Formation of shadows around compact objects as observational proofs of new gravitational physics

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SATURNALIA 2022

References

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- M. Guerrero, G. Olmo, D. Rubiera-García, DS-CG, ``Multiring images of thin accretion disk of a regular'' *Phys. Rev. D* 106 (2022) 4, 044070 [arXiv: 2205.12147 [gr-qc]].

Event Horizon Telescope

The observations of the Event Horizon Telescope [1] in 2019 have ignited the beginning of a new era for testing the spacetime structure and GR itself via the illumination of (ultra)-compact objects with electromagnetic radiation from accretion disks.

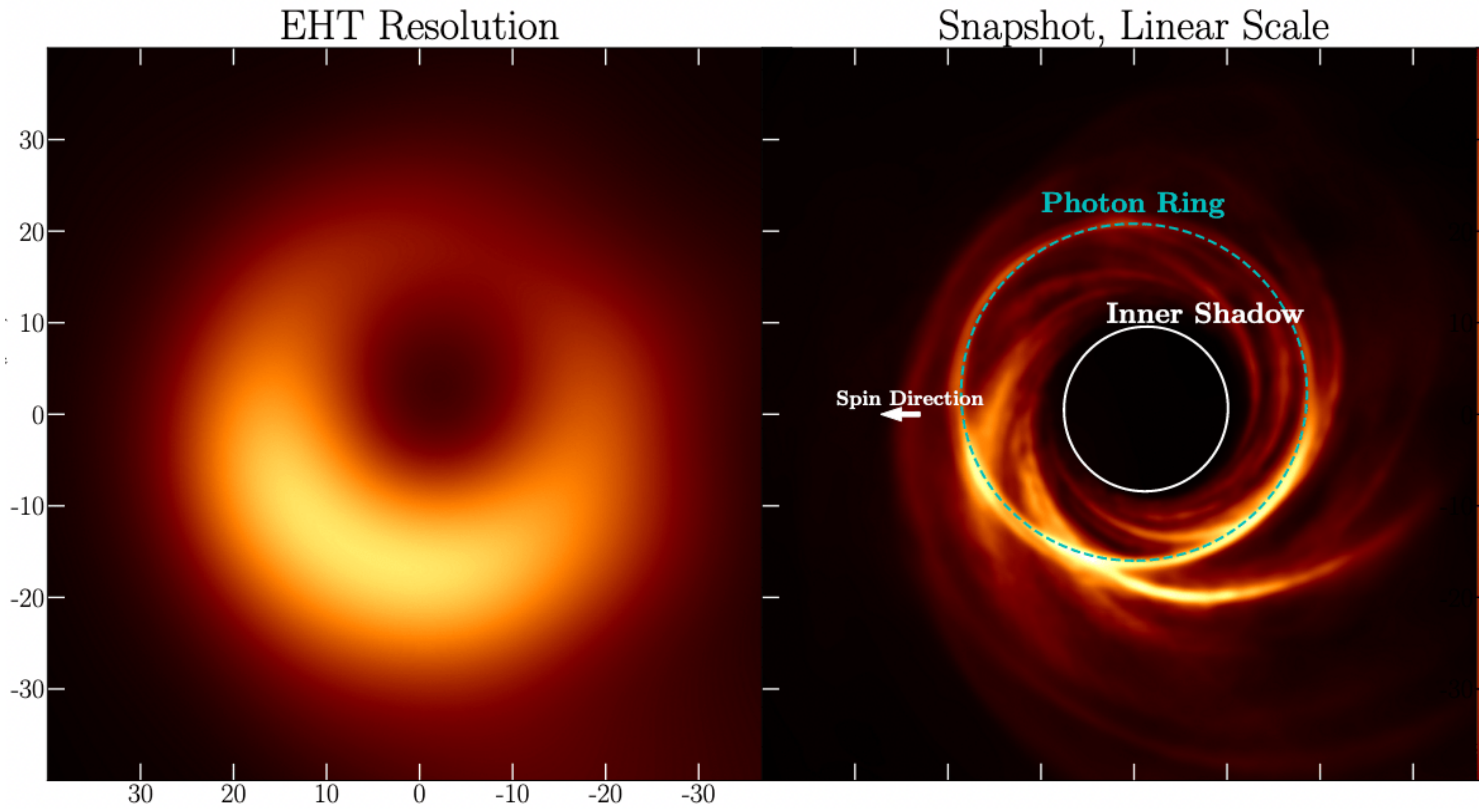


Figure: The observed EHT image (left) and the GRMHD simulated one (right). Credit from Ref.[2]

[1] Event Horizon Telescope Collaboration, Astrophys. J. Lett. **875** L5 (2019)
[2] A. Chael, M. D. Johnson and A. Lupsasca, Astrophys. J. **918** (2021) 6.

Outline

- Null geodesics and light deflection
- Ray-tracing for spherically symmetric spacetimes
- Optically thin accretion disks and intensity profiles
- Black boles’ mimickers: shadows
- Conclusions and perspectives

Null geodesics and light deflection

Given a spacetime described by the metric:

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

Null geodesics are:

$$\frac{d^2 x^\alpha(\lambda)}{d\lambda^2} + \Gamma^\alpha_{\mu\nu} \frac{dx^\mu(\lambda)}{d\lambda} \frac{dx^\nu(\lambda)}{d\lambda} = 0\,, \qquad \text{Geodesics equation.}$$

$$g_{\mu\nu} k^\mu k^\nu = 0 \longrightarrow g_{\alpha\beta} \frac{dx^\alpha(\lambda)}{d\lambda} \frac{dx^\beta(\lambda)}{d\lambda} = 0\,, \qquad \text{Constraint equation.}$$

For our purposes, we simplify the calculations by considering a spherically symmetric spacetime:

$$ds^2 = -A(r)dt^2 + B(r)dr^2 + C(r)(d\theta^2 + \sin^2\theta d\phi^2),$$

And assume that is asymptotically flat:

$$\lim_{r\to\infty} A(r) = 1,$$
$$\lim_{r\to\infty} B(r) = 1,$$
$$\lim_{r\to\infty} C(r) = r^2.$$

Null geodesics and light deflection

Symmetries: Killing vectors. A particular metric is said to be invariant under a coordinate transformation as far as:

$$g_{\mu\nu} \rightarrow g'_{\mu\nu}(x') = g_{\mu\nu}(x)$$

By applying an infinitesimal transformation:

$$x'^{\mu} = x^{\mu} + \epsilon\xi^{\mu} \ , \ \epsilon << 1$$

The equation for the Killing vectors (generators of the symmetries) is obtained from the condition on the invariance of the metric:

$$\boxed{\xi_{\mu;\nu} + \xi_{\nu;\mu} = 0}$$

These symmetries on the metric tensor provide some conserved quantities (Noether's theorem).

- **Stationary** spacetime: timelike Killing vector, invariance under time translations.
- **Static** spacetime: timelike Killing vector which is orthogonal to spacelike hypersurfaces.
- **Homegeneous** spacetime: spacelike Killing vector, invariance under spatial translations.
- **Isotropic** spacetime: spacelike Killing vector, invariance under spatial rotations.
- **Maximally symmetric** metric: for a n-dimensional spacetime, the maximum number of symmetries are:

$$\frac{1}{2}n(n+1) \xrightarrow{n=4} 10$$

Null geodesics and light deflection

Let us get back to a general spherically symmetric spacetime given by the line element:

$$ds^2 = -A(r)dt^2 + B(r)dr^2 + C(r)(d\theta^2 + \sin^2\theta d\phi^2),$$

This spacetime has four killing vectors: one associated to time translations and the other ones to spatial rotations. Hence, we can restrict the motion at a given latitude and the Killing vector fields of time translational and axial symmetries are:

$$t^\mu \partial_\mu = \partial_t \qquad\qquad \phi^\mu \partial_\mu = \partial_\phi$$

The conserved quantities associated to these symmetries are the energy and angular momentum:

$$E \equiv -g_{\mu\nu} t^\mu k^\nu = A(r)\dot{t}$$

$$L \equiv g_{\mu\nu} \phi^\mu k^\nu = C(r)\dot{\phi}$$

We define the impact parameter for a particular photon trajectory:

$$b \equiv \frac{L}{E} = \frac{C(r)\dot{\phi}}{A(r)\dot{t}}.$$

N. Tsukamoto, Phys. Rev. D 95 064035 (2017).

Null geodesics and light deflection

Trajectory equation (we assume $\theta=\pi/2$):

$$-A(r)\dot{t}^2 + B(r)\dot{r}^2 + C(r)\dot{\phi}^2 = 0 \longrightarrow \dot{r}^2 = V(r), \qquad V(r) \geq 0$$

Where

$$V(r) \equiv \frac{L^2 R(r)}{B(r)C(r)}, \qquad R(r) \equiv \frac{C(r)}{A(r)b^2} - 1.$$

We assume that R(r) has at least one positive root, such that a photon coming from infinity approaches the object and is scattered at a minimum distance r=r0, where:

$$R(r) = 0$$

And the impact parameter of such trajectory is given by:

$$b(r_0) = \frac{L}{E} = \frac{C_0\dot{\phi}_0}{A_0\dot{t}_0} = \sqrt{\frac{C_0}{A_0}}.$$

Circular trajectories:

$$\ddot{r} + \frac{1}{2}\left(\frac{B'}{B} + \frac{C'}{C}\right)\dot{r}^2 = \frac{E^2}{AB}D(r). \longrightarrow D(r_c) = 0$$

$$D(r) \equiv \frac{C'(r)}{C(r)} - \frac{A'(r)}{A(r)},$$

Null geodesics and light deflection

The potential and its derivative satisfy:

$$V(r_c) = 0 \ , \quad V'(r_c) = 0$$

This critical curve will correspond in general to a maximum of the effective potential, leading to an unstable photon circular orbit, also known as **photon sphere**.

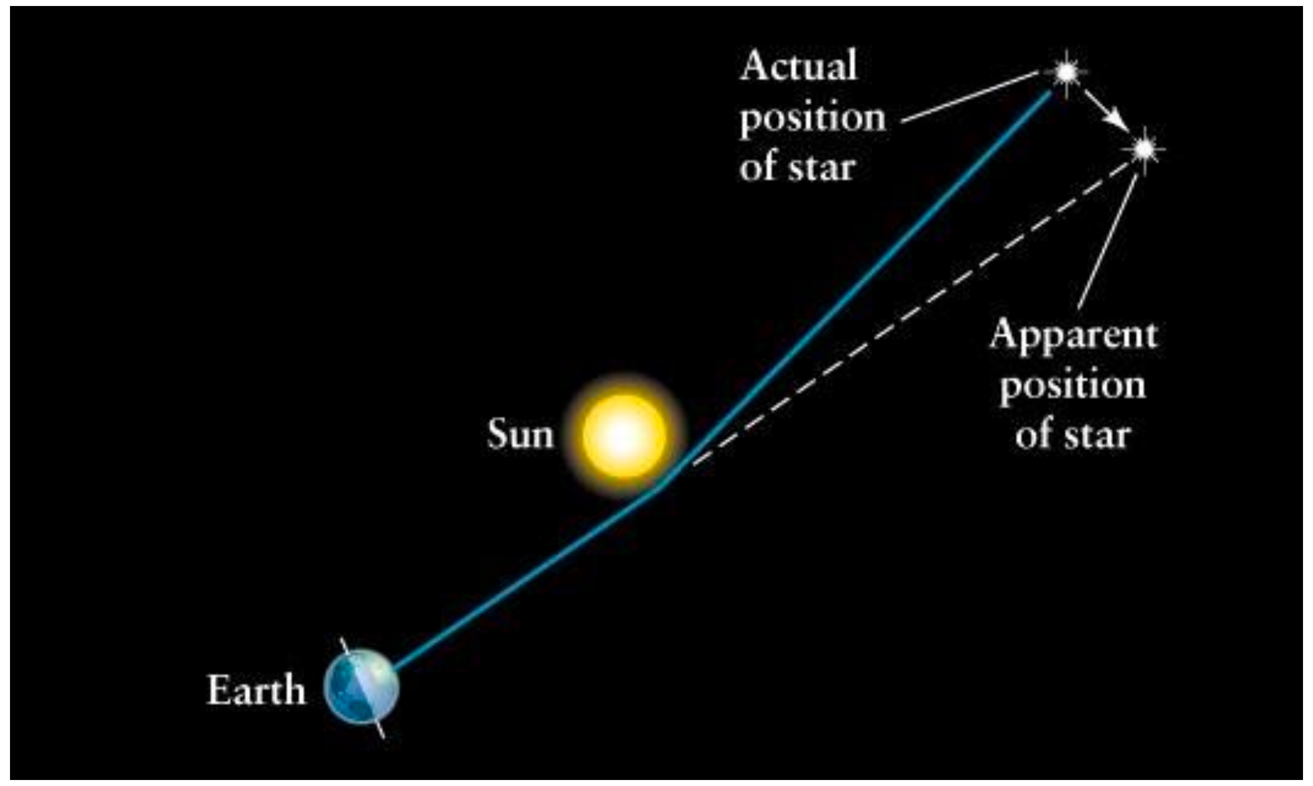
The critical impact parameter for such an orbit is given by:

$$b_c(r_c) = \lim_{r_0 \to r_c} \sqrt{\frac{C_0}{A_0}}$$

A photon with an impact parameter close to the critical one will turn around an arbitrary number of orbits before falling into the object or getting away to infinity.

We are interested on computing the light trajectories and their deflection, so by removing the affine parameter we get the following general equation for the trajectories:

$$\left(\frac{dr}{d\phi}\right)^2 = \frac{R(r)C(r)}{B(r)},$$



The deflection angle for a particular trajectory is given by:

$$\alpha(b) = I(b) - \pi \ , \quad I(b) = 2\int_{r_0(b)}^{\infty} \frac{dr}{\sqrt{\frac{R(r)C(r)}{B(r)}}}$$

Ray-tracing

The procedure for the ray-tracing is very simple: takes a photon that reaches the observer's detector (at infinity) and trace back its trajectory to find the point where the photon was originated.

Metric:

$$r^2(x) = x^2 + a^2 \ ,$$

$$ds^2 = -A(x)dt^2 + A^{-1}(x)dx^2 + r^2(x)d\Omega^2$$

Depending on the function *A(x)* and on the parameter a, this metric may describe a (regular) black hole or a (traversable) wormhole.

Trajectory equation:

$$\dot{x}^2 = \frac{1}{b^2} - V(x) \ , \qquad V(x) = \frac{A(x)}{r^2(x)} \ ,$$

Critical impact parameter:

$$b_c^2 = \frac{r^2(x_{ps})}{A(x_{ps})} \ ,$$

This gives the impact parameter for the null unstable circular orbit:

$$V_{eff}(x = x_{ps}) = 1/b_c^2, V'_{eff}(x = x_{ps}) = 0, V''_{eff}(x = x_{ps}) < 0$$

Ray-tracing

Trajectory equation:

$$\frac{d\phi}{dx} = \mp \frac{b}{r^2(x)\sqrt{1 - \frac{b^2 A(x)}{r^2(x)}}} \ ,$$

By assuming an **accretion disk** surrounding the central object, this equation gives the trace of all trajectories of the photons arriving to observer's screen emitted from the disk. We define the number of half-orbits around the central object by:

$$n = \frac{\phi}{2\pi}$$

Which is used to define the different types of emission for an optically-thin accretion disk (transparent to their own radiation):

- Direct emission: trajectories intersecting the equatorial plane just once, $n \leq 3/4$.
- Lensed emission: trajectories intersecting the equatorial plane twice, $3/4 < n \leq 5/4$
- Photon ring emission: $n > 5/4$.

The luminosity collected by the observer on the screen and the appearance of the demagnified rings will depend on the type of emission.

Optically thin accretion disks and intensity profiles

The emission for a finite-size accretion disk is described by the radiative Boltzmann equation:

$$\frac{d}{d\lambda}\left(\frac{dI_\nu}{d\nu^3}\right)=\left(\frac{j_\nu}{\nu^2}\right)-(\nu\alpha_\nu)\left(\frac{I_\nu}{\nu^3}\right) \ ,$$

General Relativistic MagnetoHydroDynamic (GRMHD) simulations solve this equation by using a pool of assumptions upon all these coefficients, while simplified analytical models can also be employed upon reasonable assumptions: zero absorptivity and monochromatic emission. Moreover, we assume an infinitesimally thin disk, for which I/ν^3 is conserved along a photon's trajectory and also an isotropic emission $I(x)$ under three models whose intensity starts and peak at three relevant surfaces:

- Model I: Innermost stable circular orbit, decaying quadratically.
- Model II: Photon sphere, decaying quadratically.
- Model III: Event Horizon (if any), decaying with a more complex function.

Every intersection with the accretion disk, light rays pick up additional brightness. However, for n>3 intersections, the corresponding light rings are so demagnified that their contribution to the total luminosity is negligible. Hence, the total intensity received by the observer is corrected by two effects: gravitational redshift and collected luminosities, such that we get:

$$I^{ob}=\sum_m A^2(x)I(x)\left.\right|_{x=x_m(b)} \ ,$$

S. E. Grill, D. E. Holz and R. Wald, Phys. Rev. D **100** 024018 (2019).

Black holes' mimickers: shadows

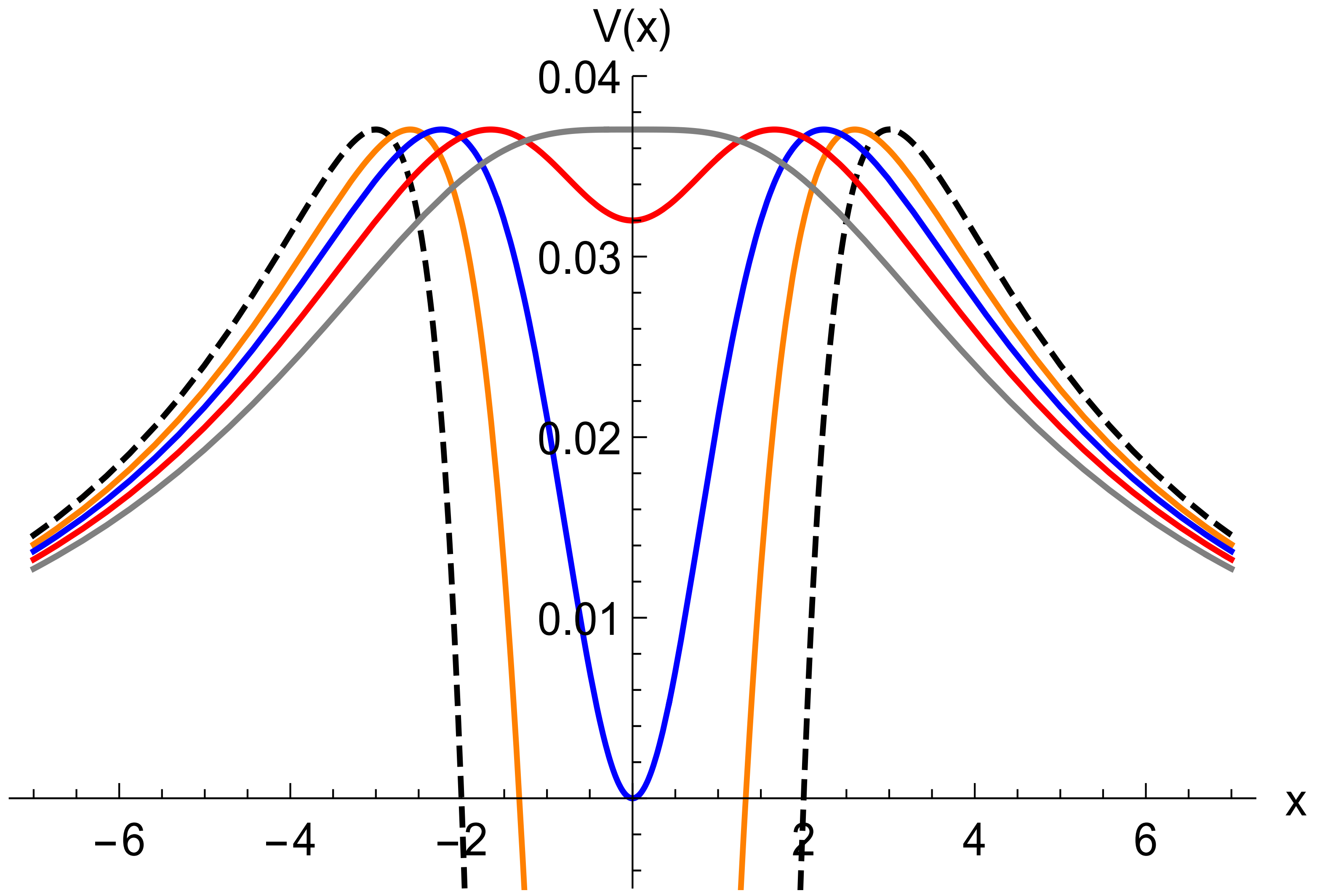
First type of models: a single photon sphere

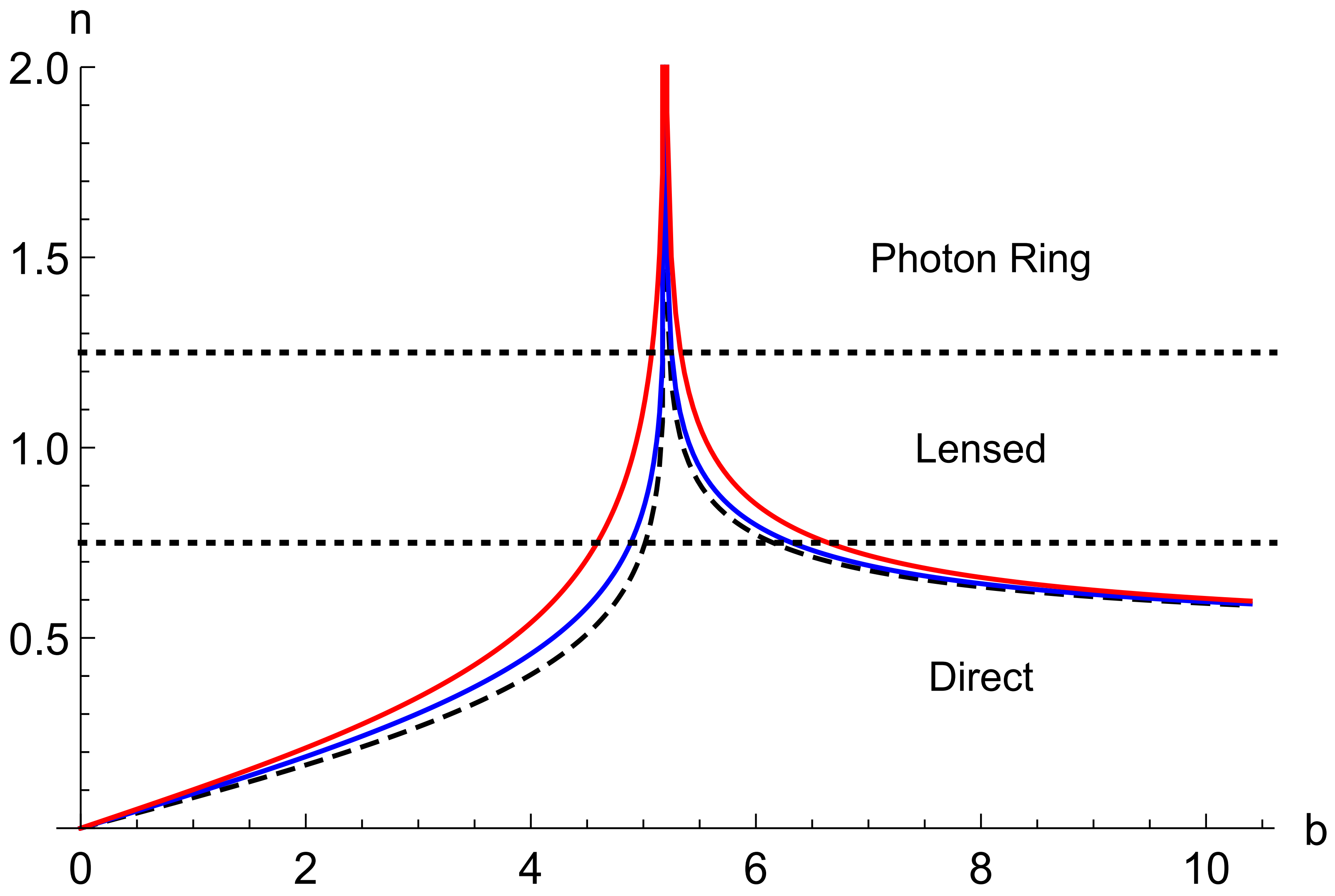
$$ds^2 = -A(x)dt^2 + B(x)dx^2 + r^2(x)d\Omega^2 \ , \qquad A(x) = B^{-1}(x) = 1 - \frac{2M}{r(x)} \ ; \ r^2(x) = x^2 + a^2 \ ,$$

Trajectory equation:

$$\dot{x}^2 = \frac{1}{b^2} - V(x) \ ,$$

- a=0 Schwarzschild black hole
- a<2M: regular Black hole
- a>2M: a traversable wormhole.



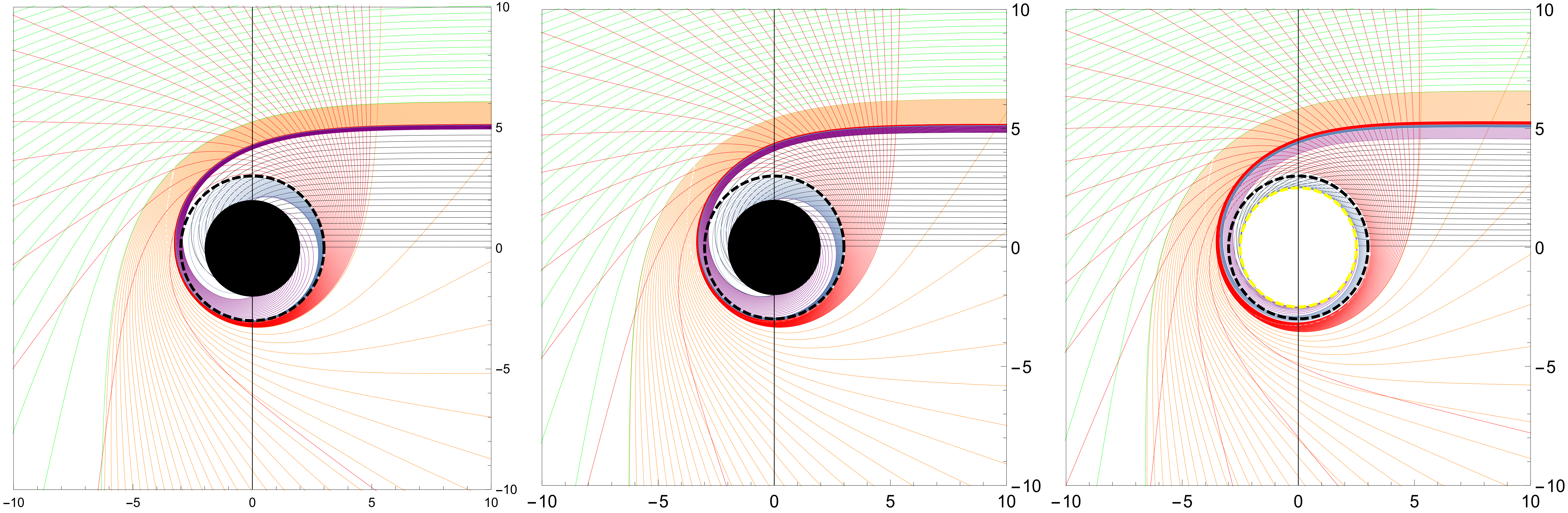


M. Guerrero, G. Olmo, D. Rubiera-García, DS-CG, JCAP **08** 036 (2021), [arXiv:2105.15073 [gr-qc]].

Black holes' mimickers: shadows

Ray-tracing

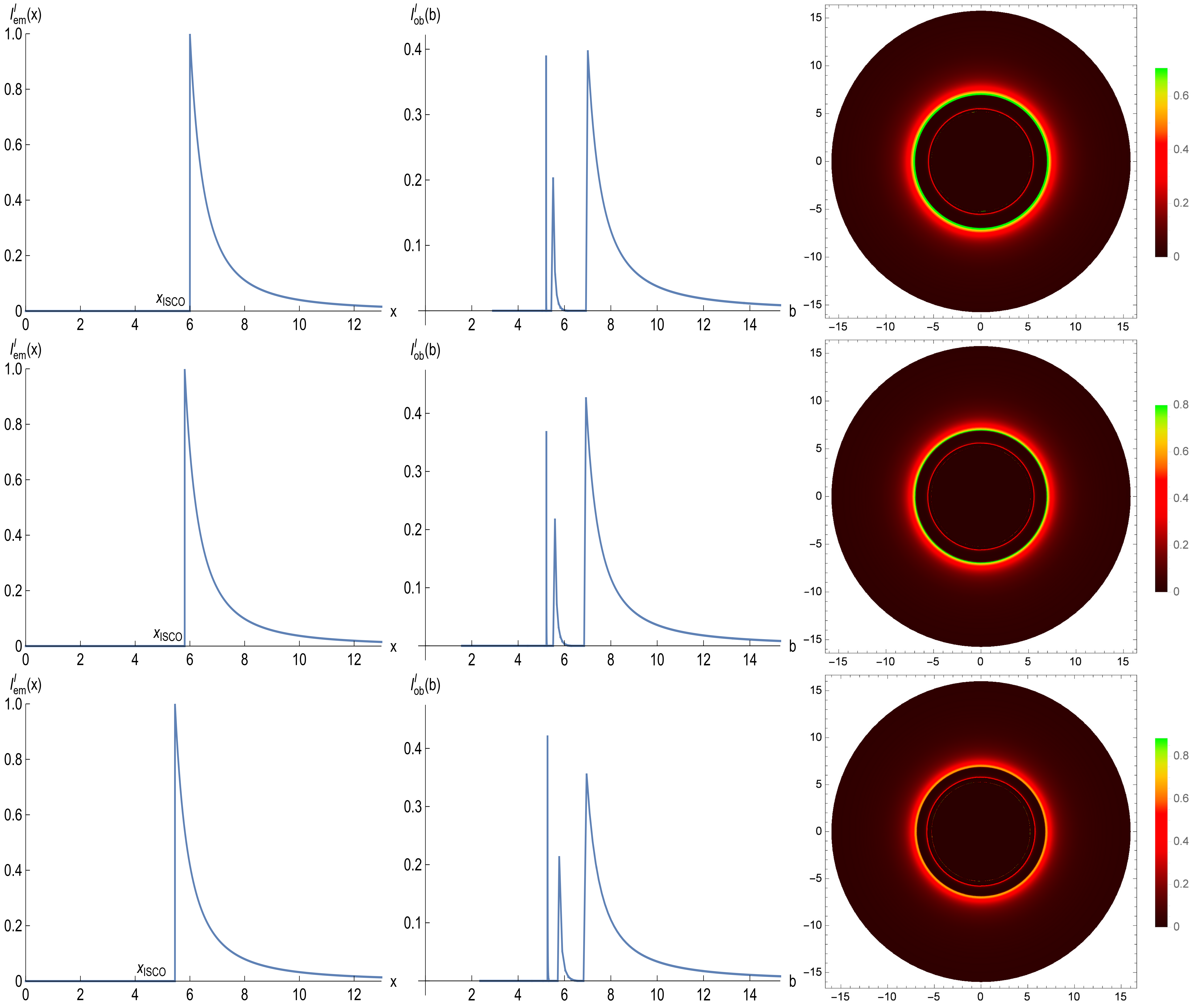
Schw. BH **Regular BH** **Wormhole**



Black holes' mimickers: shadows

Optical appearance: shadows

Model I of emission



Schw. BH

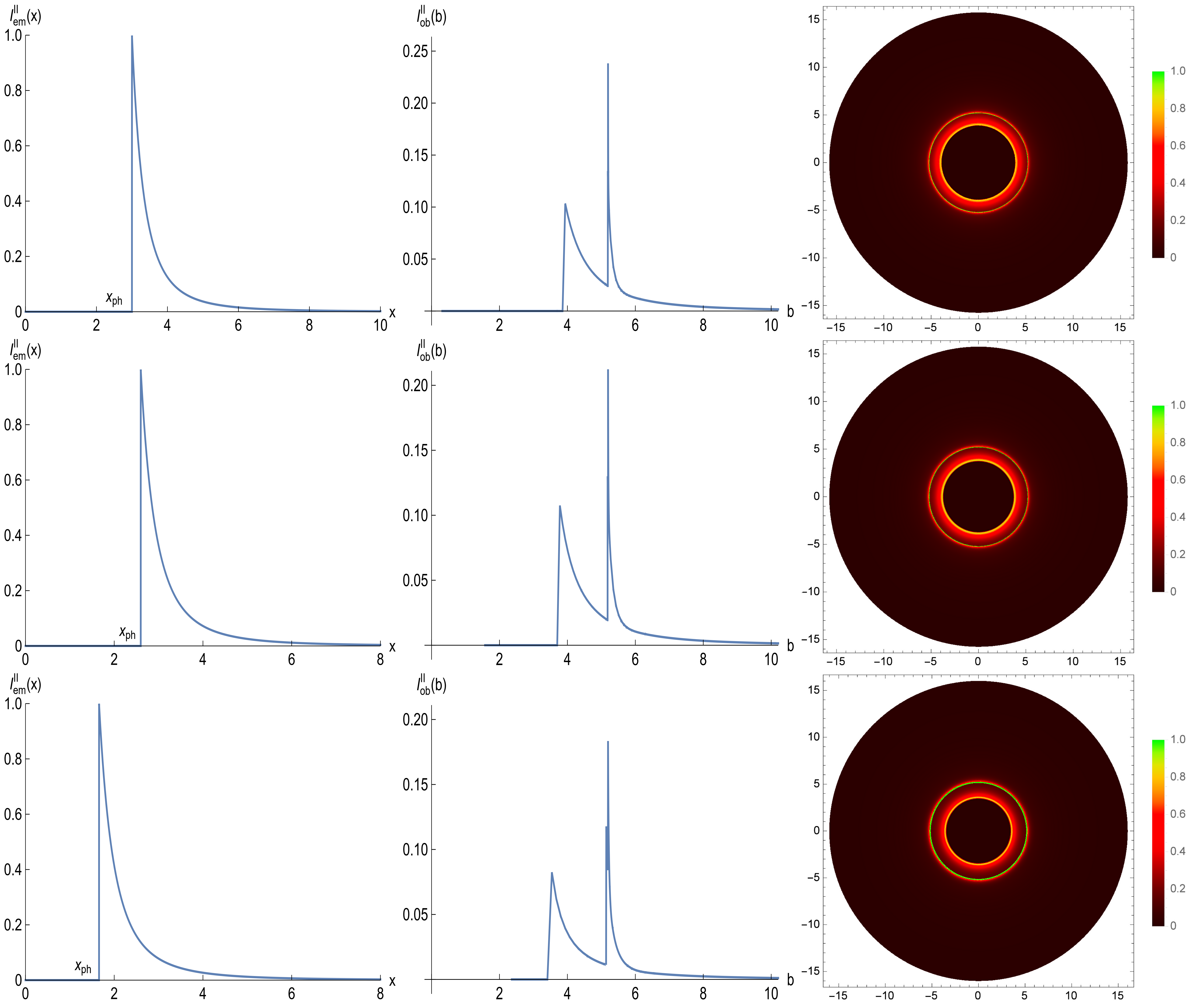
Regular BH

Wormhole

Black holes' mimickers: shadows

Optical appearance: shadows

Model II of emission



Schw. BH

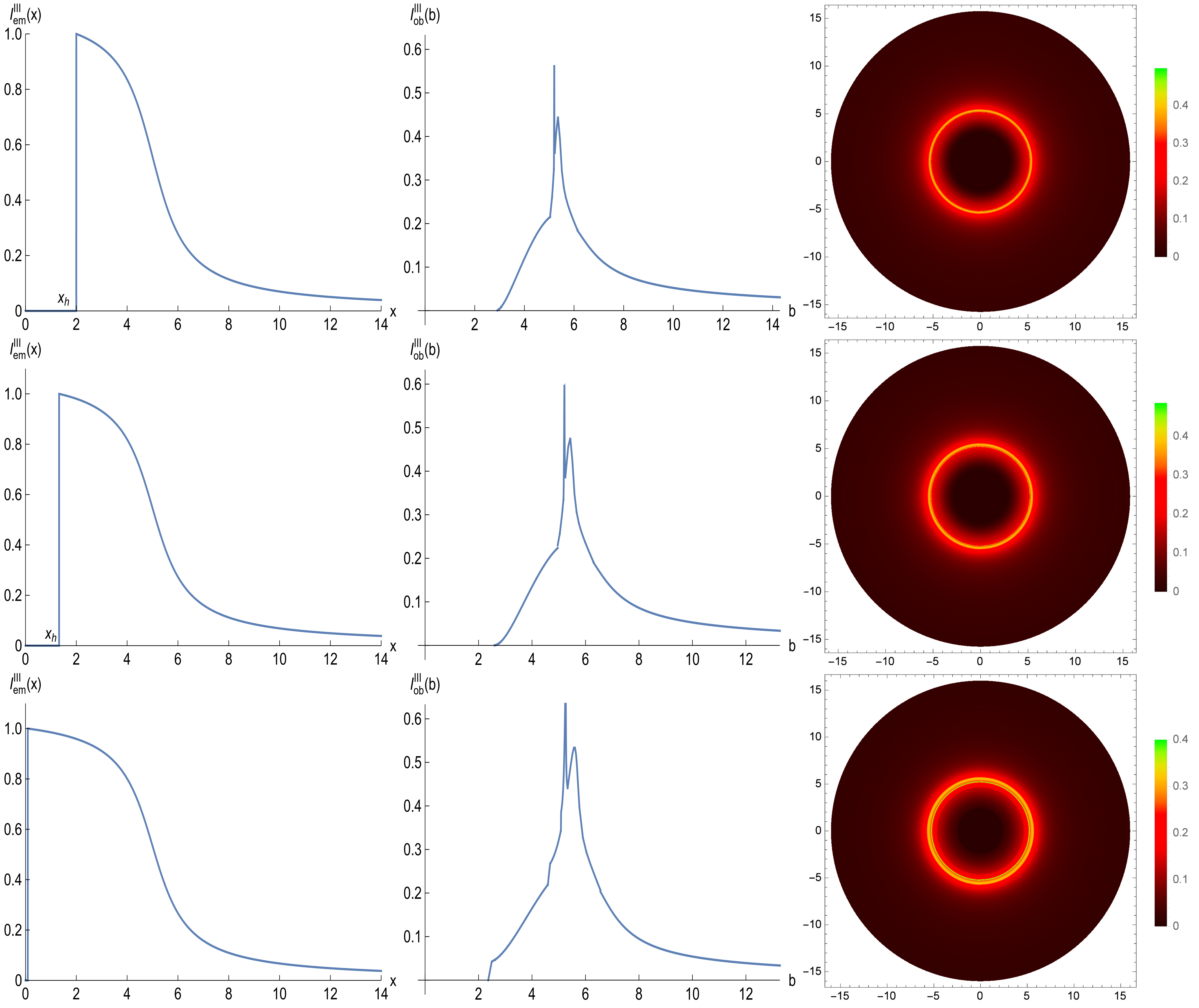
Regular BH

Wormhole

Black holes' mimickers: shadows

Optical appearance: shadows

Model III of emission



Schw. BH

Regular BH

Wormhole

Black holes' mimickers: shadows

Second type of models: double photon spheres

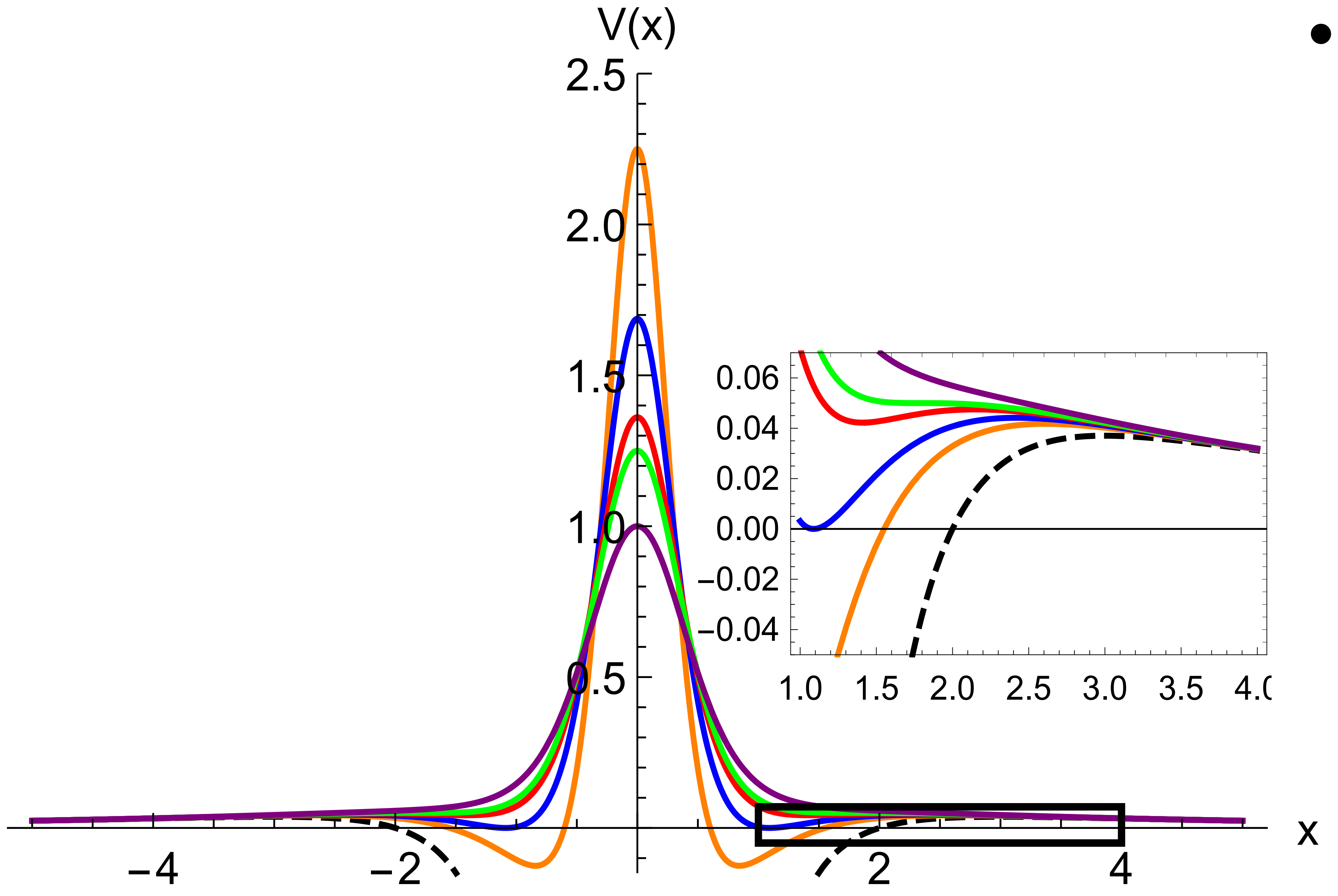
M. Guerrero, G. Olmo, D. Rubiera-García, DS-CG, arXiv:2202.03809

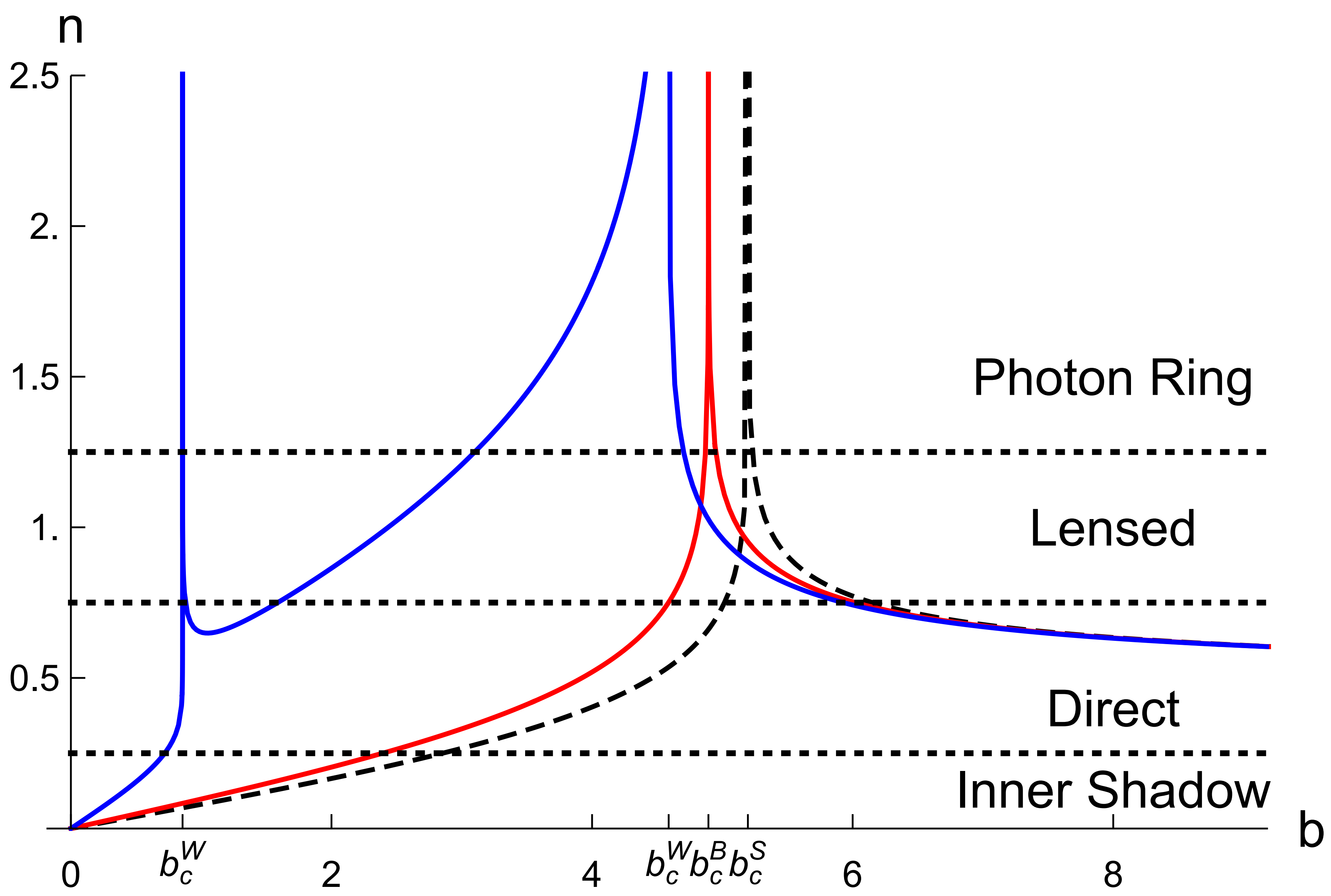
$$ds^2 = -A(x)dt^2 + B(x)dx^2 + r^2(x)d\Omega^2 \ , \qquad A(x) = B^{-1}(x) = 1 - \frac{2Mx^2}{(x^2+a^2)^{3/2}} \, ; \, r^2(x) = x^2 + a^2 \ .$$

Trajectory equation:

$$\dot{x}^2 = \frac{1}{b^2} - V(x) \ ,$$

- a=0 Schwarzschild black hole
- $0 < \frac{a}{M} < \frac{4\sqrt{3}}{9}$: two horizon black hole.
- $\frac{4\sqrt{3}}{9} < \frac{a}{M} < \frac{2\sqrt{5}}{5}$: a traversable wormhole with two photon spheres.

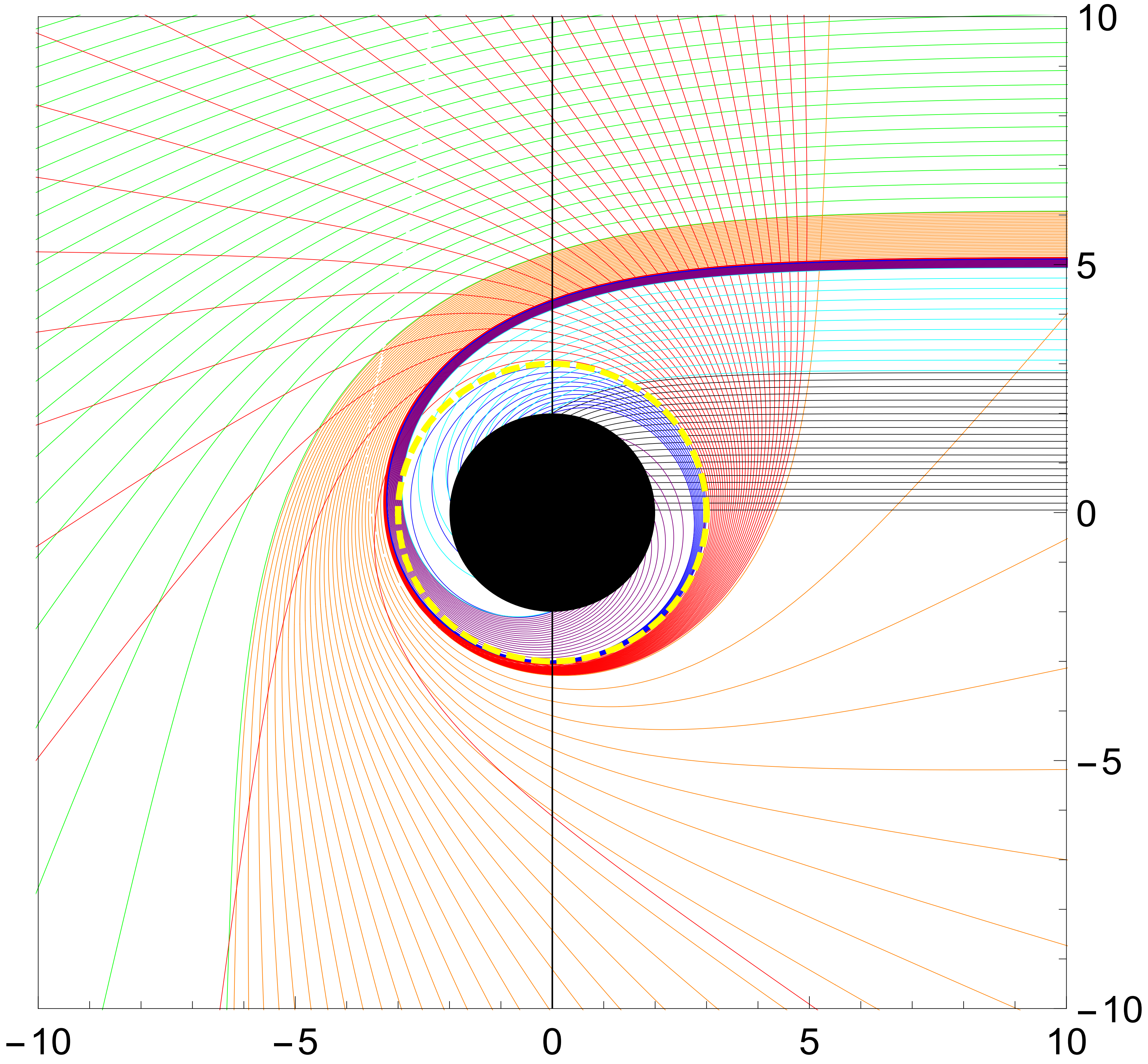




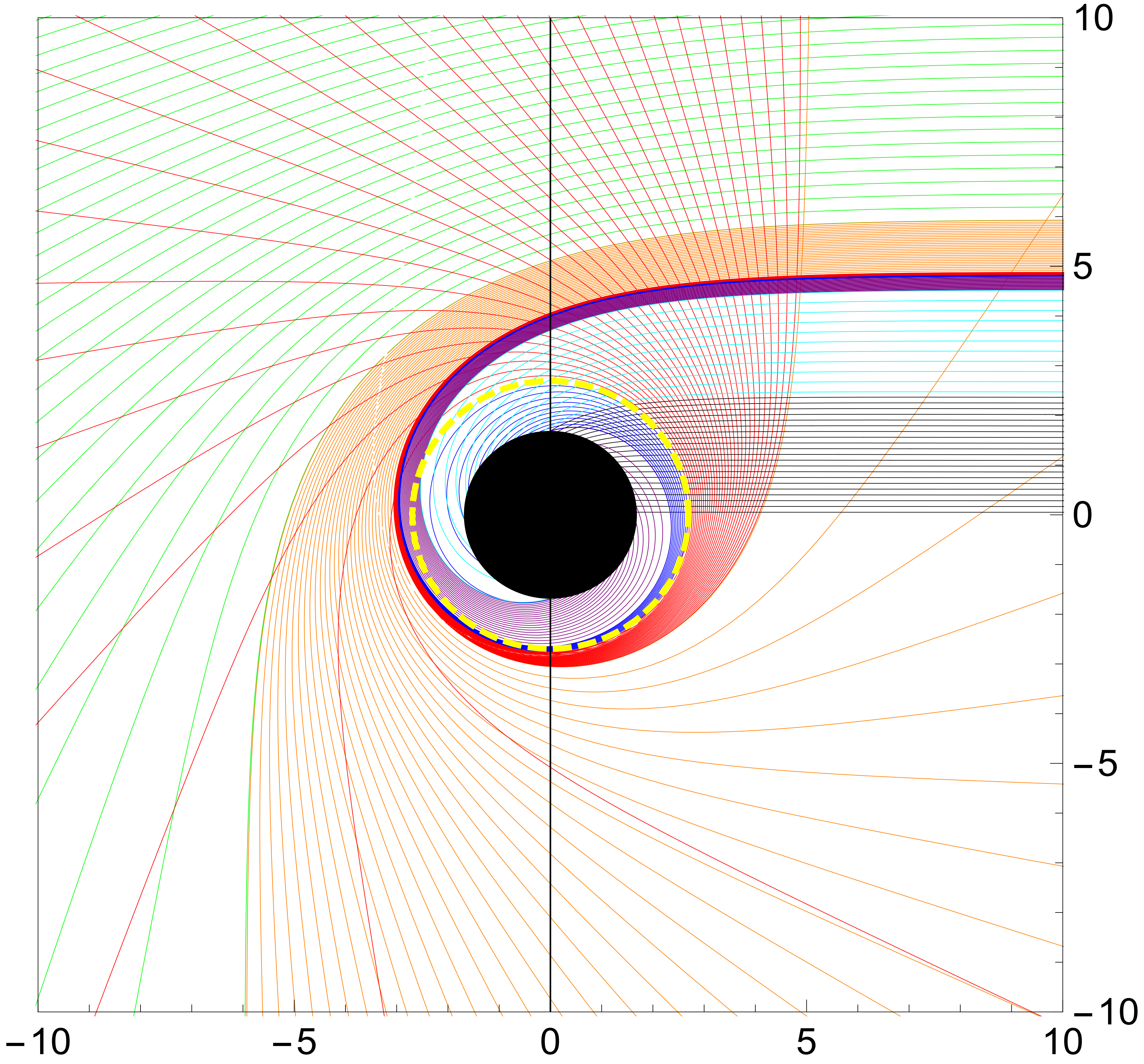
Black holes' mimickers: shadows

Ray-tracing

Schw. BH

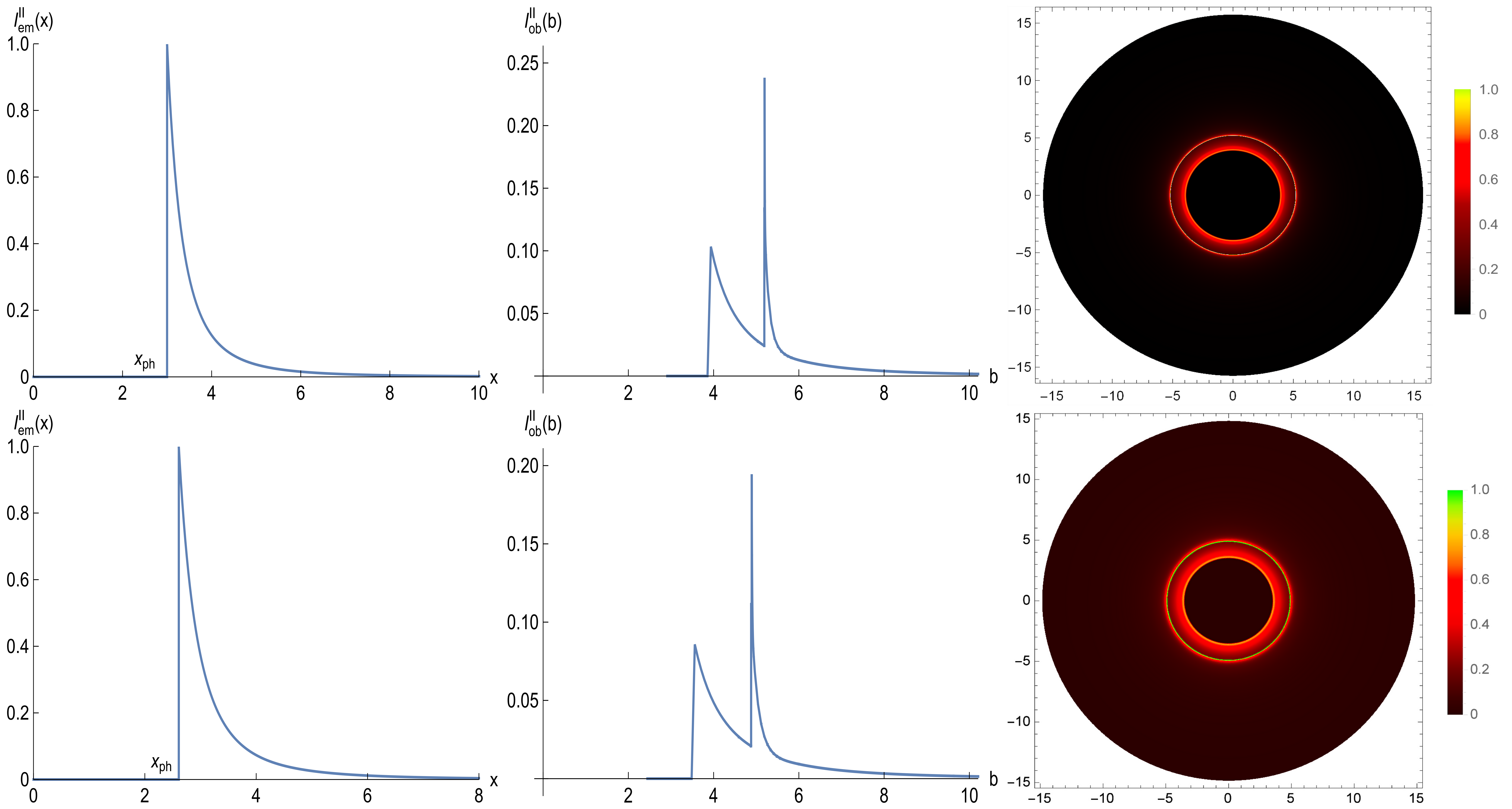


Two horizon BH



Black holes' mimickers: shadows

Optical appearance: shadows



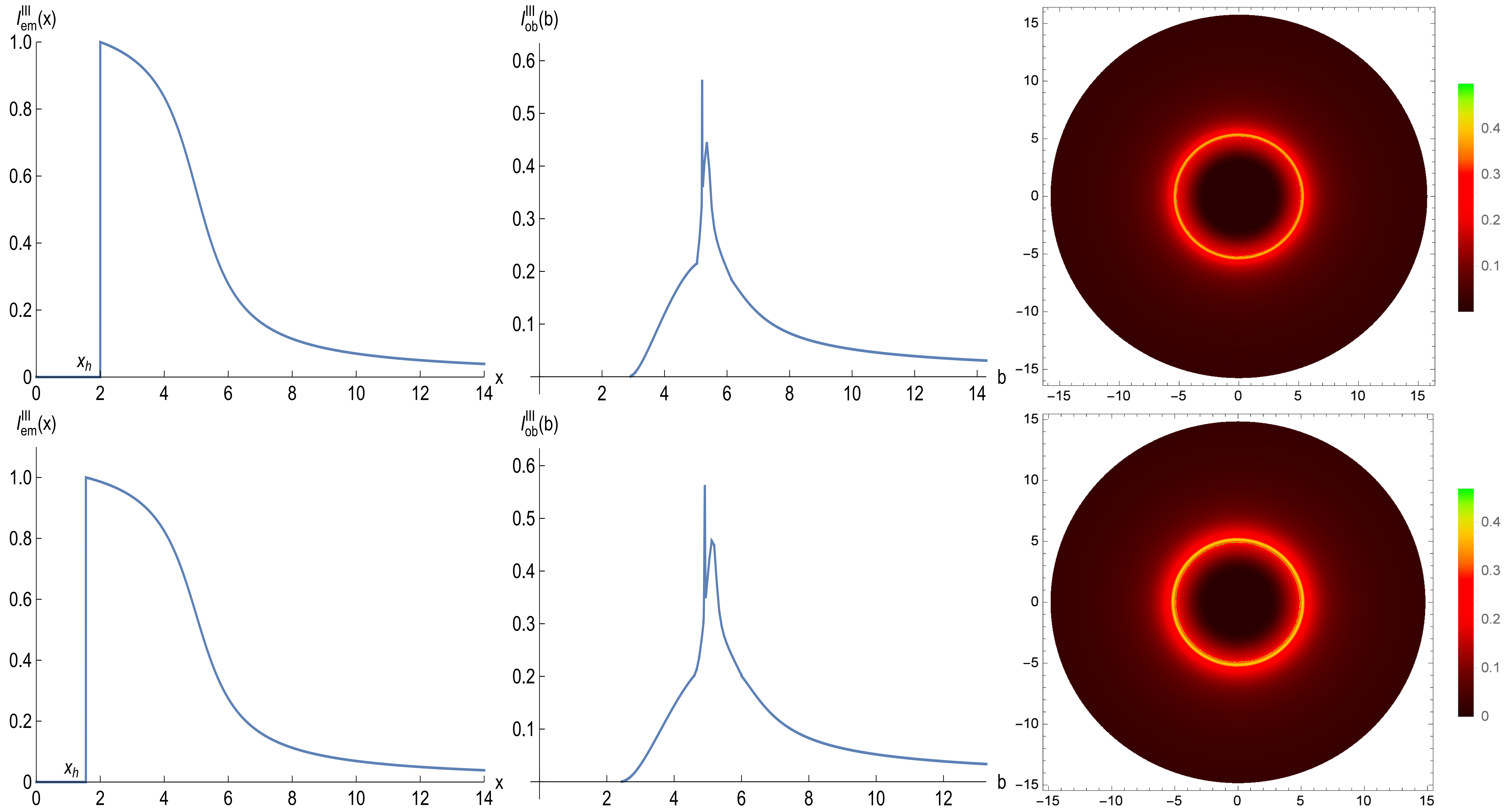
Schw. BH

Two horizon BH

Model II of emission

Black holes' mimickers: shadows

Optical appearance: shadows



Schw. BH

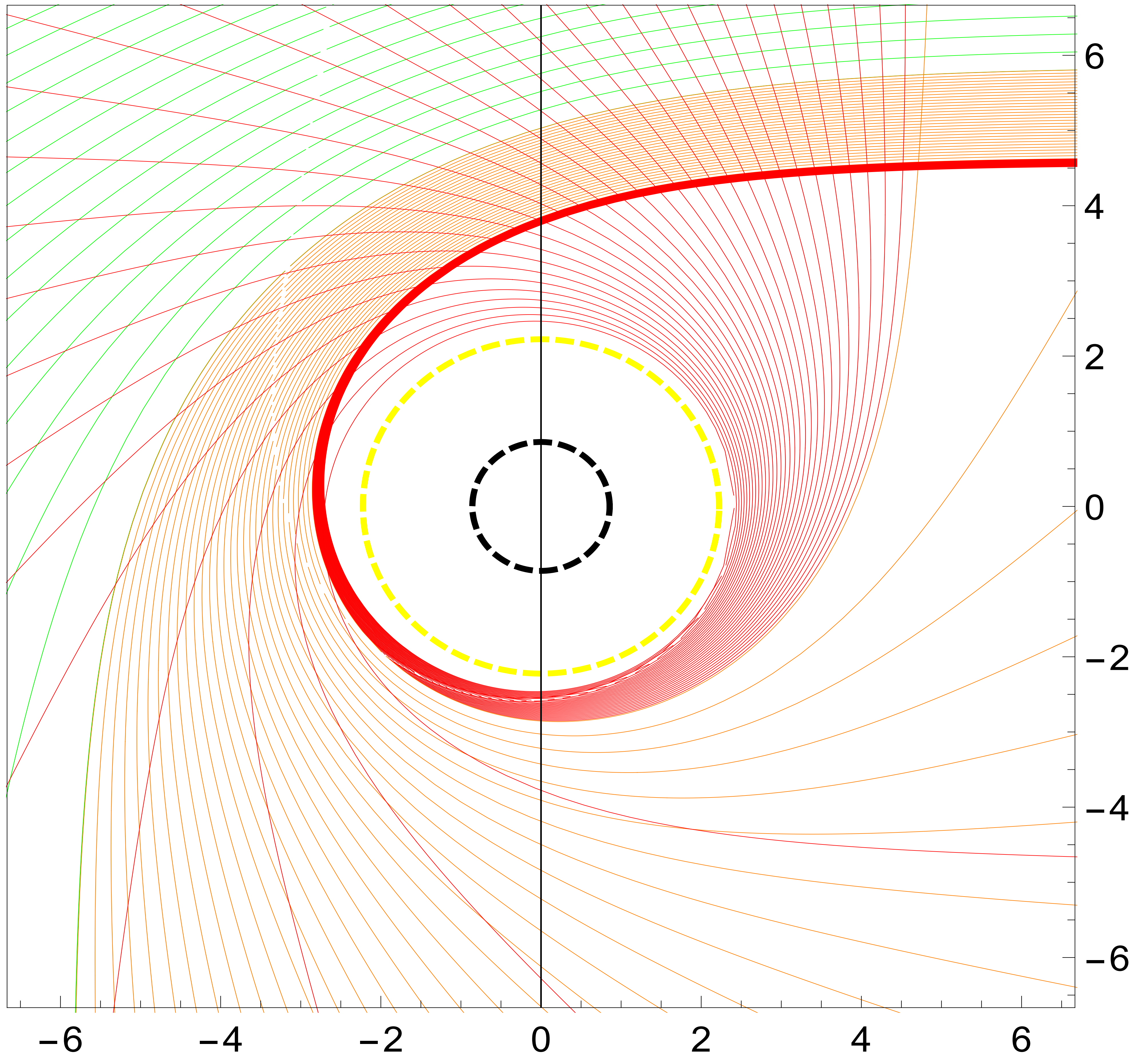
Two horizon BH

Model III of emission

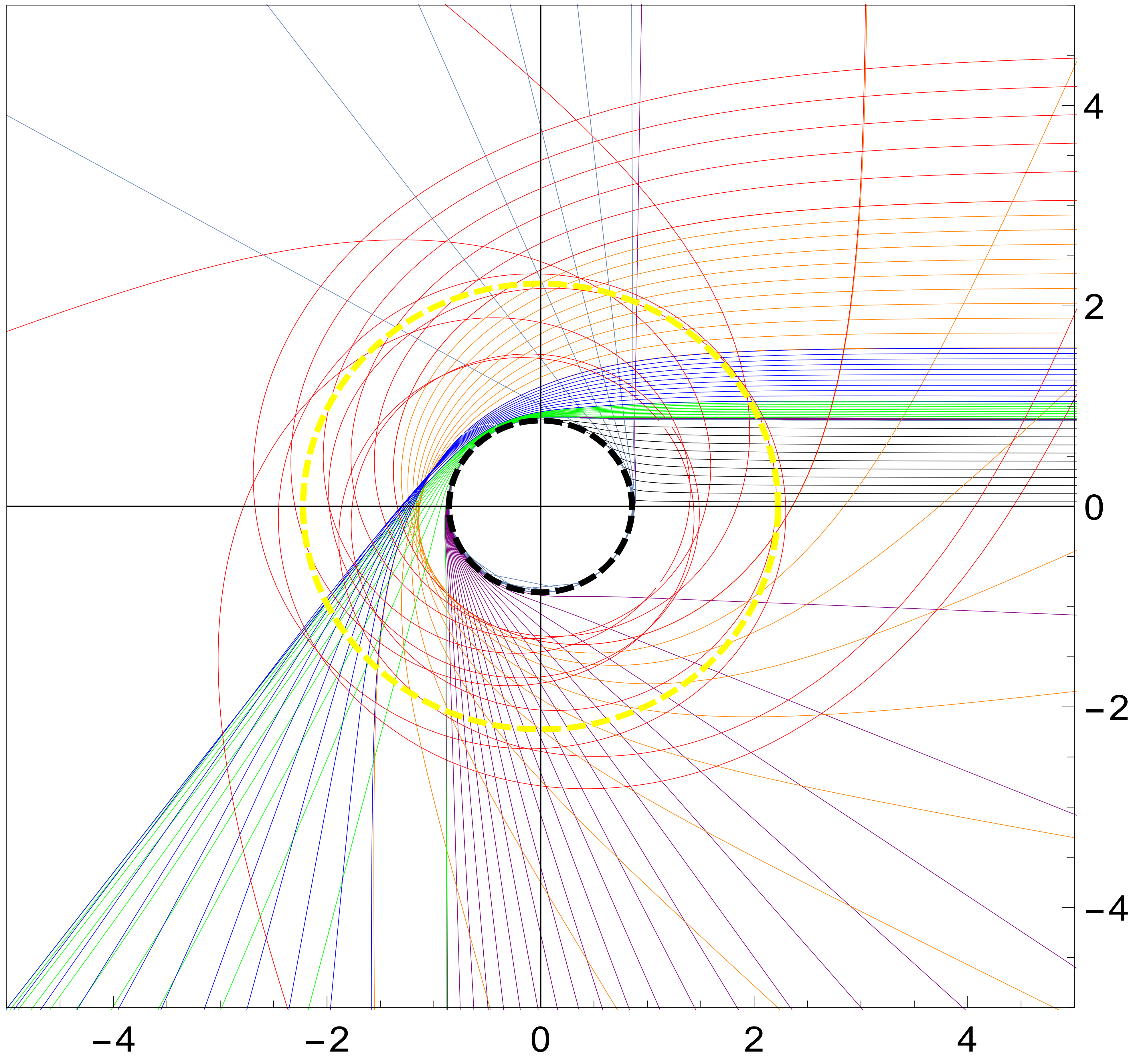
Black holes' mimickers: shadows

Ray-tracing: wormhole

Outer photon sphere



Inner photon sphere



Black holes' mimickers: shadows

Optical appearance: shadows

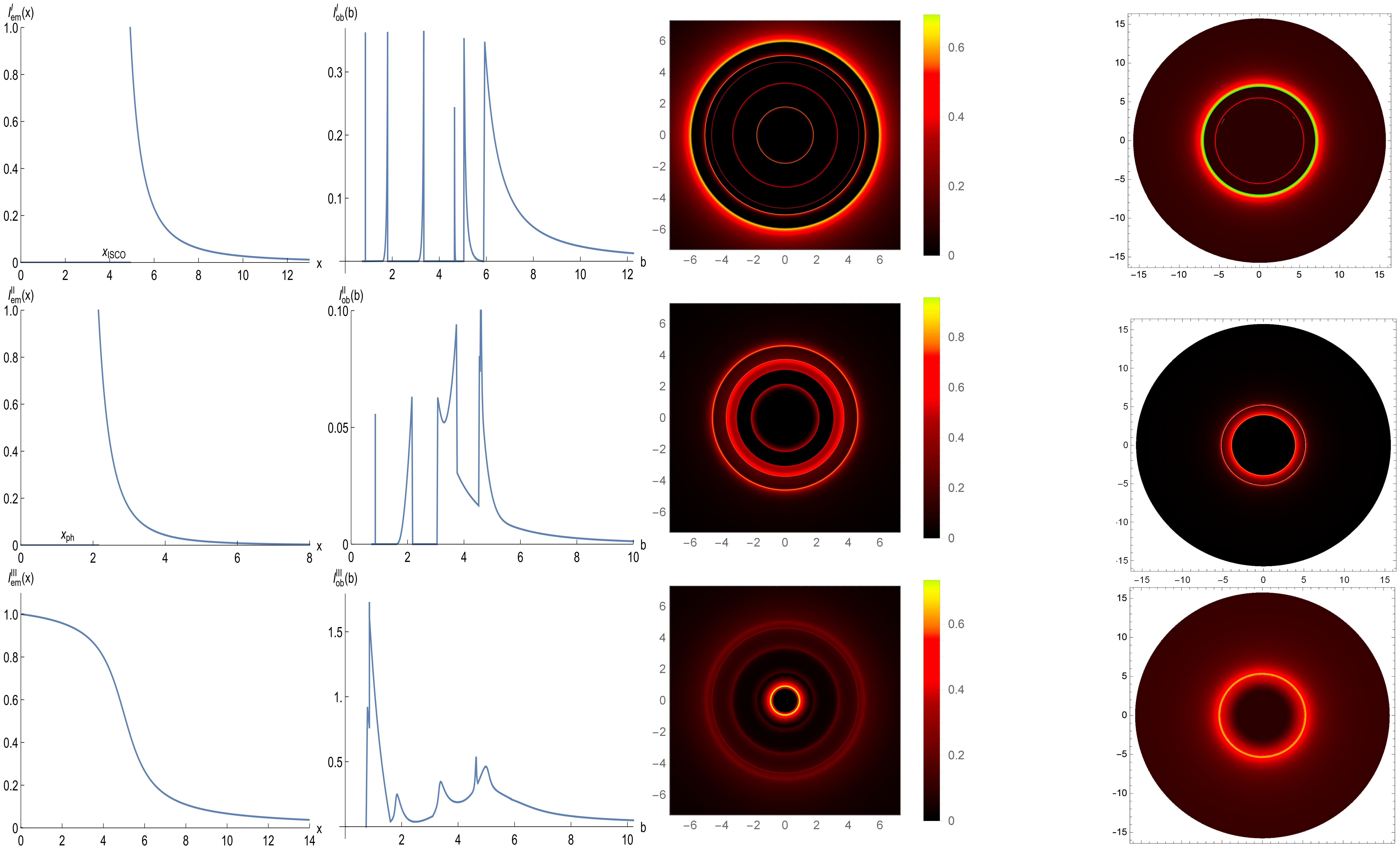
Wormhole

Schw. BH

Model I

Model II

Model III



Conclusions and perspectives

- Similar spacetime metrics characterising different objects might induce a different optical appearance for far away observers.

- We have just considered some very simple toy models of emission for the accretion disk but shown that the background geometry might be of high importance on the location and relative luminosities of the light rings.

- Nevertheless, the physics of the accretion disk might not be possible to be disentangled from the pure geometric effects.

- We have not considered rotating objects, neither inclinations of the disk relative to the observer.

- May future long base line interferometry be able to resolve the diffuse but sharp contribution from light rings associated to multiple photon spheres?

- The disentangle of the different effects associated to the background geometry might be a definite test of the Kerr-family of solutions.

Work funded by:
National Plans I+D+i (Spain): *New Strategies for testing gravity in strong-field regimes*, PID2020-117301GA-I00
Consolidated Research Unit (CyL): Mathematical Physics Group (UIC 011)

Thank you very much for your attention!

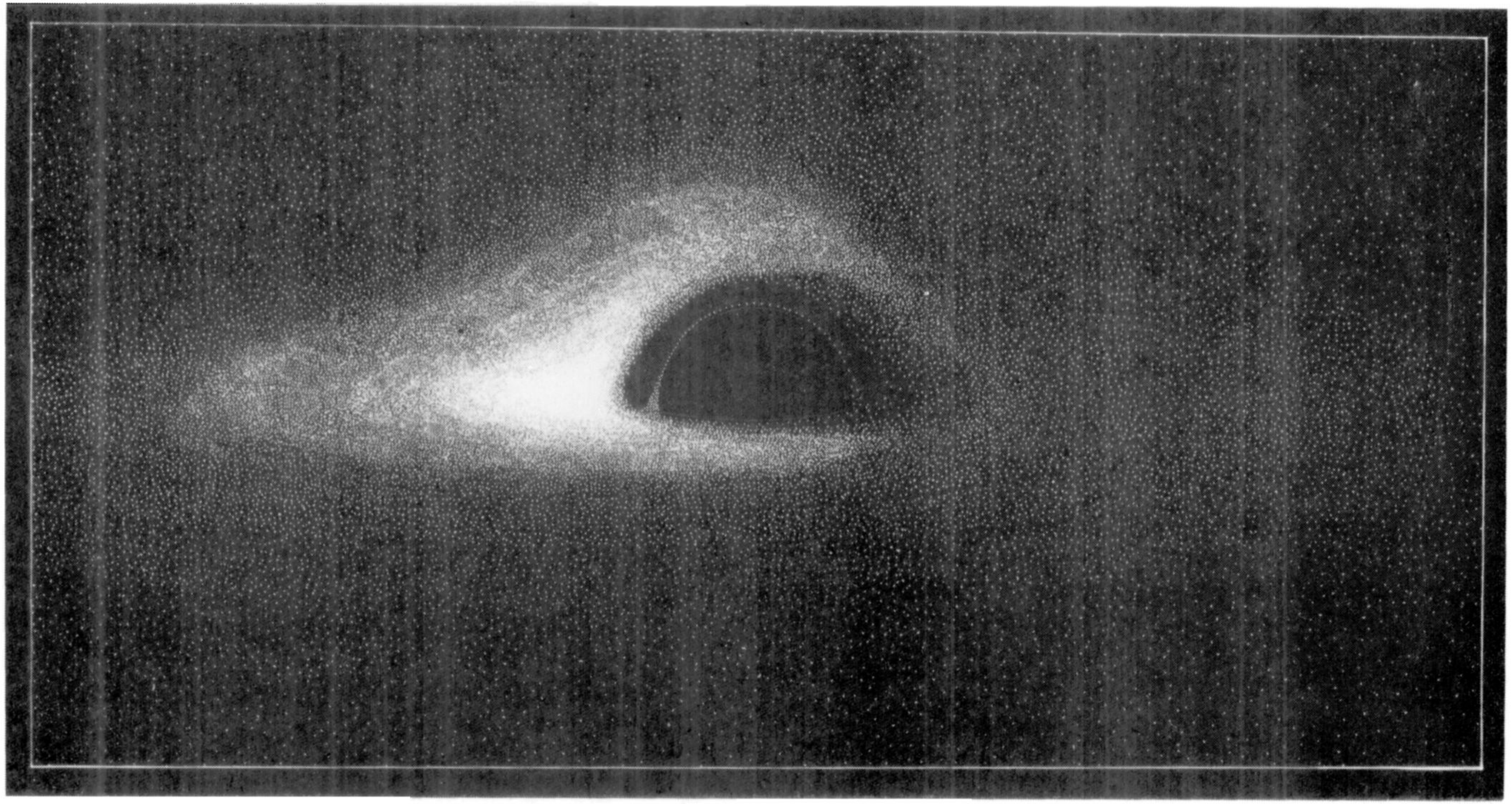


Image of a Spherical Black Hole with Thin Accretion Disk, **J. P. Luminet,** Astron. Astrophys. 75, 228 (1979).