Detecting (high frequency) gravitational waves in a box

based on 2112.11465 (PRD22) + 2303.01518 (PRD23)

(w. A. Berlin, DB, R. T. D'Agnolo, S. Ellis, R. Harnik, Y. Kahn, J. Schütte-Engel, M. Wentzel)

2312.02270 [hep-ph] P. Navarro, B. Gimeno, J. Monzón-Cabrera, A. Díaz-Morcillo

Diego Blas

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AGENCIA ESTATAL DE INVESTIGACIÓN

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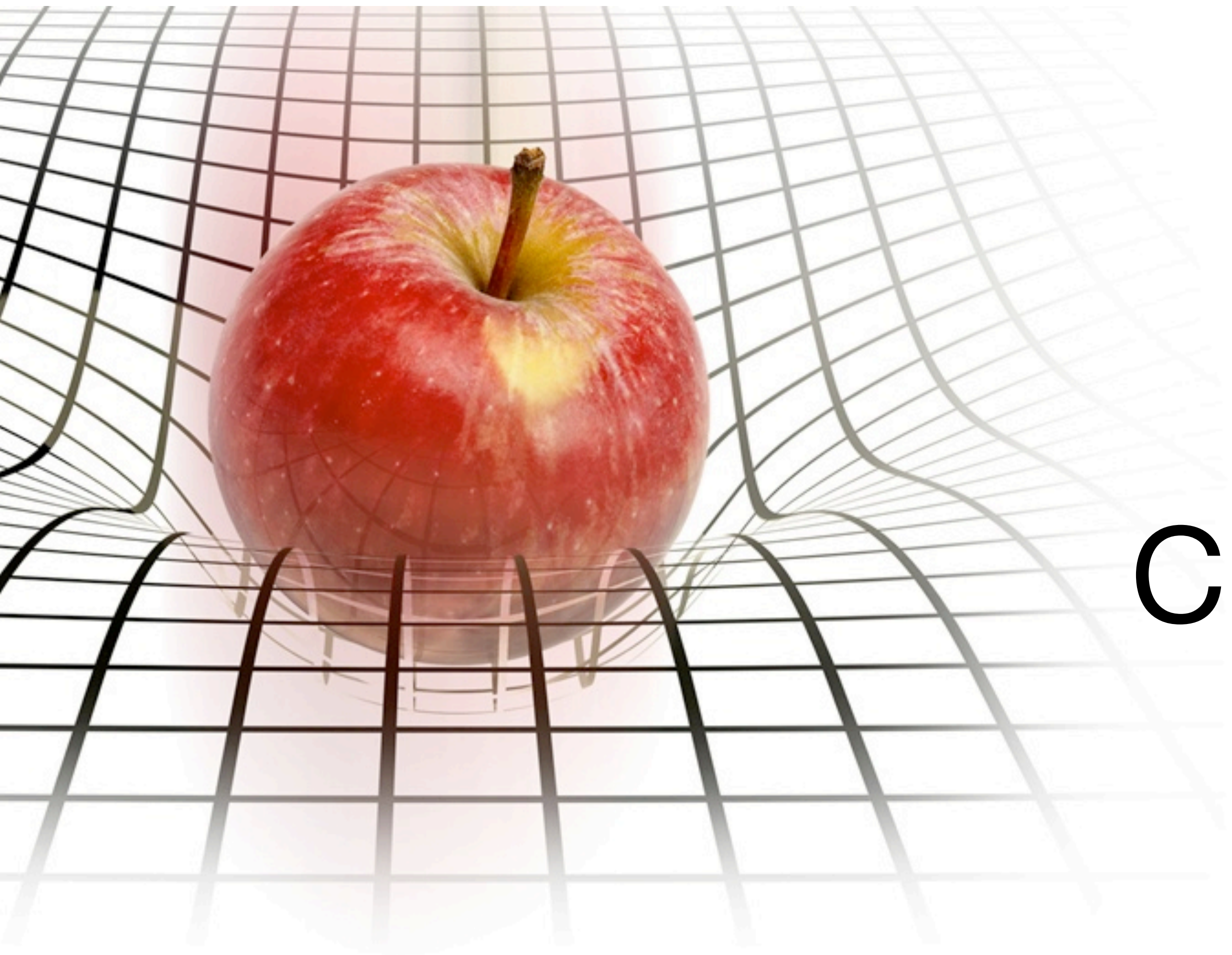
Departament de Recerca i Universitats

GWs (essentials)



Perturbations of space-time
travelling as waves of frequency f

Characterised by 2 polarizations $h_{+,\times}$ (dimensionless)

$$h_{+,\times} \overset{c=1}{\approx} h_0 \cos\left(2\pi f(t-z)+\phi\right)$$

GWs carry energy. They have **energy density**

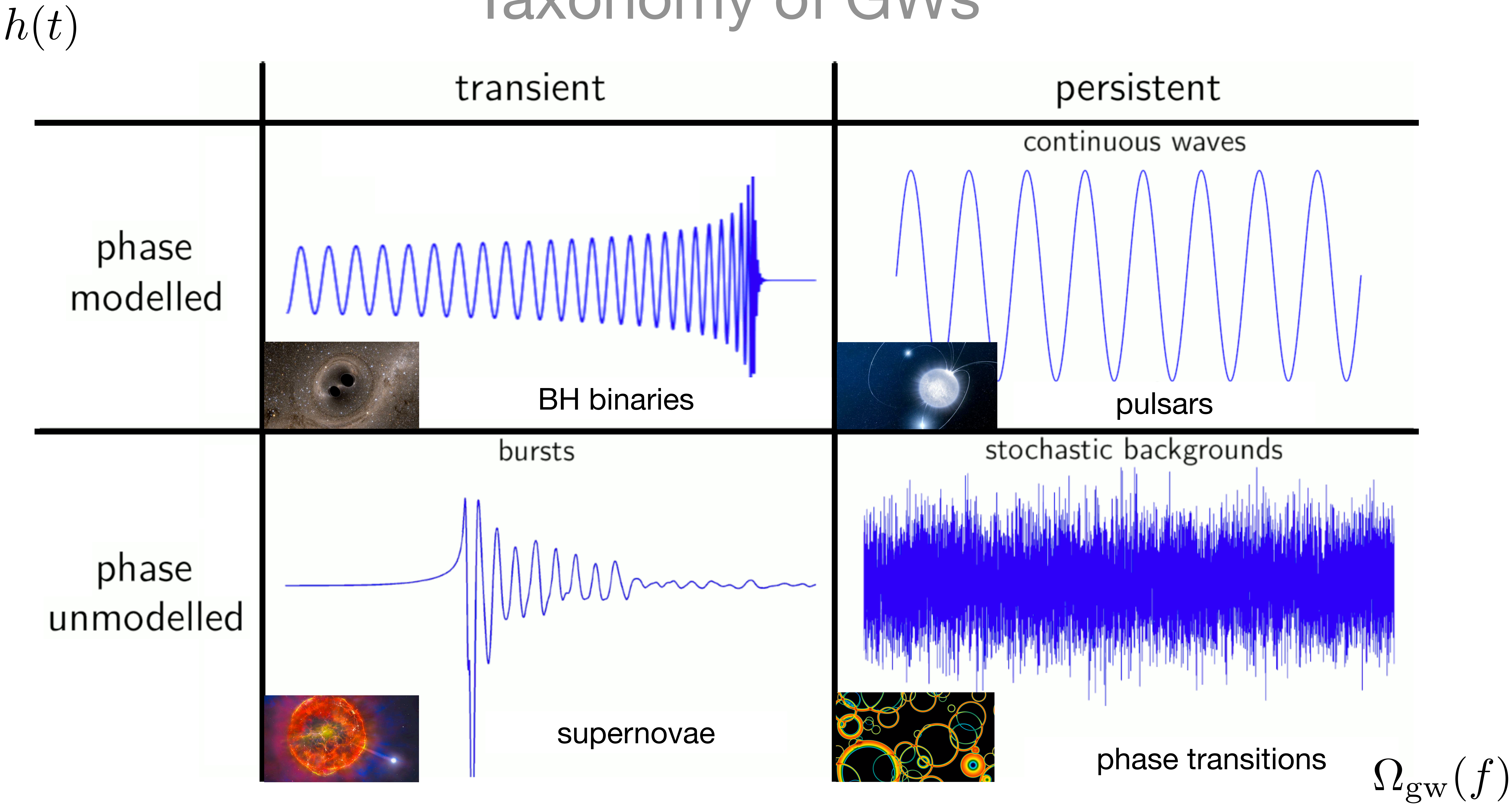
$$\rho_{\rm gw} = \frac{1}{16\pi G}\left\langle \dot{h}_+^2 + \dot{h}_\times^2 \right\rangle$$

$$\Omega_{\rm gw}(f) \equiv \frac{1}{\rho_c}\frac{\mathrm{d}\rho_{\rm gw}}{\mathrm{d}(\ln f)}$$

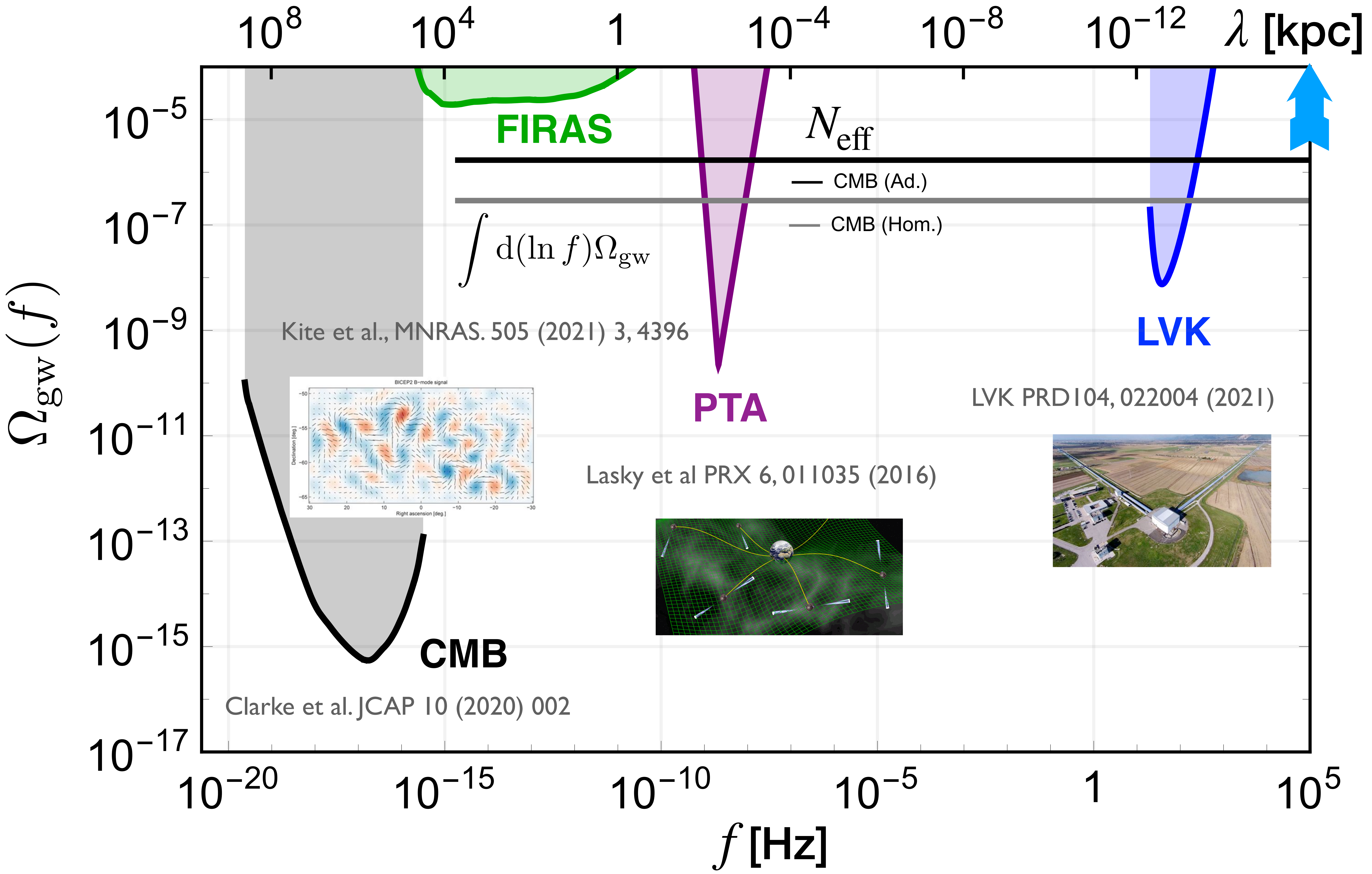
$h^2\Omega_{\rm gw}(f)$ $\quad h \approx 0.67$

$$\rho_c = 1.2\times 10^{11} M_\odot \mathrm{Mpc}^{-3}$$
$$\sim \mathrm{keV/cm}^3$$

Taxonomy of GWs

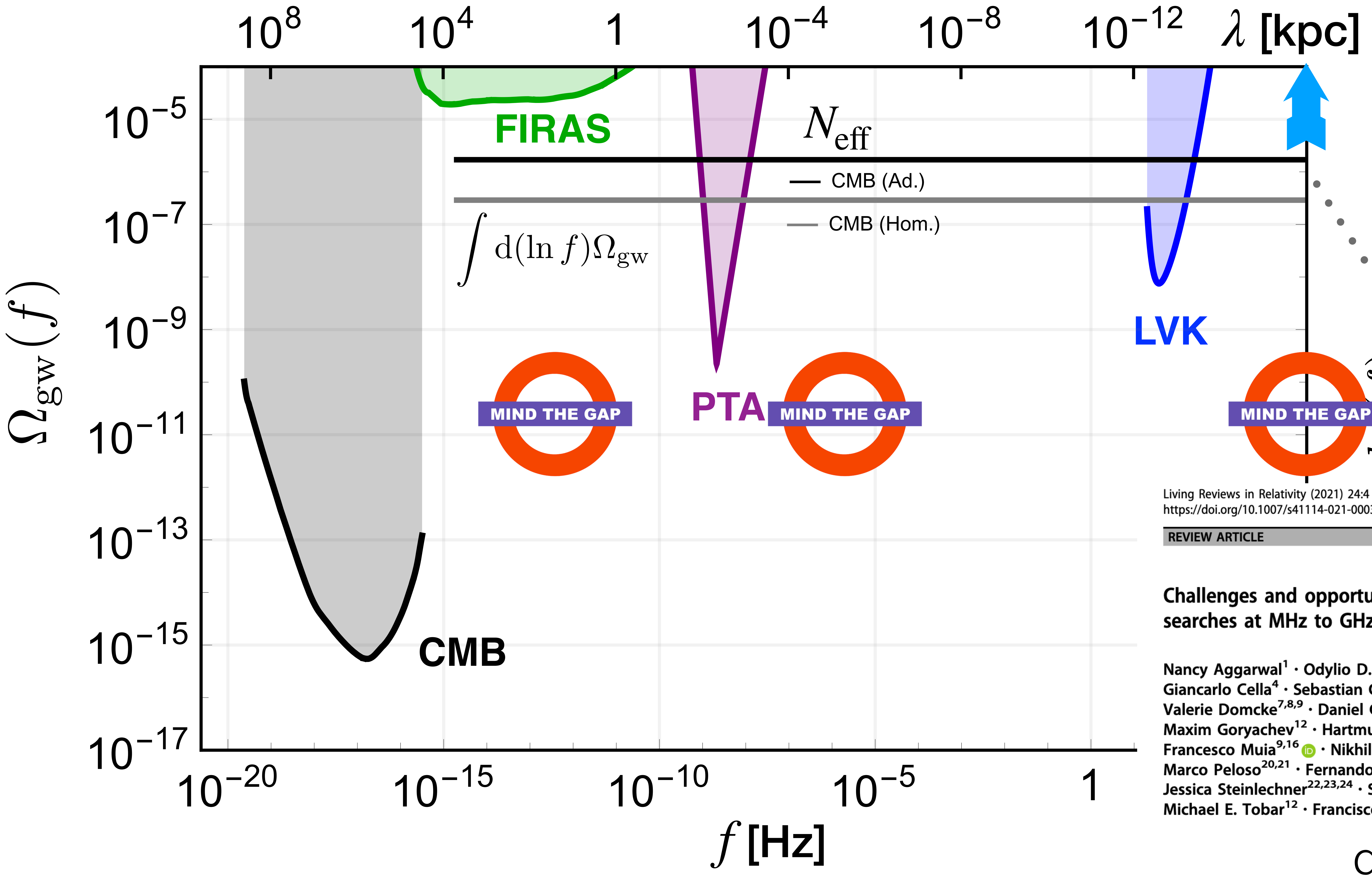


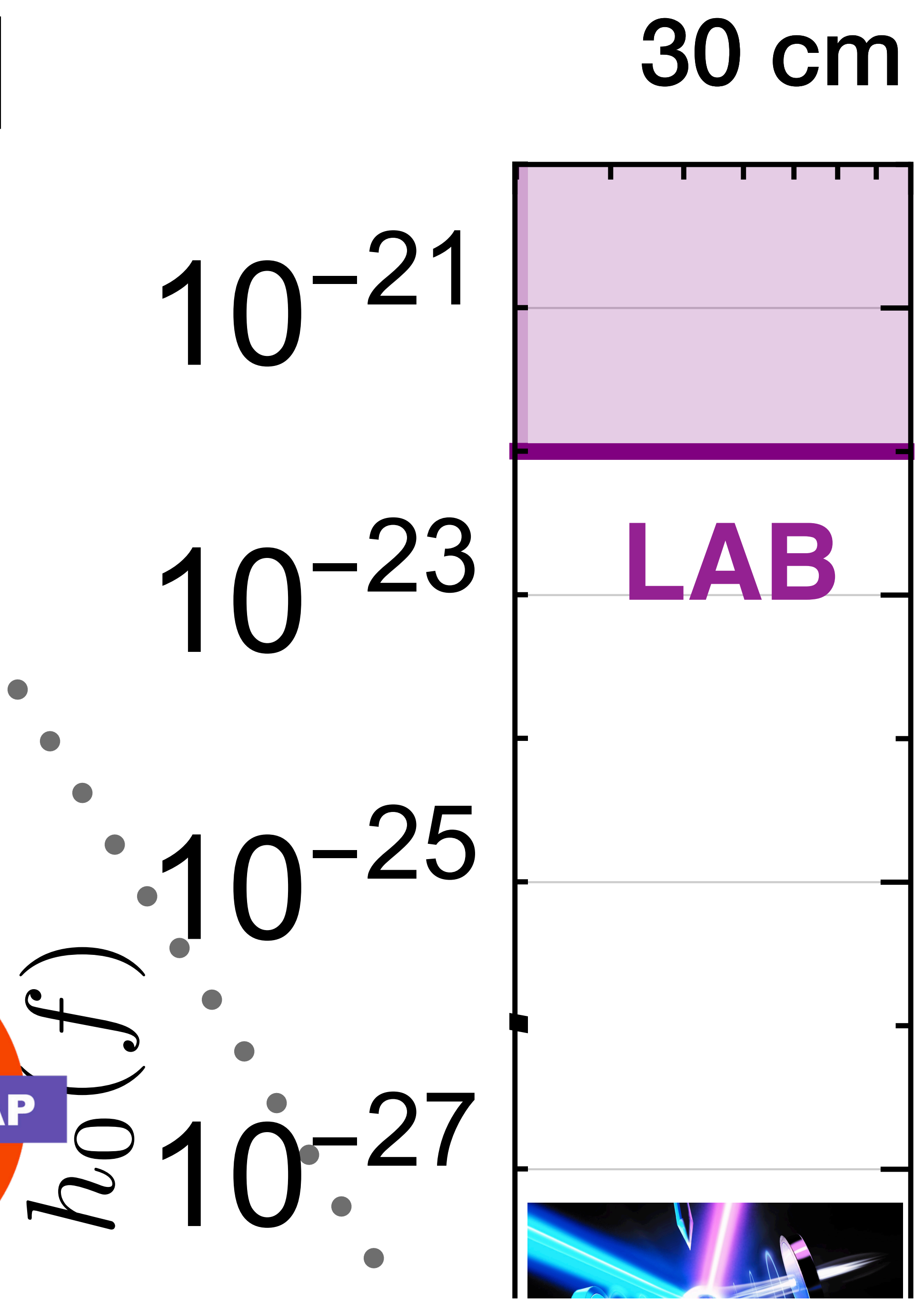
GWs soundscape today



GWs soundscape today
10^{8} 10^{4} 1 10^{-4} 10^{-8} 10^{-12} λ [kpc]
30 cm
$\Omega_{\rm gw}(f)$
10^{-5} 10^{-7} 10^{-9} 10^{-11} 10^{-13} 10^{-15} 10^{-17}
FIRAS
$N_{\rm eff}$
CMB (Ad.)
CMB (Hom.)
$\int d(\ln f)\Omega_{\rm gw}$
LVK
PTA
MIND THE GAP
MIND THE GAP
MIND THE GAP
CMB
10^{-20} 10^{-15} 10^{-10} 10^{-5} 1 10^{5}
f [Hz]
$h_0(f)$
10^{-21} 10^{-23} 10^{-25} 10^{-27} 10^{-29} 10^{-31} 10^{-33}
LAB
10^{9}

GWs soundscape today





Living Reviews in Relativity (2021) 24:4
https://doi.org/10.1007/s41114-021-00032-5

REVIEW ARTICLE

Check for updates

Challenges and opportunities of gravitational-wave searches at MHz to GHz frequencies

Nancy Aggarwal[1] · Odylio D. Aguiar[2] · Andreas Bauswein[3] · Giancarlo Cella[4] · Sebastian Clesse[5] · Adrian Michael Cruise[6] · Valerie Domcke[7,8,9] · Daniel G. Figueroa[10] · Andrew Geraci[11] · Maxim Goryachev[12] · Hartmut Grote[13] · Mark Hindmarsh[14,15] · Francesco Muia[9,16] · Nikhil Mukund[17] · David Ottaway[18,19] · Marco Peloso[20,21] · Fernando Quevedo[16] · Angelo Ricciardone[20,21] · Jessica Steinlechner[22,23,24] · Sebastian Steinlechner[22,23] · Sichun Sun[25,26] · Michael E. Tobar[12] · Francisco Torrenti[27] · Caner Ünal[28] · Graham White[29]

growing interest!

UHFGWs -> Laboratory searches

GWs interact with **every** source of energy-momentum!

in the laboratory

*Interaction GWs with **light***

*Interaction GWs with **matter external dofs***

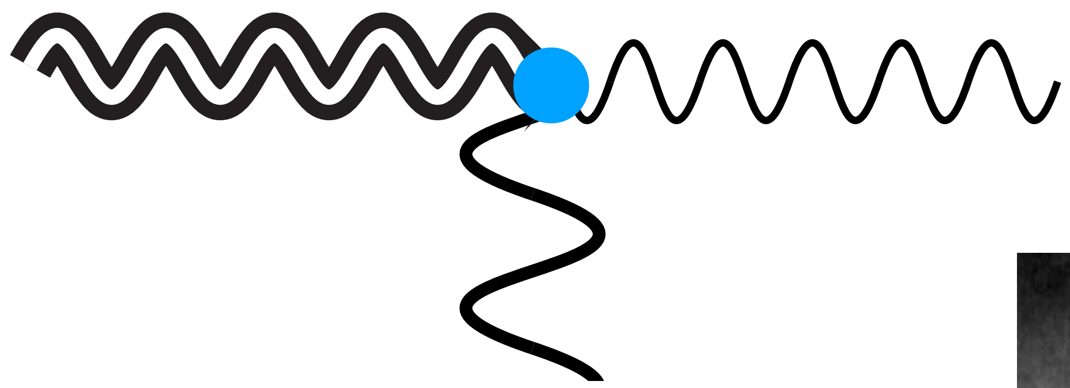
*Interaction GWs with **spin or other internal dofs***

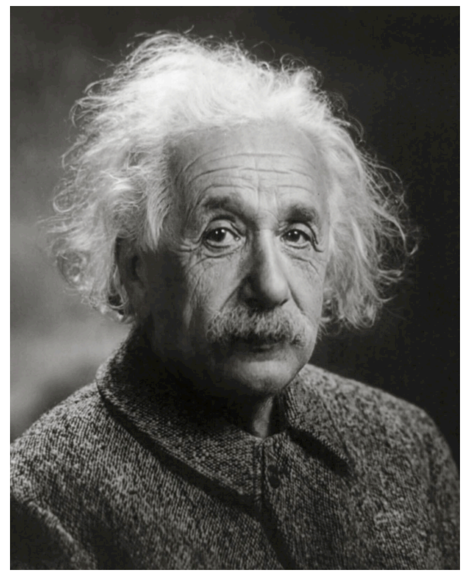
we have a lot to learn from DM searches!

GWs and EM fields

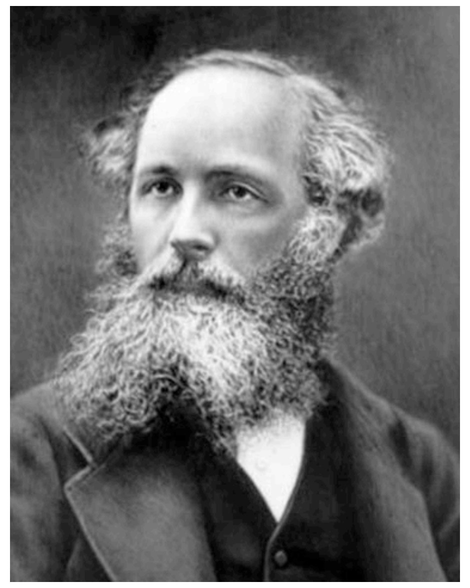
Raffelt Stodolsky 87

GWs + EM field -> EM field





$$\mathcal{L} = \sqrt{-g}\,(R + F_{\mu\nu}F^{\mu\nu}) \supset \frac{1}{2}A_\mu j^\mu_{\rm eff}(h) + \eta^{\mu\alpha}\eta^{\nu\beta}F_{\mu\nu}F_{\alpha\beta} + O(h^2)$$



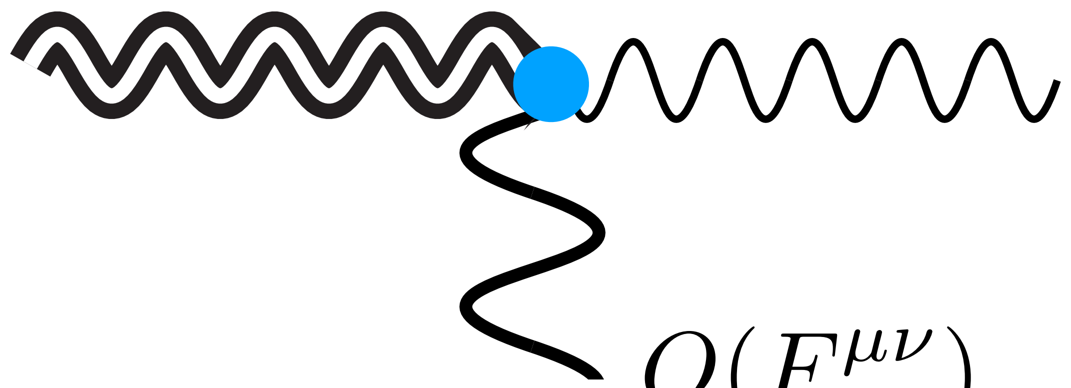
$$j^\mu_{\rm eff} = -\partial_\beta\left(\frac{1}{2}hF^{\mu\beta} + h^\beta_\alpha F^{\alpha\mu} - h^\mu_\alpha F^{\alpha\beta}\right)$$

analogy with **axions** + EM field -> EM field

(Safdi's talk)

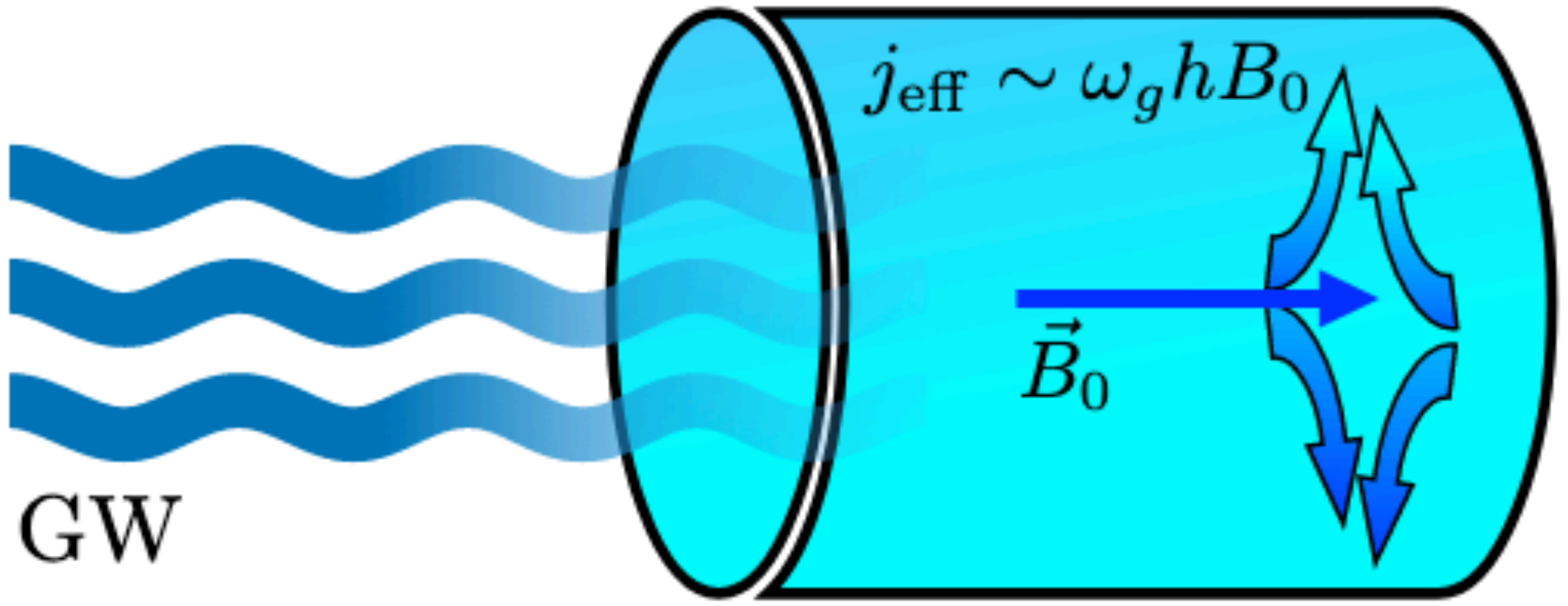
$$a\tilde{F}F$$

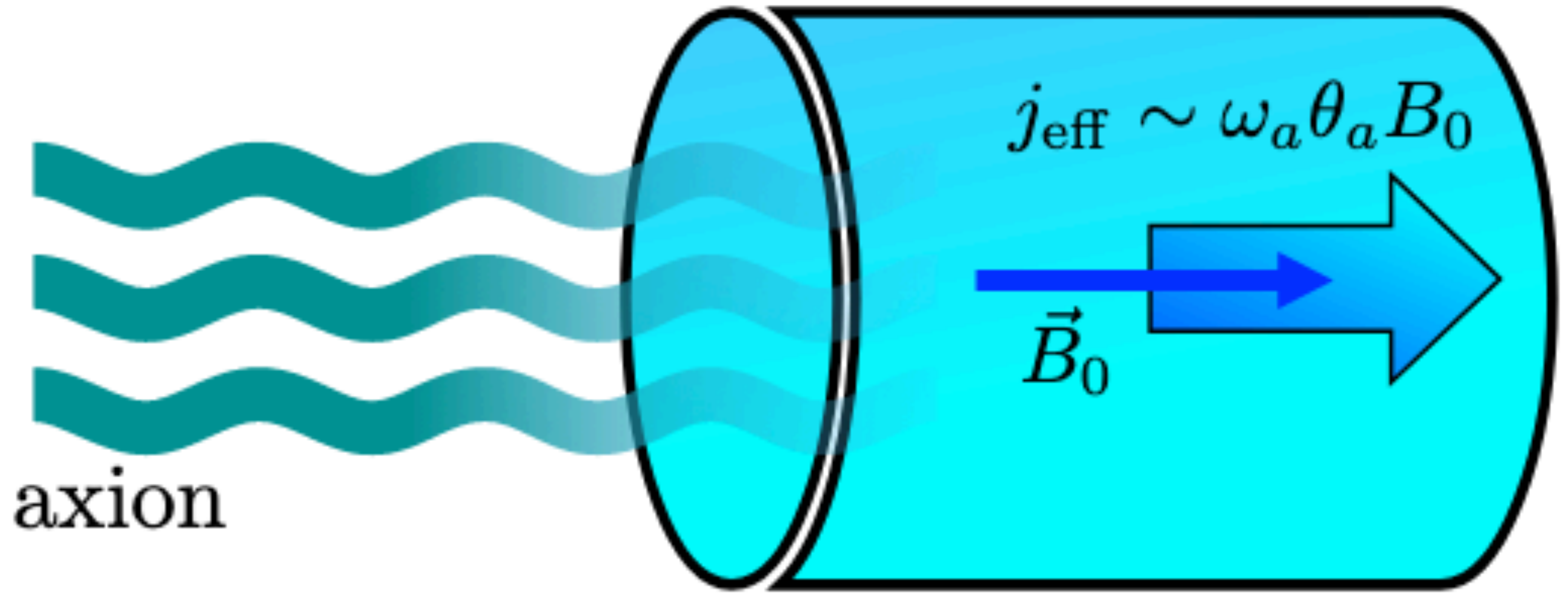
a



$O(F^{\mu\nu})$

Connection to axions





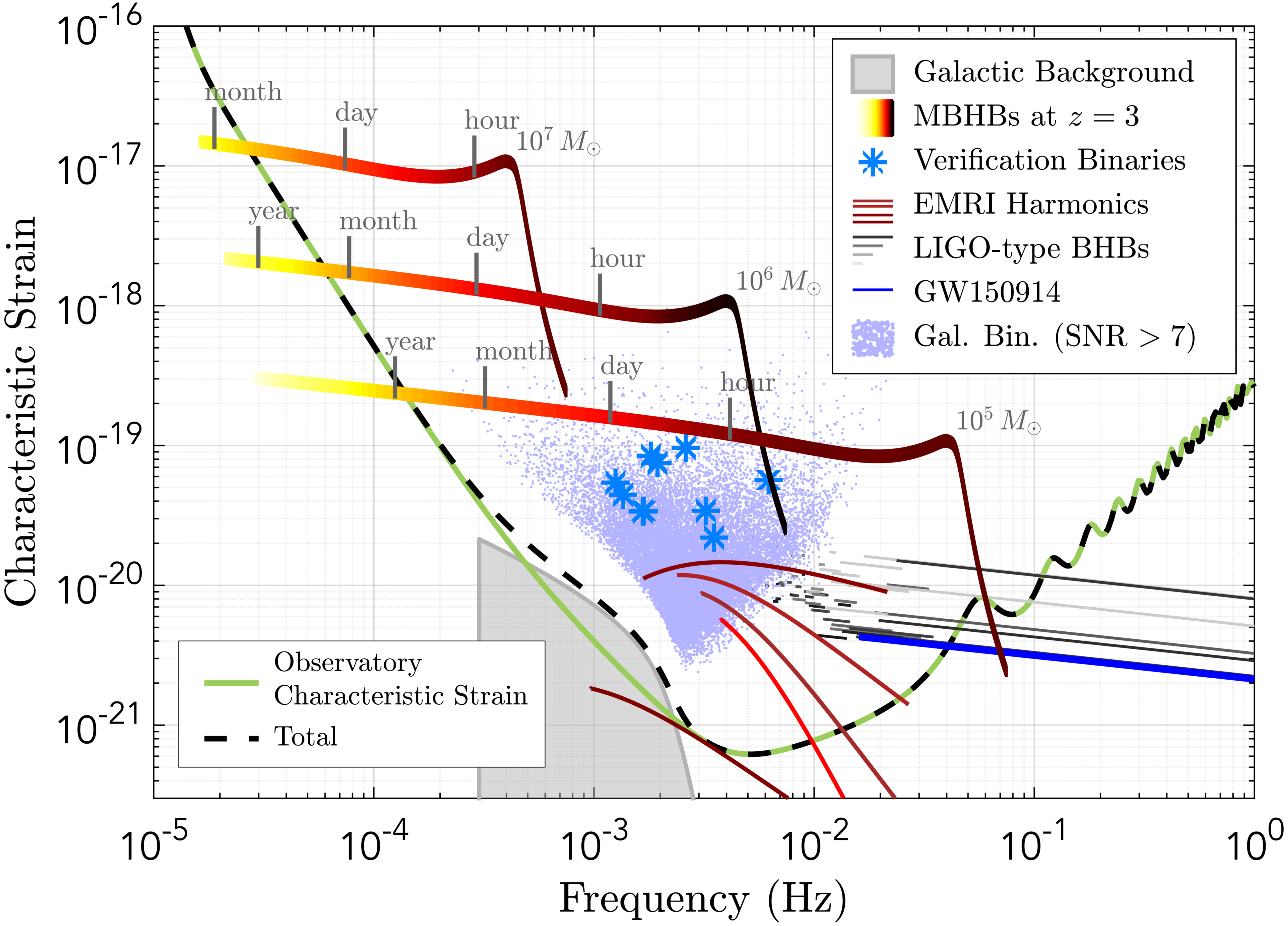
This already sets the scale of the GW we want to measure:

for resonant production (for constant $\overrightarrow{B}$) $\lambda_{gw} \approx L$

waves of (a priori) laboratory size

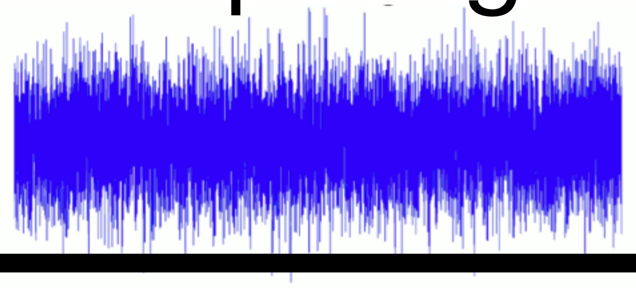
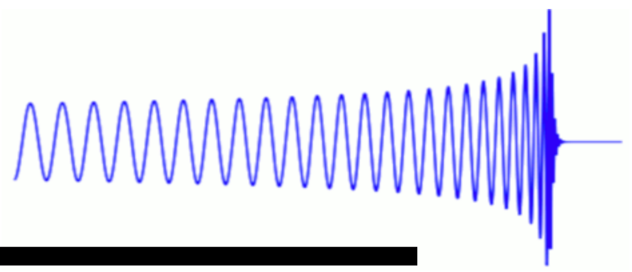
LISA Sources

Amaro-Seoane et al. 1702.00786



Sources of UHFGW ($> 10\,\mathrm{kHz}$)

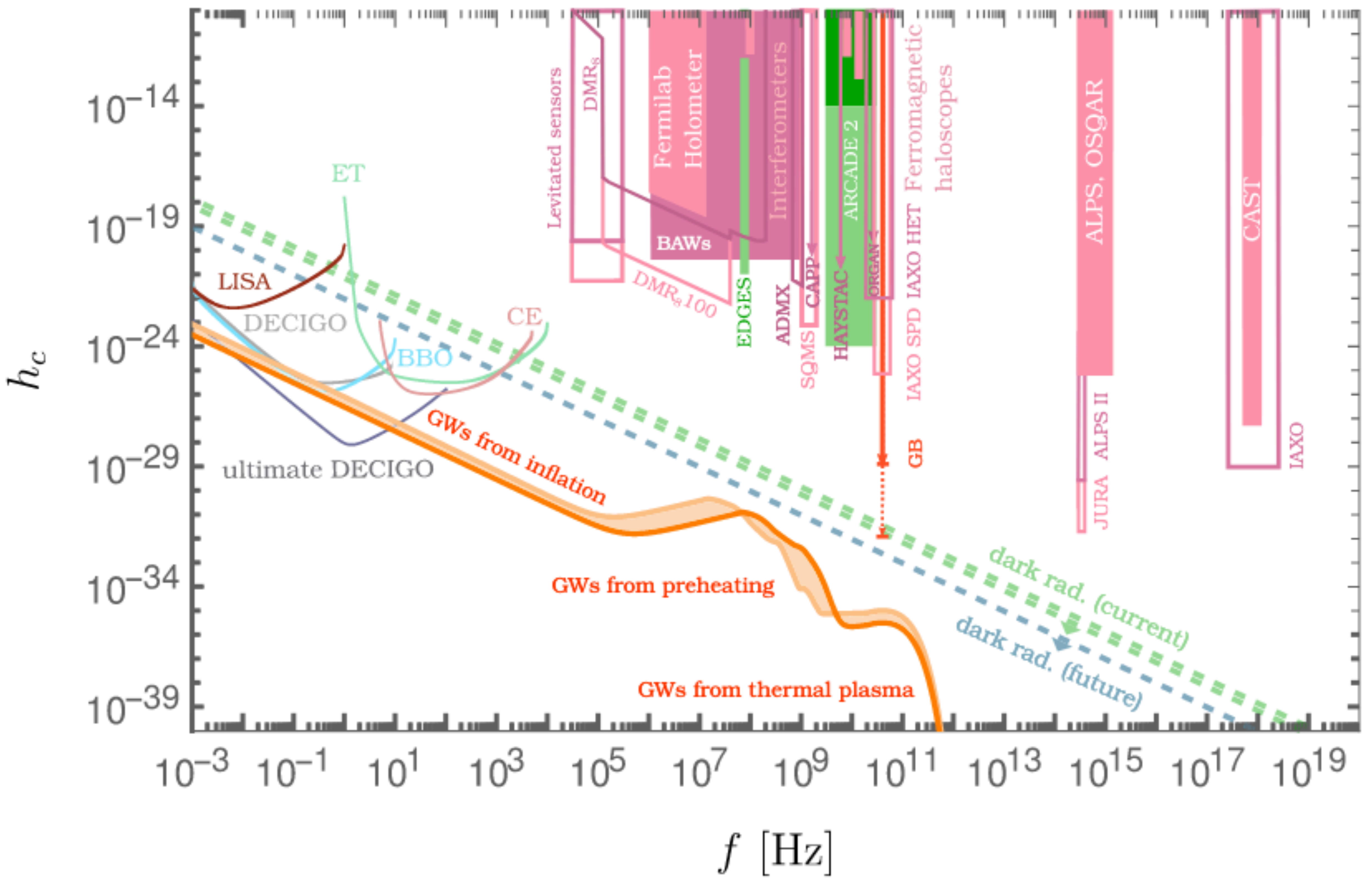
Very different picture comparing to EM spectrum

	Stochastic 	Coherent 
standard physics	Primordial plasma thermal fluctuations ✓ Ghiglieri et al JHEP 07 (2020) 092 Ringwald et al JCAP 03 (2021) 054	? ✗
	Inflation	
new physics	Phase transitions, Cosmic strings Close encounters of BHs ✓	Light PBHB Ultra-light DM Exotic objects ✓

Sources of UHFGW: spoiler!

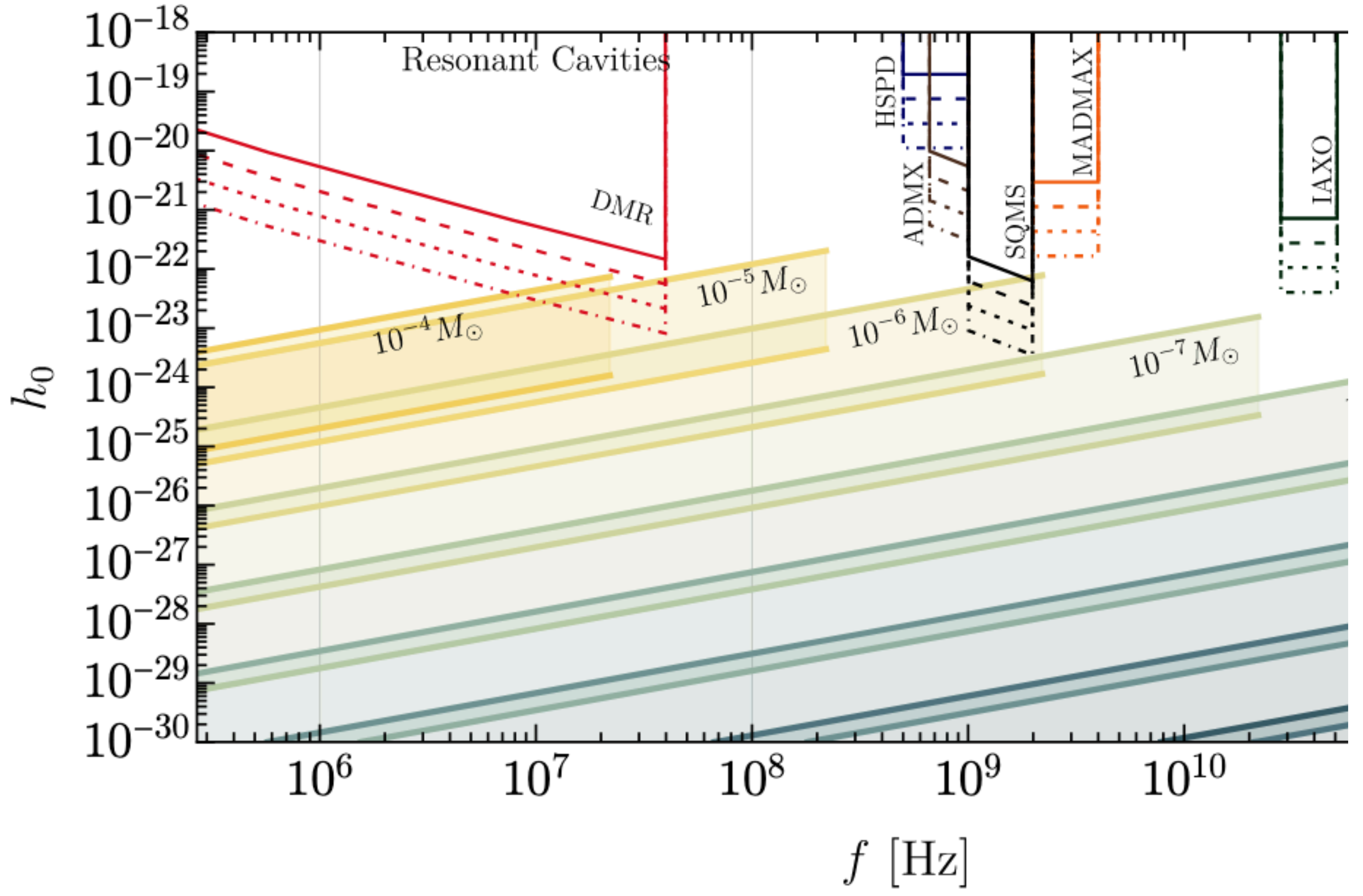
SMASH model full spectrum

Ringwald Tamarit 22



GWs from PBHs (rates 1/year)

Franciolini et al 2205.02153



Sources of UHFGW: spoiler!

SMASH model full spectrum

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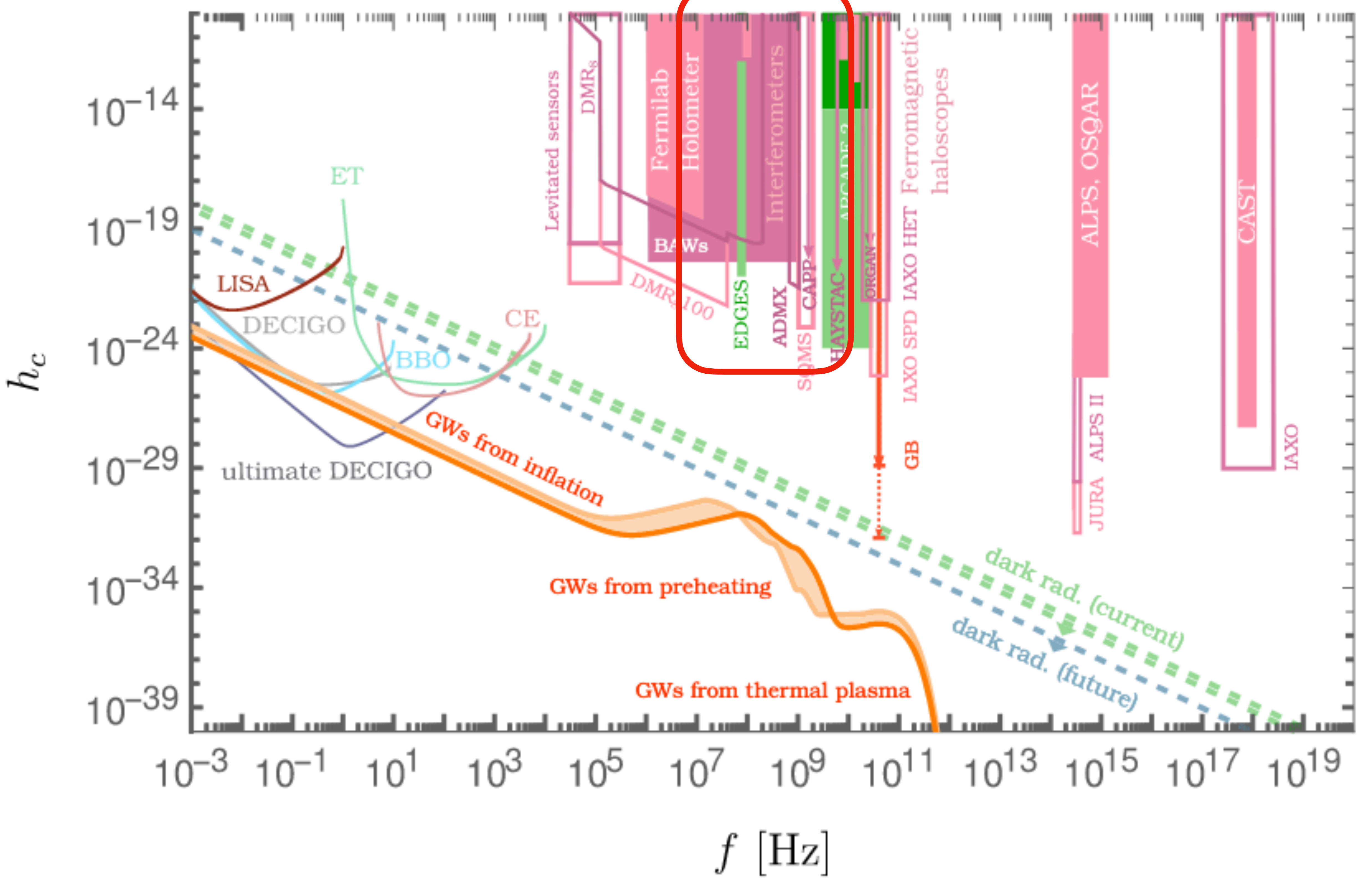

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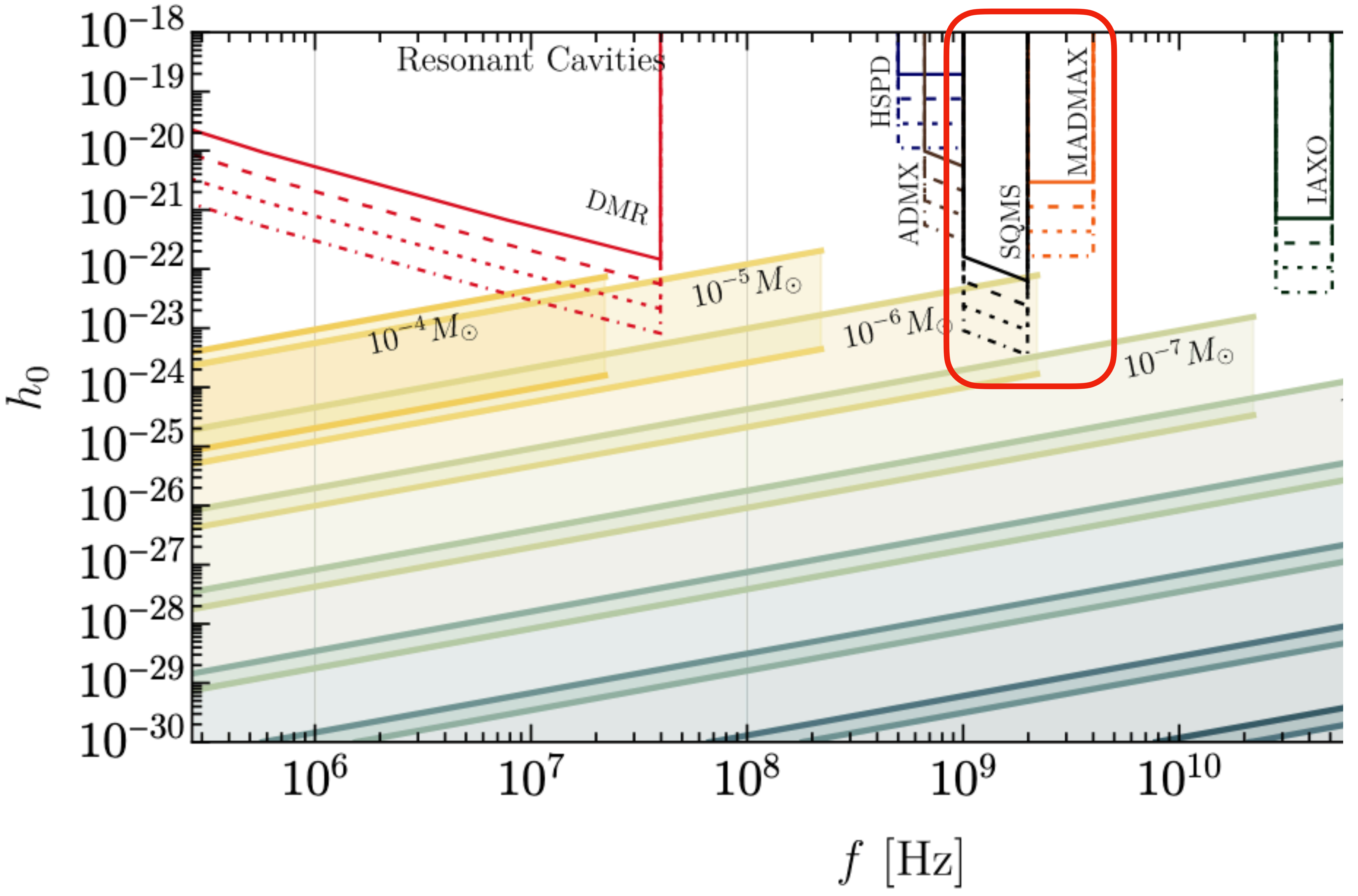
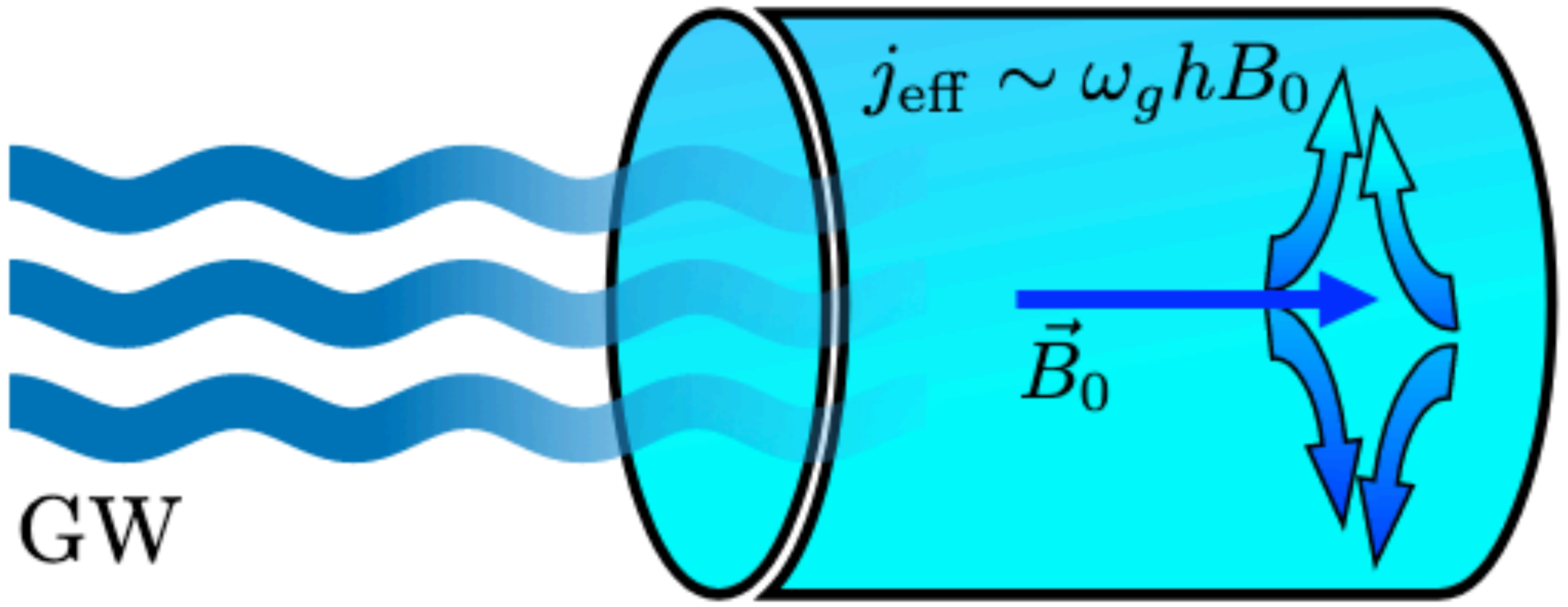


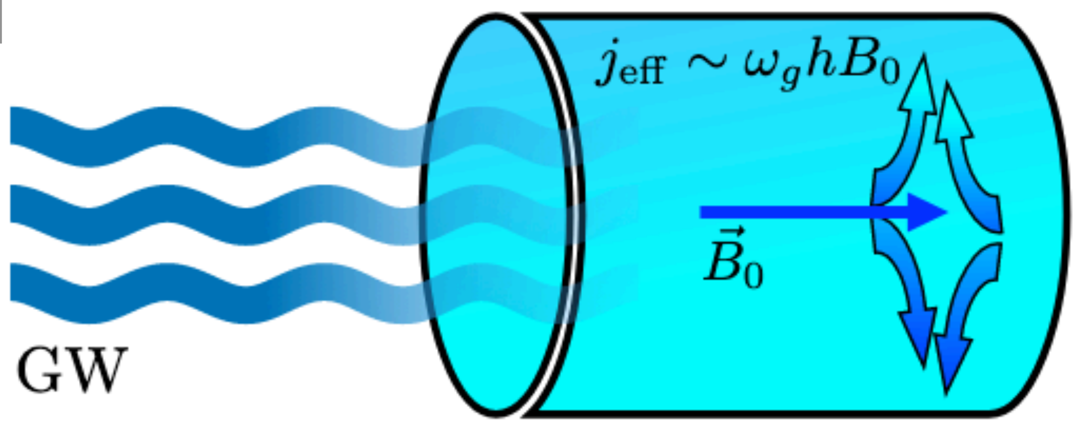
Searching for GWs with light



$$\mathcal{L} = \sqrt{-g}\,(R + F_{\mu\nu}F^{\mu\nu}) \supset \frac{1}{2}A_\mu j^\mu_{\text{eff}}(h) + \eta^{\mu\alpha}\eta^{\nu\beta}F_{\mu\nu}F_{\alpha\beta} + O(h^2)$$

$$j^\mu_{\text{eff}} = -\partial_\beta\left(\frac{1}{2}hF^{\mu\beta} + h^\beta_\alpha F^{\alpha\mu} - h^\mu_\alpha F^{\alpha\beta}\right)$$

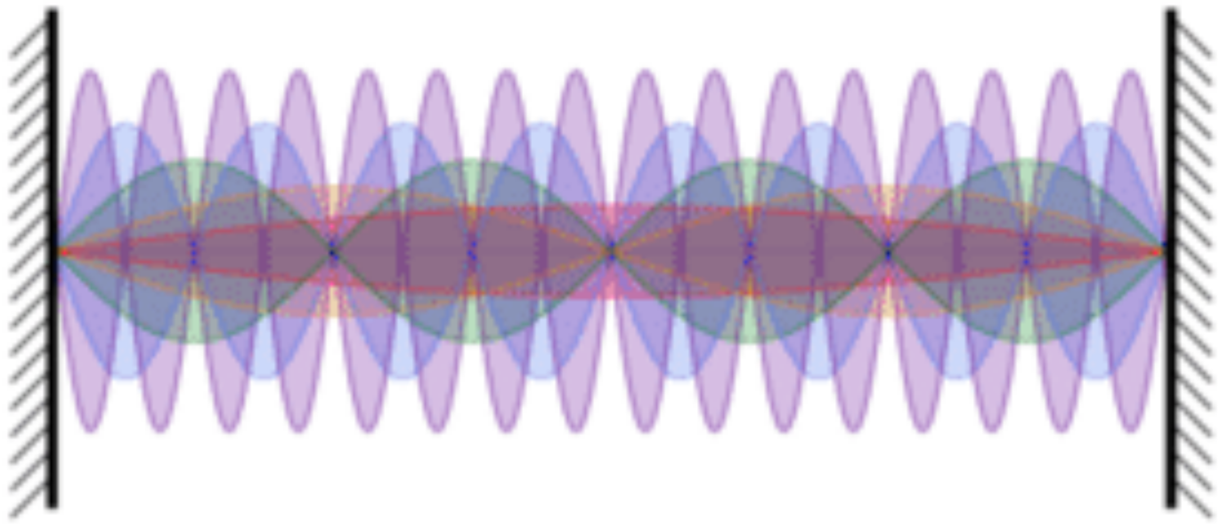
Cavity modes excitation



$$j^{\mu}_{\rm eff} = -\partial_\beta \left(\frac{1}{2} h F^{\mu\beta} + h^{\beta}_{\alpha} F^{\alpha\mu} - h^{\mu}_{\alpha} F^{\alpha\beta} \right)$$

$$\boldsymbol{E}(\boldsymbol{x}, t) = \sum \boldsymbol{E}_{sn}(\boldsymbol{x}, t) + \boldsymbol{E}_{in}(\boldsymbol{x}, t)$$

solenoidal irrotational



$$\boldsymbol{E}_{sn}(\boldsymbol{x}, t) = e_{sn}(t) \boldsymbol{E}_{sn}(\boldsymbol{x})$$
$$\boldsymbol{E}_{in}(\boldsymbol{x}, t) = e_{in}(t) \boldsymbol{E}_{in}(\boldsymbol{x})$$

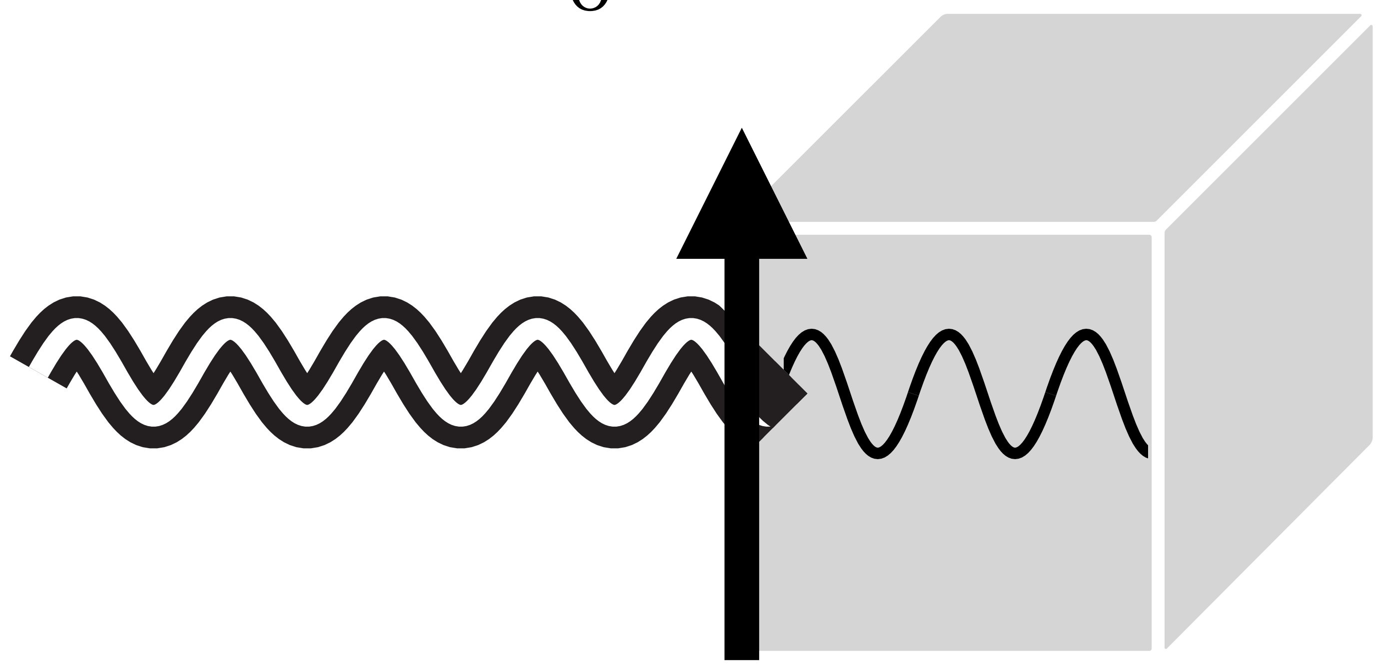
$$\left(\omega^2_{sm} + \partial^2_t + \sigma_{sm}\partial_t\right) e_{sm}(t) = e^{-i\omega_G t}\eta_{sm}$$
$$\left(\partial^2_t + \sigma_{im}\partial_t\right) e_{im}(t) = e^{-i\omega_G t}\eta_{im}$$

$$\eta \sim \int_V \mathrm{d}^3x \, E J_{eff}$$

‘source’ (here we want to maximise. It is also directional)

From axions to GWs

$$P_h \sim h_0^2$$



Du et al 2018

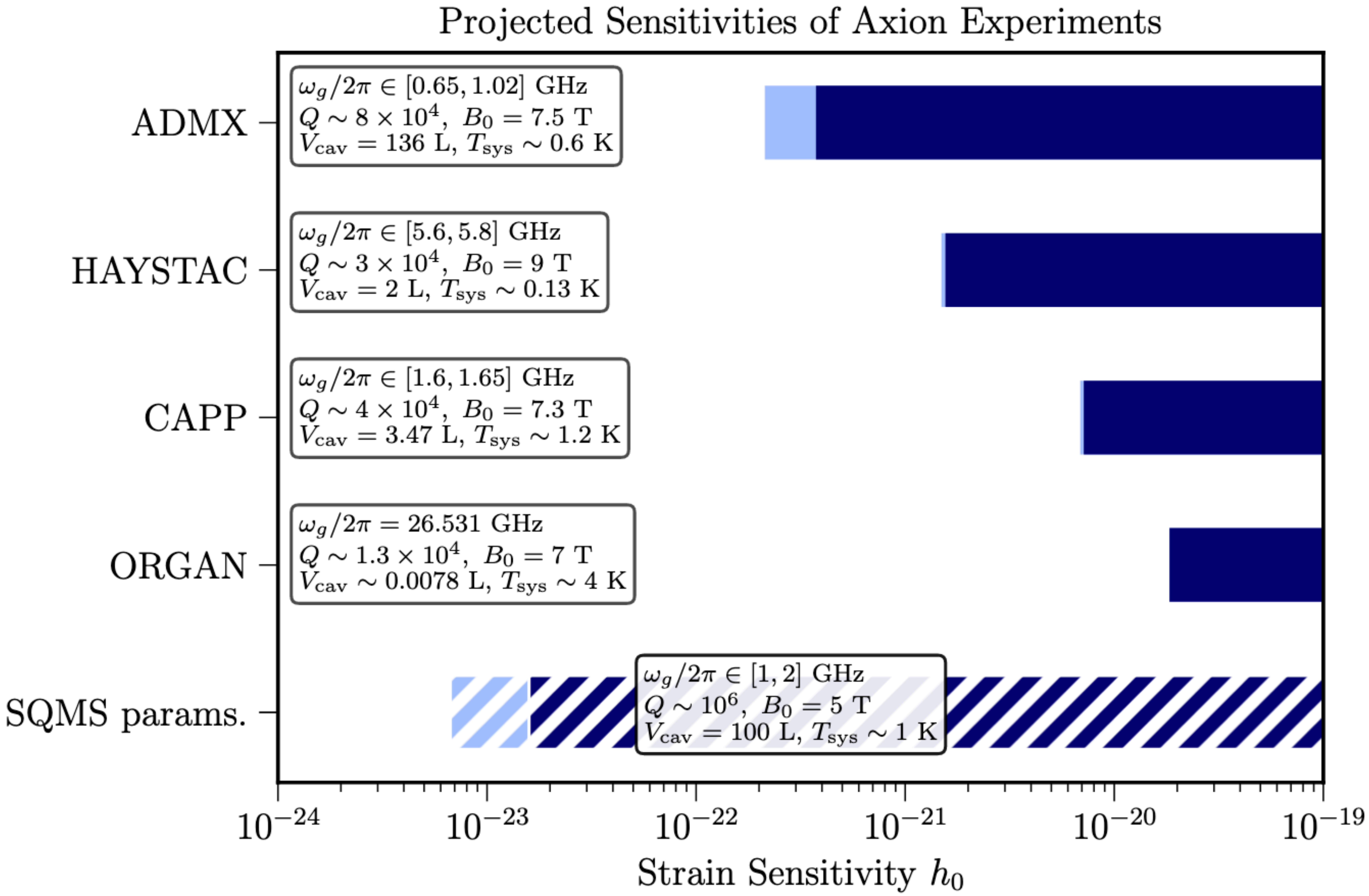
indeed: ADMX sensitivity

$$P_{\rm axion} = 1.9 \times 10^{-22}\ {\rm W} \left(\frac{V}{136l}\right)\left(\frac{B}{6.8\ {\rm T}}\right)^2\left(\frac{C}{0.4}\right)\left(\frac{g_\gamma}{0.97}\right)^2\left(\frac{\rho_{\rm a}}{0.45{\rm GeVcm^{-3}}}\right)\left(\frac{f}{650{\rm MHz}}\right)\left(\frac{Q}{50,000}\right)$$

quadratic in axion field and in *h*

$$P_h \sim h_0^2$$

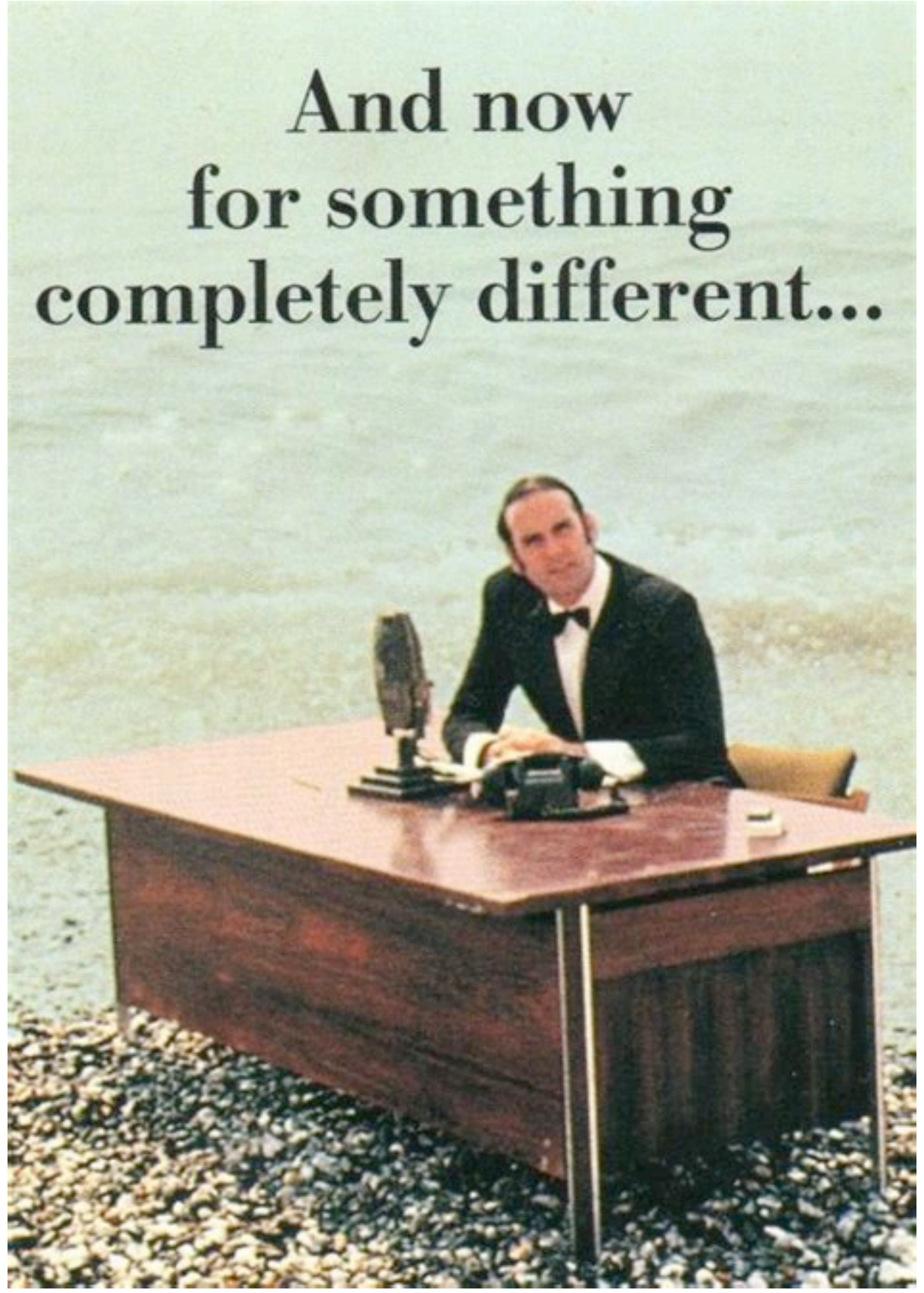
$$h_0 \gtrsim 3 \times 10^{-22} \times \left(\frac{1\ {\rm GHz}}{\omega_g/2\pi}\right)^{3/2}\left(\frac{0.1}{\eta_n}\right)\left(\frac{8\ {\rm T}}{B_0}\right)\left(\frac{0.1\ {\rm m}^3}{V_{\rm cav}}\right)^{5/6}\left(\frac{10^5}{Q}\right)^{1/2}\left(\frac{T_{\rm sys}}{1\ {\rm K}}\right)^{1/2}\left(\frac{\Delta\nu}{10\ {\rm kHz}}\right)^{1/4}\left(\frac{1\ {\rm min}}{t_{\rm int}}\right)^{1/4}$$


Projected Sensitivities of Axion Experiments
ADMX
$\omega_g/2\pi \in [0.65, 1.02]$ GHz
$Q \sim 8 \times 10^4$, $B_0 = 7.5$ T
$V_{\rm cav} = 136$ L, $T_{\rm sys} \sim 0.6$ K
HAYSTAC
$\omega_g/2\pi \in [5.6, 5.8]$ GHz
$Q \sim 3 \times 10^4$, $B_0 = 9$ T
$V_{\rm cav} = 2$ L, $T_{\rm sys} \sim 0.13$ K
CAPP
$\omega_g/2\pi \in [1.6, 1.65]$ GHz
$Q \sim 4 \times 10^4$, $B_0 = 7.3$ T
$V_{\rm cav} = 3.47$ L, $T_{\rm sys} \sim 1.2$ K
ORGAN
$\omega_g/2\pi = 26.531$ GHz
$Q \sim 1.3 \times 10^4$, $B_0 = 7$ T
$V_{\rm cav} \sim 0.0078$ L, $T_{\rm sys} \sim 4$ K
SQMS params.
$\omega_g/2\pi \in [1, 2]$ GHz
$Q \sim 10^6$, $B_0 = 5$ T
$V_{\rm cav} = 100$ L, $T_{\rm sys} \sim 1$ K
10^{-24}
10^{-23}
10^{-22}
10^{-21}
10^{-20}
10^{-19}
Strain Sensitivity h_0

A very exploratory field…

we still need to **deeply think what's better** for the future

do we move to something else ?

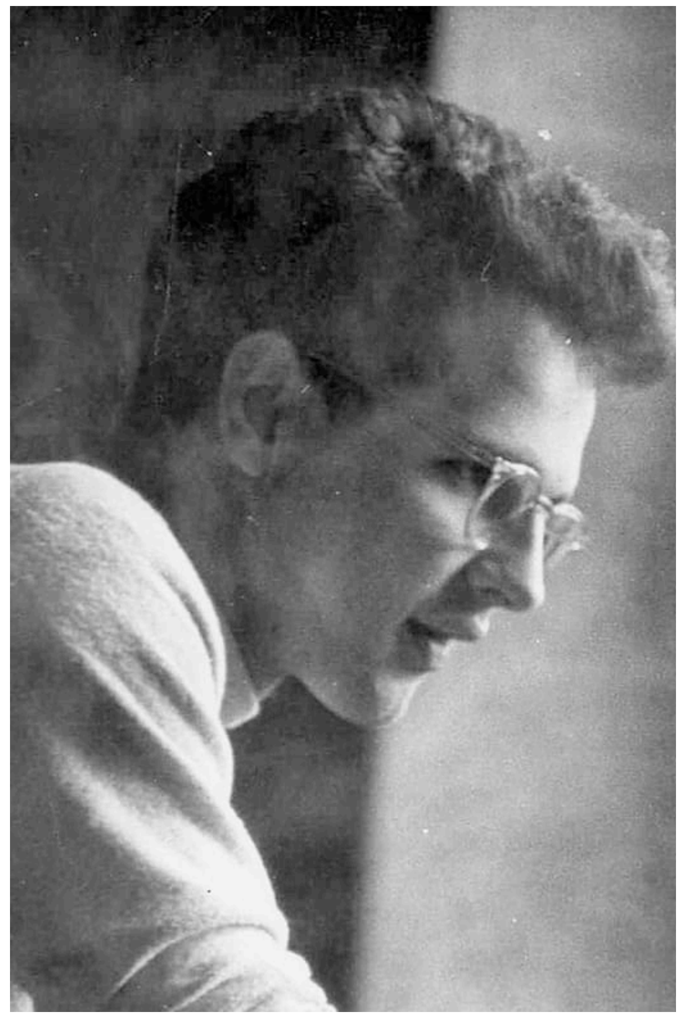


LIGO lesson

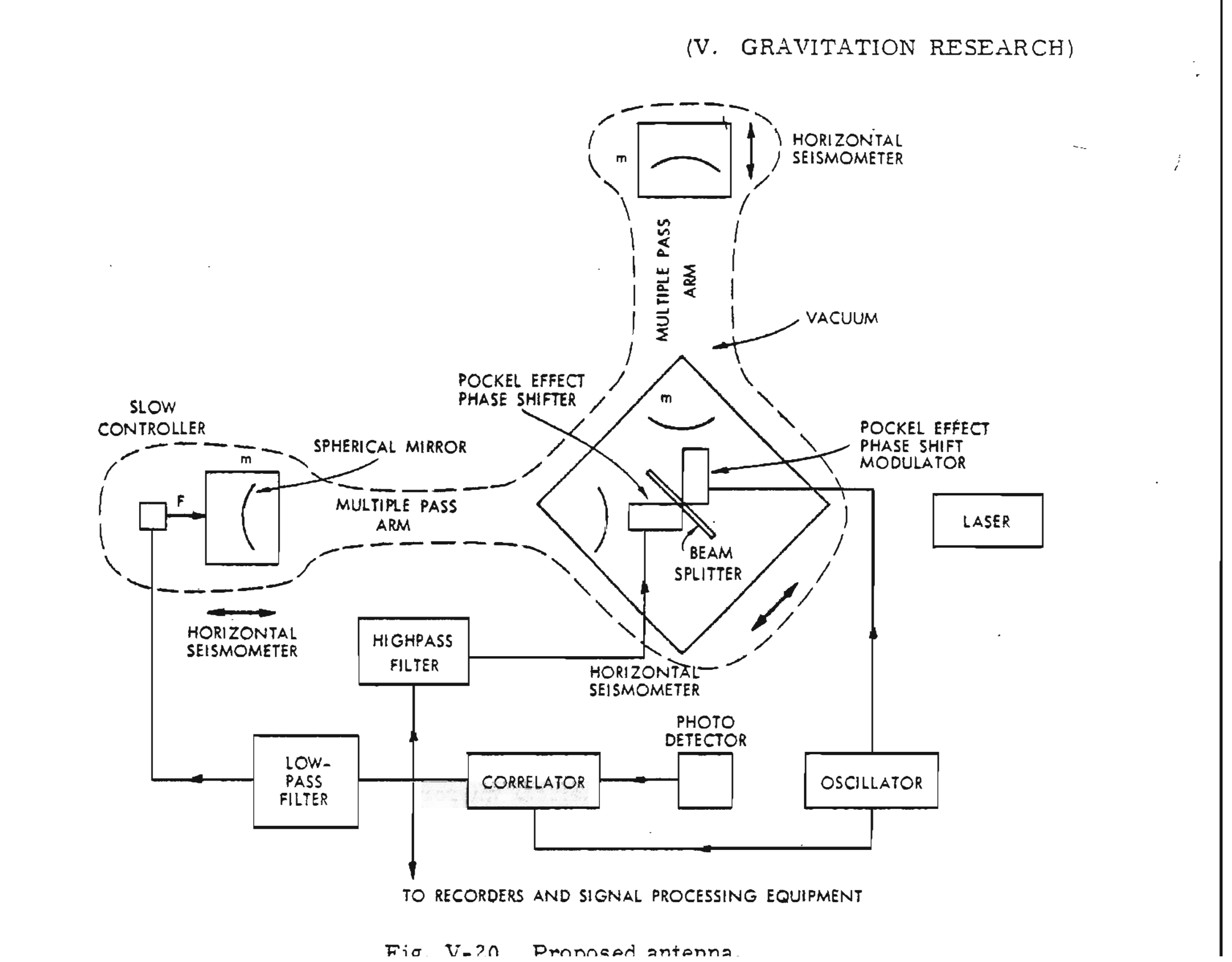
A very exploratory field…
we still need to **deeply think what's better** for the future

do we move to something else?

Rainer Weiss, ca. 1972



LIGO concept
Weiss 1972



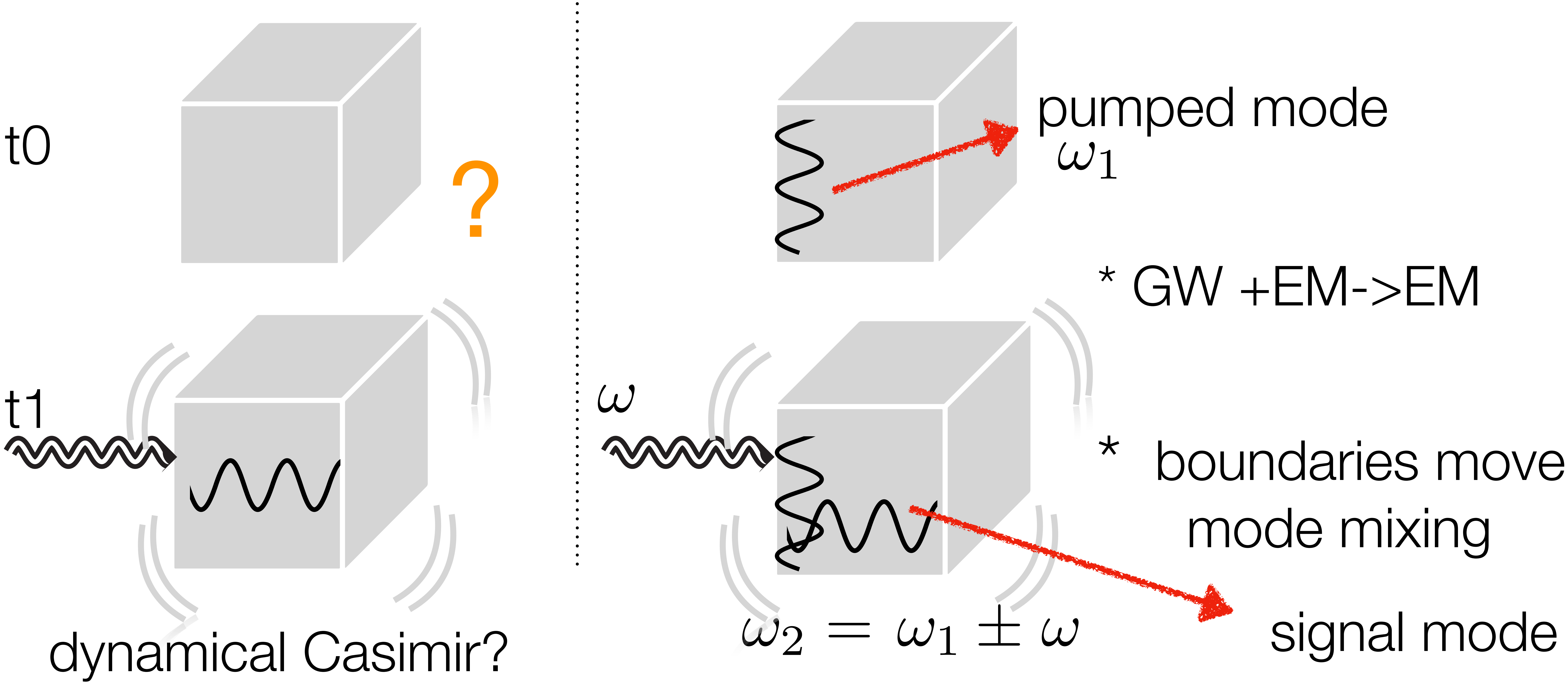
For continuous gravitational waves, the minimum detectable gravitational wave metric spectral density is then

$$h^2(f) > \frac{4}{f^2}\,\frac{\Delta x_n^2(f)}{\Delta f} \approx \frac{4 \times 10^{-33}}{f^2(\mathrm{cm})} \qquad \mathrm{Hz}^{-1}.$$

Recycling axion experiments II

Mechanical-coupling (shaking the walls)

MAGO design from CERN (gr-qc/0502054)
Berlin, DB et al 2303.01518

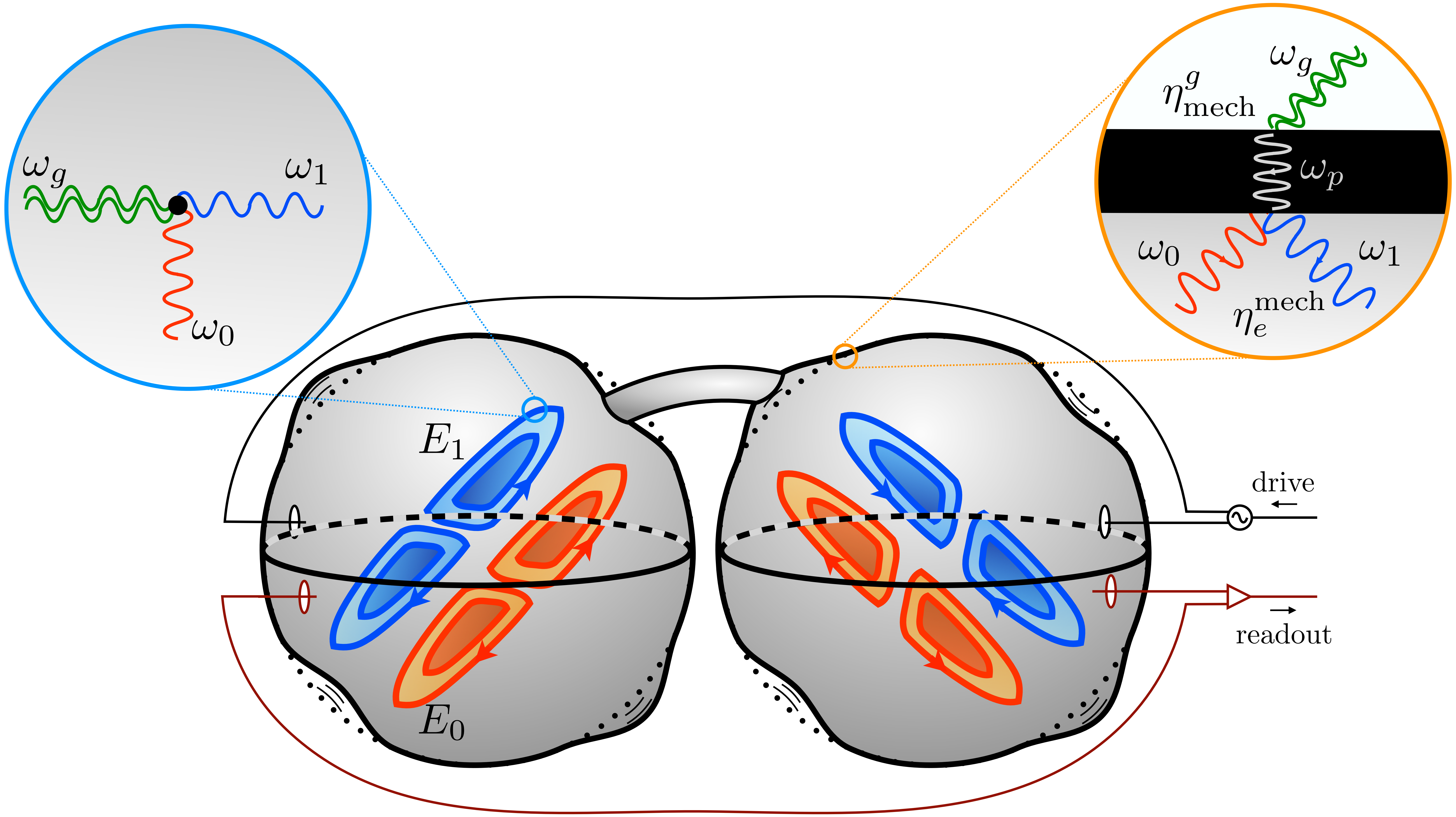


* GW +EM->EM

* boundaries move: mode mixing

MAGO set-up

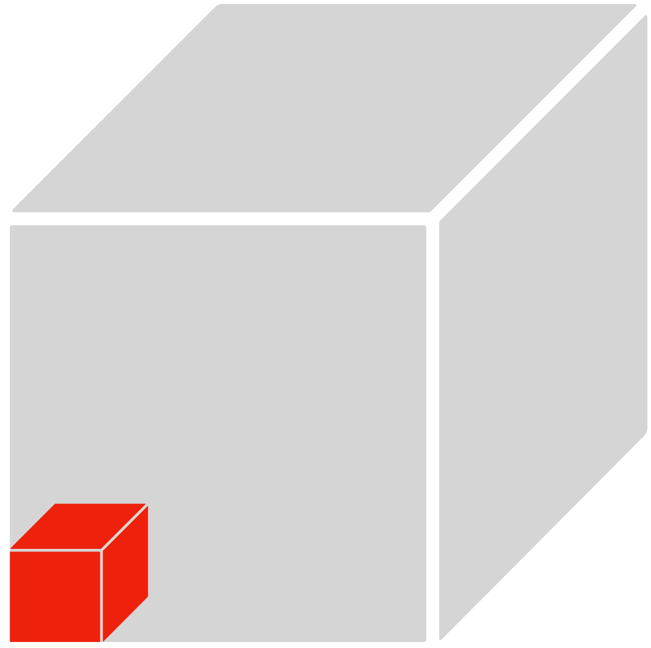
(Microwave Apparatus for Gravitational Waves Observation)



GWs exciting solids

searched for many years (Weber bars)

a solid affected by a external source (e.g. *x* direction)



$dm(x+u(x,t))$

$$dm\left(\frac{\partial^2 u}{\partial t^2} - v_s^2 \frac{\partial^2 u}{\partial x^2}\right) = dF_x(t,x),$$

$$dF_i = \frac{1}{2}\ddot{h}_{ij}^{TT} x^j\, dm$$

In terms of eigenmodes:

$$\mathbf{u}(\mathbf{x},t) = u_p(t)\mathbf{u}_p(\mathbf{x})$$

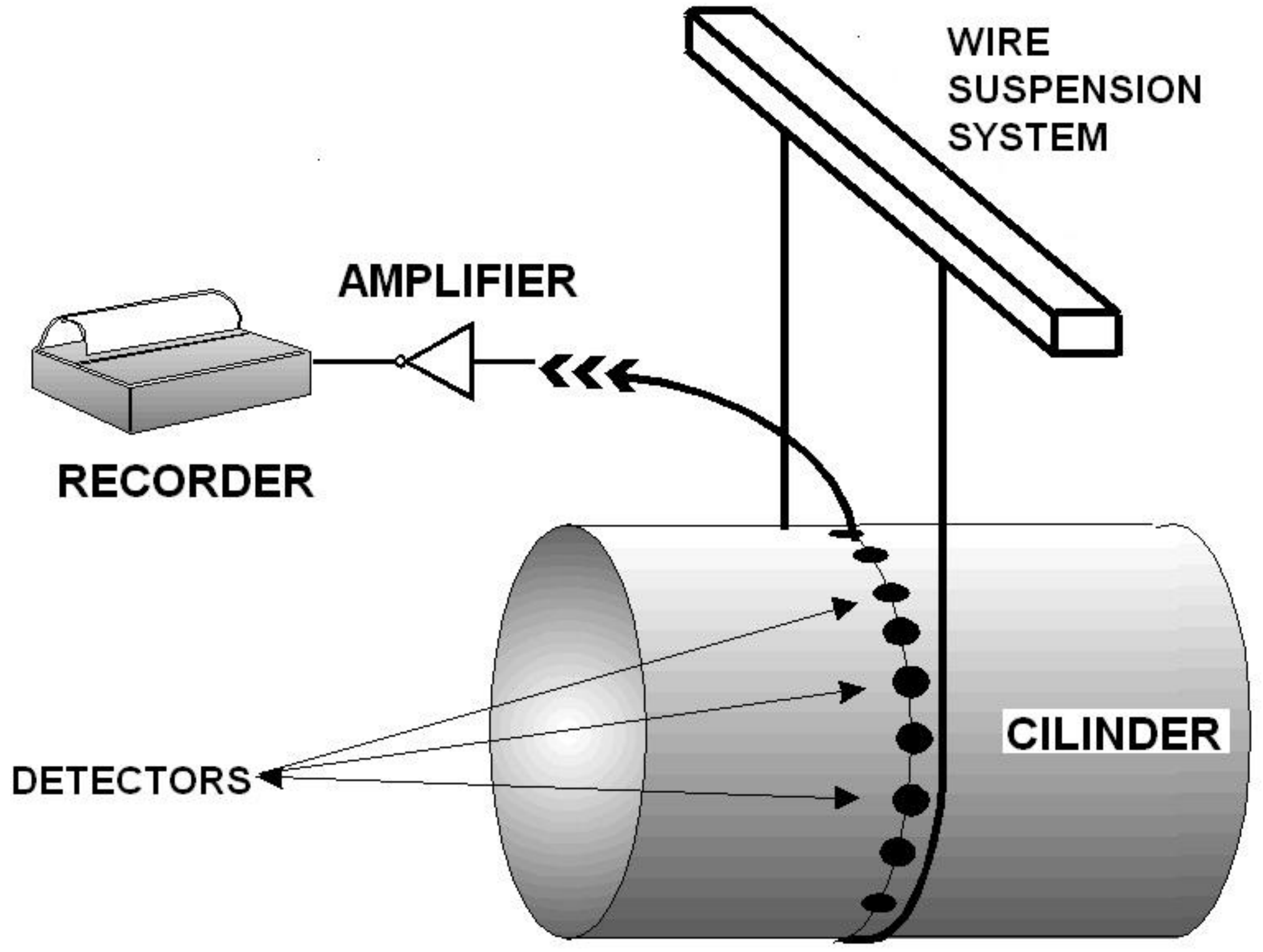
GWs exciting solids

$$\mathbf{u}(\mathbf{x},t) = u_p(t)\mathbf{u}_p(\mathbf{x})$$

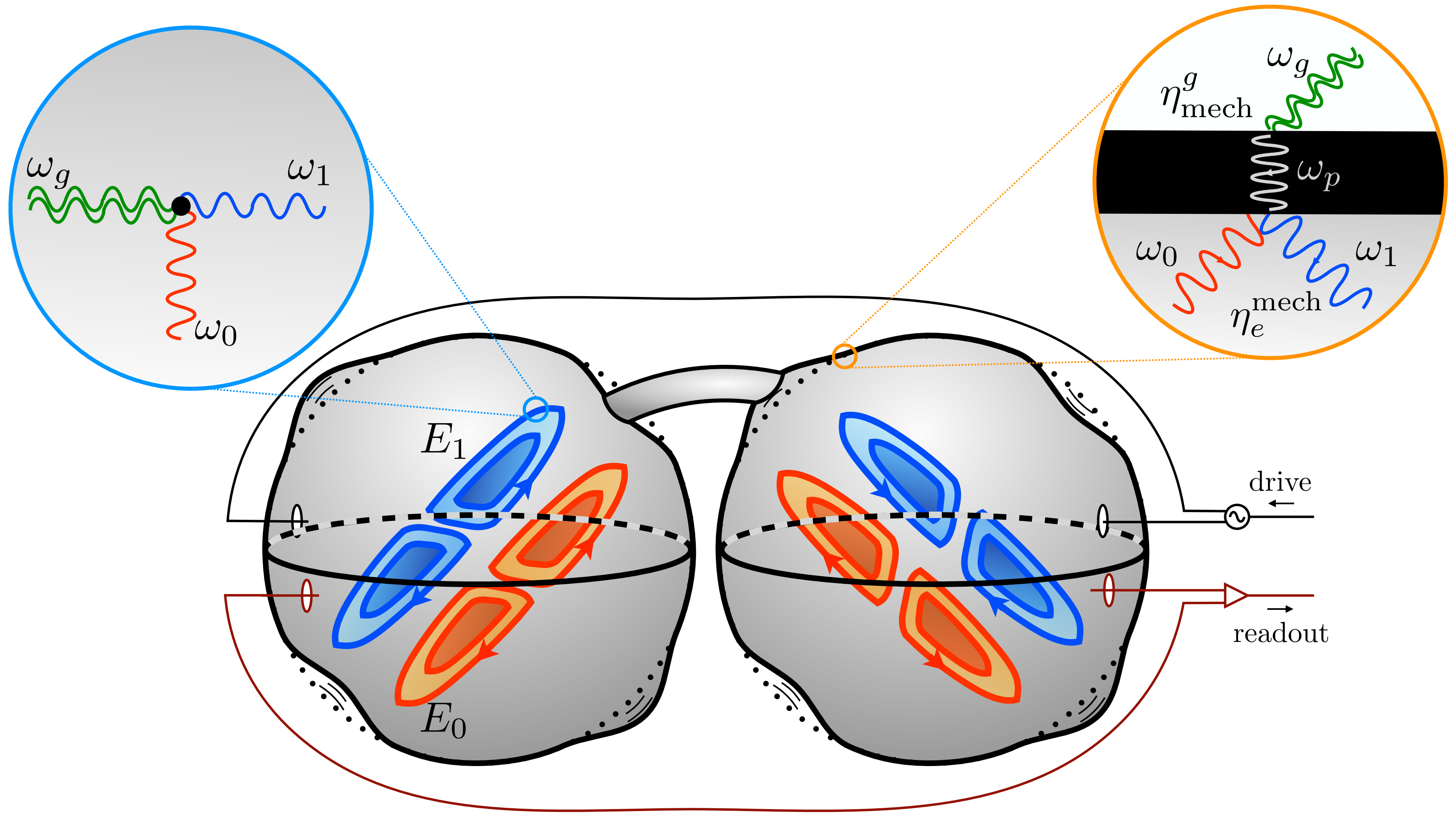
$$\ddot{u}_p + \frac{\omega_p}{Q_p}\dot{u}_p + \omega_p^2 u_p \simeq -\frac{1}{2}\omega_g^2 V_{\rm cav}^{1/3}\eta^g_{\rm mech}\ h_0 e^{i\omega_g t}$$

$$\eta^g_{\rm mech} = \frac{\hat{h}_{ij}^{\rm TT}}{V_{\rm cav}^{1/3}V_{\rm shell}}\int_{V_{\rm shell}} d^3\mathbf{x} U_p^{*i} x^j$$

this rings the solid (Weber bars)



MAGO set-up: read-out from mode mixing



$$\boldsymbol{E}_{sn}(\boldsymbol{x},t) = e_{sn}(t)\,\boldsymbol{E}_{sn}(\boldsymbol{x})$$

$$\left(\partial_t^2 + \omega_n^2\right) e_n \simeq -\omega_n e_m \frac{\int_{\Delta V} d^3\mathbf{x}\,(\omega_n \boldsymbol{E}_m \cdot \boldsymbol{E}_n^* - \omega_m \boldsymbol{B}_m \cdot \boldsymbol{B}_n^*)}{\int_{V_0} d^3\mathbf{x}\,|\boldsymbol{E}_n|^2}$$

mixing

modes are not orthogonal in $\boldsymbol{\Delta V}$

$$P_{\rm sig} \propto h^2_{+,\times} \left|\eta_u^{(\rm mech.\ )}\right|^2 \left|\tilde{\eta}_e^{(\rm mech.\ )}\right|^2 P_{in} \times \begin{cases} \frac{\omega_h^4}{\left(\omega_h^2-\omega_m^2\right)^2} & (|\omega_h - \omega_m| \gg \omega_m/Q_m) \\ Q_m^2 & (|\omega_h - \omega_m| \ll \omega_m/Q_m) \end{cases}$$

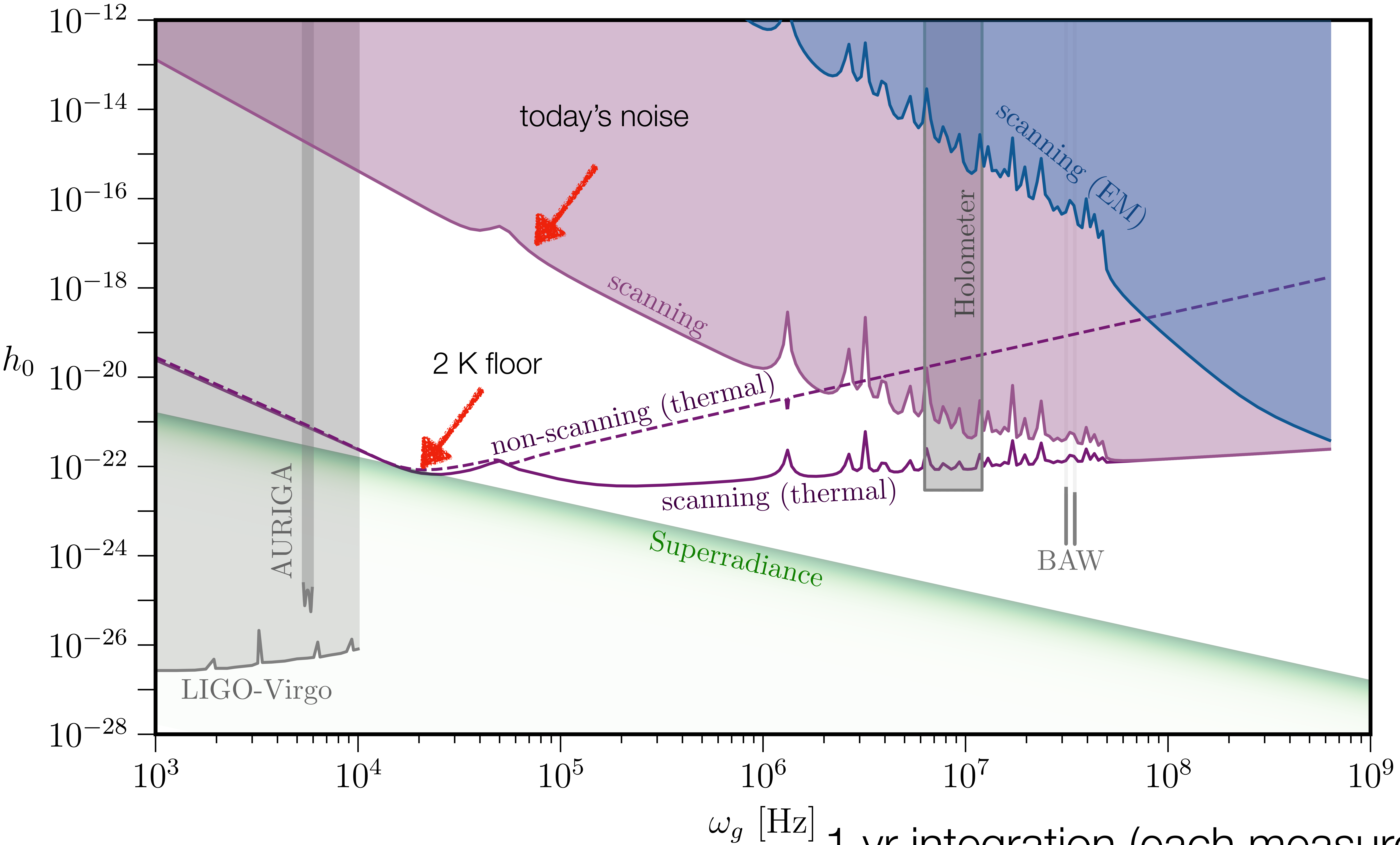
off-resonance

on-resonance

$Q_m \sim 10^6$

Estimates

A. Berlin, DB, R.T. D'Agnolo, S. Ellis, R. Harnik, Y. Kahn, J. Schütte-Engel, M.Wentze 2303.01518



1 yr integration (each measurement ms)!

A word on sources GW coupling to spin

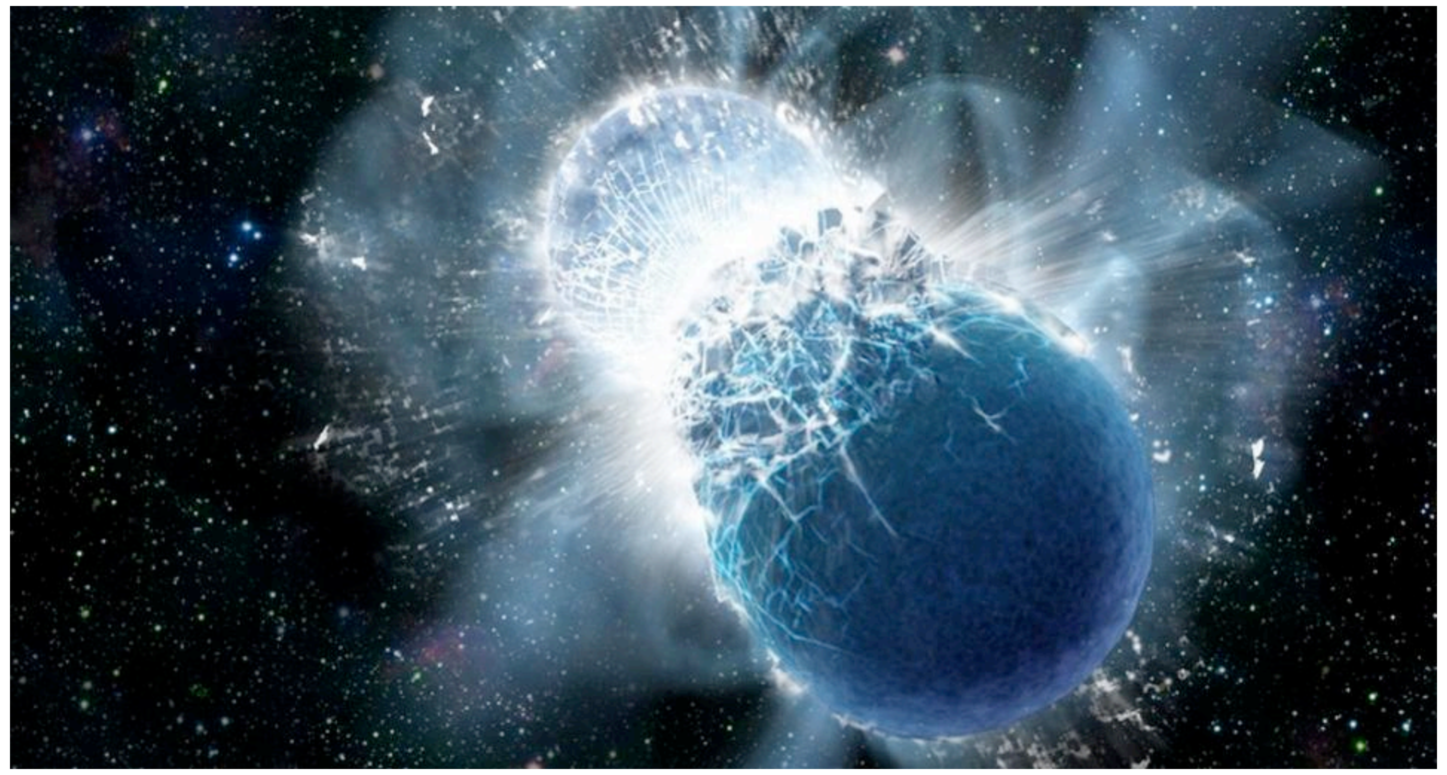
e.g. Ito and Soda 20

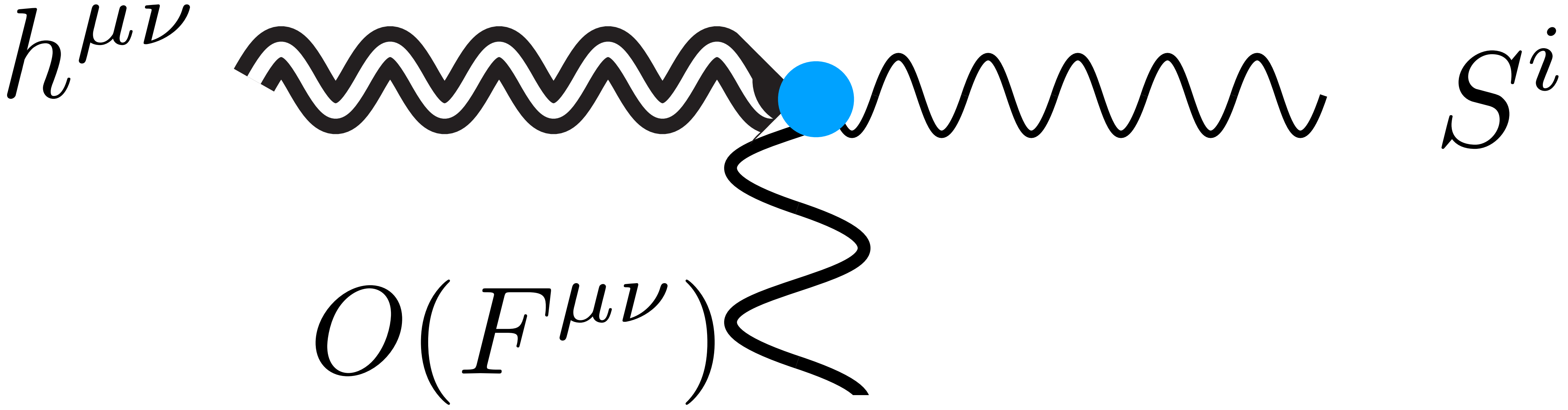
Dirac equation in GR $\quad i\gamma^{\hat{\alpha}} e^{\mu}_{\hat{\alpha}} \left(\partial_\mu - \Gamma_\mu - ieA_\mu\right)\psi = m\psi \ ,$

NR limit

$$H_{\text{tot}} = -\mu_B \left(2\delta_{za} + Q_{za}\right) B_z \sum_i \hat{S}^a_{(i)} - \sum_{i,j} J_{ij} \hat{\boldsymbol{S}}_{(i)} \cdot \hat{\boldsymbol{S}}_{(j)}$$

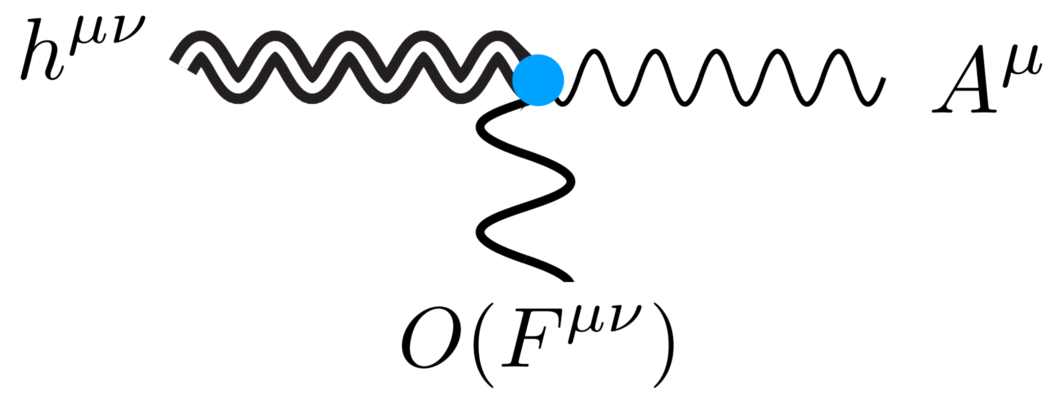
Linear in GW $\quad Q_{ij} \supset -\frac{2}{3}\delta_{ij}\ddot{h}_{kl}\Big|_{\boldsymbol{x}=0} x^k x^l$





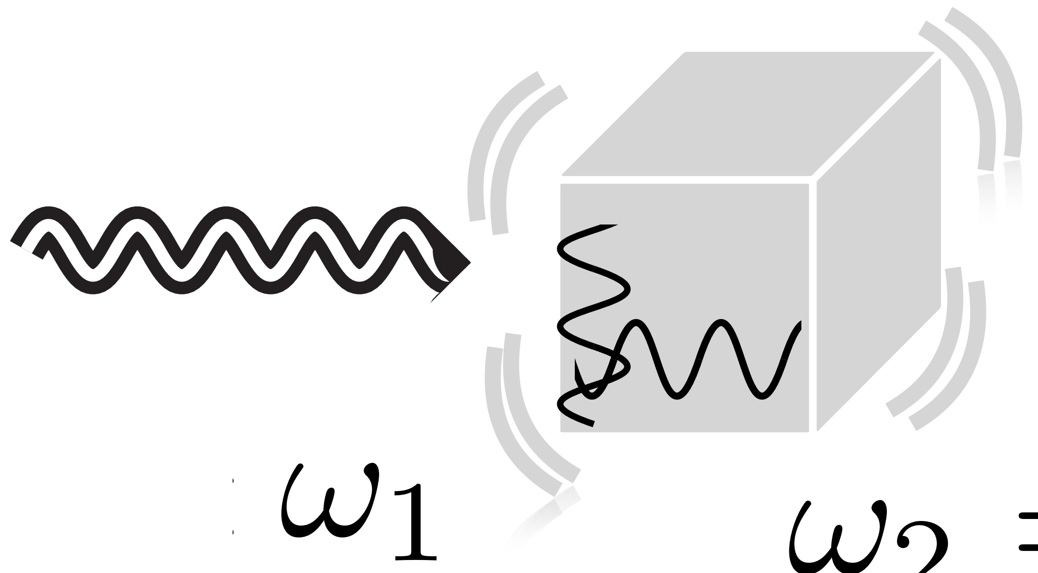
Summary and outlook

- SRF cavities are a mature technology to look for GWs at GHz either
  - Coupling to photons in a cavity



- 'ADMX' like $\omega = \omega_g$
- Heterodyne $\omega_2 = \omega_g \pm \omega_1$

  - Ringing the cavity and generating mode mixing



$$\omega_2 = \omega \pm \omega_1$$

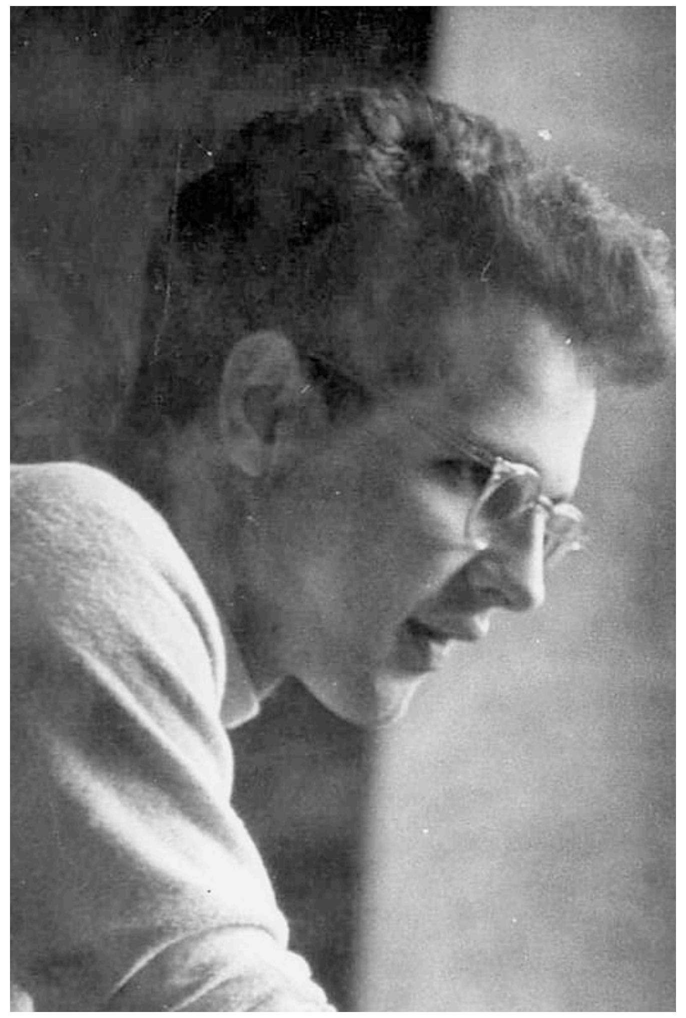
- Heterodyne
- Casimir?

✳ Reach of $h_0 \sim 10^{-23}$ possible (100 kHz-GHz), though far from known signals

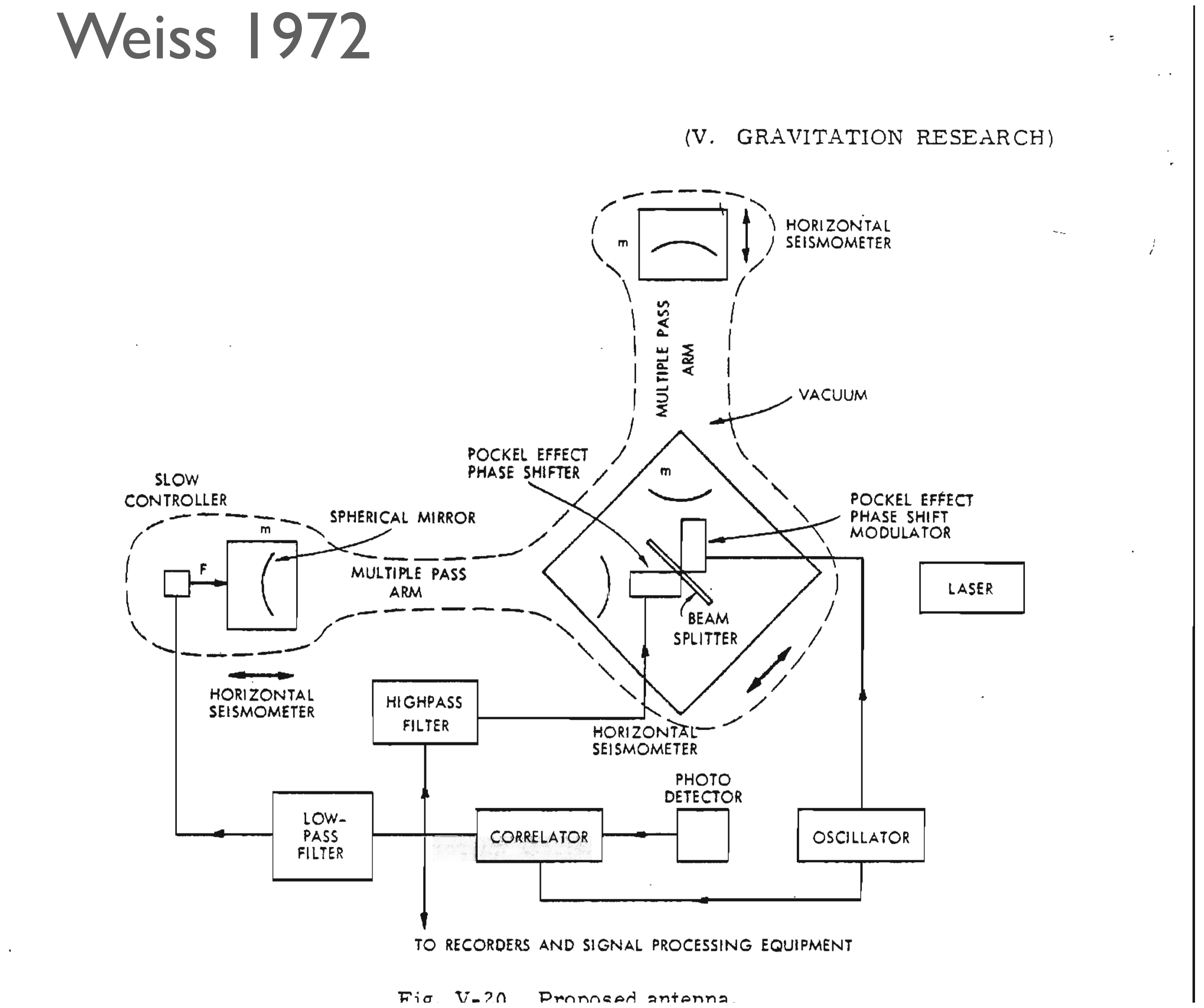
Outlook

A very exploratory field…
we still need to **deeply think what's better** for this endeavour

Rainer Weiss, ca. 1972



LIGO concept
Weiss 1972

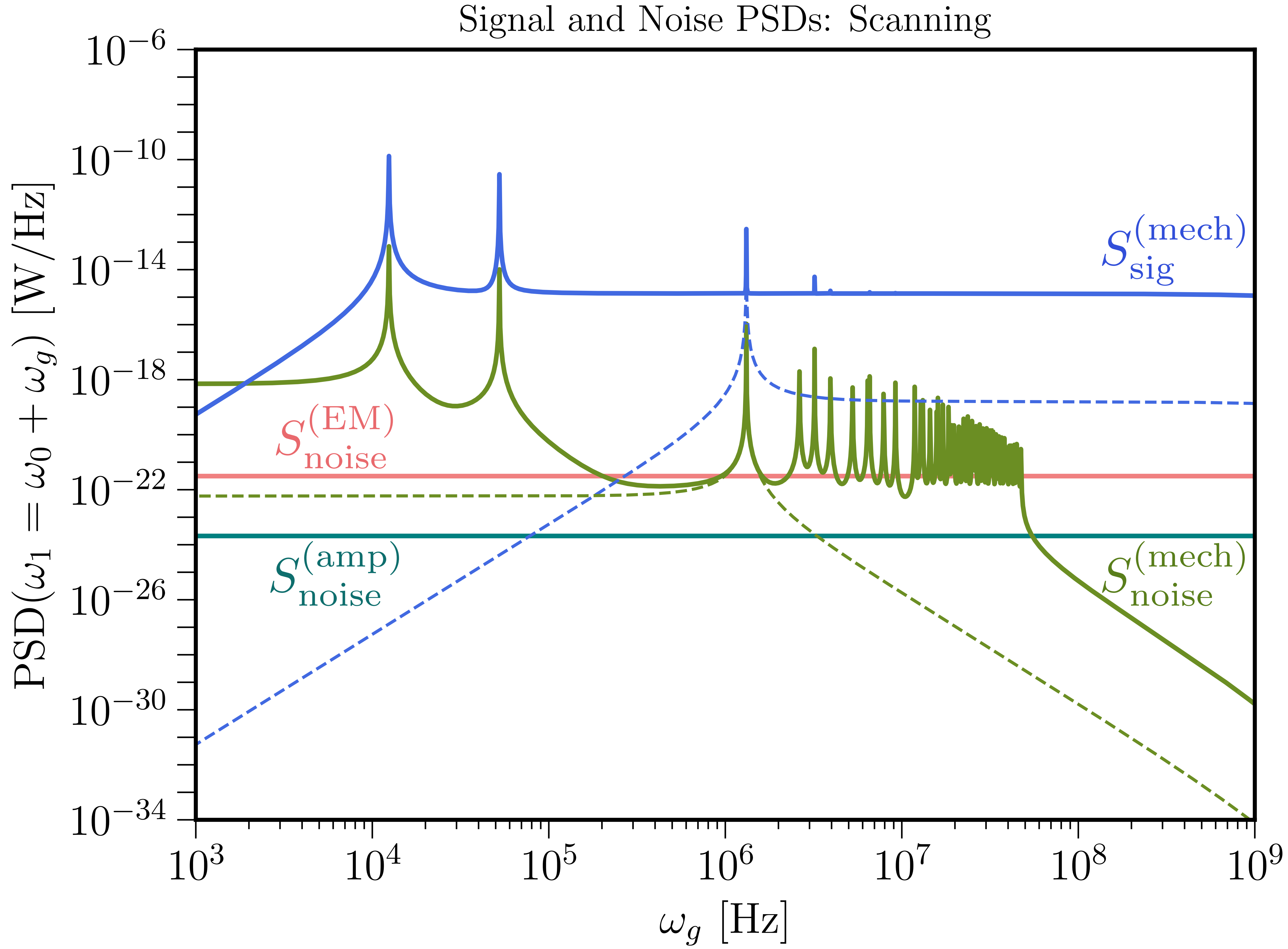


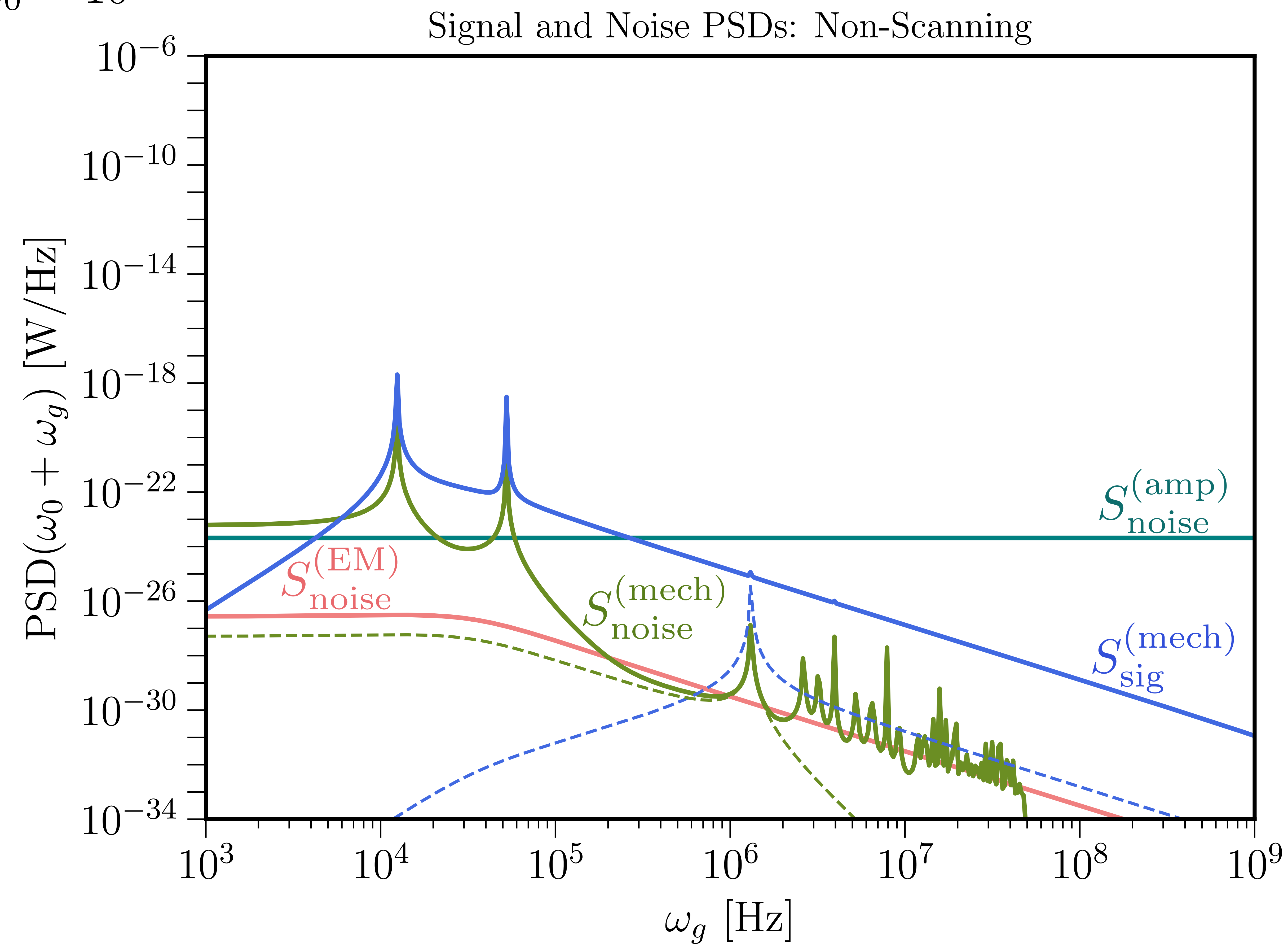
For continuous gravitational waves, the minimum detectable gravitational wave metric spectral density is then

$$h^2(f) > \frac{4}{\ell^2}\frac{\Delta x_n^2(f)}{\Delta f} \approx \frac{4\times 10^{-33}}{\ell^2(\mathrm{cm})} \qquad \mathrm{Hz}^{-1}.$$

Response

$h_0 \sim 10^{-20}$





$$\omega_1 - \omega_0 = \omega_g$$

$$\omega_1 - \omega_0 = \min \omega_{\mathrm{p}} \sim 10\,\mathrm{kHz}$$