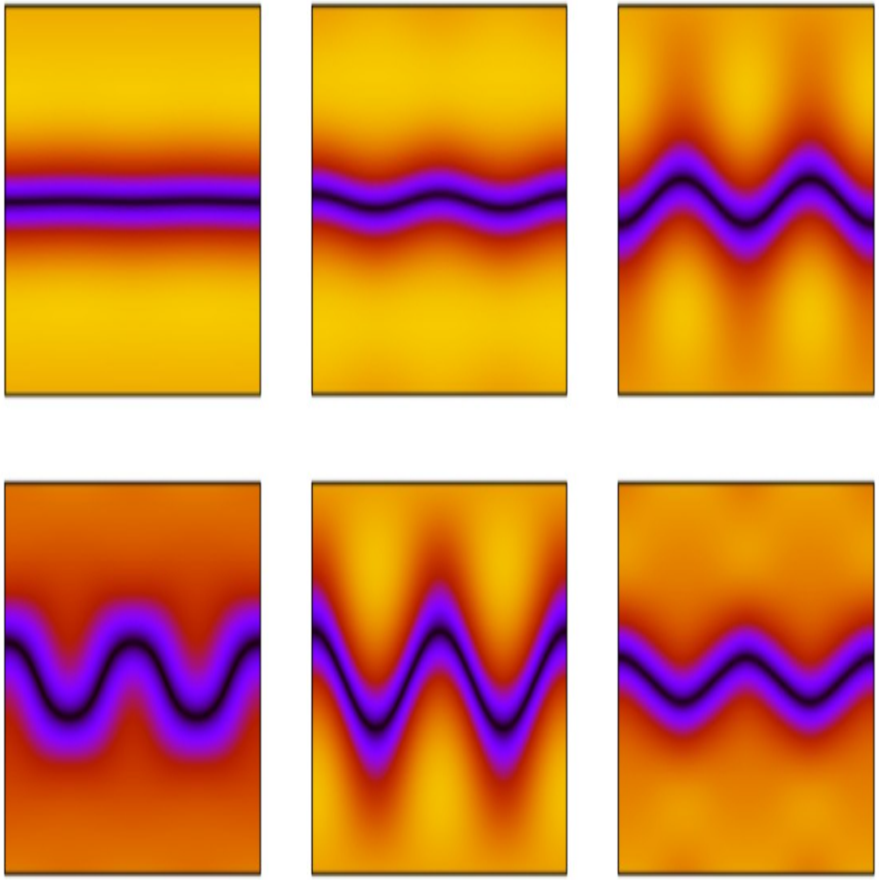
When D is big:

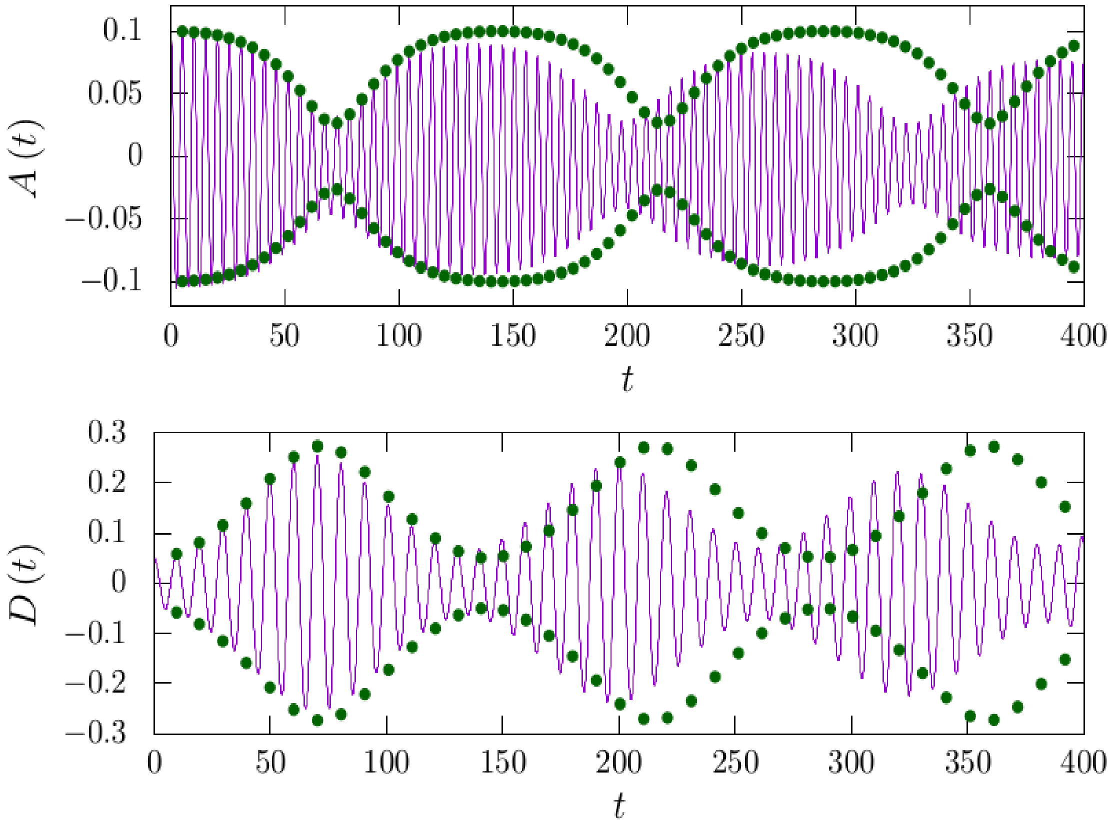
$$\ddot{D} + \omega_0^2 D = 0 \rightarrow D = \hat{D}\cos(\omega_0 t)$$

$$\ddot{A} + \frac{3}{2}A + \frac{3C_2}{2}D^2 = 0$$



4.1. Parametric resonances in field theory simulations





4. DYNAMICS OF EXCITED DOMAIN WALL STRINGS
4.1. Parametric resonances in field theory simulations

$$S=\int d^2\xi\sqrt{-\gamma}\left[(\alpha+\beta\mathcal{R})\,\theta-\mu+\frac{1}{2}\gamma^{ab}\partial_a\theta\partial_b\theta-V\left(\theta\right)\right]$$

$\mathcal{R}$: Ricci scalar of the string worldsheet

$$\boxed{\partial_a\left[\sqrt{-\gamma}\left(M^{ab}-2\beta F^{ab}\right)\partial_b X^\rho\right]=0}$$

Einstein tensor $(=0)$

$$M^{ab}=2\left(\alpha+\beta\theta\right)G^{ab}+\mu\gamma^{ab}+T^{ab}\,,$$

$$F^{ab}=\gamma^{ac}\gamma^{bd}\nabla_c\nabla_d\theta-\gamma^{ab}\nabla_c\left(\gamma^{cd}\nabla_d\theta\right)\,.$$

$$X^1=\psi\left(t,y\right)=D\left(t\right)\cos\left(\omega_0 y\right)$$

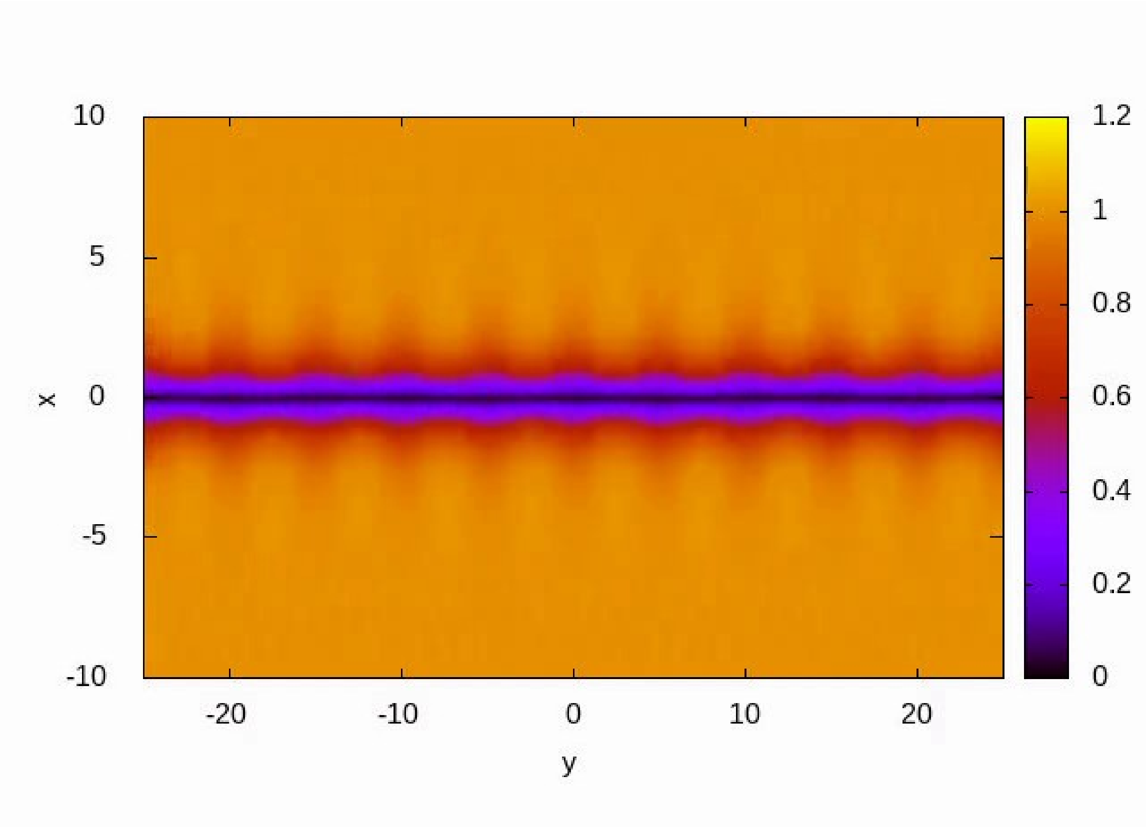
$$\ddot{D}+\omega_0^2\left(1+\frac{2\beta}{\mu}\ddot{\theta}\right)D=0$$

Instability for

$\omega_0=\Omega_1/2$

4. DYNAMICS OF EXCITED DOMAIN WALL STRINGS

4.1. Parametric resonances in field theory simulations



$|\phi|/\eta$

4.1. Parametric resonances in field theory simulations

$$\phi\left(t,x,y\right)=\phi_K\left(x\right)+A\left(t\right)f_1\left(x\right)\cos\left(k_s y\right)+D_1\left(t\right)f_0\left(x\right)\cos\left(k_1 y\right)+D_2\left(t\right)f_0\left(x\right)\cos\left(k_2 y\right)$$

$$\Big\downarrow \mathcal{L}$$

$$L\left(A,\dot{A},D_1,\dot{D}_1,D_2,\dot{D}_2\right)$$

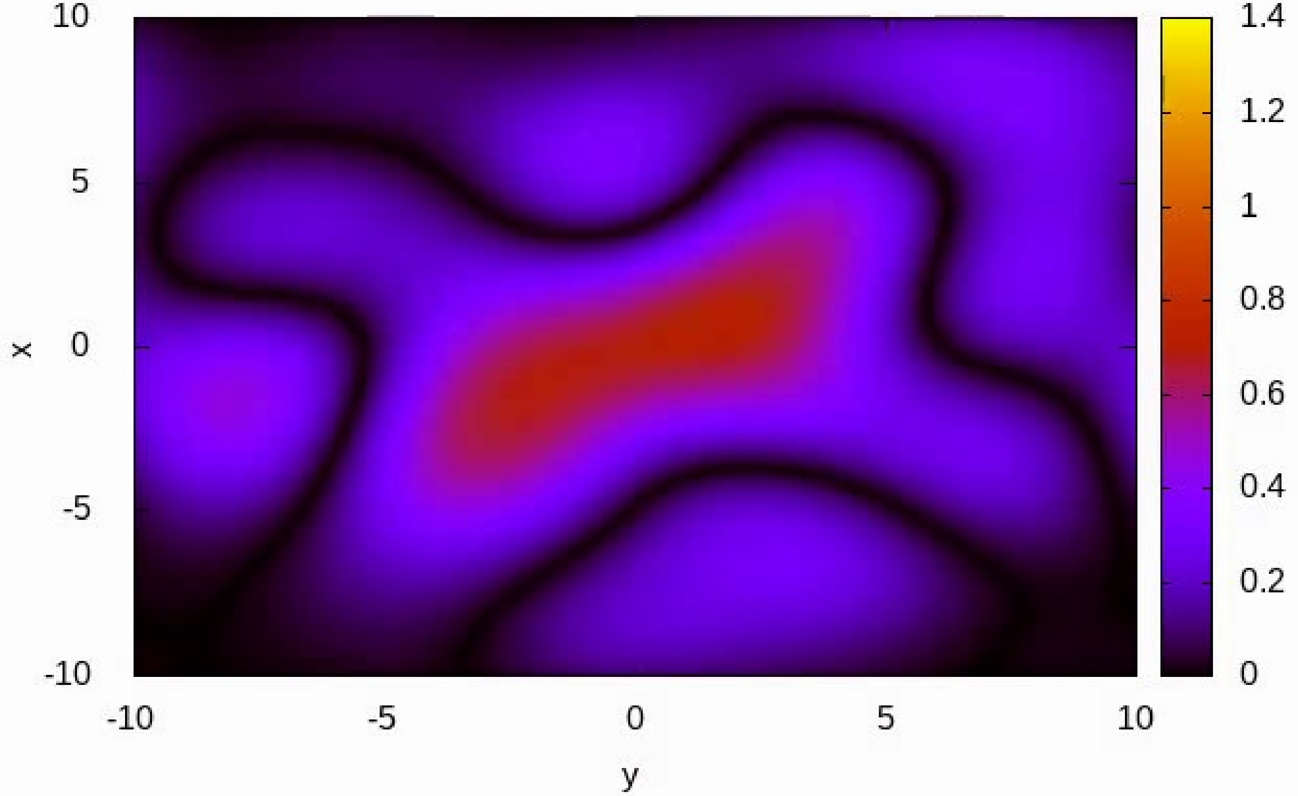
$$\Big\downarrow$$

$$\begin{aligned}
\ddot{A}(t)+\frac{3}{2}\left[1+k_s^2\right]A(t)+C_3D_1(t)D_2(t)\ &=\ 0\ ,\\
\ddot{D}_1(t)+k_1^2D_1(t)+C_3A(t)D_2(t)\ &=\ 0\ ,\\
\ddot{D}_2(t)+k_2^2D_2(t)+C_3A(t)D_1(t)\ &=\ 0\ ,
\end{aligned}$$

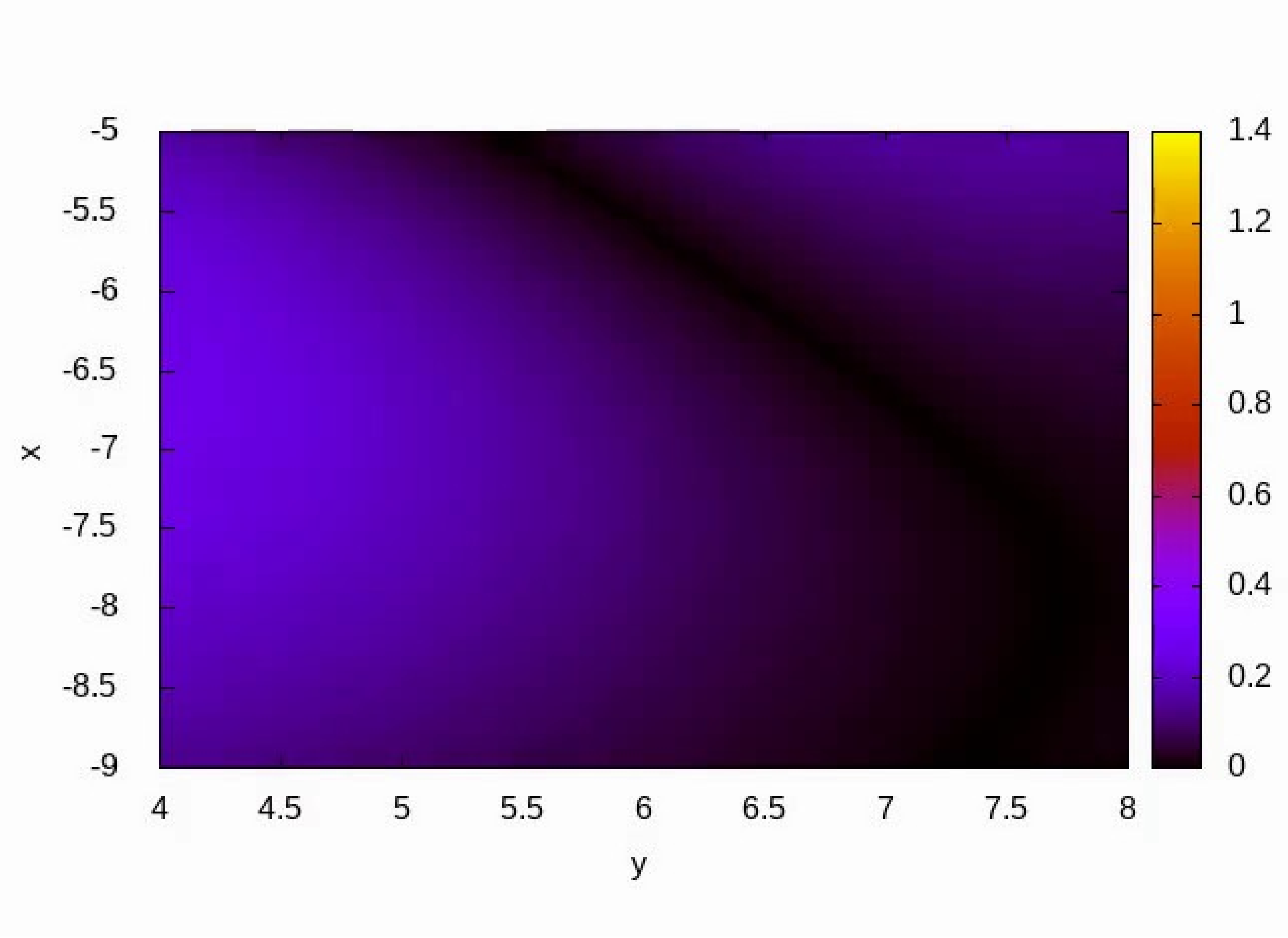
Mettler, 1967

$$\text{Amplification for } k_1=\frac{\omega_s+k_s}{2} \text{ and } k_2=\frac{\omega_s-k_s}{2}, \text{ with } \omega_s=\sqrt{\frac{3}{2}+k_s^2}$$

4. DYNAMICS OF EXCITED DOMAIN WALL STRINGS
4.1. Parametric resonances in field theory simulations



$|\phi|/\eta$

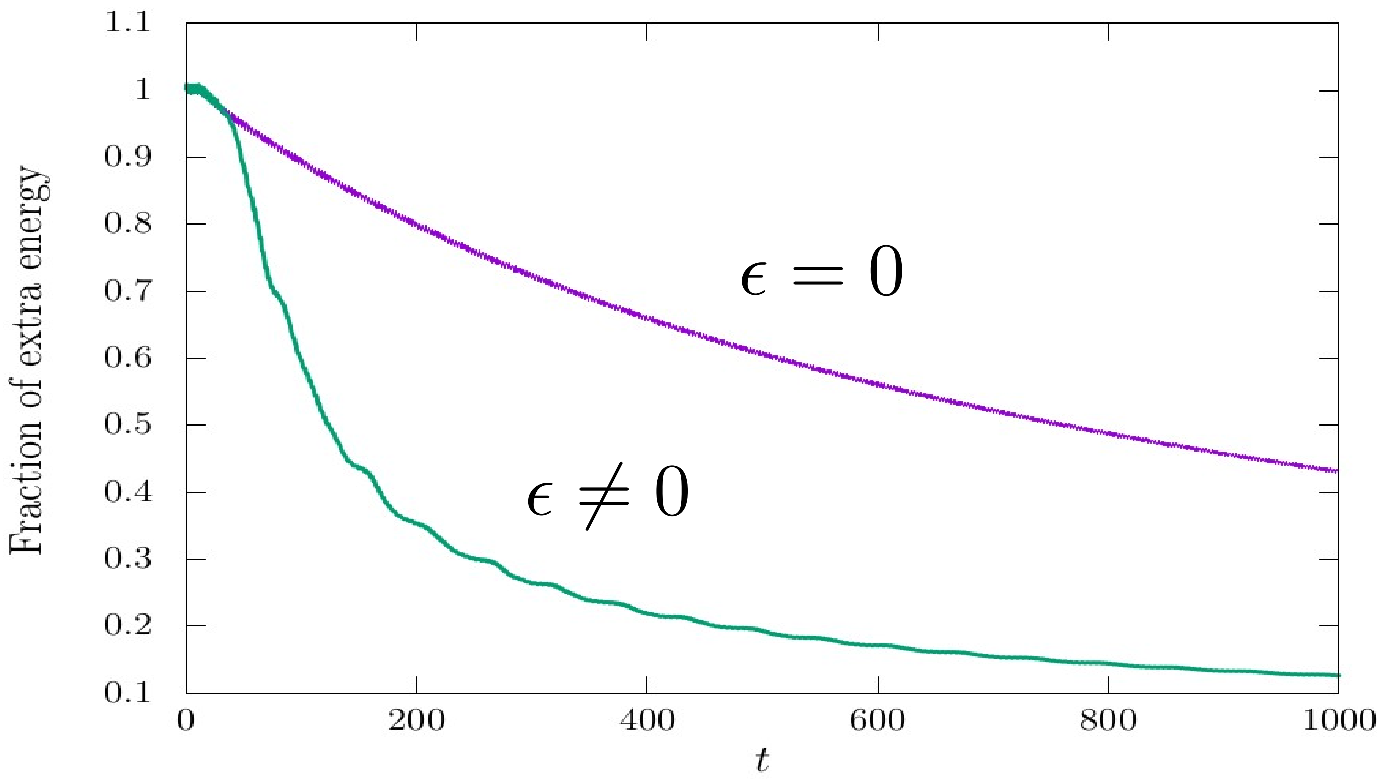


$$|\phi|/\eta$$

4.1. Parametric resonances in field theory simulations

The energy stored in the shape mode decays faster due to the resonance phenomena.

$$\phi\left(t=0,x,y\right)=\phi_K\left(x\right)+\hat{A}f_1\left(x\right)+\epsilon\hat{D}f_0\left(x\right)\cos\left(\frac{\Omega_1}{2}y\right)$$



4.2. Dynamical excitation of the shape mode

The shape mode can play a relevant role in the dynamics of the strings for $t < \tau$.

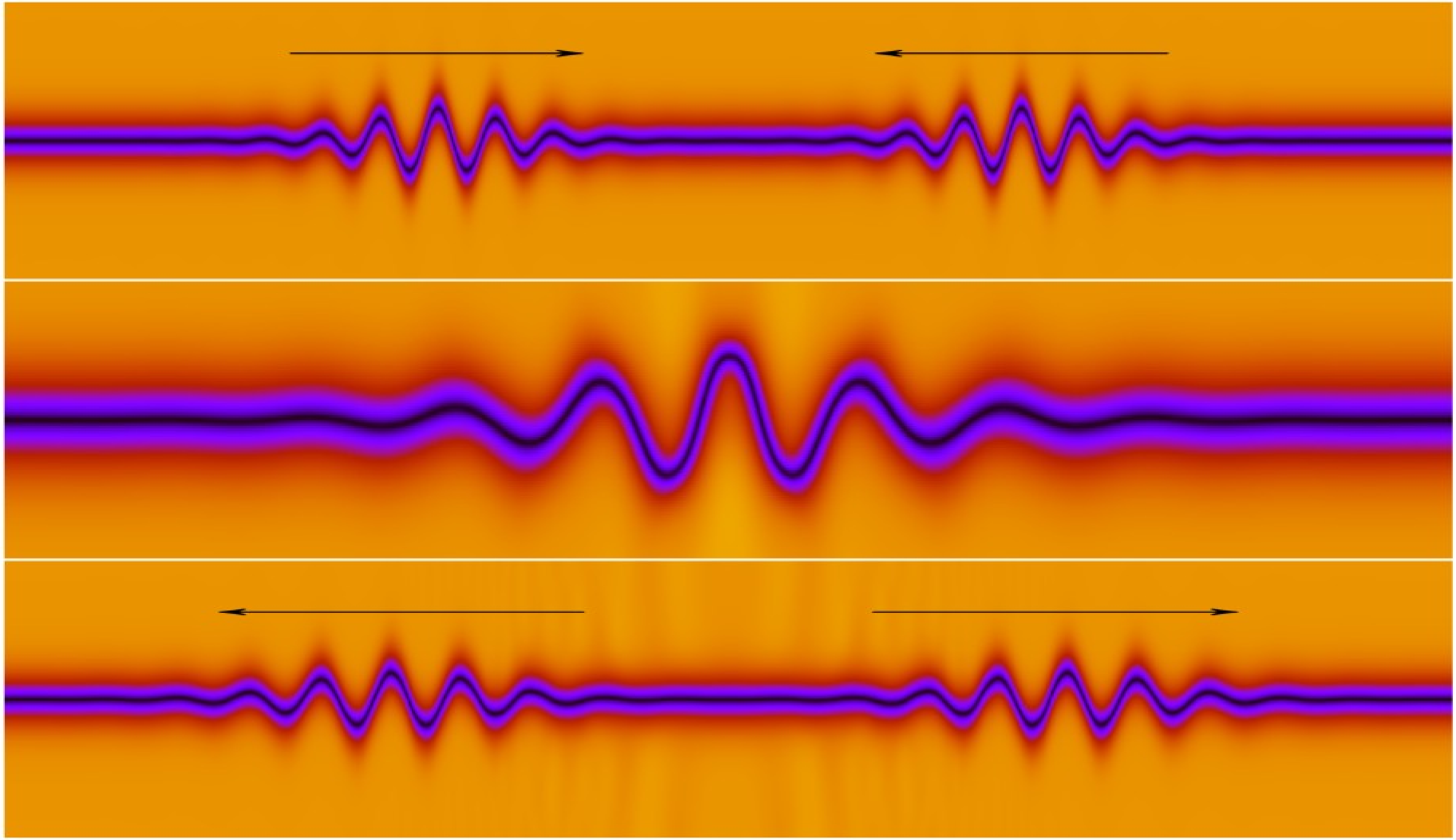
Decay time scale of the shape mode

However, we should not expect it to be important at cosmological time scales $t >> \tau$.

Can the shape mode be reexcited?

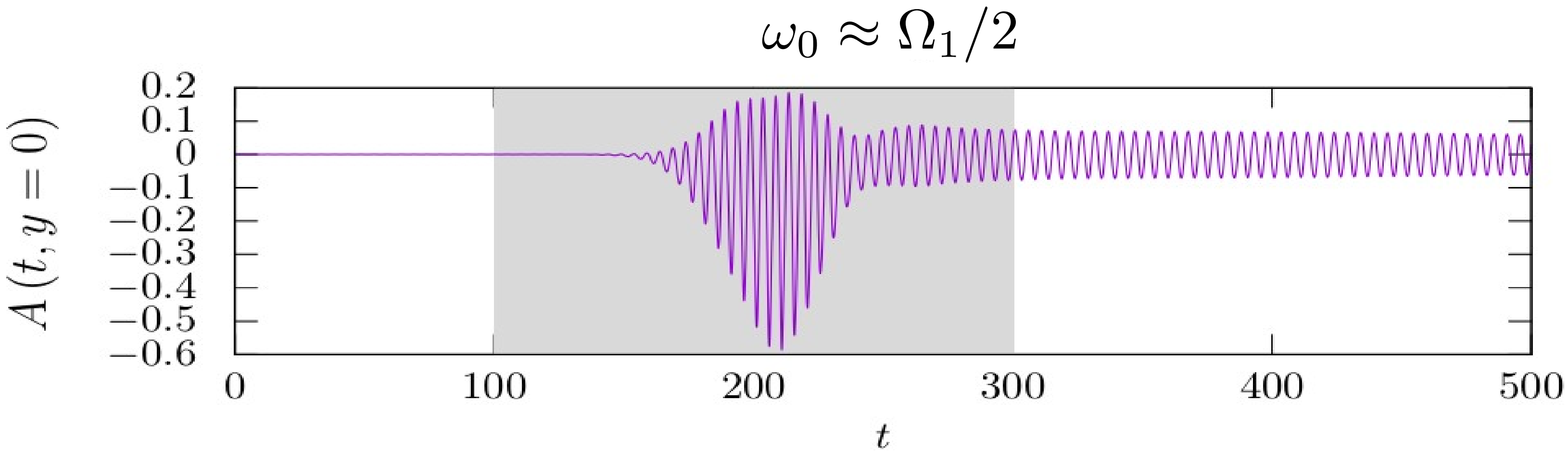
4. DYNAMICS OF EXCITED DOMAIN WALL STRINGS

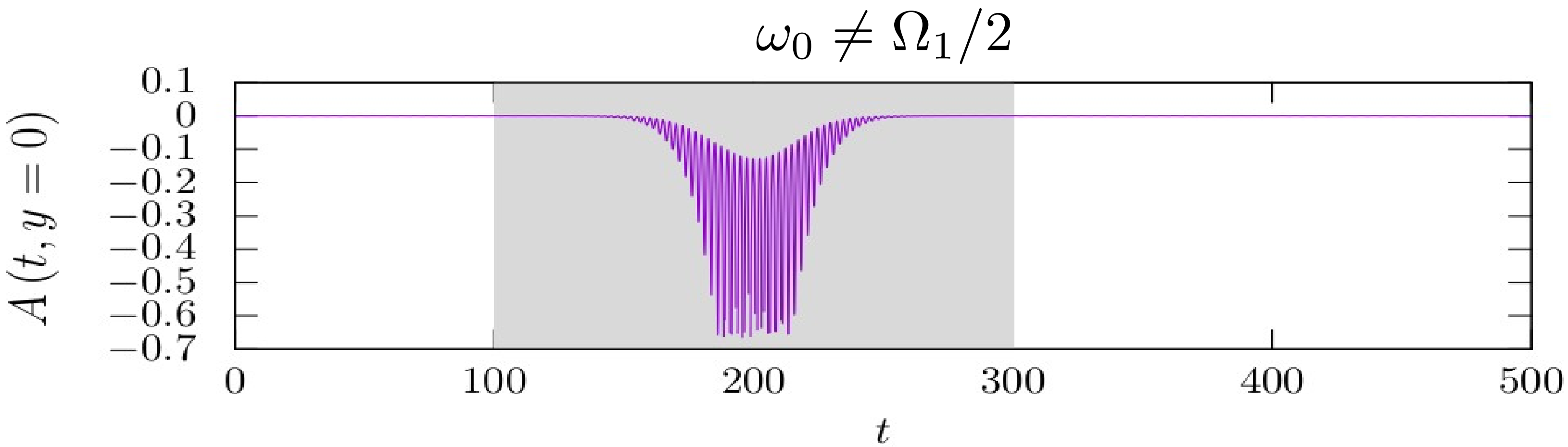
4.2. Dynamical excitation of the shape mode



4.2. Dynamical excitation of the shape mode

Only wiggles of frequency $\omega_0 \approx \Omega_1/2$ can repopulate the shape mode.

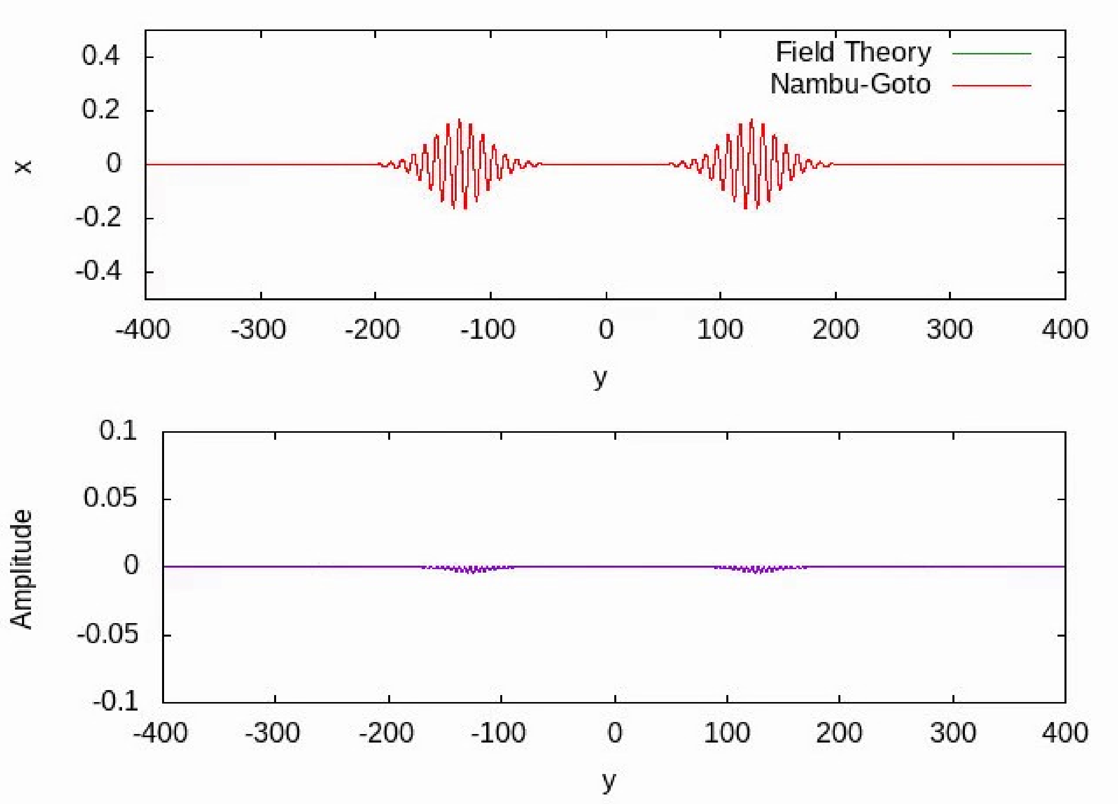




4. DYNAMICS OF EXCITED DOMAIN WALL STRINGS

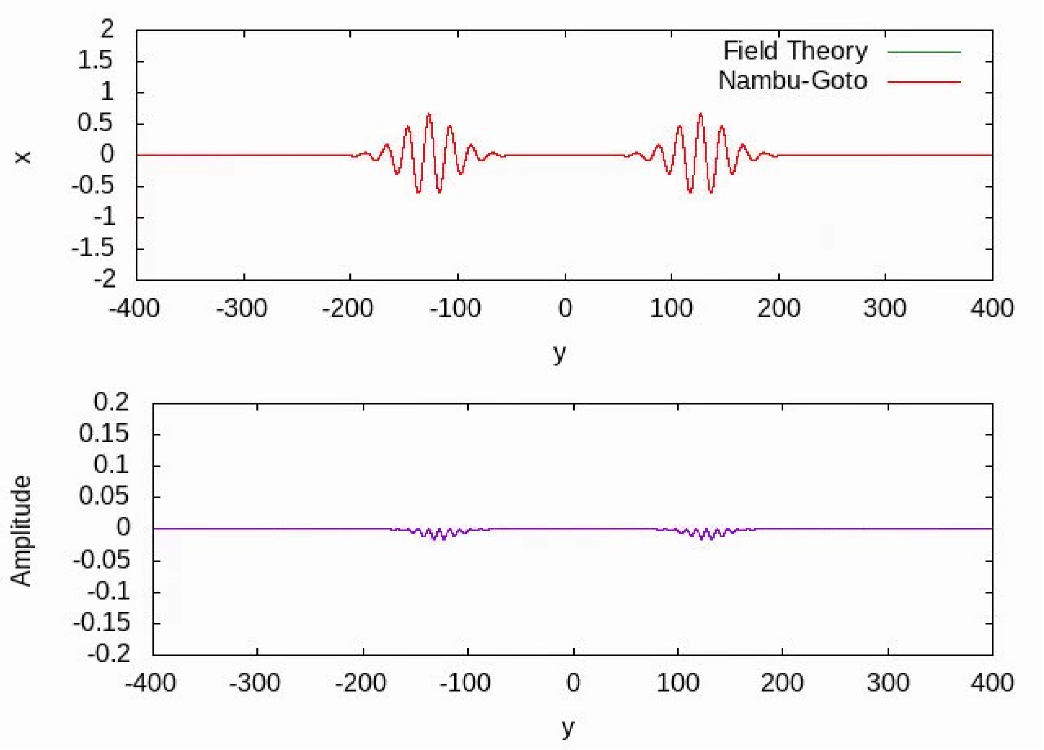
4.2. Dynamical excitation of the shape mode

$$\omega_0 \approx \Omega_1/2$$



4. DYNAMICS OF EXCITED DOMAIN WALL STRINGS
4.2. Dynamical excitation of the shape mode

$$\omega_0 \neq \Omega_1/2$$



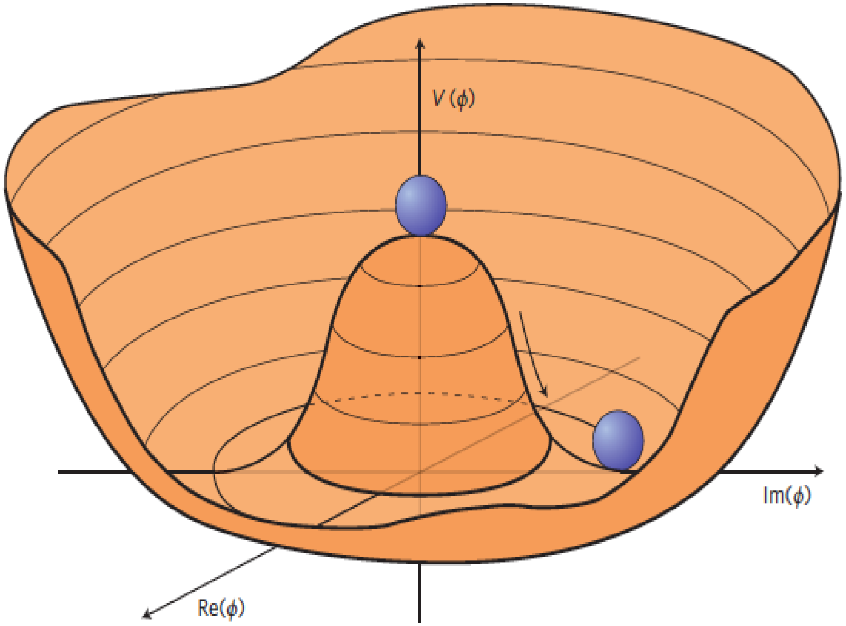
5. Dynamics of excited global strings
5.1. Static solution and bound states

Complex scalar field in 3+1 dimensions:

$$V\left(|\phi|\right) = \frac{\lambda}{4}\left(|\phi|^2 - \eta^2\right)^2$$

$$S = \int d^4x \left[\partial_\mu \phi^* \partial^\mu \phi - V\left(|\phi|\right)\right]$$

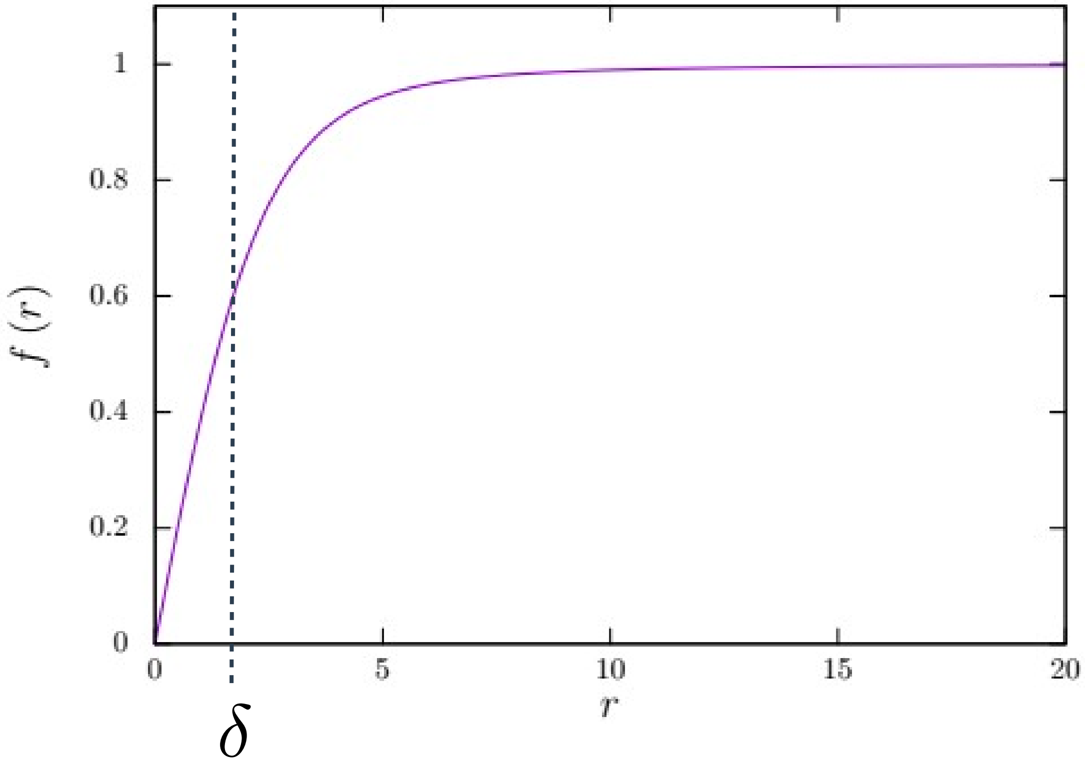
$$\boxed{\ddot{\phi} - \nabla^2\phi + \frac{\lambda}{2}\left(|\phi|^2 - \eta^2\right)\phi = 0}$$



5. Dynamics of excited global strings
5.1. Static solution and bound states

Static global string solution: $\phi_s(r,\theta) = \eta f(r)\, e^{iN\theta} \quad , \quad N = 1$

($N > 1$ solutions are unstable)



The presence of other strings at an average distance $r = R$ prevents the energy from diverging.

$$\mu_s \approx \mu_{\text{core}} + 2\pi\eta^2 \log\left(\frac{R}{\delta}\right)$$

5. Dynamics of excited global strings
5.1. Static solution and bound states

$$\boxed{\phi_s\left(r,\theta\right)=\eta f\left(r\right)e^{i\theta}}$$

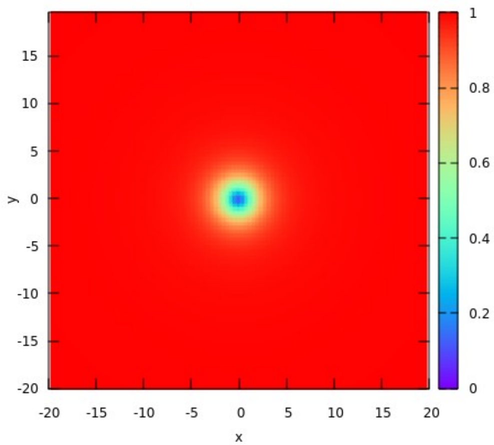
$$\phi=\frac{\varphi}{\sqrt{2}}e^{i\frac{\alpha}{\eta}}$$

Expanding about the vacuum,

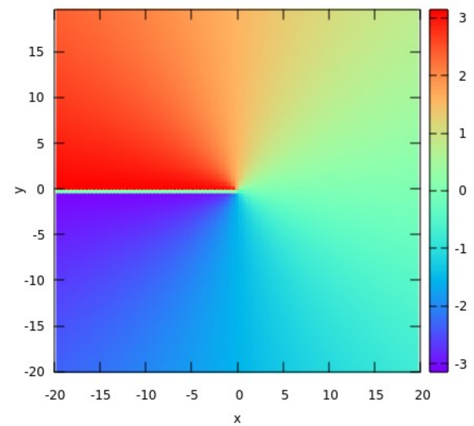
$$\varphi=\sqrt{2}\eta+\xi\ \ ,\ \ \alpha=\eta\theta+\chi$$

$$\mathcal{L}=\frac{1}{2}\partial_\mu\xi\partial^\mu\xi+\frac{1}{2}\partial_\mu\chi\partial^\mu\chi-\frac{1}{2}\lambda\eta^2\xi^2$$

Radial part



Angular part



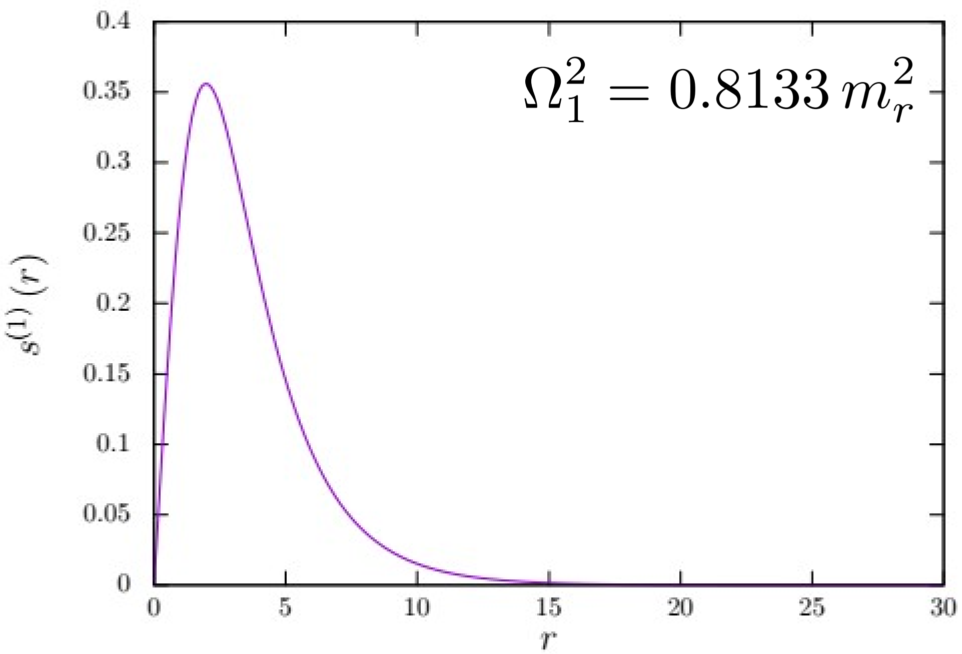
→ Radial perturbations are massive: $m_r=\sqrt{\lambda}\eta$

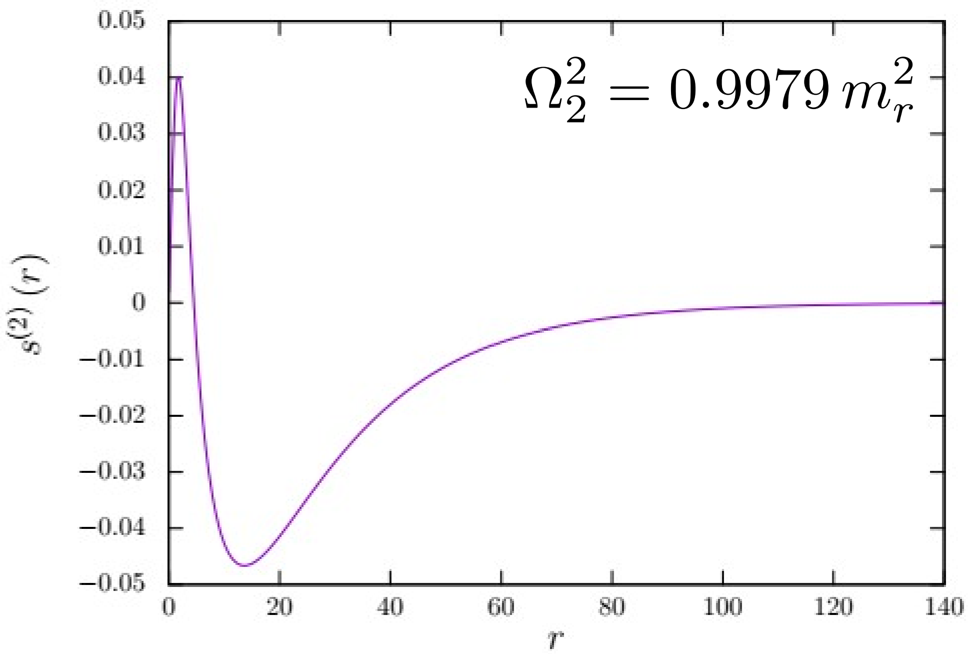
→ Angular perturbations are massless

5. Dynamics of excited global strings
5.1. Static solution and bound states

Radial perturbations: $\phi\left(r,\theta,z,t\right)=\eta\left[f\left(r\right)+s^{(j)}\left(r\right)\cos\left(\omega_j t-k_j z\right)\right]e^{i\theta}$

$$\boxed{-\frac{d^2 s^{(j)}}{dr^2}-\frac{1}{r}\frac{ds^{(j)}}{dr}+U\left(r\right)s^{(j)}\left(r\right)=\Omega_j^2 s^{(j)}\left(r\right)}\ ,\quad U\left(r\right)/m_r^2=\frac{1}{2}\left[3f^2\left(r\right)-1\right]+\frac{1}{r^2}\ ,\quad \Omega_j^2=\omega_j^2-k_j^2$$

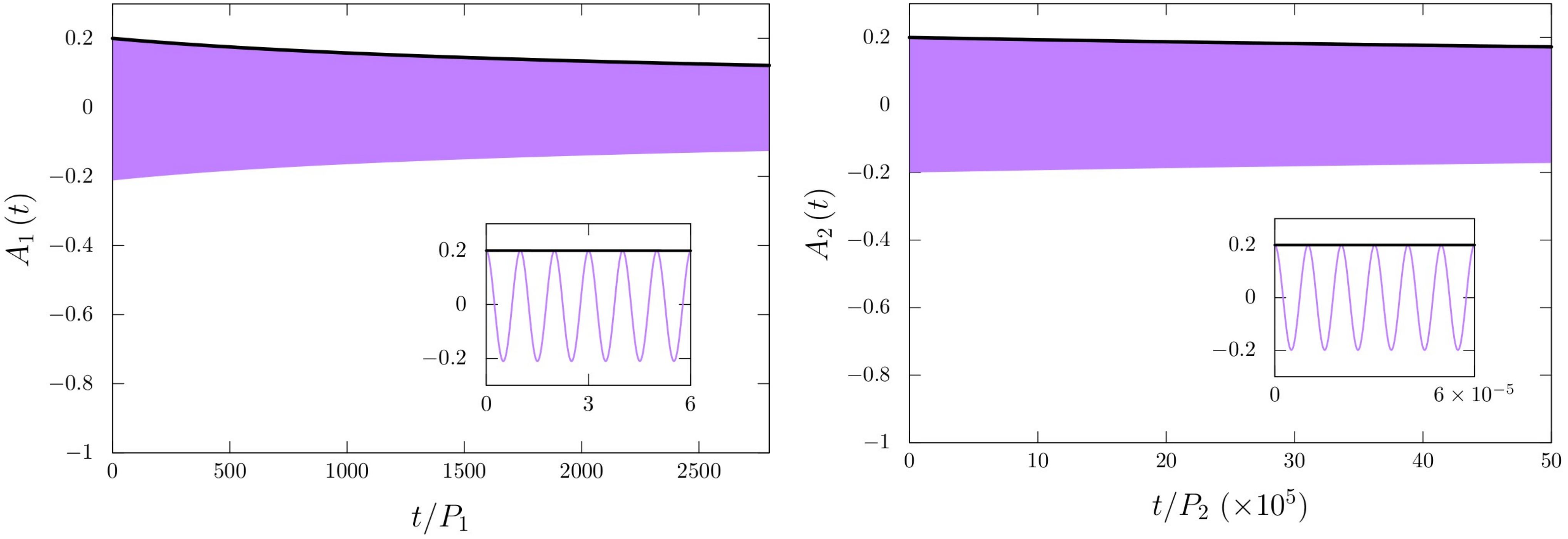




5. Dynamics of excited global strings
5.1. Static solution and bound states

$$\phi\left(r,\theta,t\right)=\phi_s\left(r,\theta\right)+s^{(j)}\left(r\right)A_j\left(t\right)e^{i\theta}+R\left(r,\theta,t\right),\quad R\left(r,\theta,t\right):\ \text{radiation field}$$

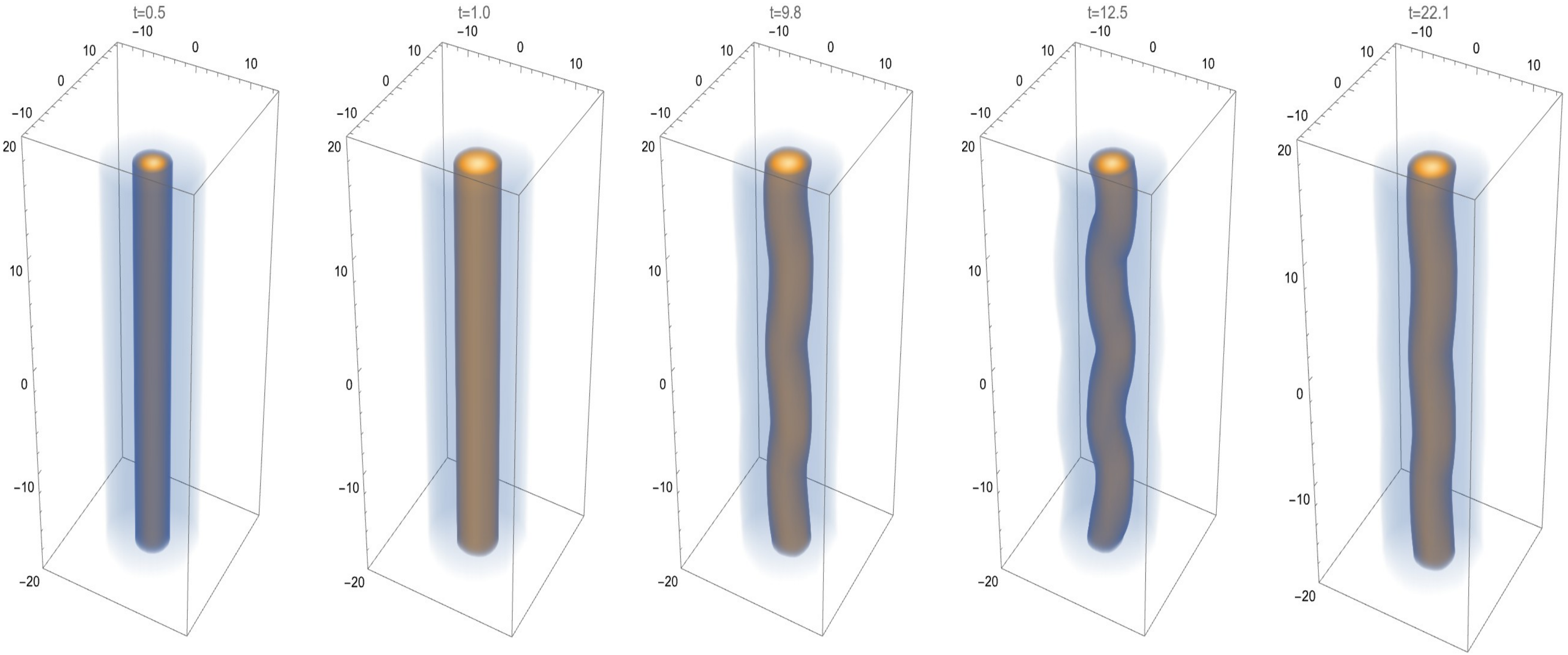
$$\boxed{\hat{A}_j^{-2}\left(t\right)=\hat{A}_j^{-2}\left(0\right)+\alpha_j t}\qquad \alpha_1=0.00218\ ,\ \ \alpha_2=2.77\times10^{-7}$$

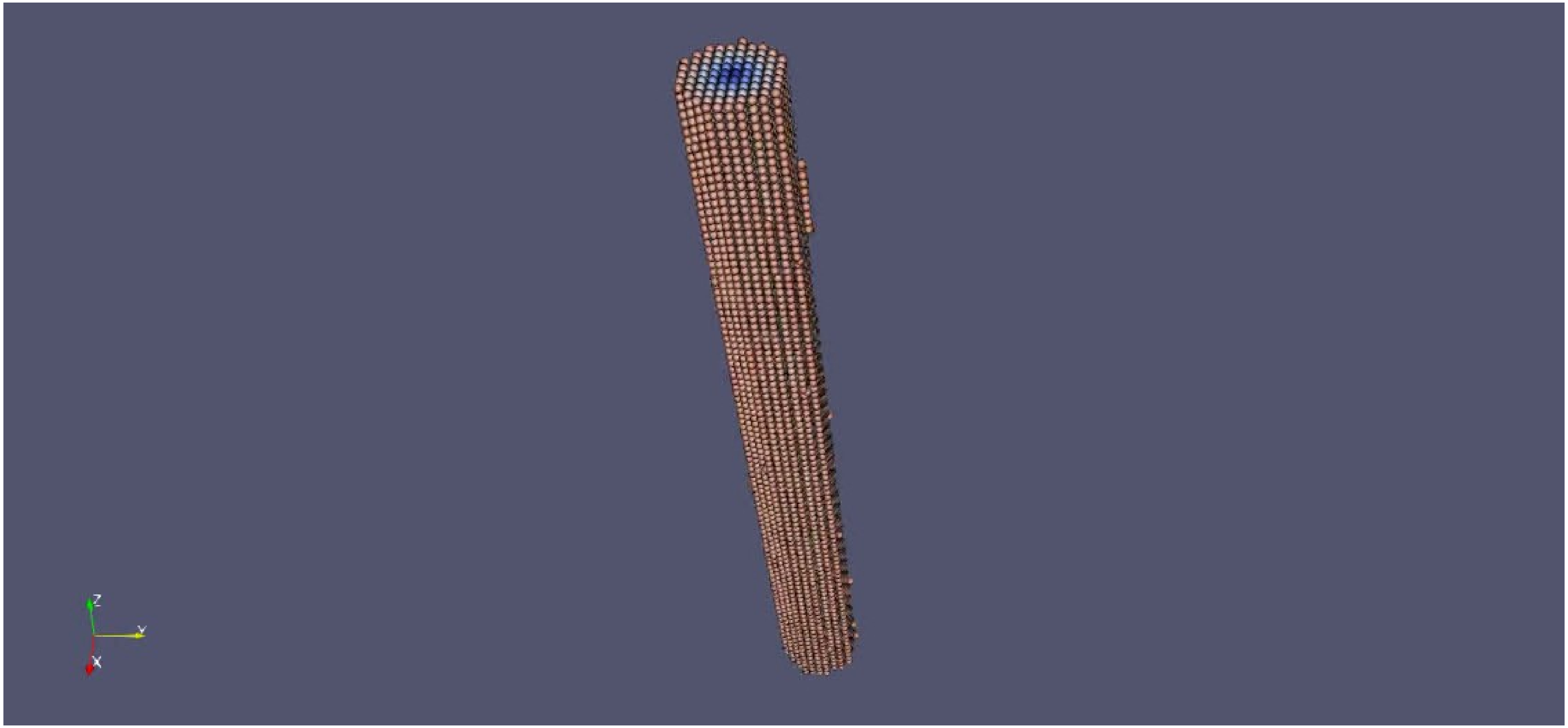


5. Dynamics of excited global strings

5.2. Parametric resonances in field theory simulations

Oscillations of the shape mode can amplify the zero modes $(\partial_x \phi_s, \partial_y \phi_s)$

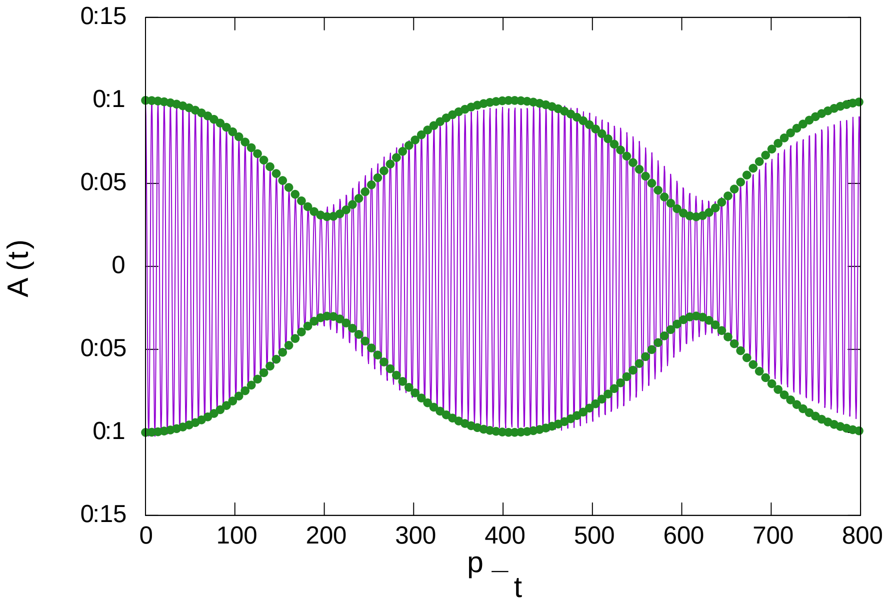


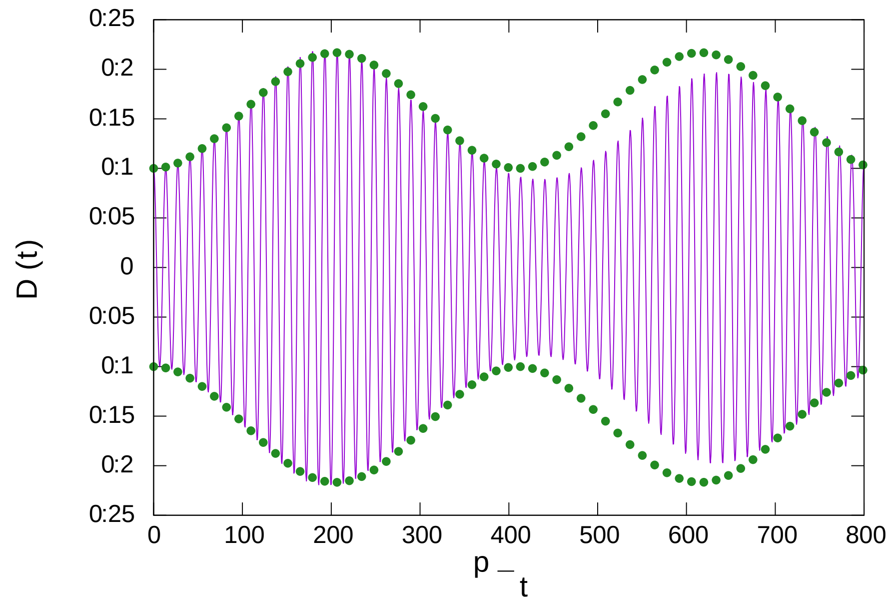

Z
Y
X

5.2. Parametric resonances in field theory simulations

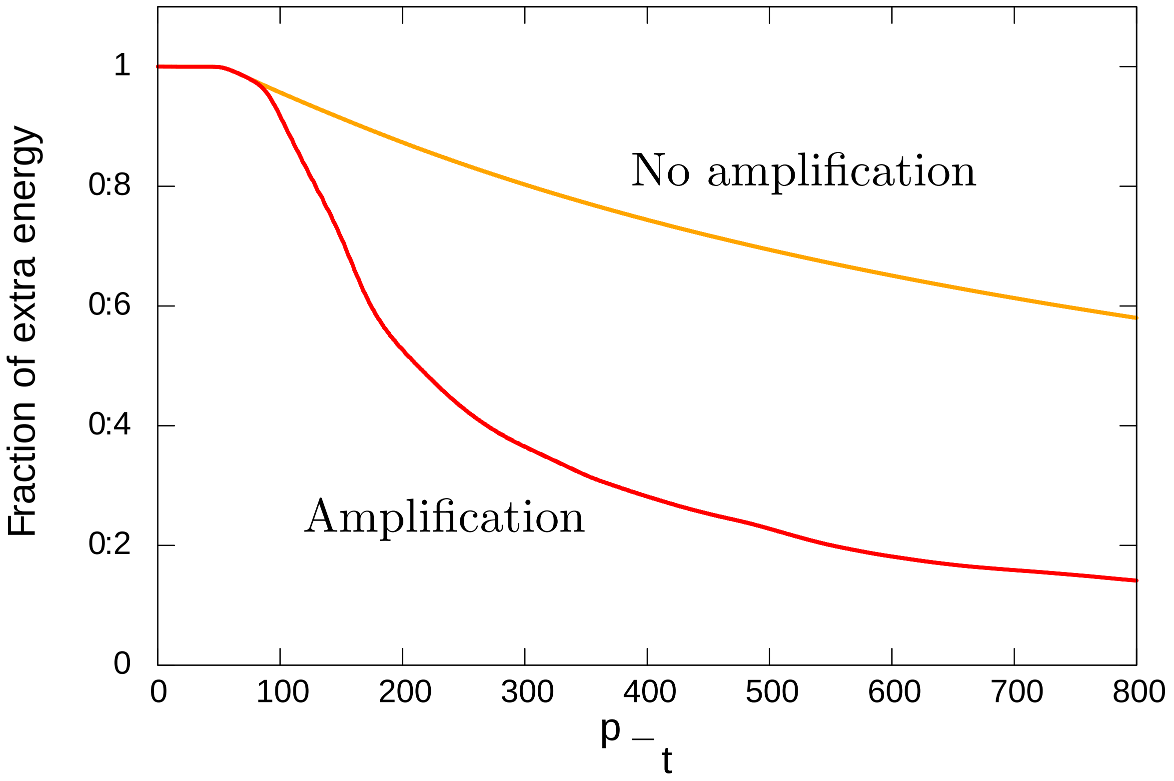
$$\ddot{A}(t) + \left(w_s^2 + I_{AD,4}\ D(t)^2\right) A(t) + \frac{3}{2} I_{A,3} A(t)^2 + 2 I_{A,4} A(t)^3 + \frac{1}{2} I_{AD,3} D(t)^2 = 0\ ,$$

$$\ddot{D}(t) + \left(w_z^2 + \frac{I_{D,2}}{I_D(R)} + \frac{I_{AD,3}}{I_D(R)} A(t) + \frac{I_{AD,4}}{I_D(R)} A(t)^2\right) D(t) + 2 \frac{I_{D,4}}{I_D(R)} D(t)^3 = 0\ .$$





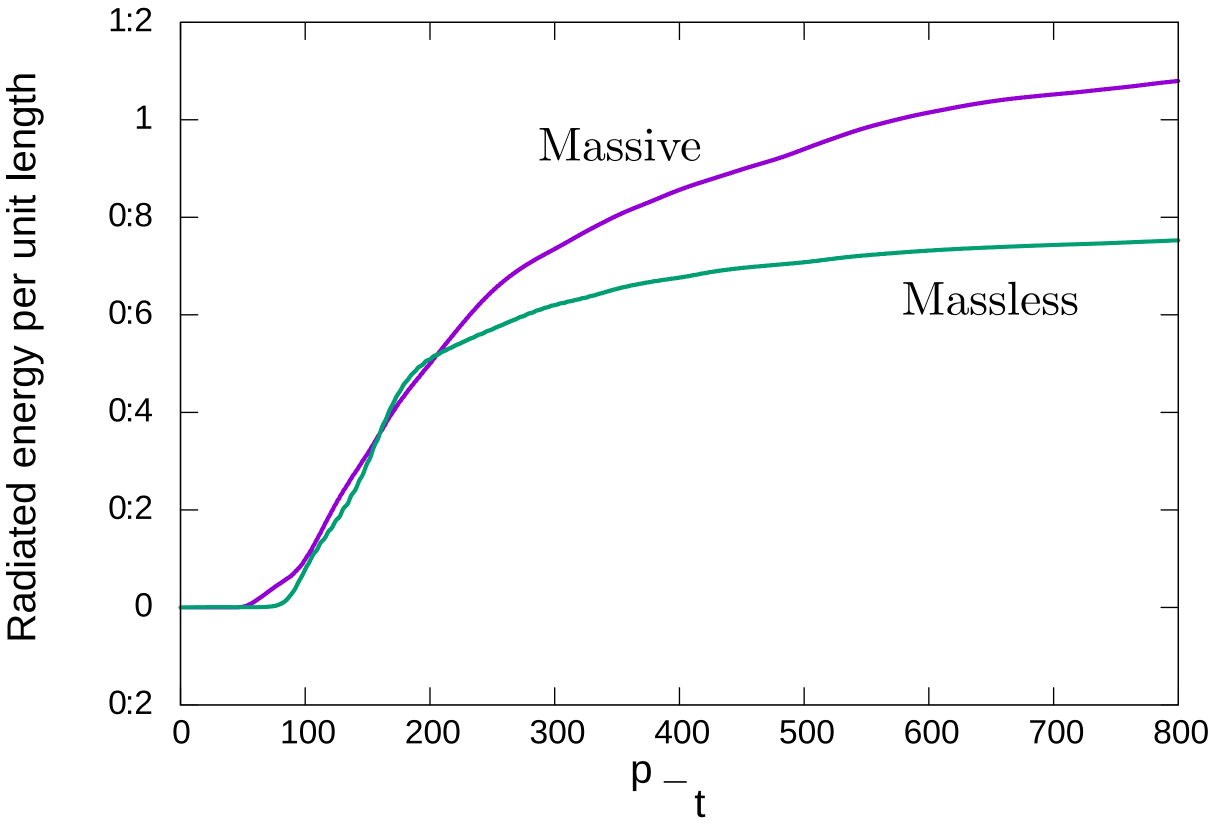
The energy stored in the shape mode decays faster when the amplification takes place



5. Dynamics of excited global strings

5.2. Parametric resonances in field theory simulations

Massless radiation is emitted as a consequence of the resonance



6. CONCLUSIONS

• Non-linear interactions between the internal modes give rise to resonance phenomena which are not captured by the Nambu-Goto action. These processes are responsible for a faster decay of the energy stored in the shape mode.

Domain wall strings

• The Nambu-Goto action provides a good description of the dynamics as long as the shape mode is not excited and there are no regions of high curvature.

• Presumably, the shape mode does not play a relevant role at late times in a cosmological setting. Collisions of wiggles do not repopulate it unless the frequency of the waves is half the frequency of the shape mode.

Global strings

• The amplification of zero mode perturbations enhances the emission of massless radiation. This could be relevant for calculations of axionic dark matter abundance from cosmic string networks.

Future plans:

- Can string reconnections excite the shape mode?
- Spectrum of perturbations and role of bound states for local strings.

THANK YOU FOR YOUR ATTENTION