Detecting (high frequency) gravitational waves in a box

based on 2112.11465 (PRD22) + 2303.01518 (PRD23)

(w. A. Berlin, DB, R. T. D'Agnolo, S. Ellis, R. Harnik, Y. Kahn, J. Schütte-Engel, M. Wentzel)

2312.02270 [hep-ph] P. Navarro, B. Gimeno, J. Monzón-Cabrera, A. Díaz-Morcillo

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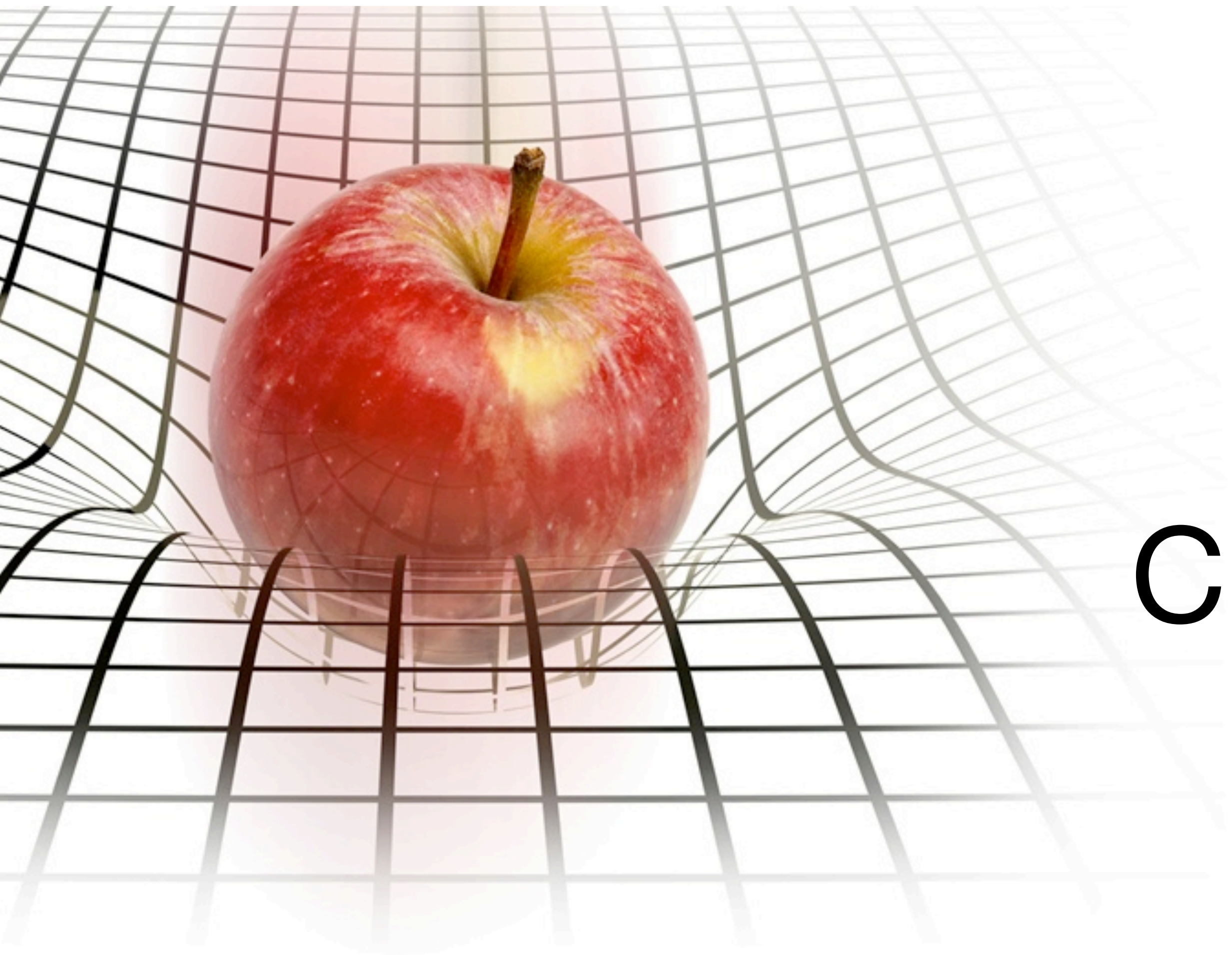
GWs (essentials)



Perturbations of space-time
travelling as waves of frequency f

Characterised by 2 polarizations $h_{+,\times}$ (dimensionless)

$$h_{+,\times} \overset{c=1}{\approx} h_0 \cos\left(2\pi f (t - z) + \phi\right)$$

GWs carry energy. They have **energy density**

$$\rho_{\rm gw} = \frac{1}{16\pi G} \left\langle \dot{h}_+^2 + \dot{h}_\times^2 \right\rangle$$

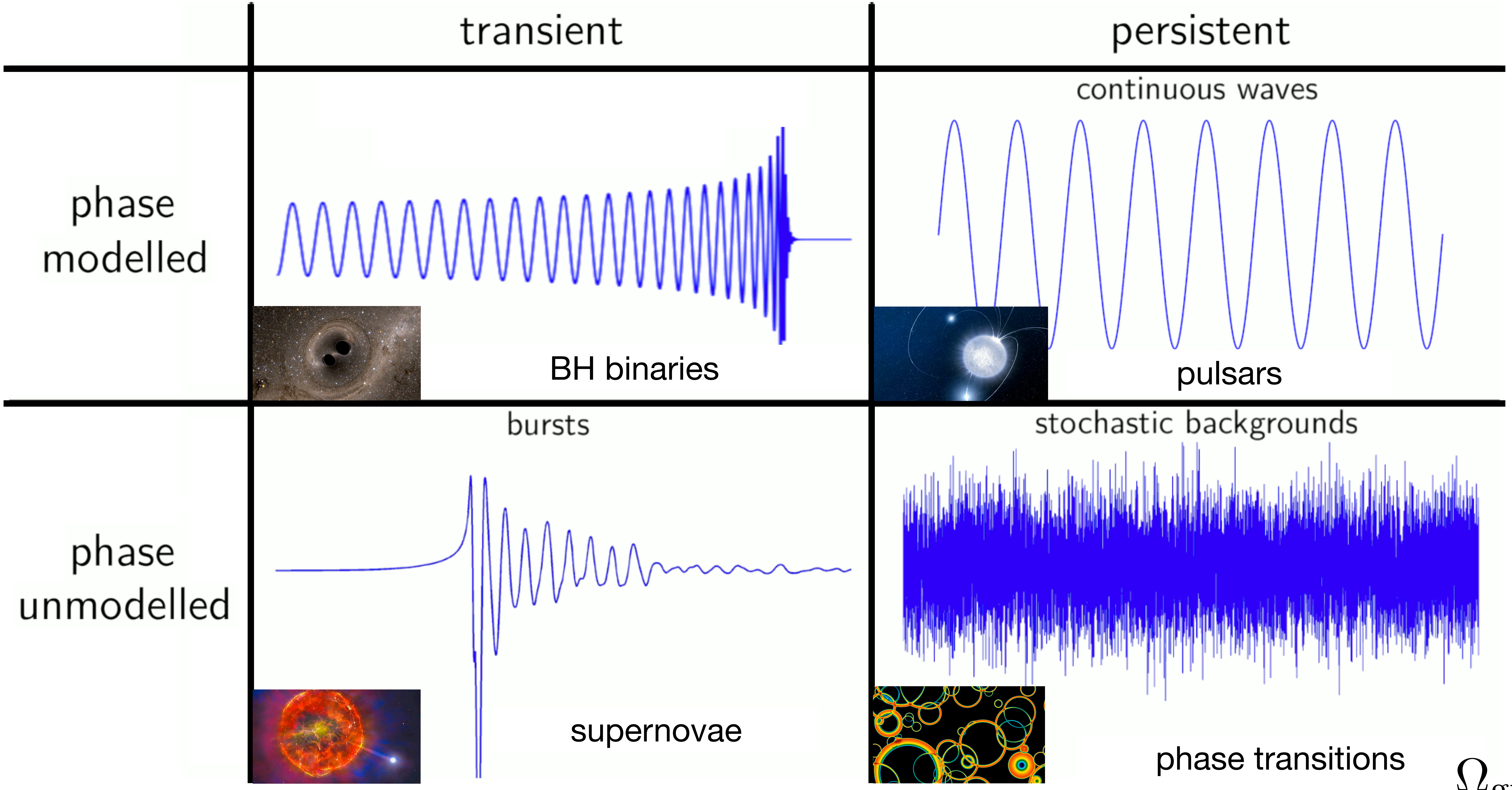
$$\Omega_{\rm gw}(f) \equiv \frac{1}{\rho_c} \frac{{\rm d}\rho_{\rm gw}}{{\rm d}(\ln f)}$$

$$h^2 \Omega_{\rm gw}(f) \qquad h \approx 0.67$$

$$\rho_c = 1.2 \times 10^{11} M_\odot {\rm Mpc}^{-3}$$
$$\sim {\rm keV}/{\rm cm}^3$$

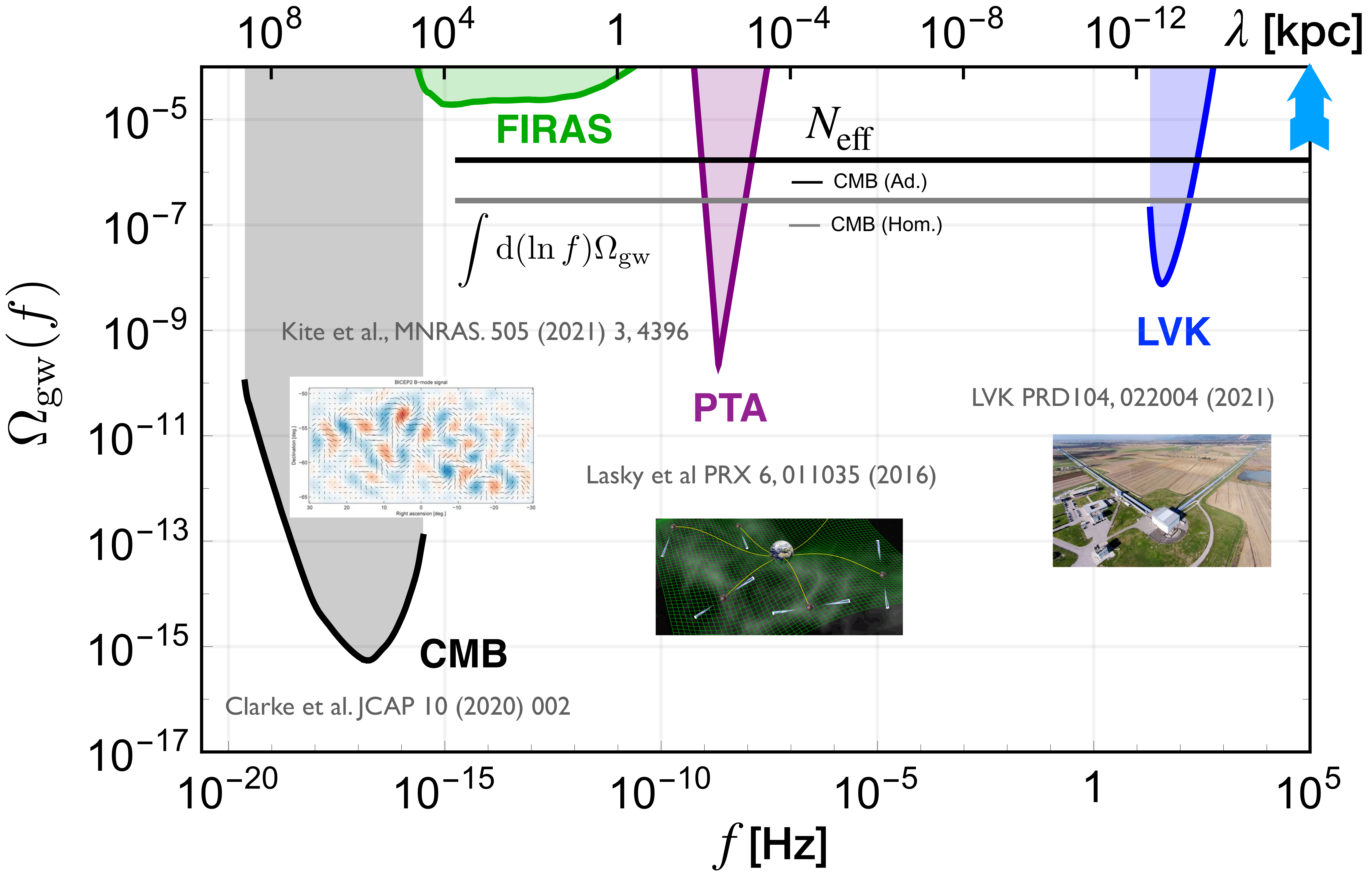
Taxonomy of GWs

$h(t)$



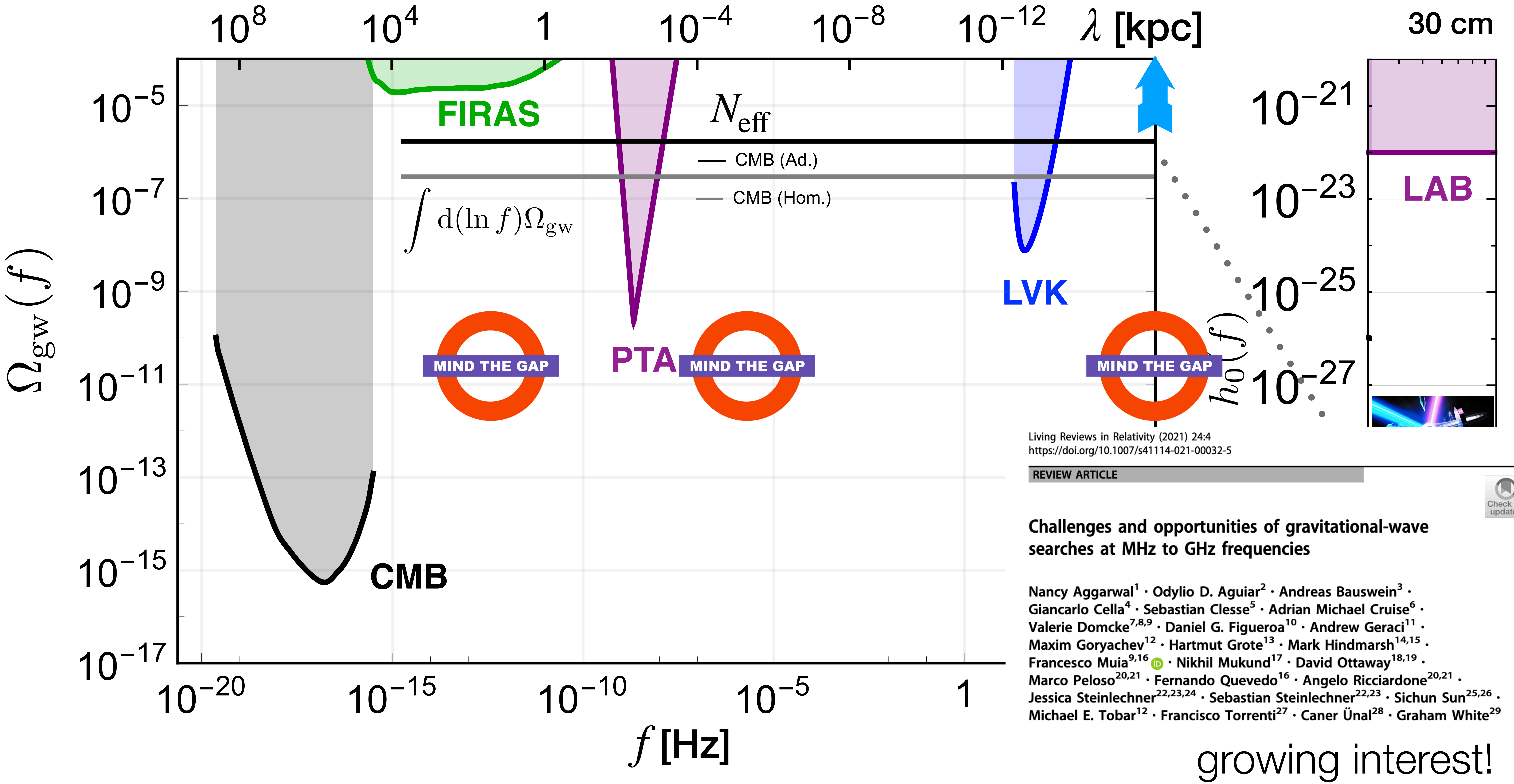
$\Omega_{\rm gw}(f)$

GWs soundscape today



GWs soundscape today
10^8 10^4 1 10^{-4} 10^{-8} 10^{-12} λ [kpc]
30 cm
FIRAS
$N_{\rm eff}$
CMB (Ad.)
CMB (Hom.)
$\int d(\ln f)\Omega_{\rm gw}$
PTA
LVK
LAB
MIND THE GAP
MIND THE GAP
MIND THE GAP
CMB
$\Omega_{\rm gw}(f)$
$h_0(f)$
10^{-5} 10^{-7} 10^{-9} 10^{-11} 10^{-13} 10^{-15} 10^{-17}
10^{-21} 10^{-23} 10^{-25} 10^{-27} 10^{-29} 10^{-31} 10^{-33}
10^{-20} 10^{-15} 10^{-10} 10^{-5} 1 10^5 10^9
f [Hz]

GWs soundscape today



Living Reviews in Relativity (2021) 24:4
https://doi.org/10.1007/s41114-021-00032-5

REVIEW ARTICLE

Check for updates

Challenges and opportunities of gravitational-wave searches at MHz to GHz frequencies

Nancy Aggarwal[1] · Odylio D. Aguiar[2] · Andreas Bauswein[3] · Giancarlo Cella[4] · Sebastian Clesse[5] · Adrian Michael Cruise[6] · Valerie Domcke[7,8,9] · Daniel G. Figueroa[10] · Andrew Geraci[11] · Maxim Goryachev[12] · Hartmut Grote[13] · Mark Hindmarsh[14,15] · Francesco Muia[9,16] · Nikhil Mukund[17] · David Ottaway[18,19] · Marco Peloso[20,21] · Fernando Quevedo[16] · Angelo Ricciardone[20,21] · Jessica Steinlechner[22,23,24] · Sebastian Steinlechner[22,23] · Sichun Sun[25,26] · Michael E. Tobar[12] · Francisco Torrenti[27] · Caner Ünal[28] · Graham White[29]

growing interest!

UHFGWs -> Laboratory searches

GWs interact with **every** source of energy-momentum!

in the laboratory

*Interaction GWs with **light***

*Interaction GWs with **matter external dofs***

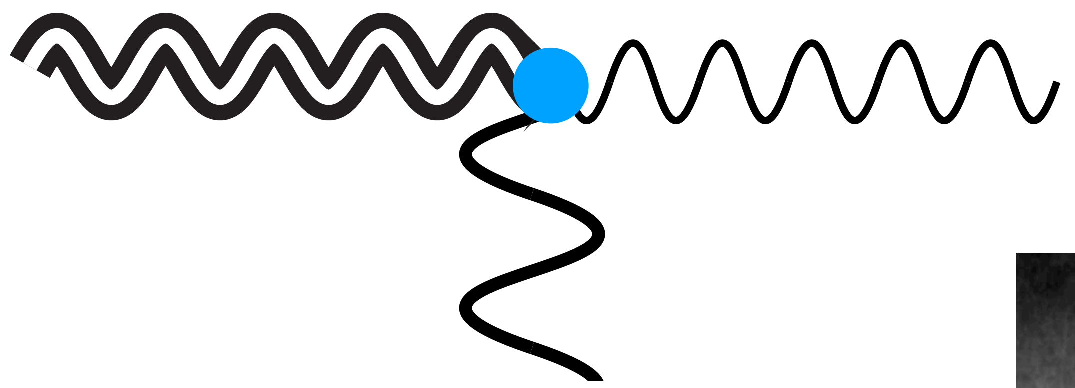
*Interaction GWs with **spin or other internal dofs***

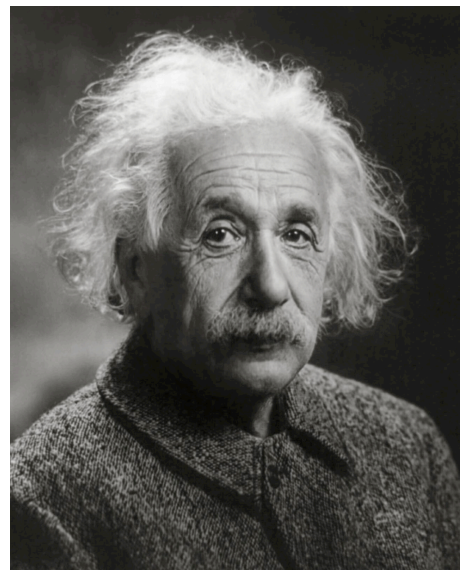
we have a lot to learn from DM searches!

GWs and EM fields

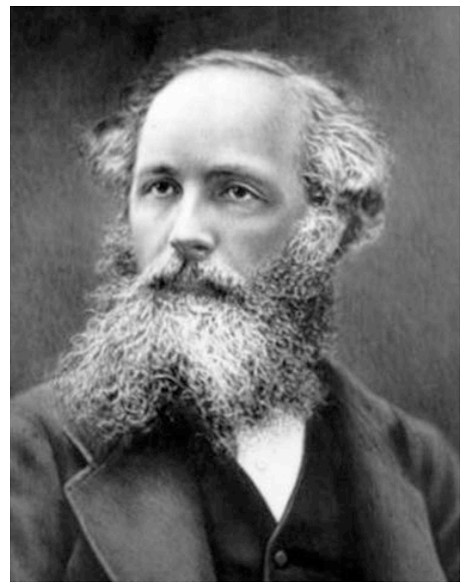
Raffelt Stodolsky 87

GWs + EM field -> EM field





$$\mathcal{L} = \sqrt{-g}\,(R + F_{\mu\nu}F^{\mu\nu}) \supset \frac{1}{2}A_\mu j^\mu_{\text{eff}}(h) + \eta^{\mu\alpha}\eta^{\nu\beta}F_{\mu\nu}F_{\alpha\beta} + O(h^2)$$

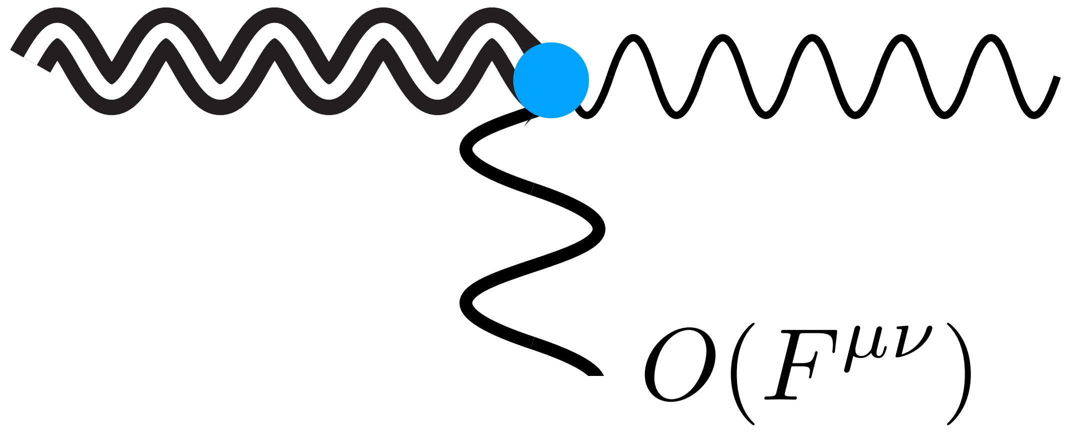


$$j^\mu_{\text{eff}} = -\partial_\beta\left(\frac{1}{2}hF^{\mu\beta} + h^\beta_\alpha F^{\alpha\mu} - h^\mu_\alpha F^{\alpha\beta}\right)$$

analogy with **axions** + EM field -> EM field

$$a\tilde{F}F$$

$$a$$



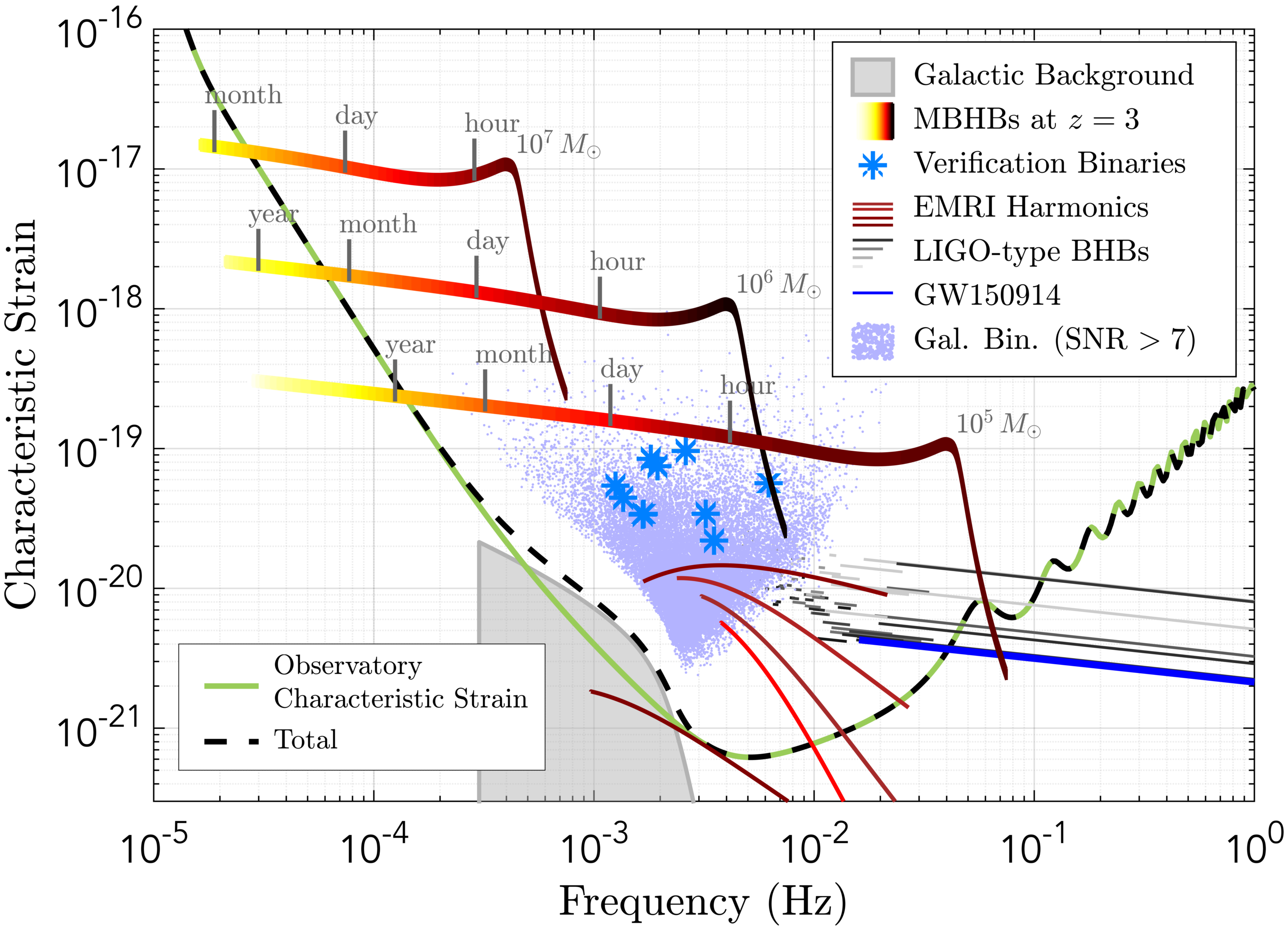
Connection to axions

This already sets the scale of the GW we want to measure:
for resonant production (for constant $\overrightarrow{B}$) $\lambda_{gw} \approx L$

waves of (a priori) laboratory size

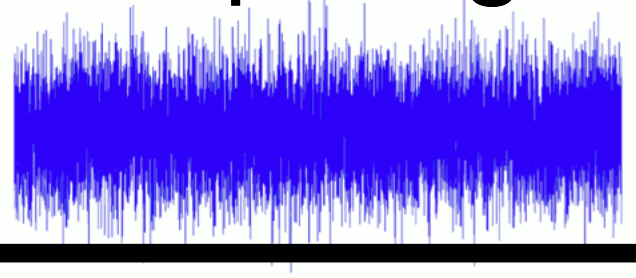
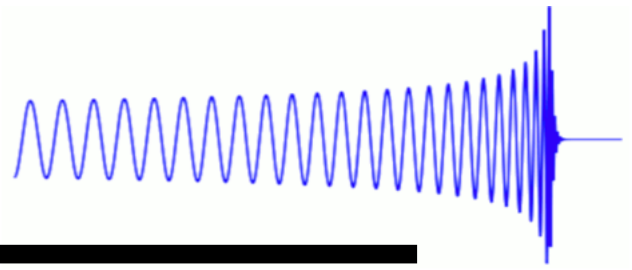
LISA Sources

Amaro-Seoane et al. 1702.00786



Sources of UHFGW ($> 10\,\mathrm{kHz}$)

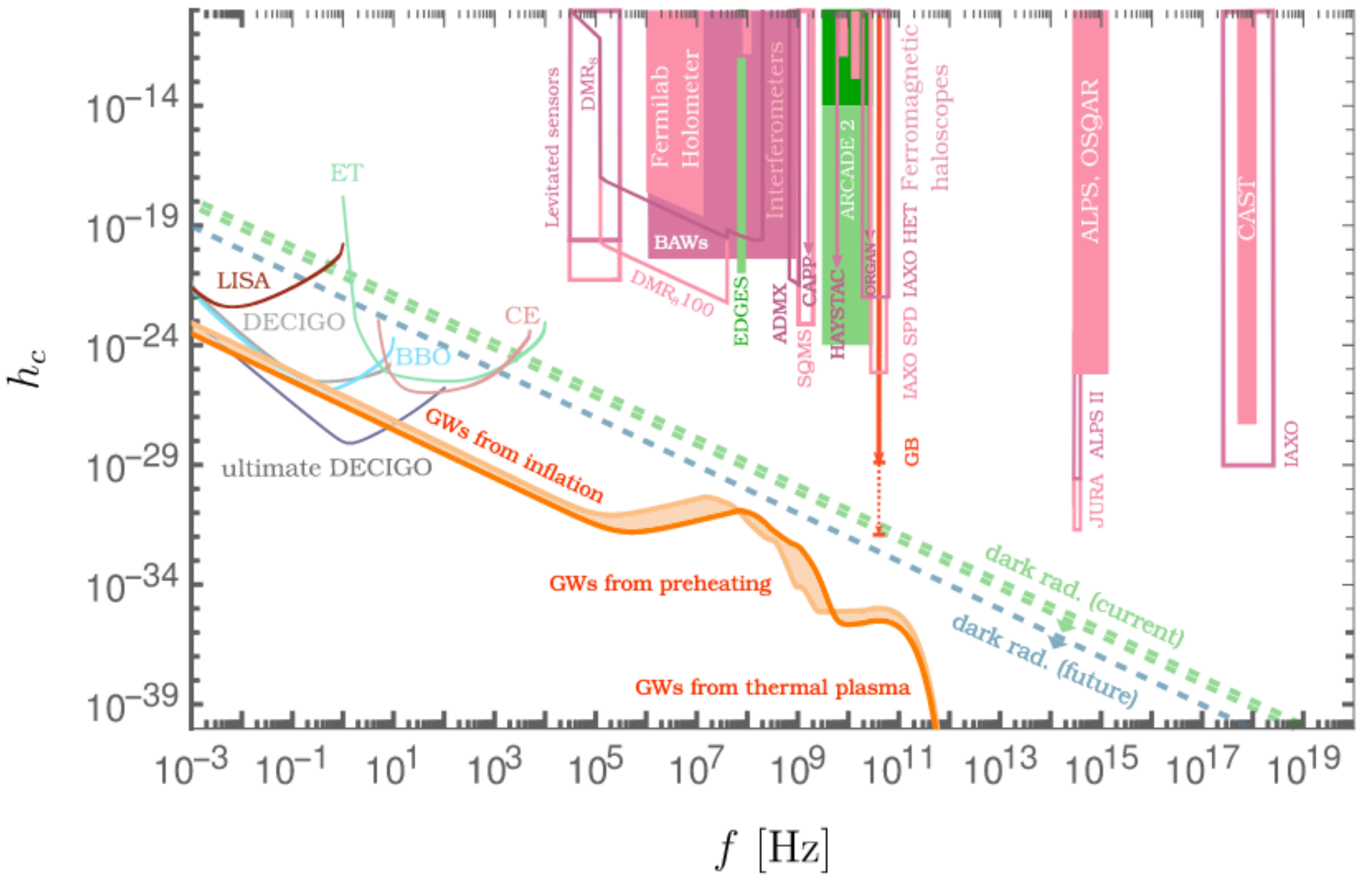
Very different picture comparing to EM spectrum

	Stochastic 	Coherent 
standard physics	Primordial plasma thermal fluctuations ✓ Ghiglieri et al JHEP 07 (2020) 092 Ringwald et al JCAP 03 (2021) 054	? ✗
	Inflation	
new physics	Phase transitions, Cosmic strings Close encounters of BHs ✓	Light PBHB Ultra-light DM Exotic objects ✓

Sources of UHFGW: spoiler!

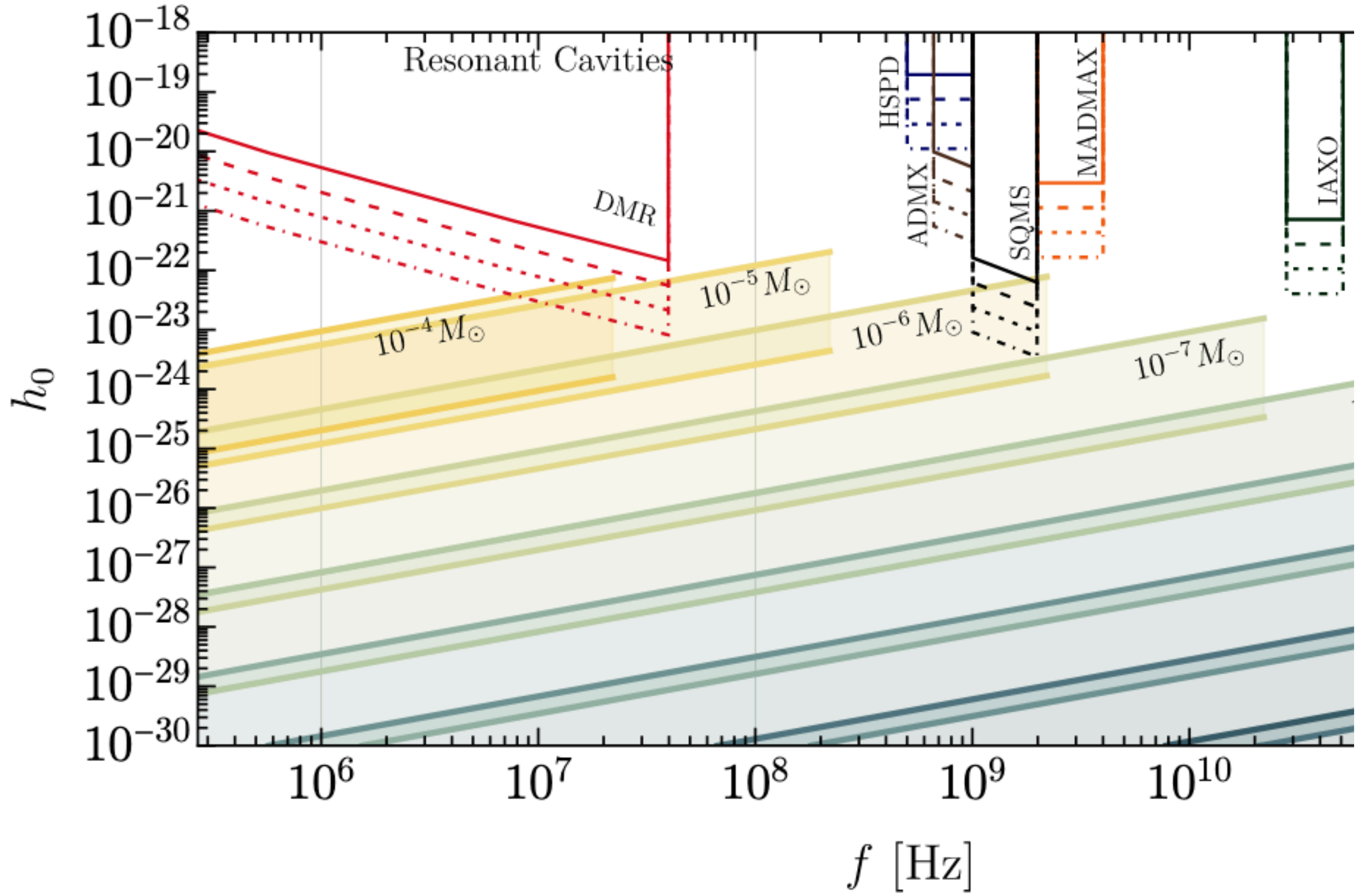
SMASH model full spectrum

Ringwald Tamarit 22



GWs from PBHs (rates 1/year)

Franciolini et al 2205.02153



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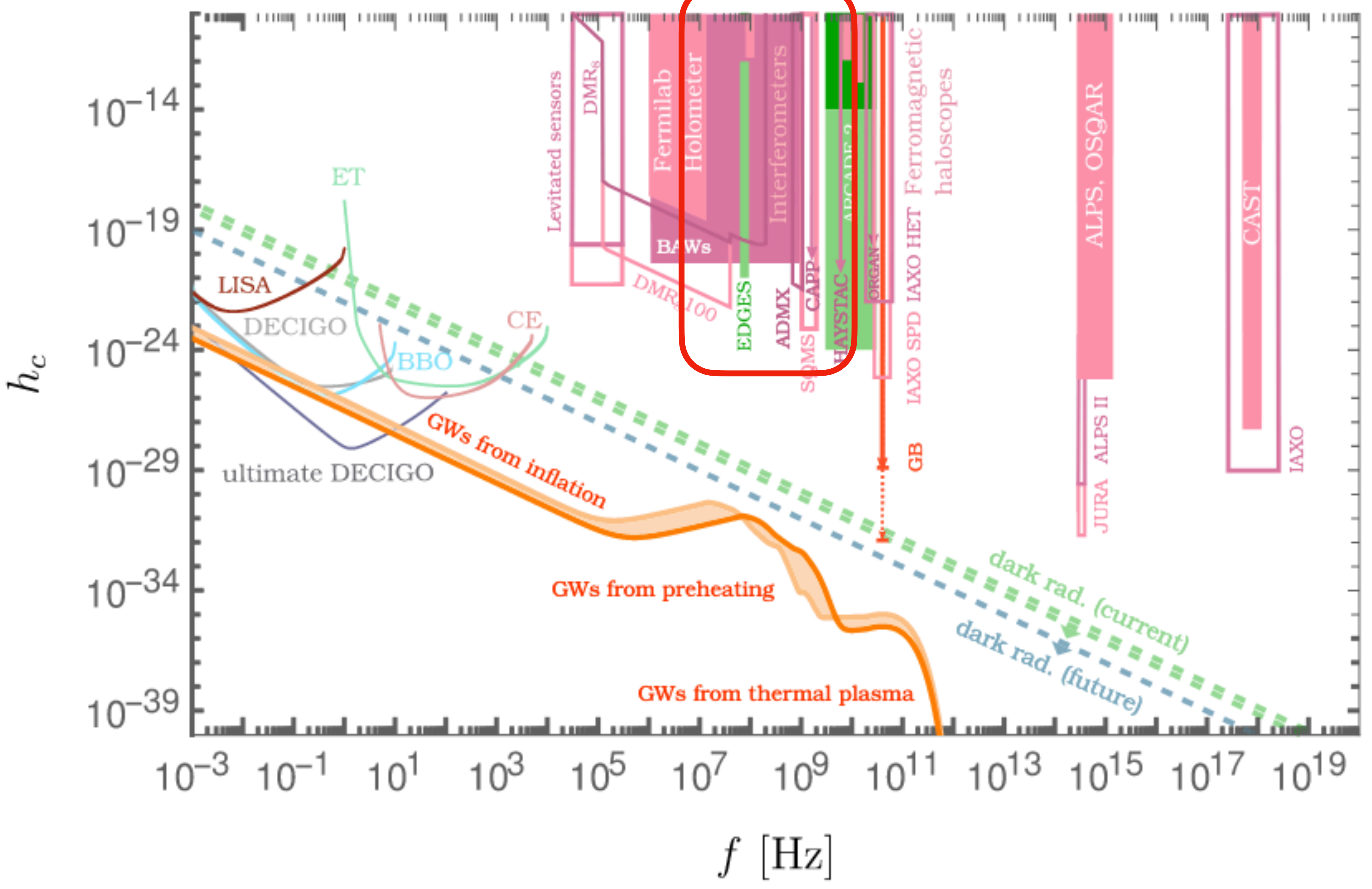

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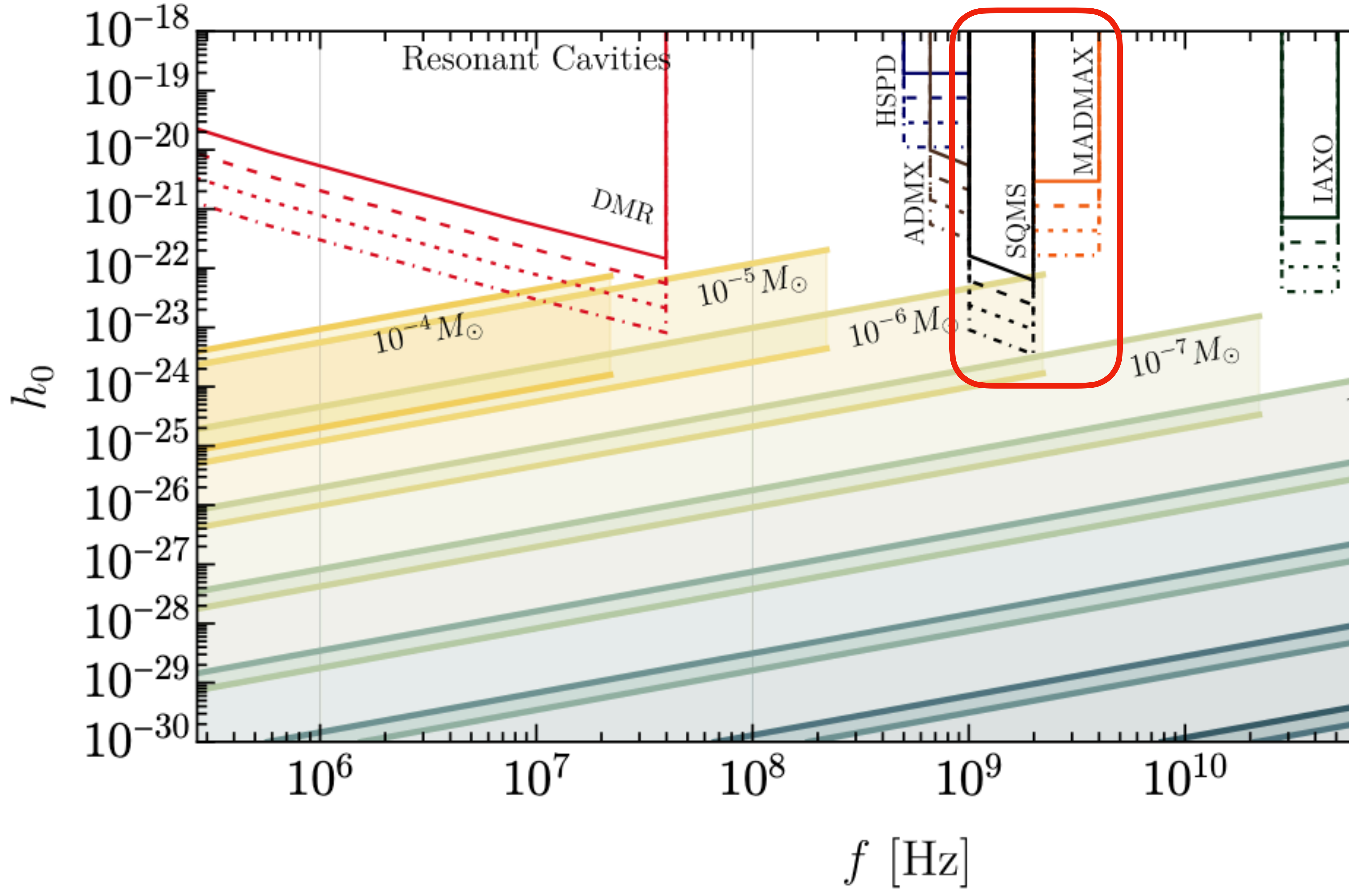
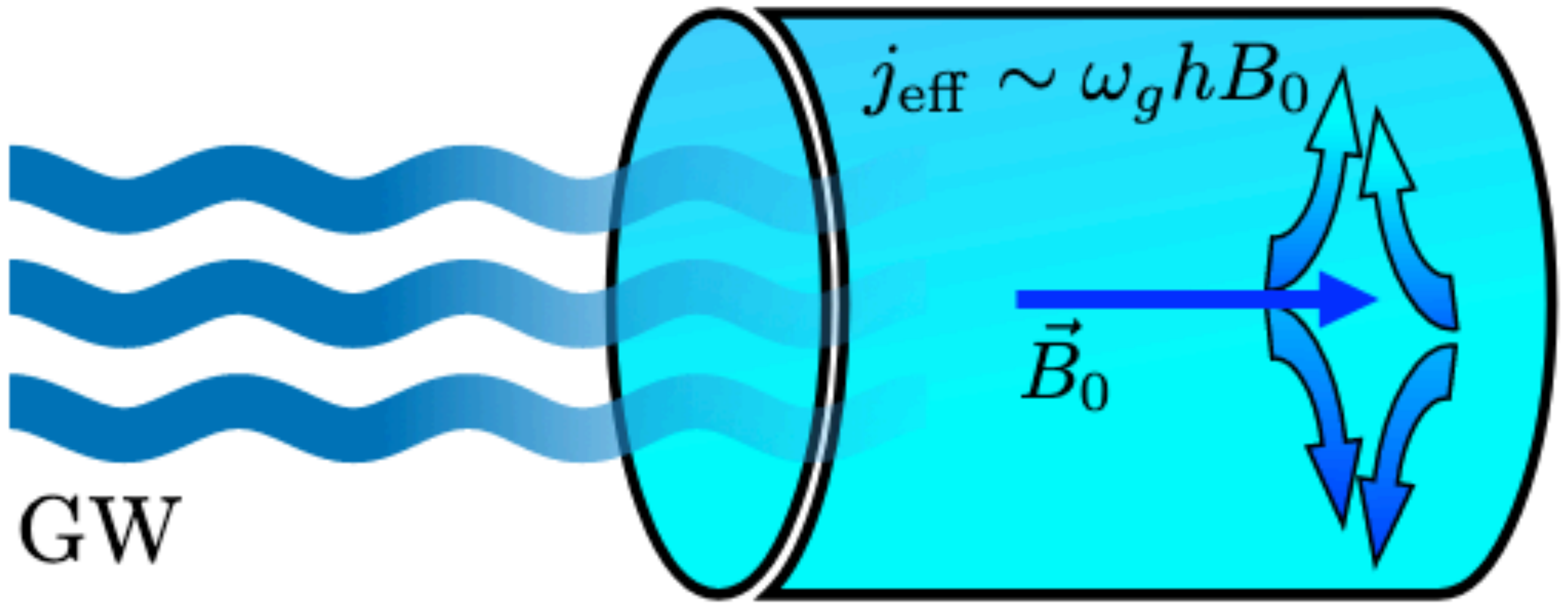


Searching for GWs with light



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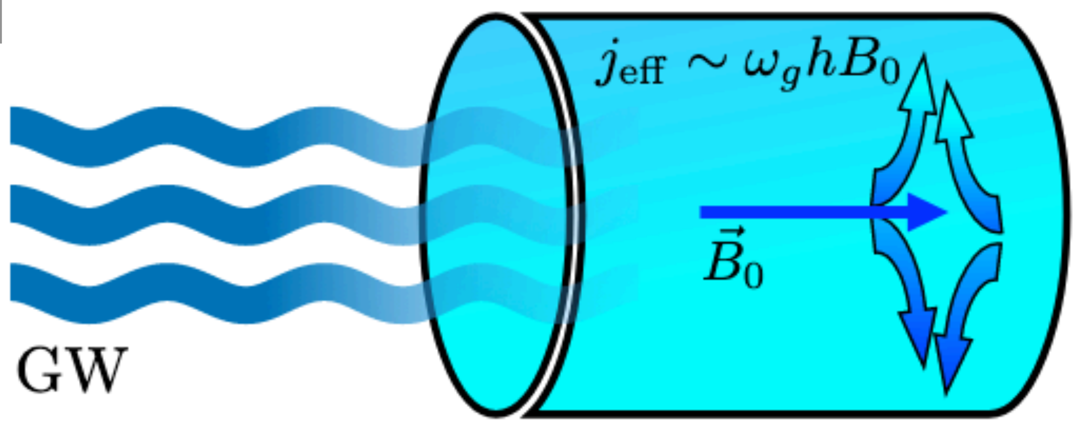
$$j^\mu_{\rm eff} = -\partial_\beta\left(\frac{1}{2}hF^{\mu\beta} + h^\beta_\alpha F^{\alpha\mu} - h^\mu_\alpha F^{\alpha\beta}\right)$$

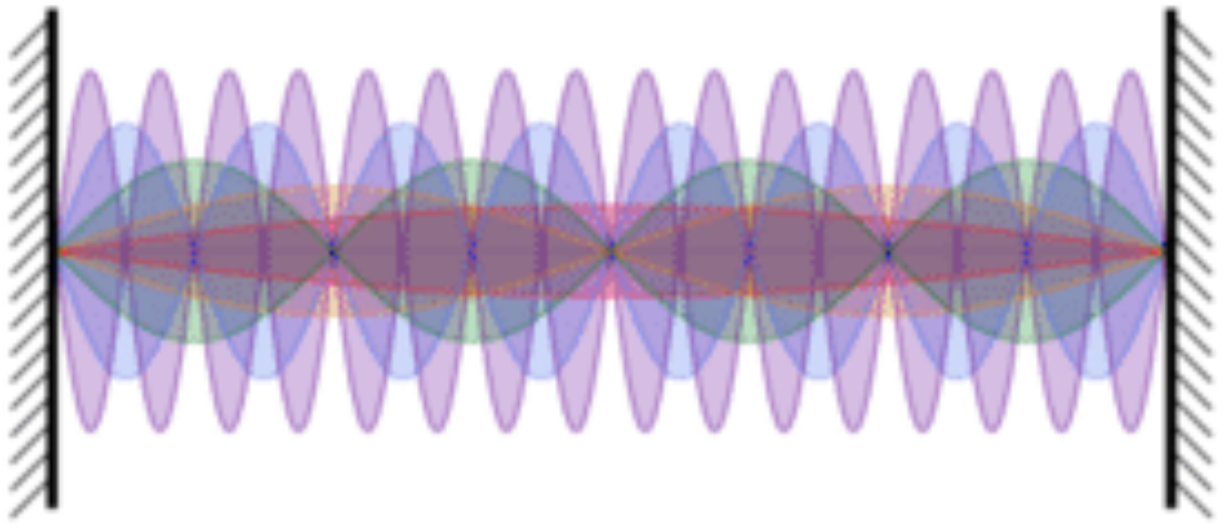
Cavity modes excitation



$$j^{\mu}_{\rm eff} = -\partial_{\beta}\left(\frac{1}{2}hF^{\mu\beta} + h^{\beta}_{\alpha}F^{\alpha\mu} - h^{\mu}_{\alpha}F^{\alpha\beta}\right)$$

$$\boldsymbol{E}(\boldsymbol{x},t) = \sum \boldsymbol{E}_{sn}(\boldsymbol{x},t) + \boldsymbol{E}_{in}(\boldsymbol{x},t)$$

solenoidal irrotational



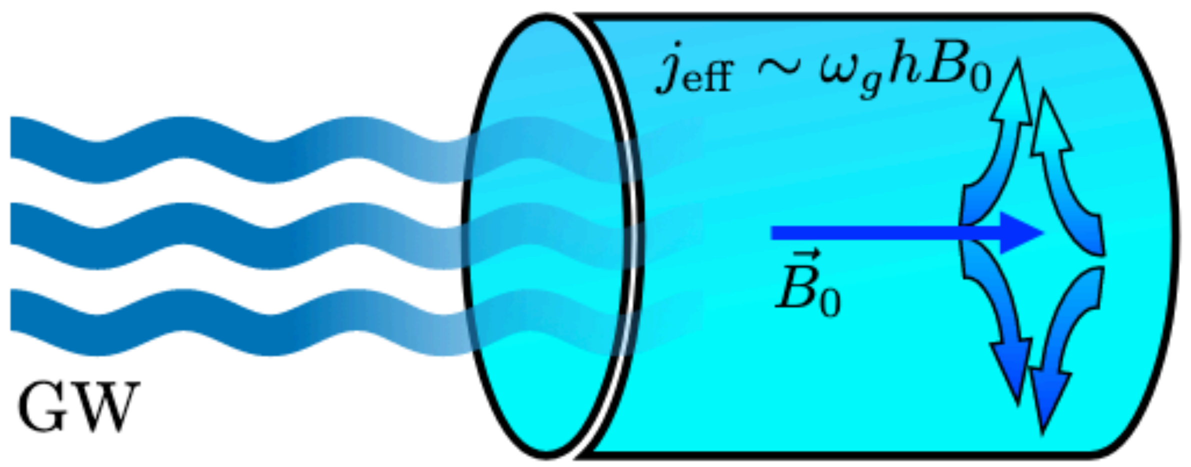
$$\boldsymbol{E}_{sn}(\boldsymbol{x},t) = e_{sn}(t)\boldsymbol{E}_{sn}(\boldsymbol{x})$$
$$\boldsymbol{E}_{in}(\boldsymbol{x},t) = e_{in}(t)\boldsymbol{E}_{in}(\boldsymbol{x})$$

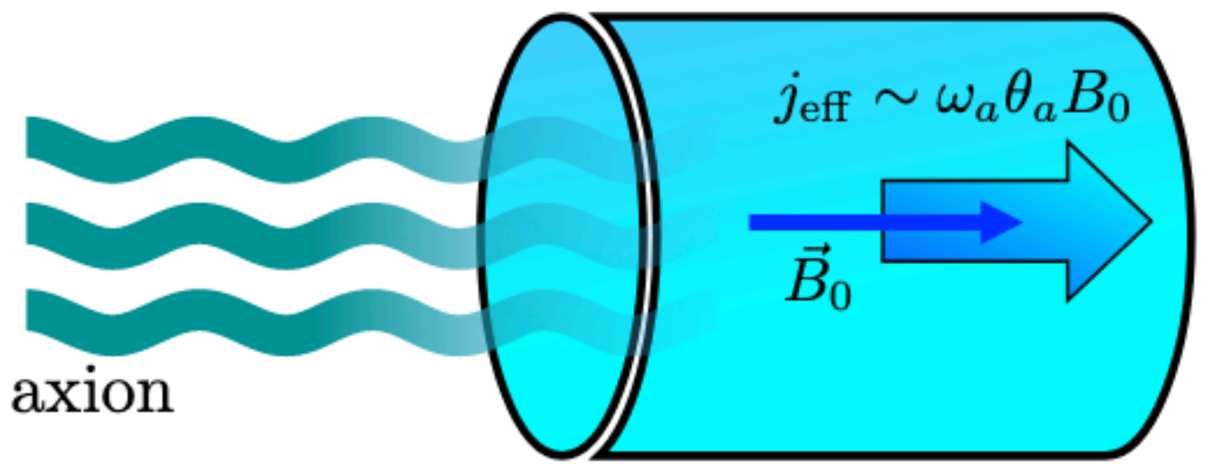
$$\left(\omega^2_{sm} + \partial^2_t + \sigma_{sm}\partial_t\right)e_{sm}(t) = e^{-i\omega_G t}\eta_{sm}$$
$$\left(\partial^2_t + \sigma_{im}\partial_t\right)e_{im}(t) = e^{-i\omega_G t}\eta_{im}$$

$$\eta \sim \int_V \mathrm{d}^3x\, E J_{eff}$$

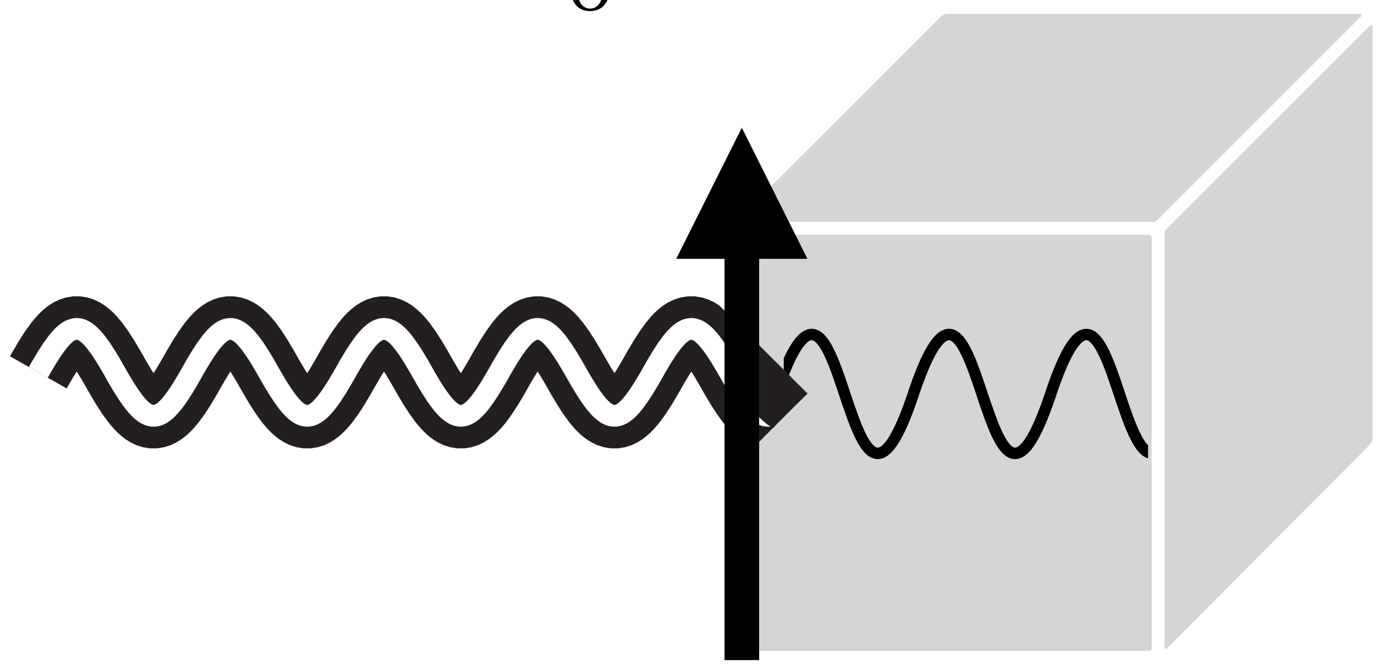
'source' (here we want to maximise. It is also directional)

From axions to GWs





$$P_h \sim h_0^2$$



Du et al 2018

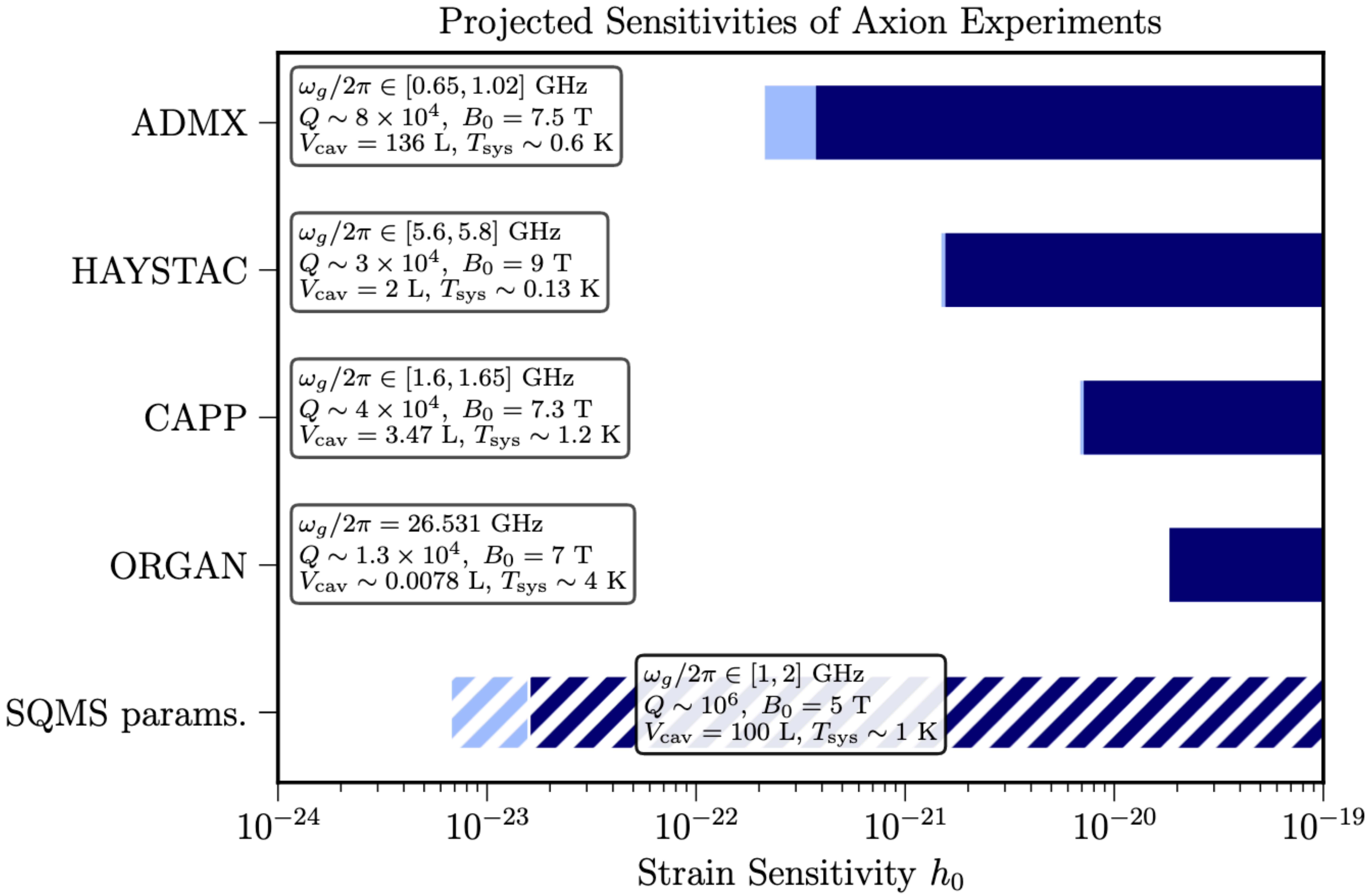
indeed: ADMX sensitivity

$$P_{\rm axion} = 1.9 \times 10^{-22}\ \mathrm{W} \left(\frac{V}{136l}\right)\left(\frac{B}{6.8\ \mathrm{T}}\right)^2\left(\frac{C}{0.4}\right)\left(\frac{g_\gamma}{0.97}\right)^2\left(\frac{\rho_{\rm a}}{0.45\mathrm{GeVcm}^{-3}}\right)\left(\frac{f}{650\mathrm{MHz}}\right)\left(\frac{Q}{50,000}\right)$$

quadratic in axion field and in *h*

$$P_h \sim h_0^2$$

$$h_0 \gtrsim 3 \times 10^{-22} \times \left(\frac{1\ \mathrm{GHz}}{\omega_g/2\pi}\right)^{3/2}\left(\frac{0.1}{\eta_n}\right)\left(\frac{8\ \mathrm{T}}{B_0}\right)\left(\frac{0.1\ \mathrm{m}^3}{V_{\rm cav}}\right)^{5/6}\left(\frac{10^5}{Q}\right)^{1/2}\left(\frac{T_{\rm sys}}{1\ \mathrm{K}}\right)^{1/2}\left(\frac{\Delta\nu}{10\ \mathrm{kHz}}\right)^{1/4}\left(\frac{1\ \mathrm{min}}{t_{\rm int}}\right)^{1/4}$$

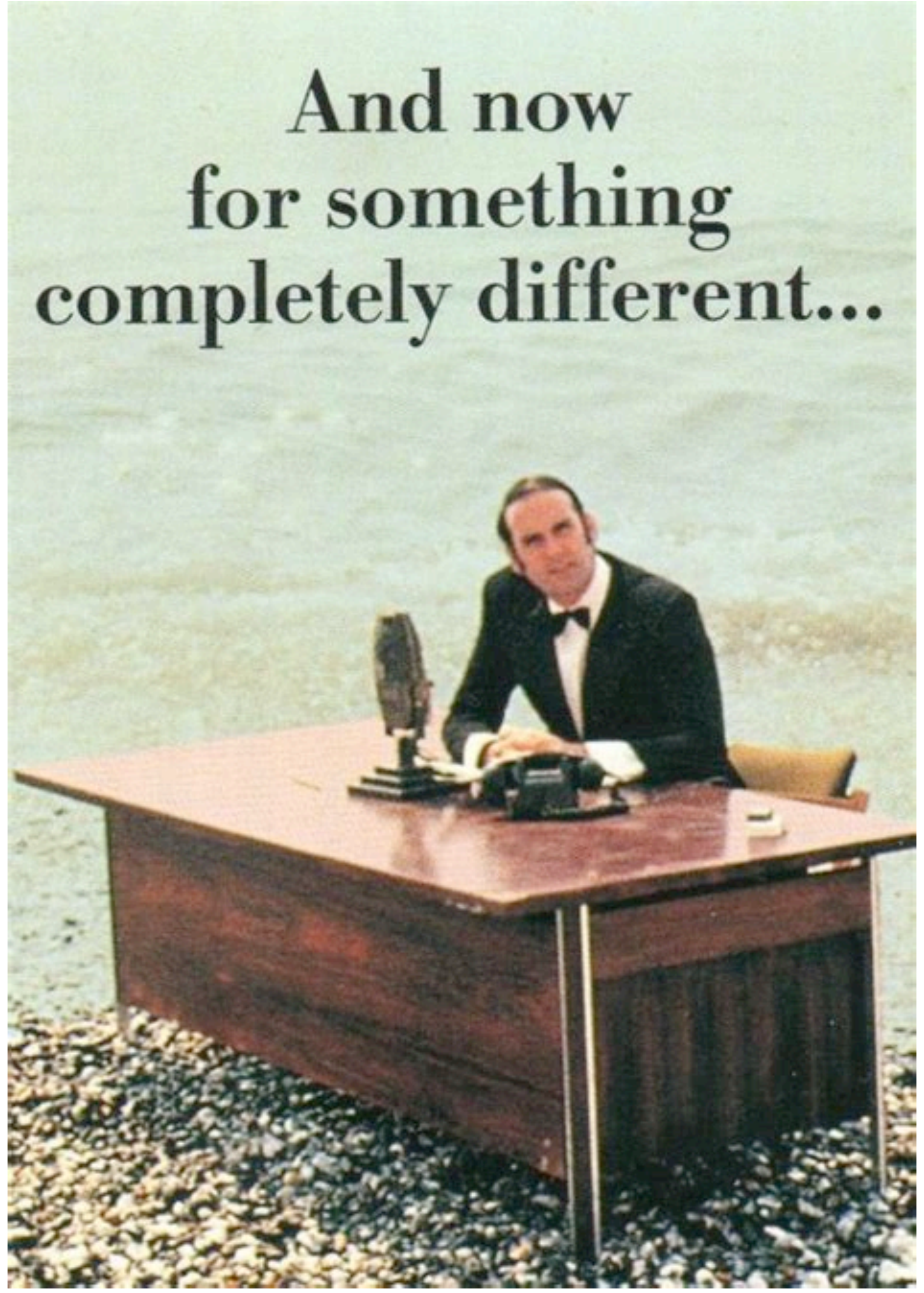


Projected Sensitivities of Axion Experiments
ADMX
$\omega_g/2\pi \in [0.65, 1.02]$ GHz
$Q \sim 8 \times 10^4,\ B_0 = 7.5$ T
$V_{\rm cav} = 136$ L, $T_{\rm sys} \sim 0.6$ K
HAYSTAC
$\omega_g/2\pi \in [5.6, 5.8]$ GHz
$Q \sim 3 \times 10^4,\ B_0 = 9$ T
$V_{\rm cav} = 2$ L, $T_{\rm sys} \sim 0.13$ K
CAPP
$\omega_g/2\pi \in [1.6, 1.65]$ GHz
$Q \sim 4 \times 10^4,\ B_0 = 7.3$ T
$V_{\rm cav} = 3.47$ L, $T_{\rm sys} \sim 1.2$ K
ORGAN
$\omega_g/2\pi = 26.531$ GHz
$Q \sim 1.3 \times 10^4,\ B_0 = 7$ T
$V_{\rm cav} \sim 0.0078$ L, $T_{\rm sys} \sim 4$ K
SQMS params.
$\omega_g/2\pi \in [1, 2]$ GHz
$Q \sim 10^6,\ B_0 = 5$ T
$V_{\rm cav} = 100$ L, $T_{\rm sys} \sim 1$ K
10^{-24}
10^{-23}
10^{-22}
10^{-21}
10^{-20}
10^{-19}
Strain Sensitivity h_0

A very exploratory field…

we still need to **deeply think what's better** for the future

do we move to something else ?



LIGO lesson

A very exploratory field…
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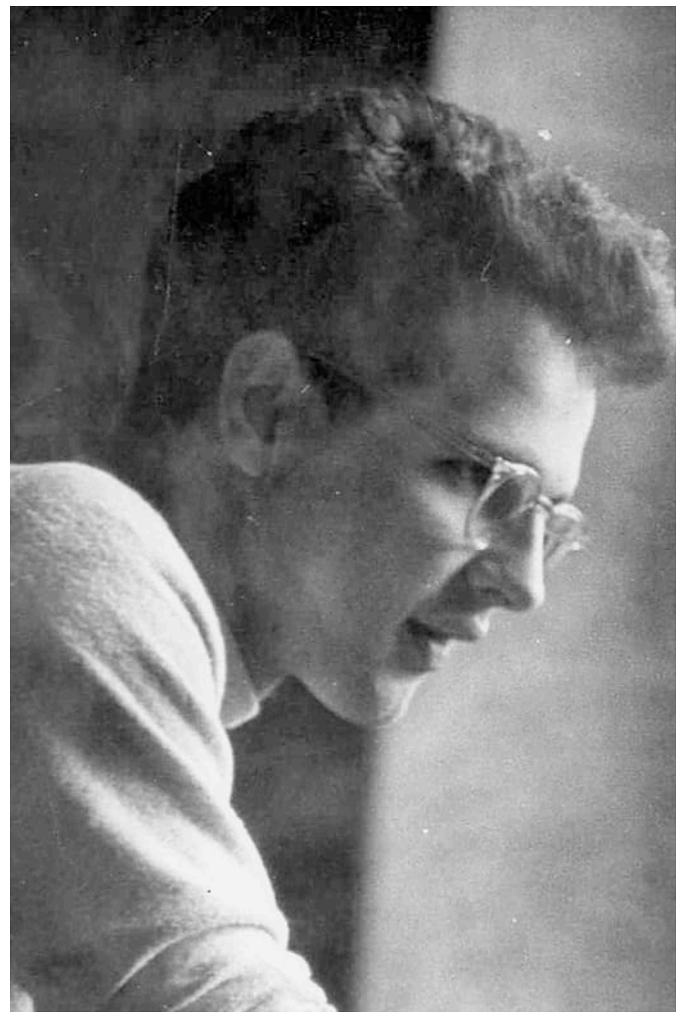
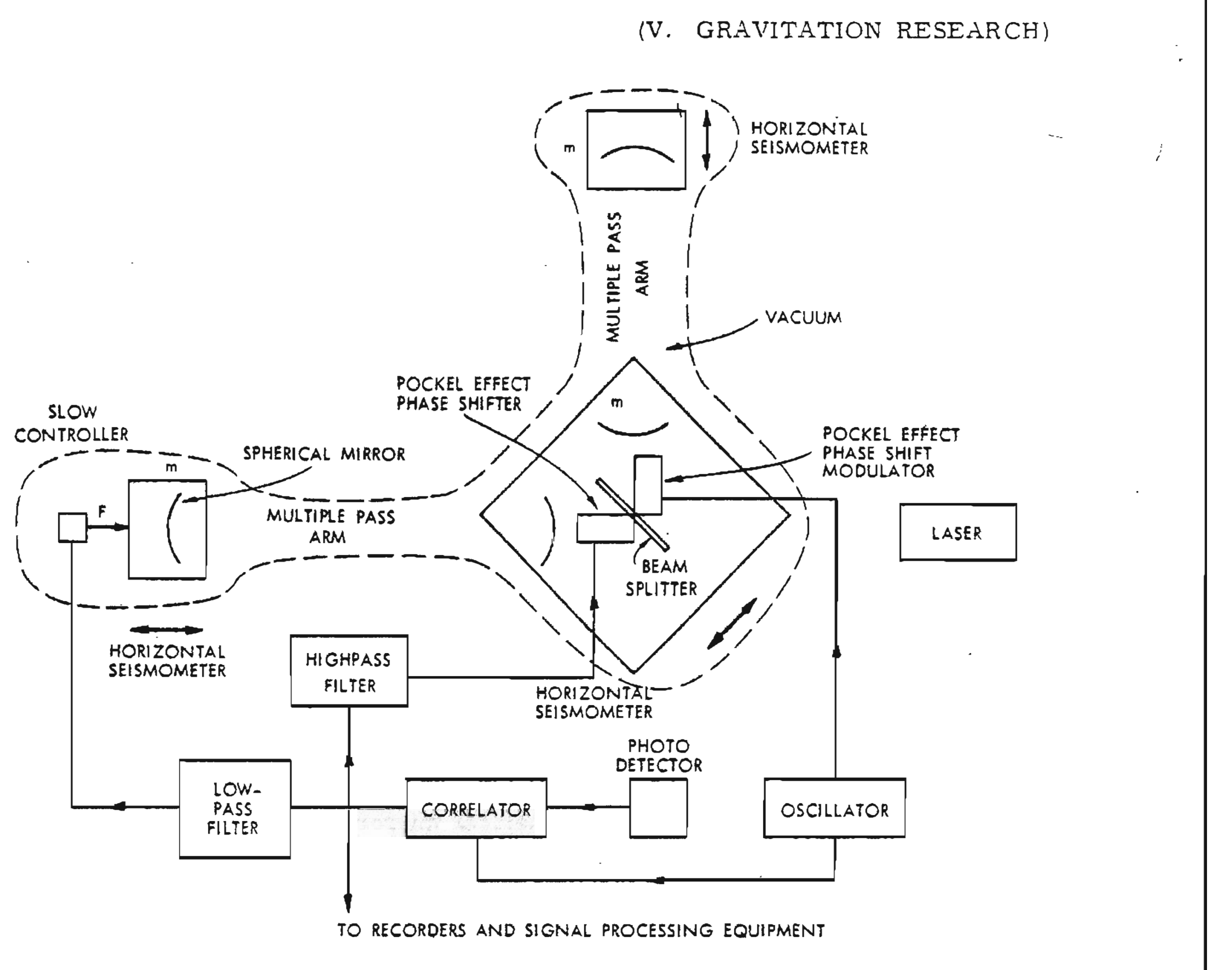
do we move to something else ?

Rainer Weiss, ca. 1972



LIGO concept
Weiss 1972



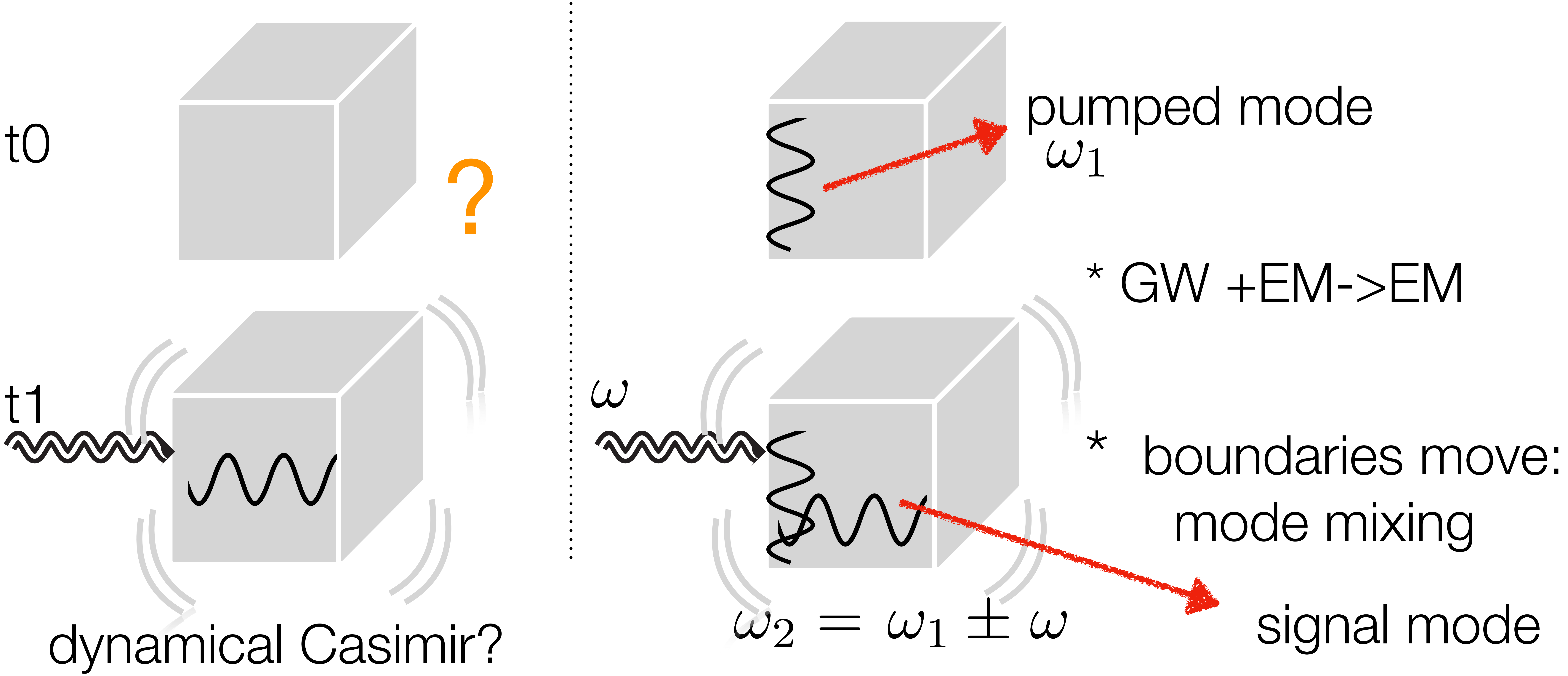
For continuous gravitational waves, the minimum detectable gravitational wave metric spectral density is then

$$h^2(f) > \frac{4}{\ell^2}\frac{\Delta x_n^2(f)}{\Delta f} \approx \frac{4 \times 10^{-33}}{\ell^2(\mathrm{cm})} \qquad \mathrm{Hz}^{-1}.$$

Recycling axion experiments II

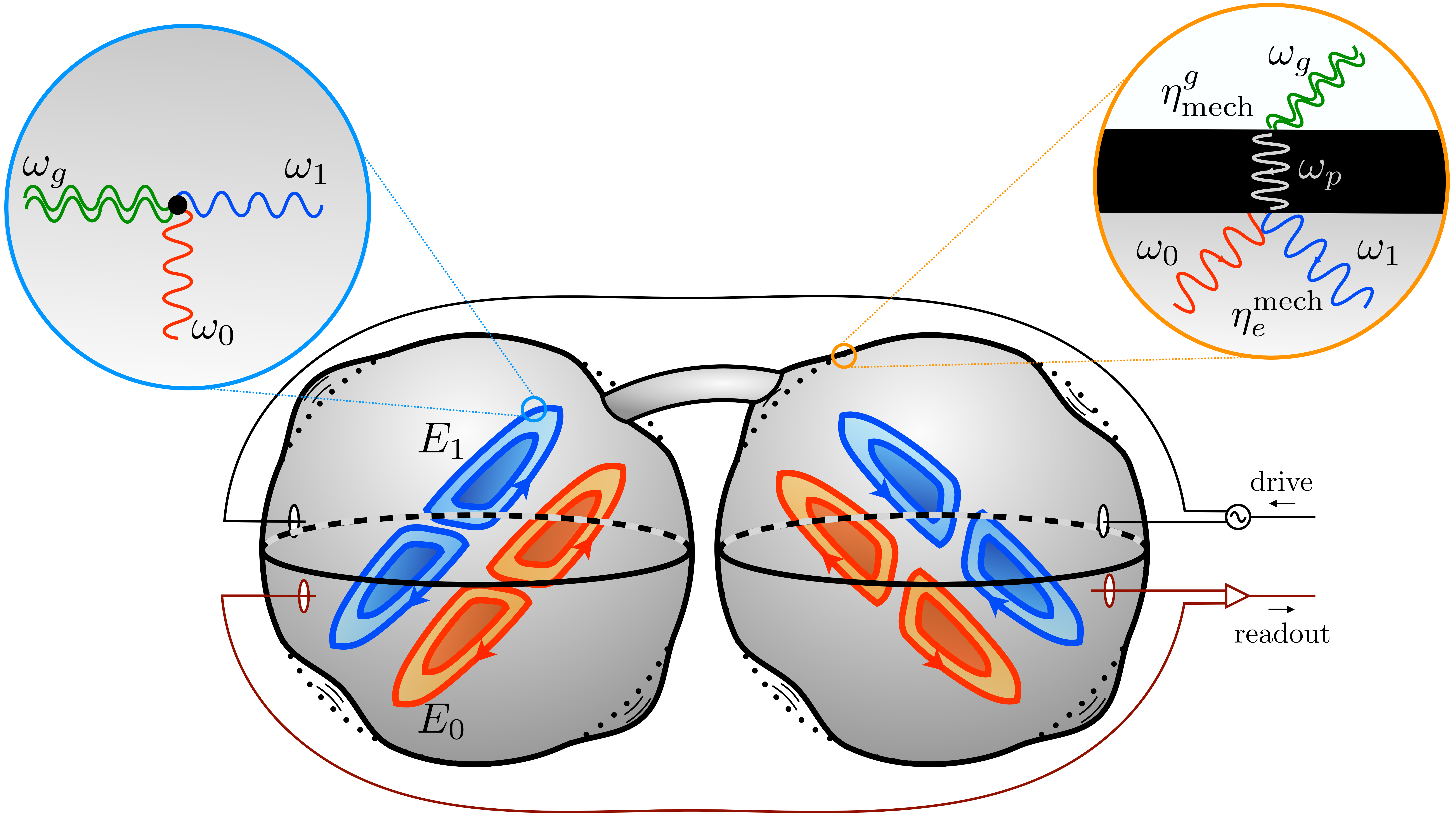
Mechanical-coupling (shaking the walls)

MAGO design from CERN (gr-qc/0502054)
Berlin, DB et al 2303.01518



MAGO set-up

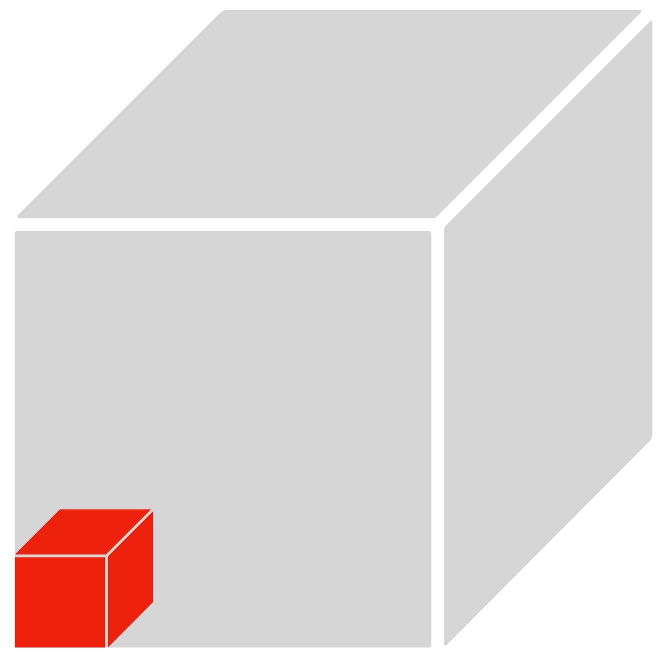
(Microwave Apparatus for Gravitational Waves Observation)



GWs exciting solids

searched for many years (Weber bars)

a solid affected by a external source (e.g. *x* direction)



$dm(x+u(x,t))$

$$dm\left(\frac{\partial^2 u}{\partial t^2} - v_s^2\frac{\partial^2 u}{\partial x^2}\right) = dF_x(t,x),$$

$$dF_i = \frac{1}{2}\ddot{h}_{ij}^{TT}x^j dm$$

In terms of eigenmodes:

$$\mathbf{u}(\mathbf{x},t) = u_p(t)\mathbf{u}_p(\mathbf{x})$$

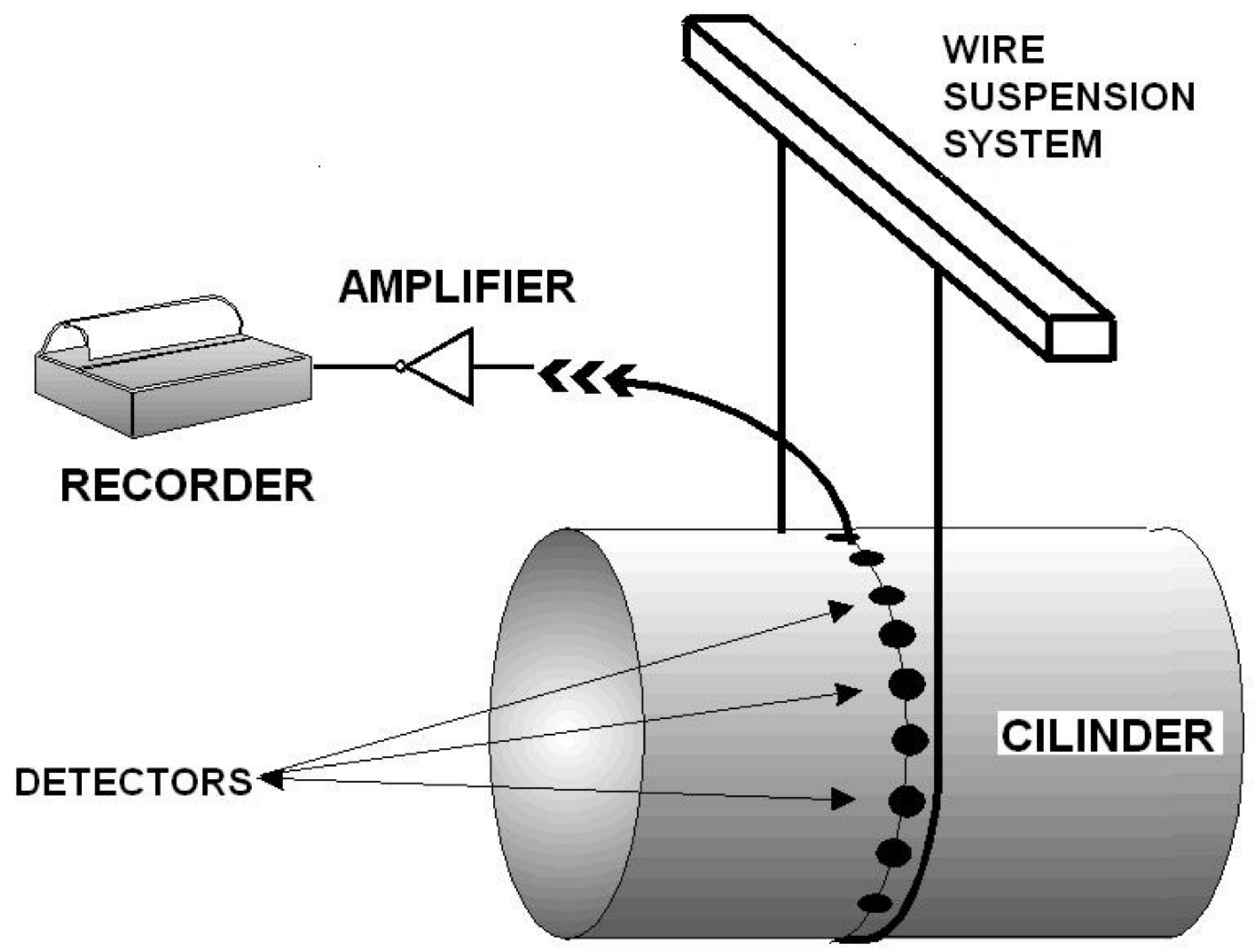
GWs exciting solids

$$\mathbf{u}(\mathbf{x},t) = u_p(t)\mathbf{u}_p(\mathbf{x})$$

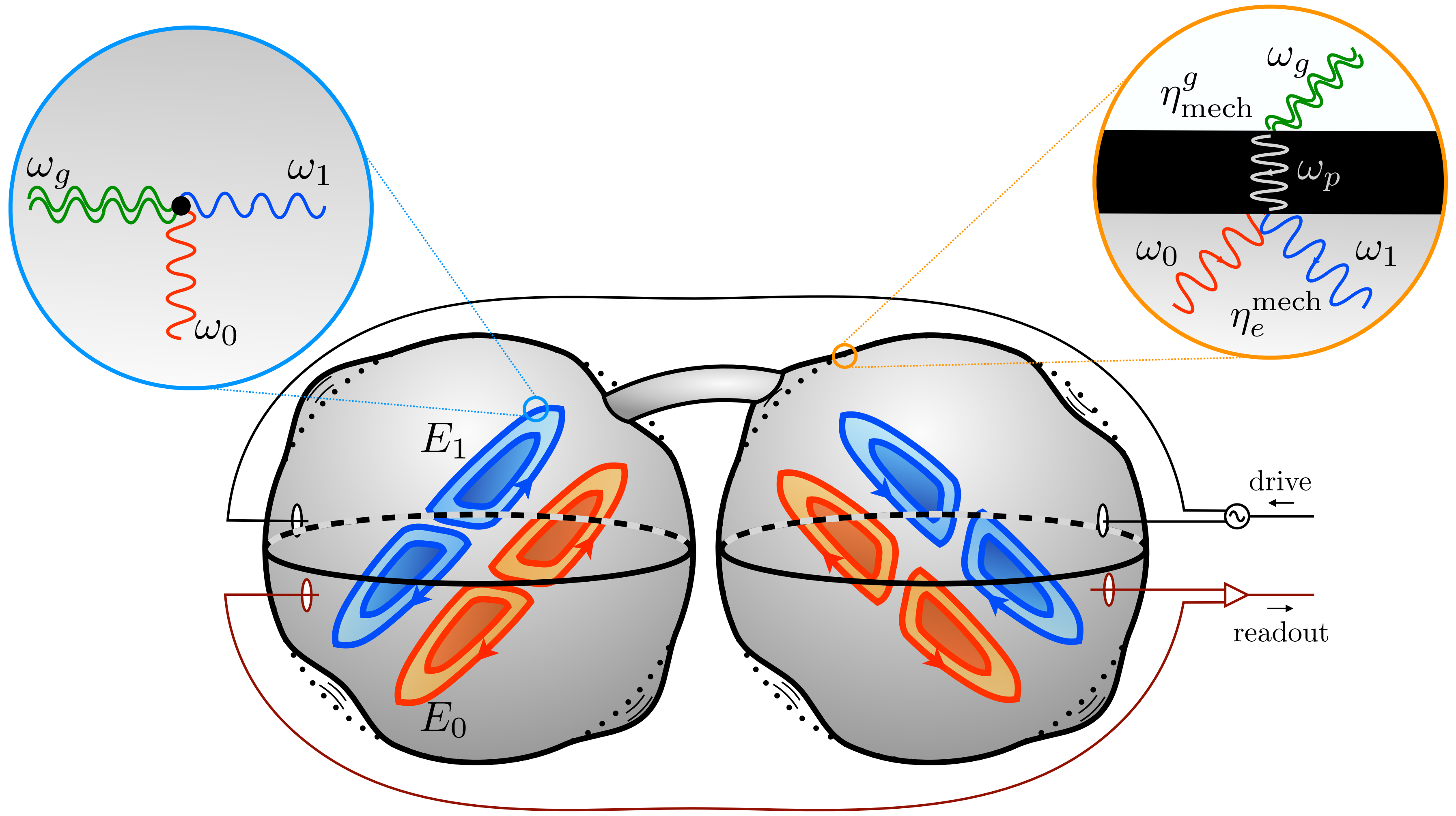
$$\ddot{u}_p + \frac{\omega_p}{Q_p}\dot{u}_p + \omega_p^2 u_p \simeq -\frac{1}{2}\omega_g^2 V_{\rm cav}^{1/3}\eta^g_{\rm mech}\ h_0 e^{i\omega_g t}$$

$$\eta^g_{\rm mech} = \frac{\hat{h}^{\rm TT}_{ij}}{V_{\rm cav}^{1/3}V_{\rm shell}}\int_{V_{\rm shell}} d^3\mathbf{x} U_p^{*i} x^j$$

this rings the solid (Weber bars)



MAGO set-up: read-out from mode mixing



$$\boldsymbol{E}_{sn}(\boldsymbol{x}, t) = e_{sn}(t)\boldsymbol{E}_{sn}(\boldsymbol{x})$$

$$\left(\partial_t^2 + \omega_n^2\right) e_n \simeq -\omega_n e_m \frac{\int_{\Delta V} d^3\mathbf{x}\,(\omega_n \boldsymbol{E}_m \cdot \boldsymbol{E}_n^* - \omega_m \boldsymbol{B}_m \cdot \boldsymbol{B}_n^*)}{\int_{V_0} d^3\mathbf{x}\,|\boldsymbol{E}_n|^2}$$

mixing

modes are not orthogonal in ΔV

$$P_{\rm sig} \propto h^2_{+,\times} \left|\eta_u^{(\rm mech.\ )}\right|^2 \left|\tilde{\eta}_e^{(\rm mech.\ )}\right|^2 P_{in} \times \begin{cases} \frac{\omega_h^4}{\left(\omega_h^2 - \omega_m^2\right)^2} & (|\omega_h - \omega_m| \gg \omega_m/Q_m) \\ Q_m^2 & (|\omega_h - \omega_m| \ll \omega_m/Q_m) \end{cases}$$

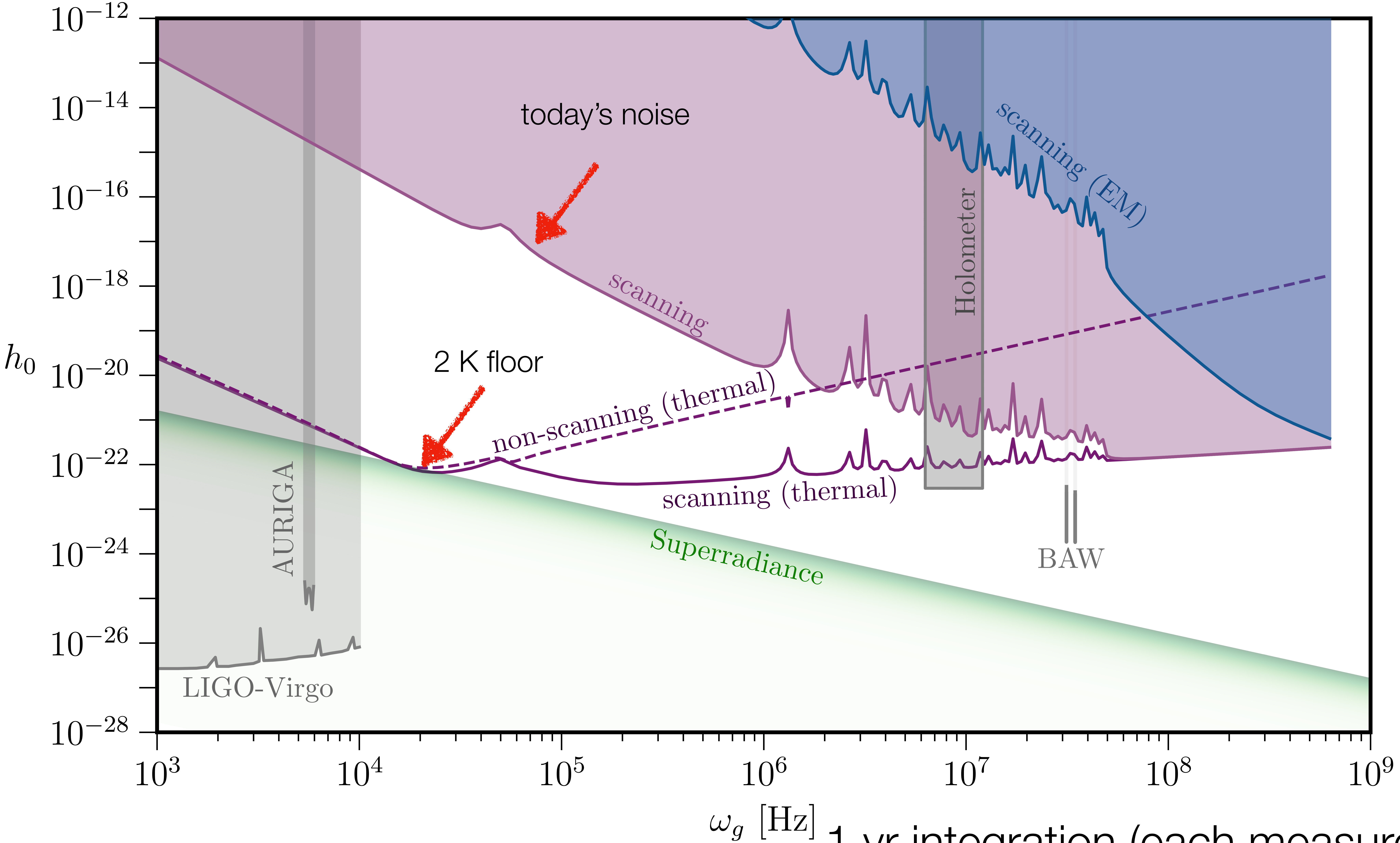
off-resonance

on-resonance

$Q_m \sim 10^6$

Estimates

A. Berlin, DB, R.T. D'Agnolo, S. Ellis, R. Harnik,
Y. Kahn, J. Schütte-Engel, M. Wentze 2303.01518



1 yr integration (each measurement ms)!

A word on sources GW coupling to spin

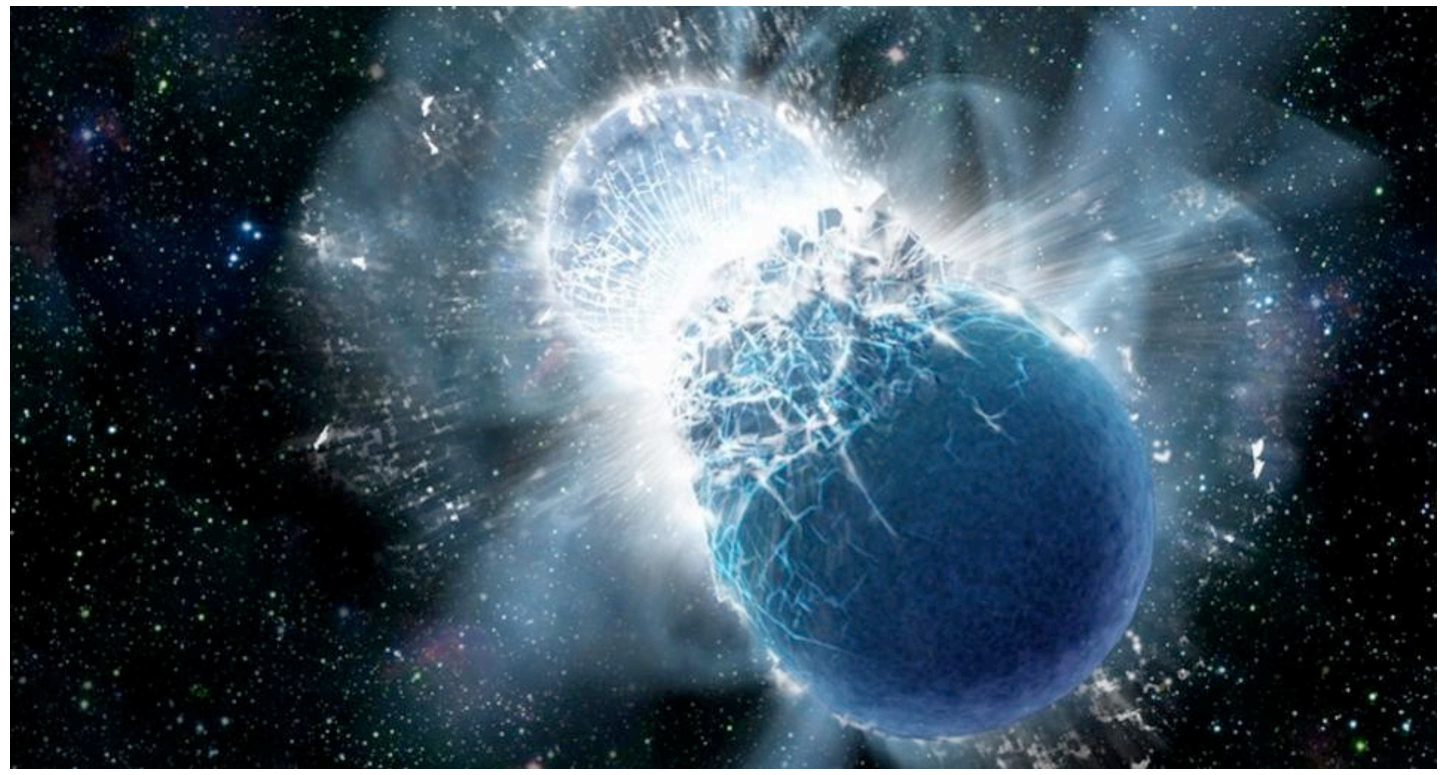
e.g. Ito and Soda 20

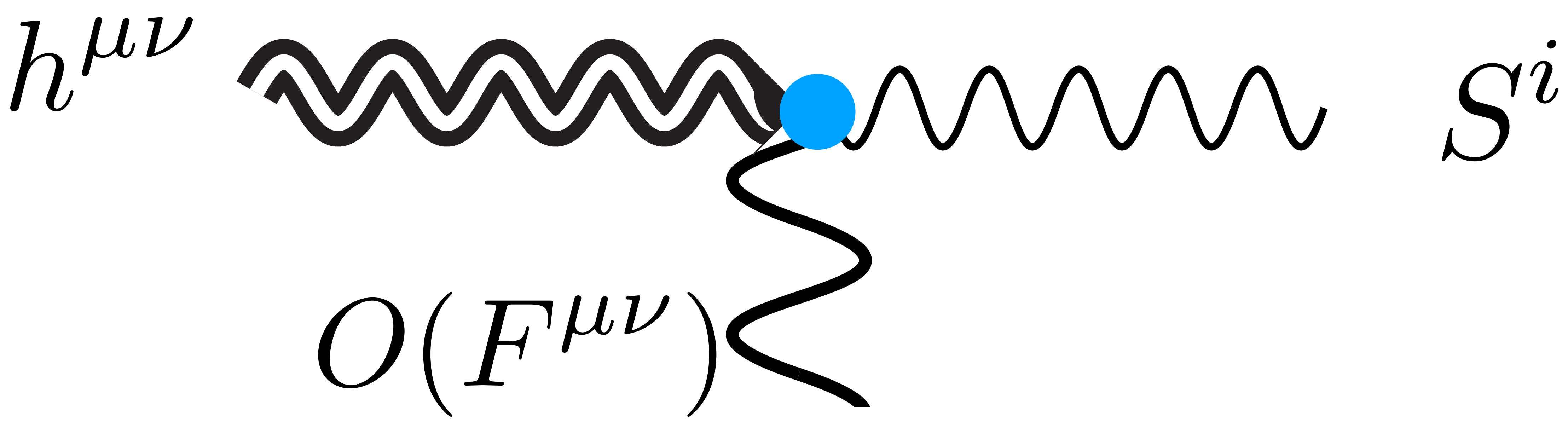
Dirac equation in GR $i\gamma^{\hat{\alpha}} e^{\mu}_{\hat{\alpha}} \left(\partial_\mu - \Gamma_\mu - ieA_\mu\right)\psi = m\psi$,

NR limit

$$H_{\rm tot} = -\mu_B \left(2\delta_{za} + Q_{za}\right) B_z \sum_i \hat{S}^a_{(i)} - \sum_{i,j} J_{ij} \hat{\boldsymbol{S}}_{(i)} \cdot \hat{\boldsymbol{S}}_{(j)}$$

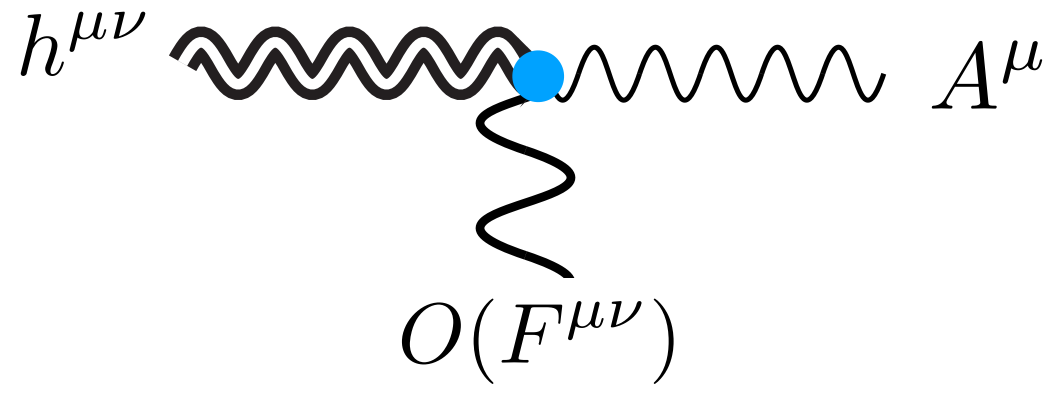
Linear in GW $Q_{ij} \supset -\frac{2}{3}\delta_{ij}\ddot{h}_{kl}\Big|_{\boldsymbol{x}=0} x^k x^l$





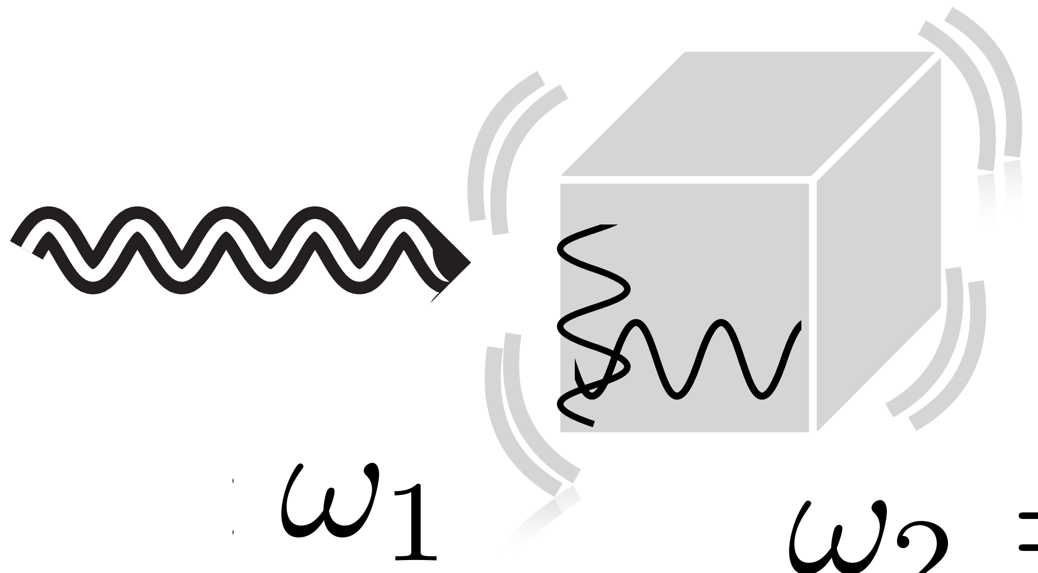
Summary and outlook

- SRF cavities are a mature technology to look for GWs at GHz either
  - Coupling to photons in a cavity



- 'ADMX' like $\omega = \omega_g$
- Heterodyne $\omega_2 = \omega_g \pm \omega_1$

  - Ringing the cavity and generating mode mixing



$$\omega_2 = \omega \pm \omega_1$$

- Heterodyne
- Casimir?

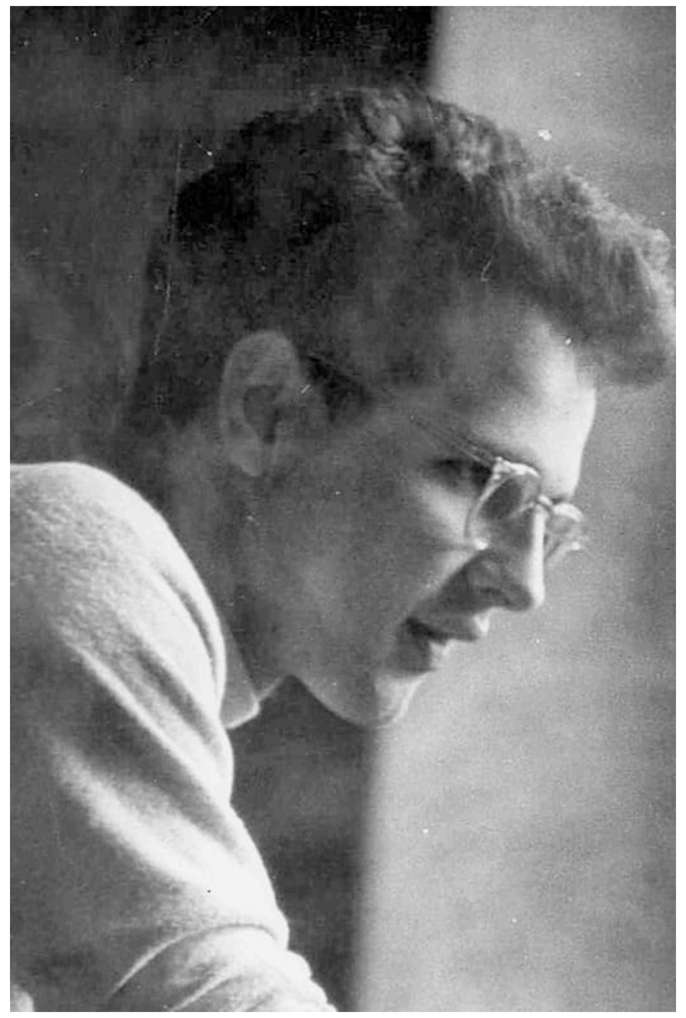
* Reach of $h_0 \sim 10^{-23}$ possible (100 kHz-GHz), though far from known signals

Outlook

A very exploratory field…

we still need to **deeply think what's better** for this endeavour

Rainer Weiss, ca. 1972



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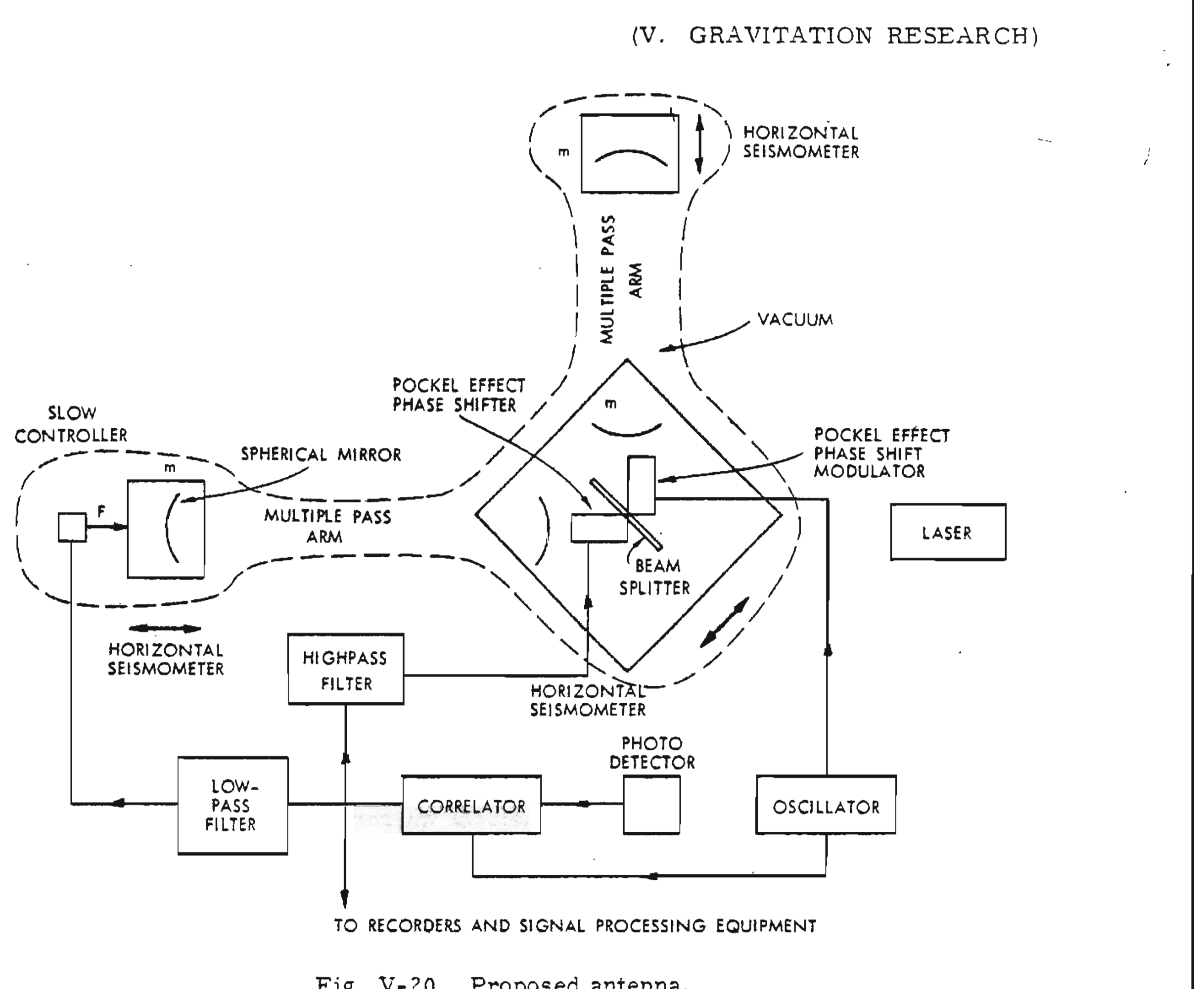

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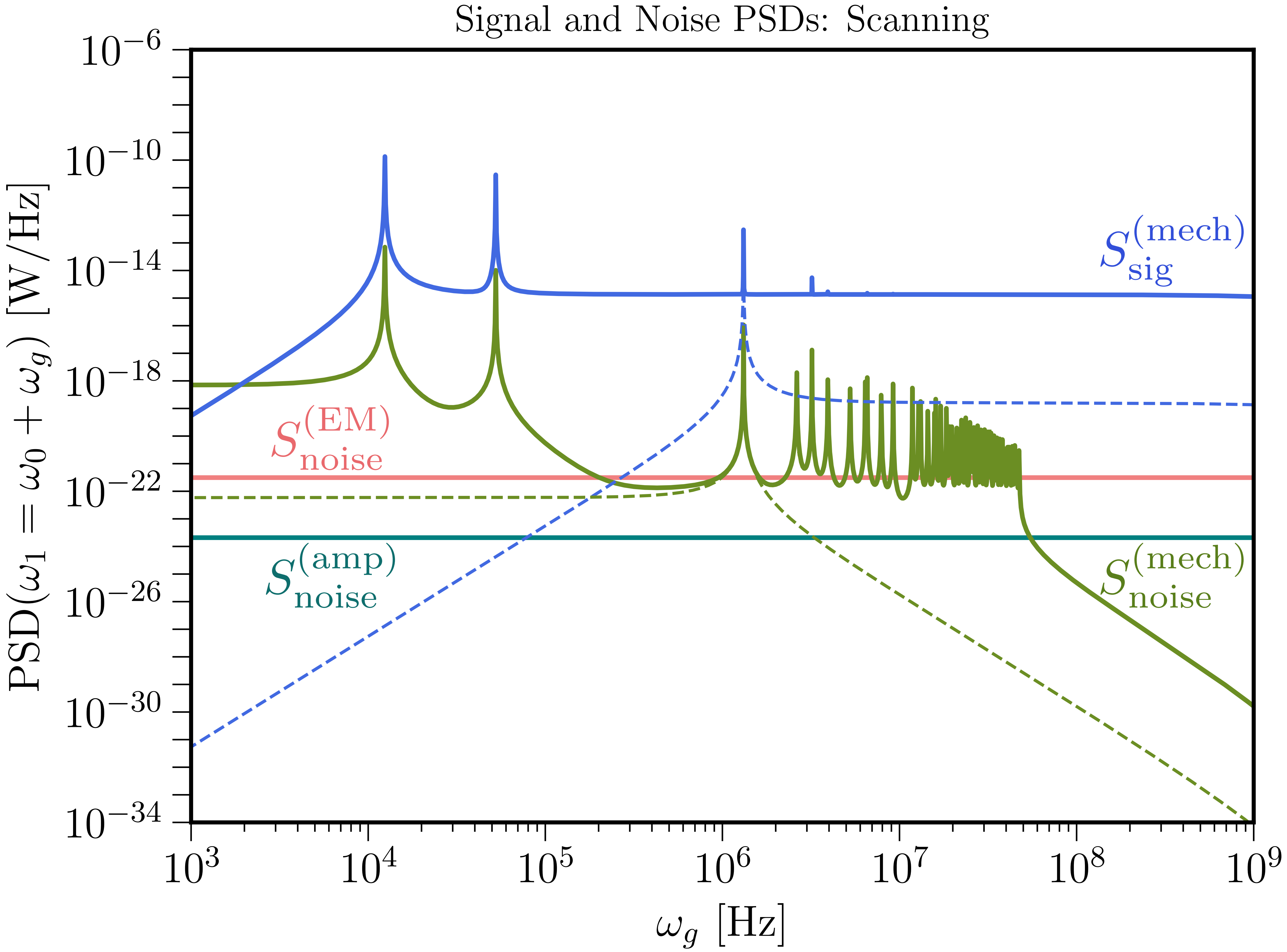
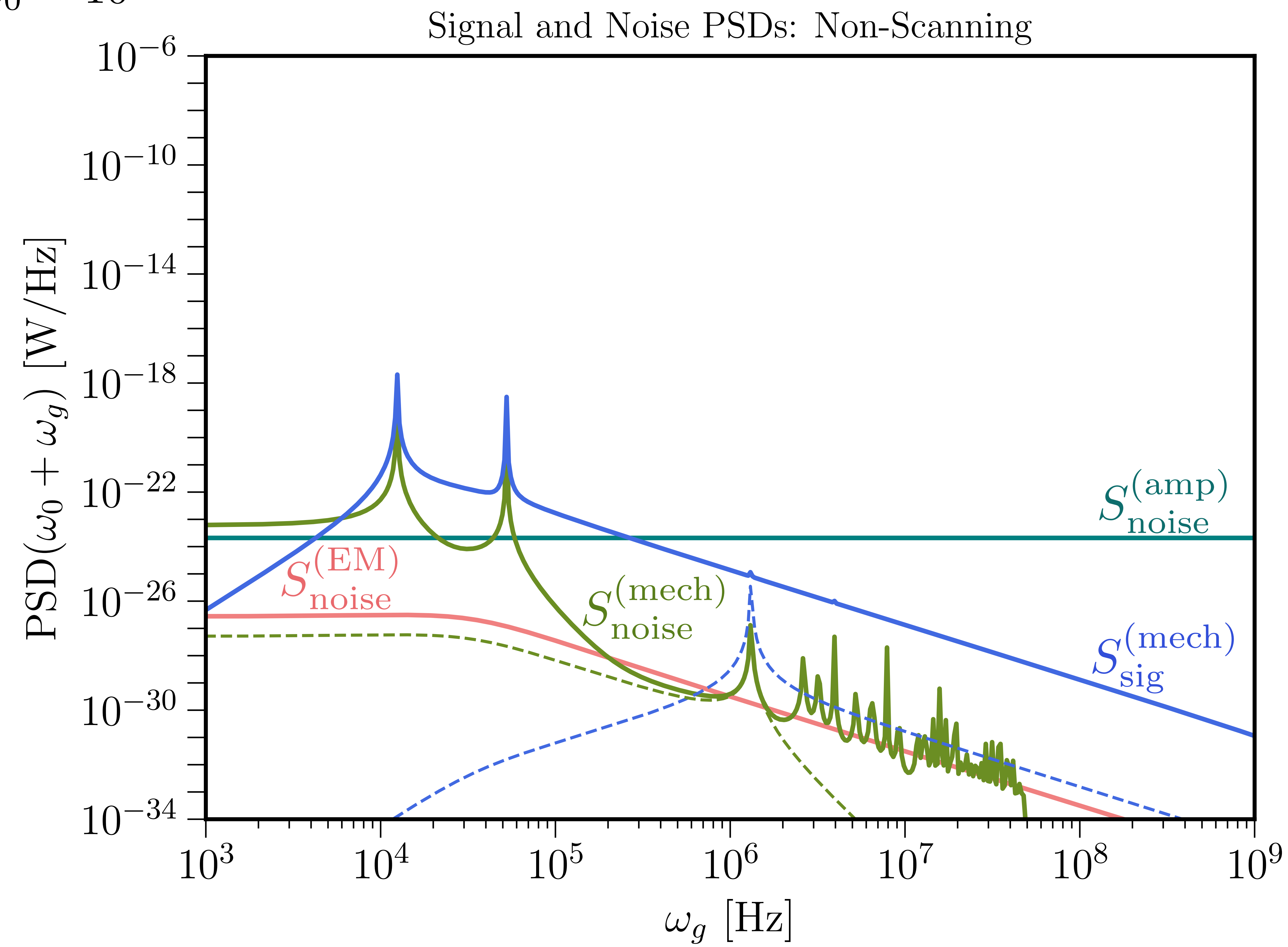
$$h^2(f) > \frac{4}{\ell^2}\frac{\Delta x_n^2(f)}{\Delta f} \approx \frac{4 \times 10^{-33}}{\ell^2(\mathrm{cm})} \quad \mathrm{Hz}^{-1}.$$

Response

$h_0 \sim 10^{-20}$



$$\omega_1 - \omega_0 = \omega_g$$



$$\omega_1 - \omega_0 = \min \omega_{\rm p} \sim 10\,\mathrm{kHz}$$